FACULTY OF ELECTRICAL
ENGINEERING
UNIVERSITY OF WEST BOHEMIA

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

ESF II – project of the University of West Bohemia
reg. no. CZ.02.2.69/0.0/0.0/18_056/0013239

Theory of Electrical Machines 2

KEV/TS2

Karel Hruska / r20201116

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Winding Leakage Inductance

Winding Leakage Inductance

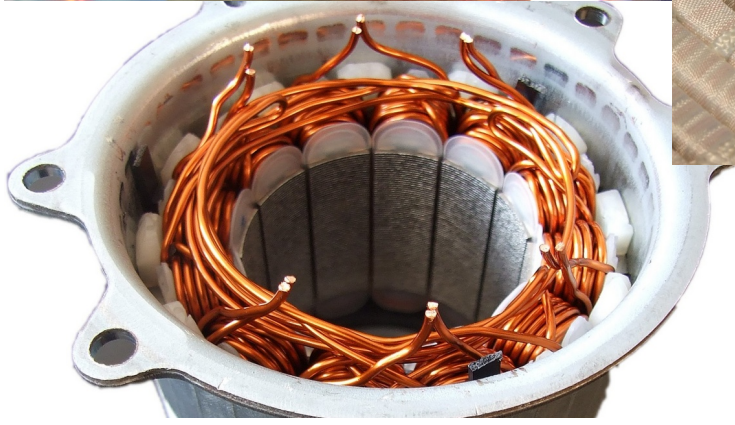
- Winding
  - #def: A Winding is an ordered system of conductors used for excitation of a magnetic field.
    => it is used for generation of a magnetic field (motors) or its reception (generators)
  - Commonly it is placed in slots, but there are also slotless variants.

Winding Leakage Inductance











Winding Leakage Inductance

- Winding
  - #def: A Winding is an ordered system of conductors used for excitation of a magnetic field.
    => it is used for generation of a magnetic field (motors) or its reception (generators)
  - Commonly it is placed in slots, but there are also slotless variants.
- A current flowing through the winding generates magnetic flux
  - A part of the magnetic flux goes through the air gap
    => main (magnetizing) magnetic flux
  - A part encloses through slots
    => leakage magnetic flux

Winding Leakage Inductance

- If there is a relation

$$\Psi = L\,I \qquad \text{or} \qquad L = \frac{\Psi}{I}$$

  then each magnetic flux is bound to an inductance.
- Main magnetic flux

  => main (magnetizing) inductance
- Leakage magnetic flux

  => leakage inductance
- By calculation it is possible to determine the leakage inductance by solution of the magnetic field in a slot of an electric machine

Winding Leakage Inductance

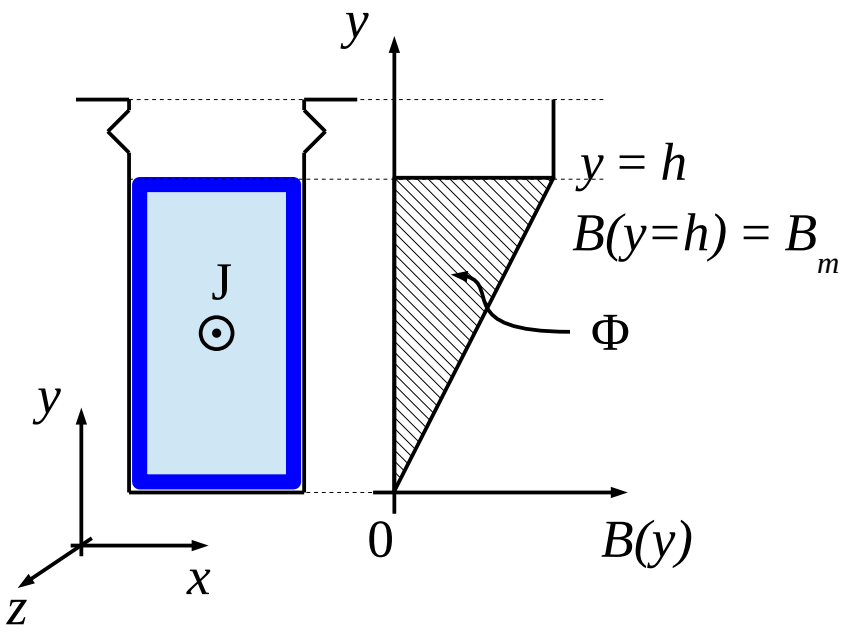
- Calculation of a leakage inductance:
  - Presumptions:
    - The winding is formed by conductors placed in a slot*
    - The iron has infinite permeability ($\mu_{Fe} \rightarrow \infty$)
    - The magnetic flux lines enter the slot perpendicularly to the slot walls
    - The flux lines in the slot do not make curves
    - The length of the slot is much greater than its width and height



*) The transformer window may be then considered as a limit case of a slot ;-)

Winding Leakage Inductance

- Determination of a magnetic flux through one conductor:
  1. „Trivial“ approach:



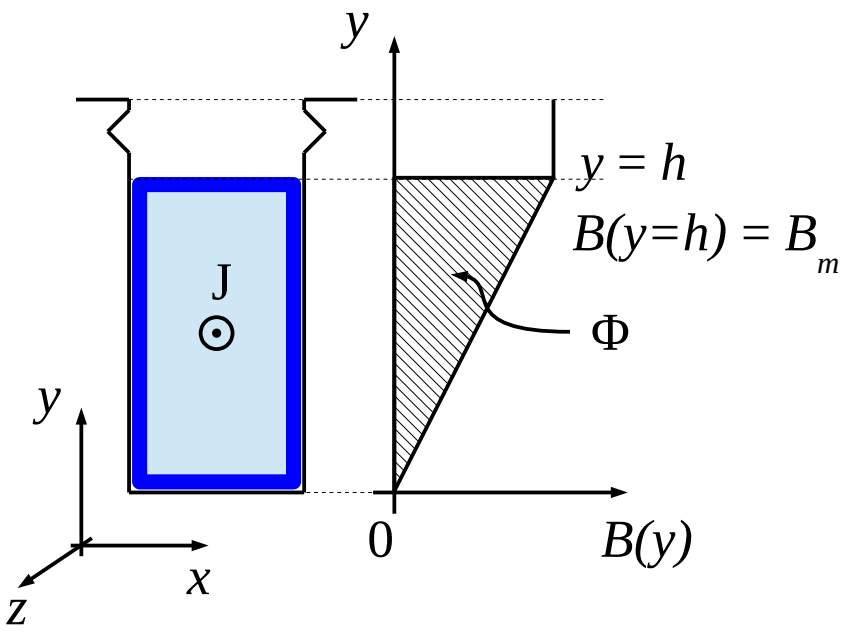
Slot width: $b_d\ [m]$

Conductor height: $h\ [m]$

Slot length: $l\ [m]$

Winding Leakage Inductance

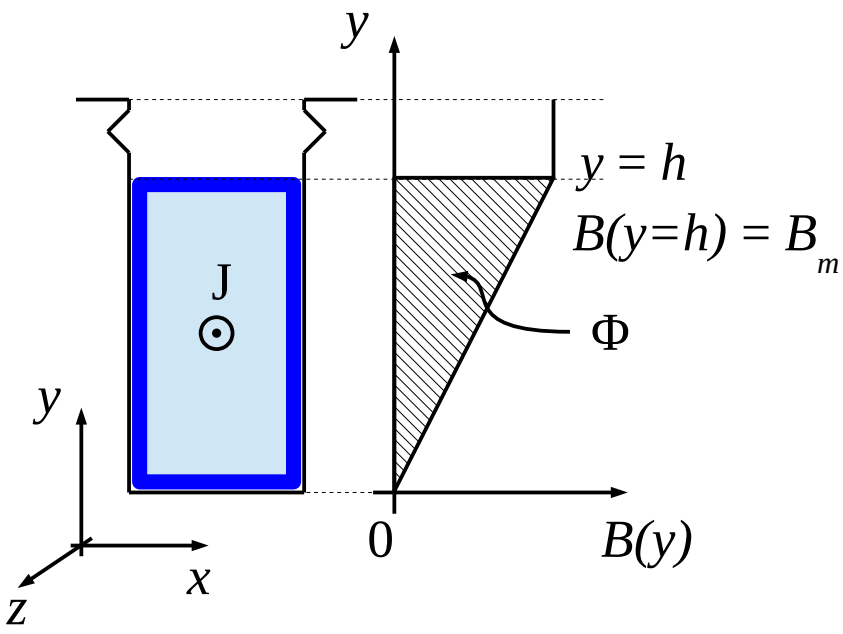
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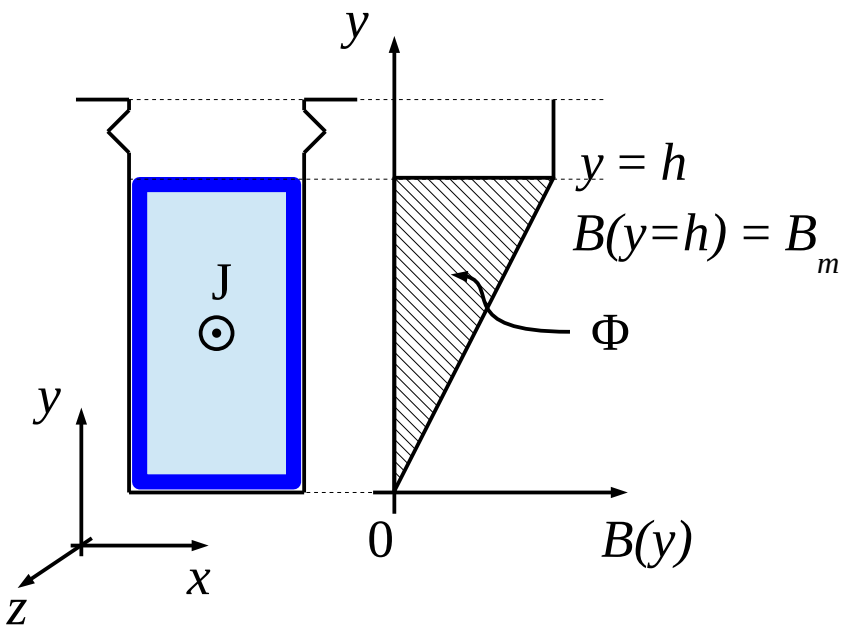
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$$\Phi = \int_S B \, dS$$



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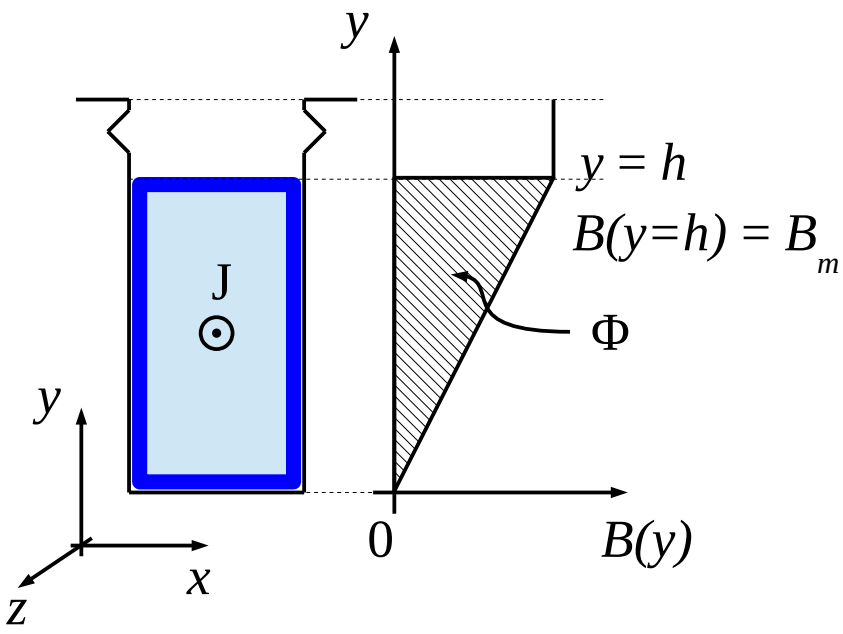


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$$\Phi = \int_0^h \int_0^l B \, dy \, dz$$

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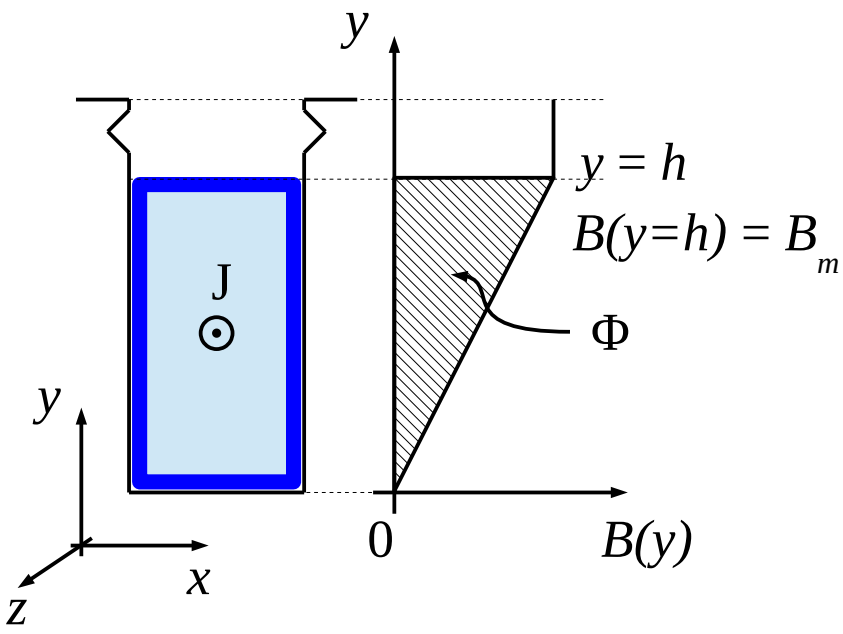
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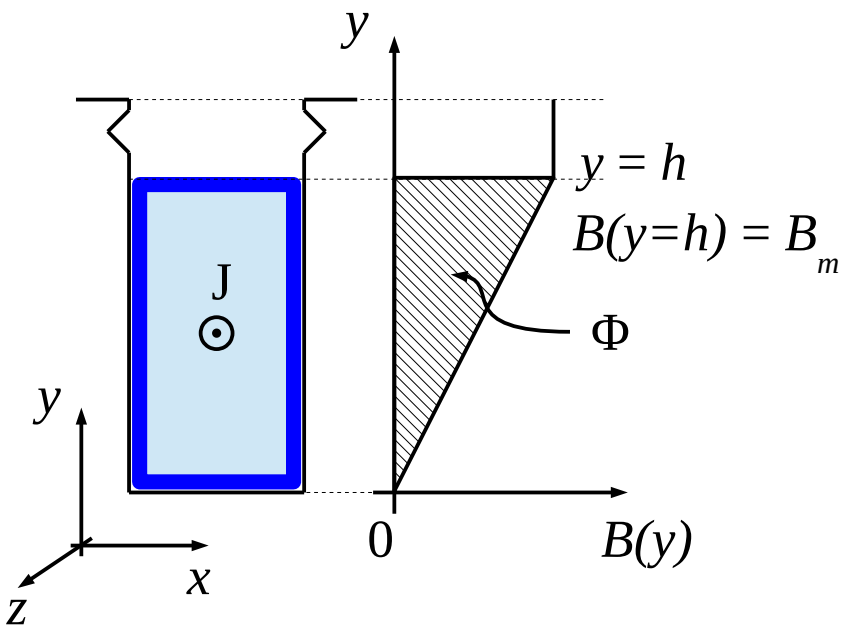
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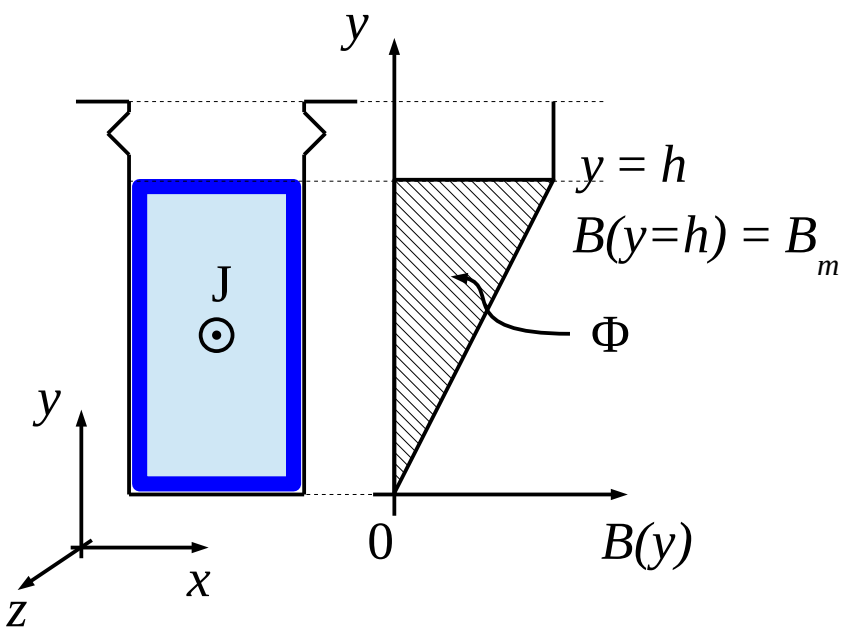
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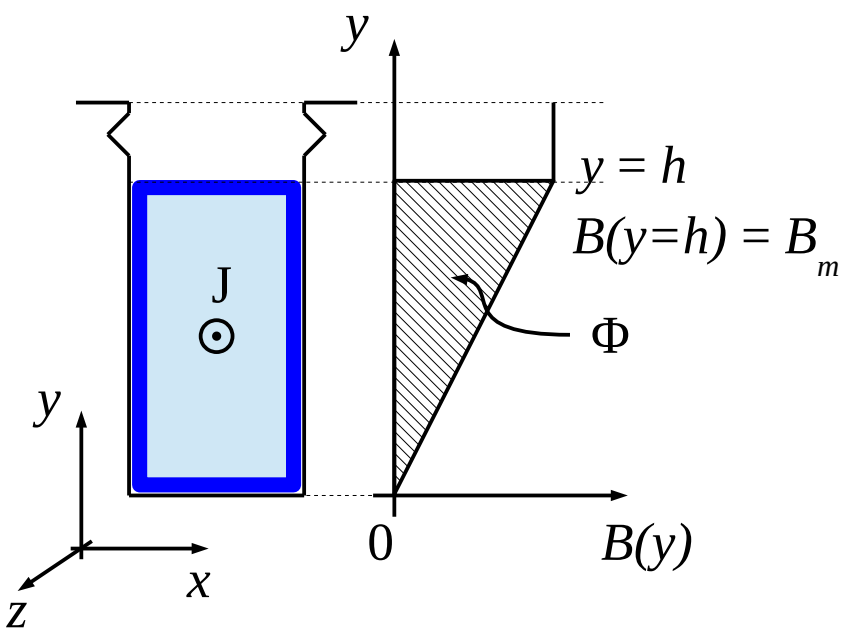
...B_m may be determined on base of equality of magnetomotive forces:



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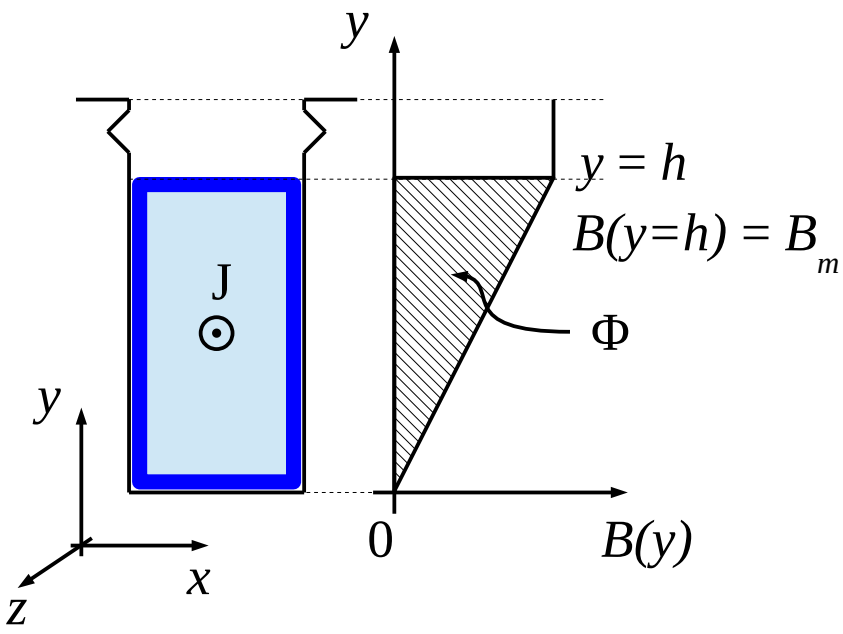
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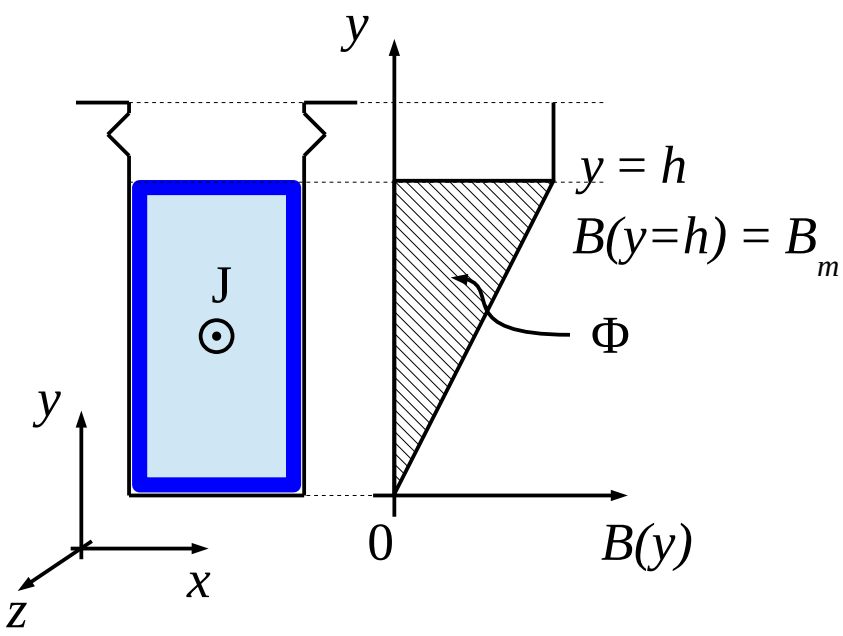
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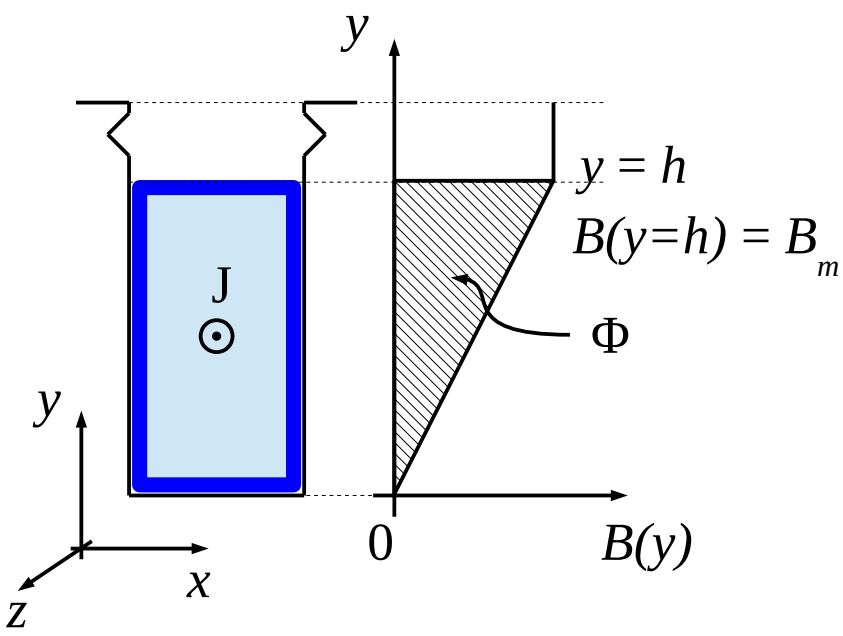
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$$H_m = J \, h$$

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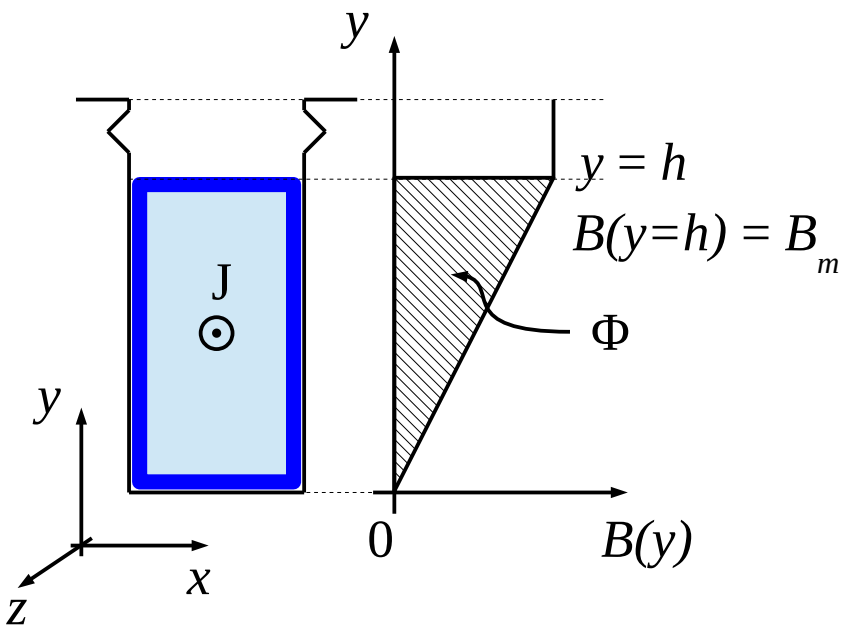
$$H_m = J \, h$$

$$B_m = \mu_0 H_m = \mu_0 J \, h$$

Winding Leakage Inductance

- Determination of a magnetic flux through one conductor:

  1. „Trivial“ approach:



...now let’s constitute to

$$\Phi = \frac{B_m h l}{2}$$

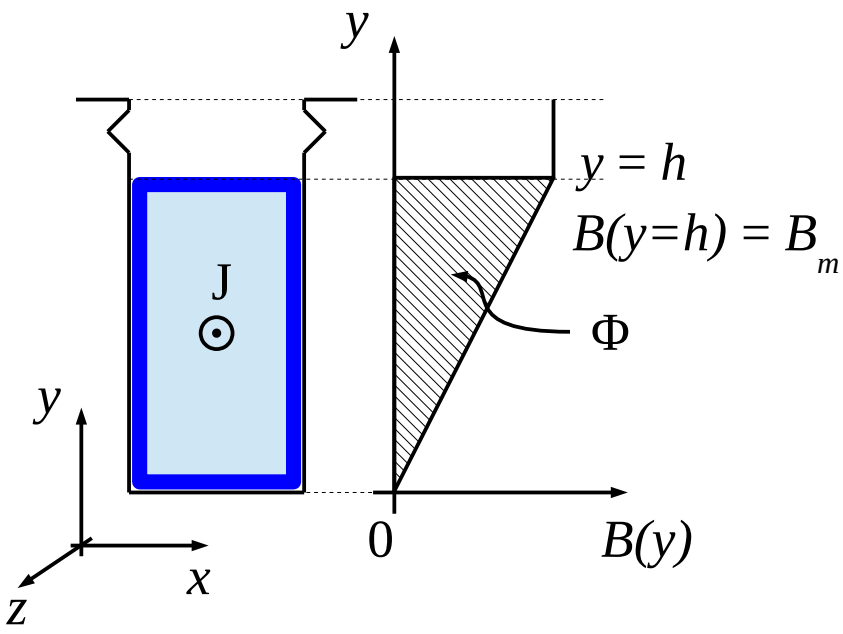
by B_m:

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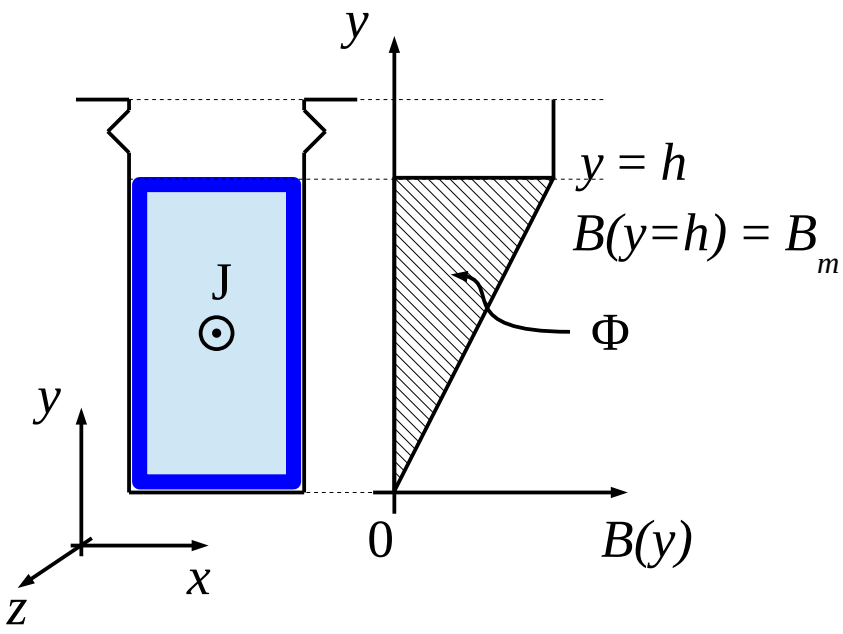
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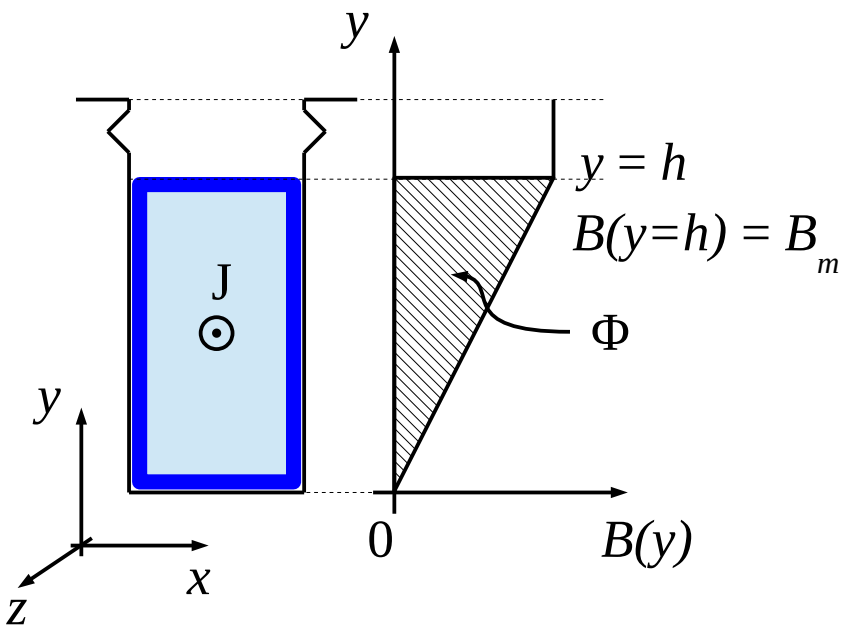
$$B_m = \mu_0 J h$$

Then we obtain:

$$\Phi = \frac{1}{2} \mu_0 J h^2 l$$

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Winding Leakage Inductance

- Determination of a magnetic flux through one conductor:
  2. Analytic approach:
  - The magnetic flux density at the bottom of the slot is zero

$$B(y=0) = 0$$

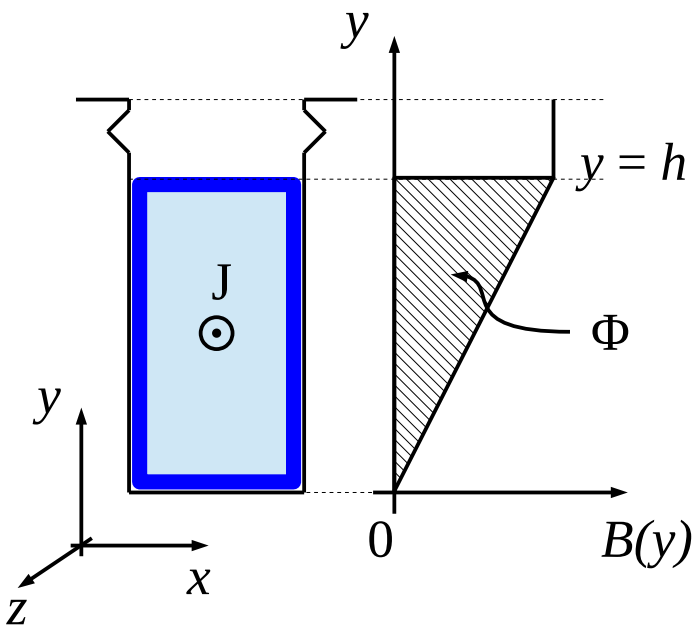
  - The magnetic flux density at the top of the slot is maximal

$$B(y=h) = B_m$$

  - Presumption: The magnetic flux density rises linearly from the bottom of the slot, however it is necessary to consider its actual value in each y-position

Winding Leakage Inductance

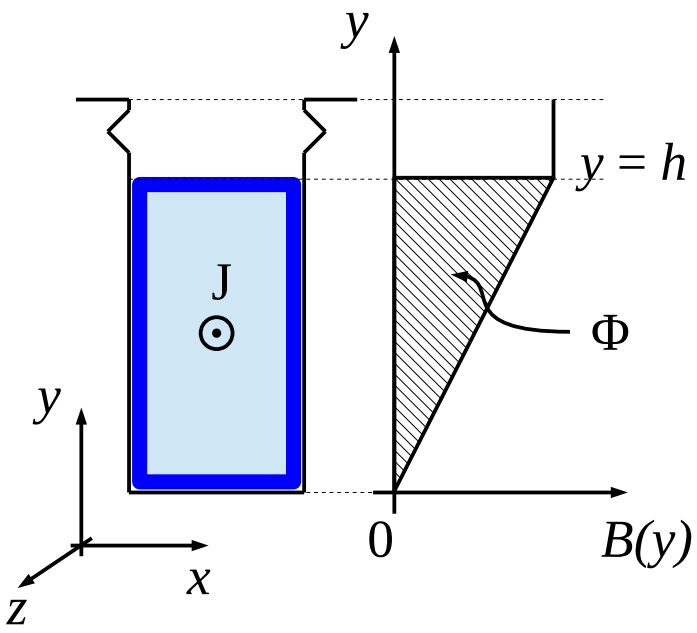
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Winding Leakage Inductance

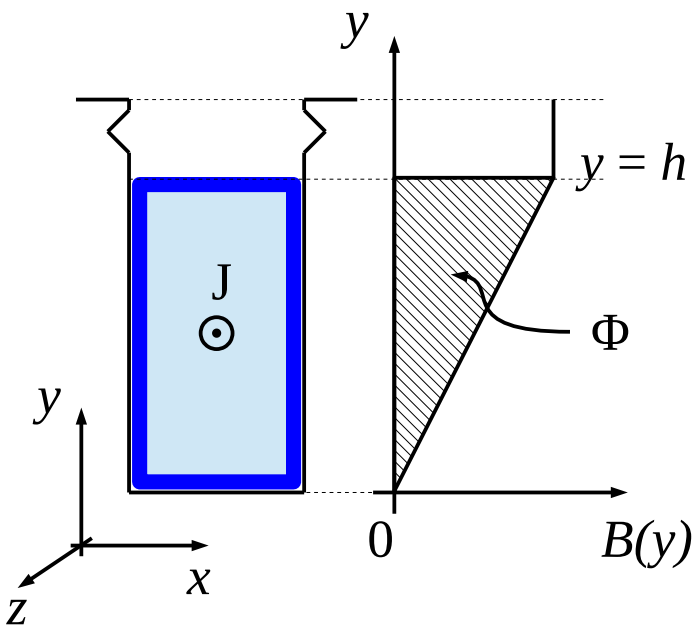
- Determination of a magnetic flux through one conductor:
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Magnetic flux density in specific y-position:



Winding Leakage Inductance

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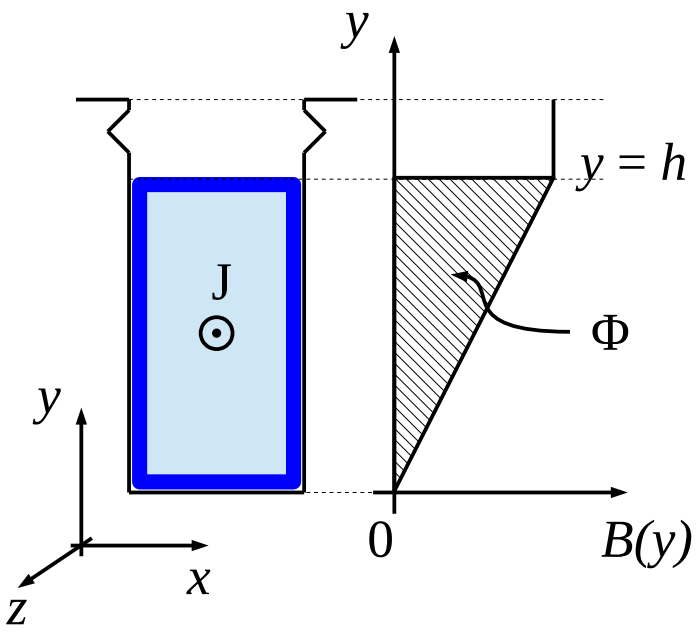


Magnetic flux density in specific y-position:

$$B(y)=\mu_0 H(y)=\mu_0 J\, y$$

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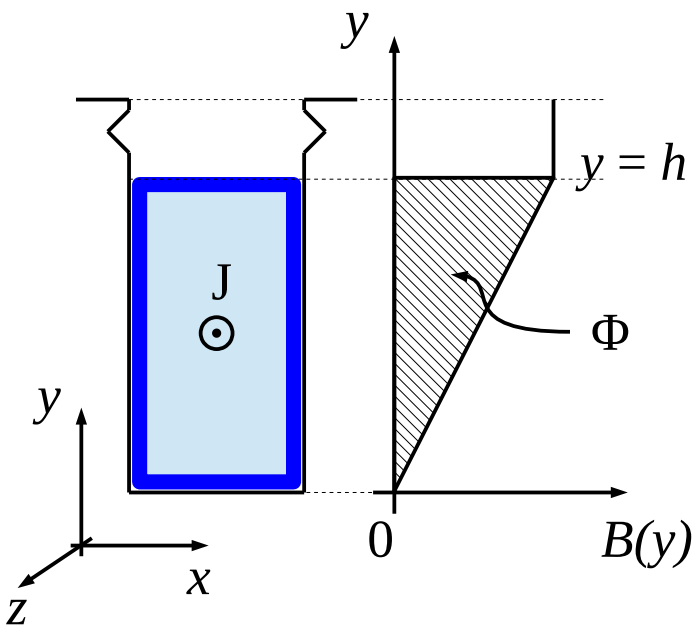
Magnetic flux density in specific y-position:

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But, at this level the magnetic flux interacts only with a part of the conductor:

Winding Leakage Inductance

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  2. Analytic approach:



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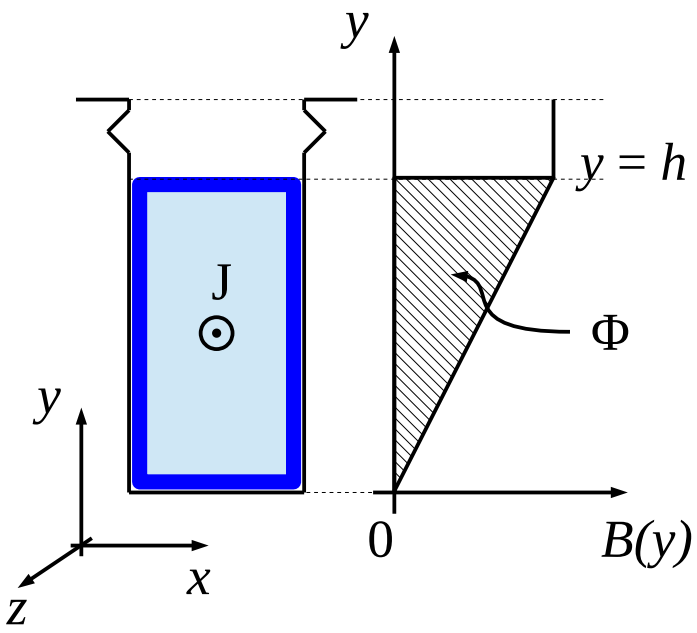
$$B(y)=\mu_0 H(y)=\mu_0 J\, y$$

But, at this level the magnetic flux interacts only with a part of the conductor:

$$\Phi' = \int_S \frac{y}{h} B(y)\, dS$$

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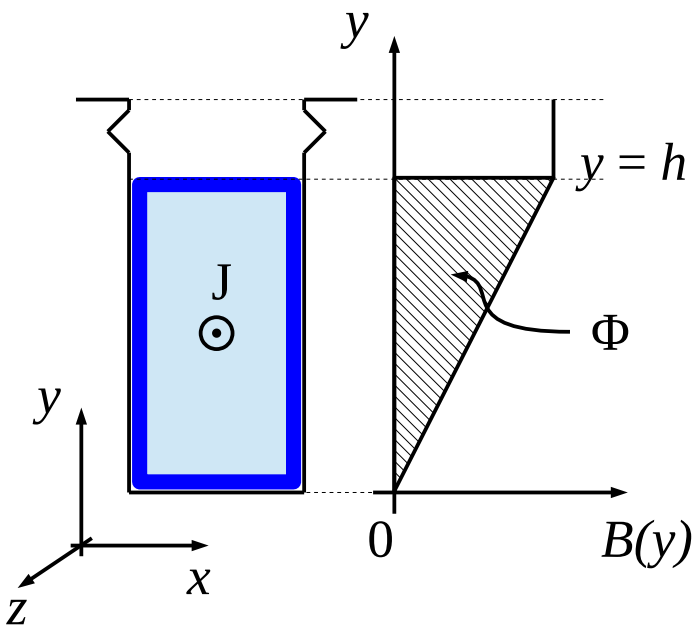
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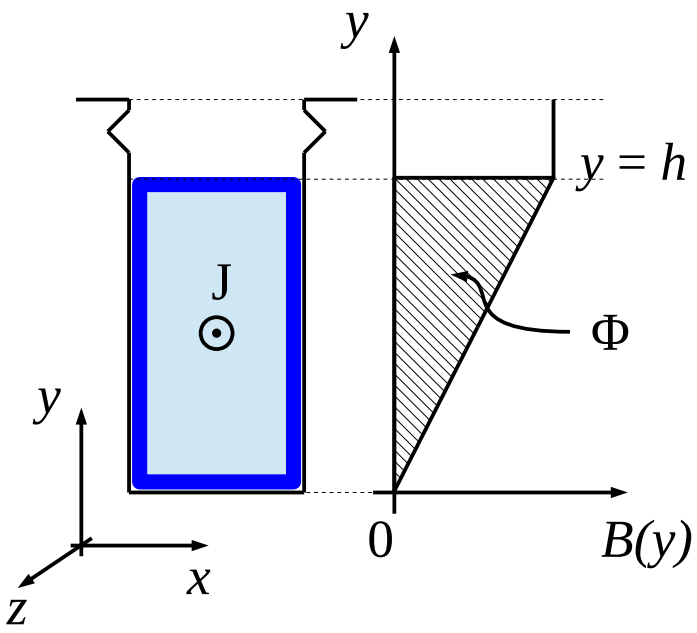


Now let's solve the integral:

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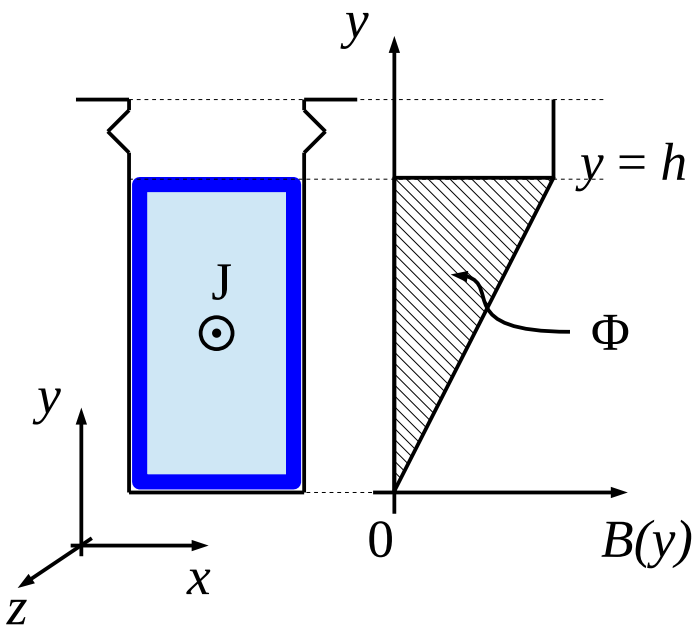
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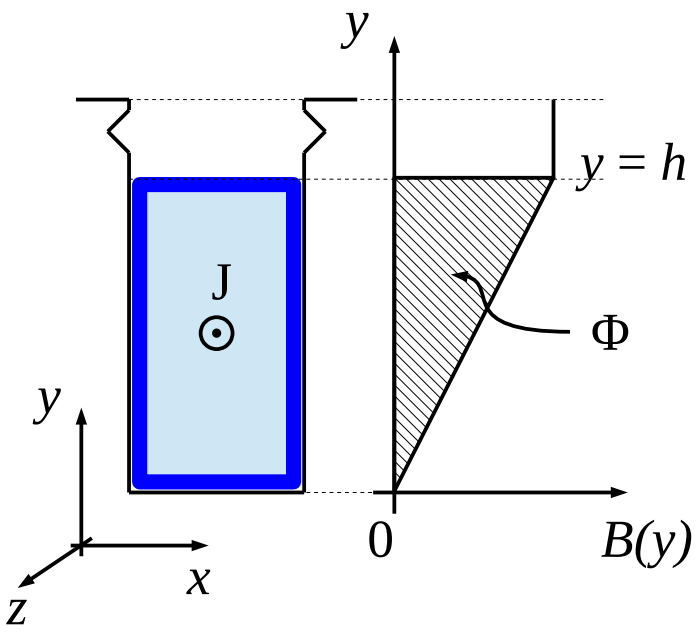
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...which is in contradiction with previous result!

Winding Leakage Inductance

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Winding Leakage Inductance

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  2. Analytic approach:

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  The ratio of both results is:

$$k = \frac{\Phi'}{\Phi} = \frac{\frac{1}{3}\mu_0 J h^2}{\frac{1}{2}\mu_0 J h^2} = \frac{2}{3}$$

  … coefficient of partial interaction of a triangular magnetic field.

Winding Leakage Inductance

- Using Φ’ it is possible to determine the leakage inductance of a conductor in a slot:

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Coefficients of Partial Interaction

Coefficients of Partial Interaction

- Differ for different typed of magnetic fields:
  - Triangular ascending field
  - Triangular descending field
  - Rectangular field (a conductor without current; air)
  - General field – area of slot opening
- A simplified determination of coefficients of partial interaction may be performed on base of induced voltages
- Induced voltage into a coil in a slot:

$$U_i = \frac{\pi}{\sqrt{2}} N_c \Phi' f$$

Coefficients of Partial Interaction

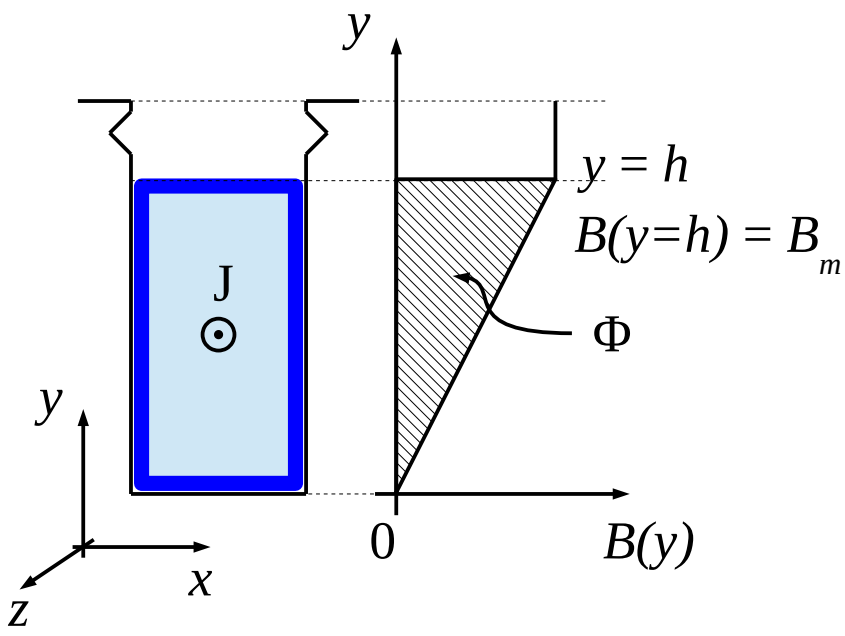
- Thus for a constant frequency we can write down

$$U_i \sim N_c \Phi' = k N_c \Phi$$

- To be able to integrate the whole induced voltage in a coil side, it is necessary to:
  - a) integrate the amount of winding turns according to magnetic flux
    - In place at y a part of magnetic flux $d\Phi$ interacts with $N(y)$ coil turns and it creates voltage of dU_i => $dU_i(y) \sim N(y) d\Phi$
  - b) integrate the magnetic flux according to the amount of coil turns
    - In a place at y with a part of dN winding turns interacts with the magnetic flux $\Phi(y)$ resulting in dU_i => $dU_i(y) \sim \Phi(y) dN$

Coefficients of Partial Interaction

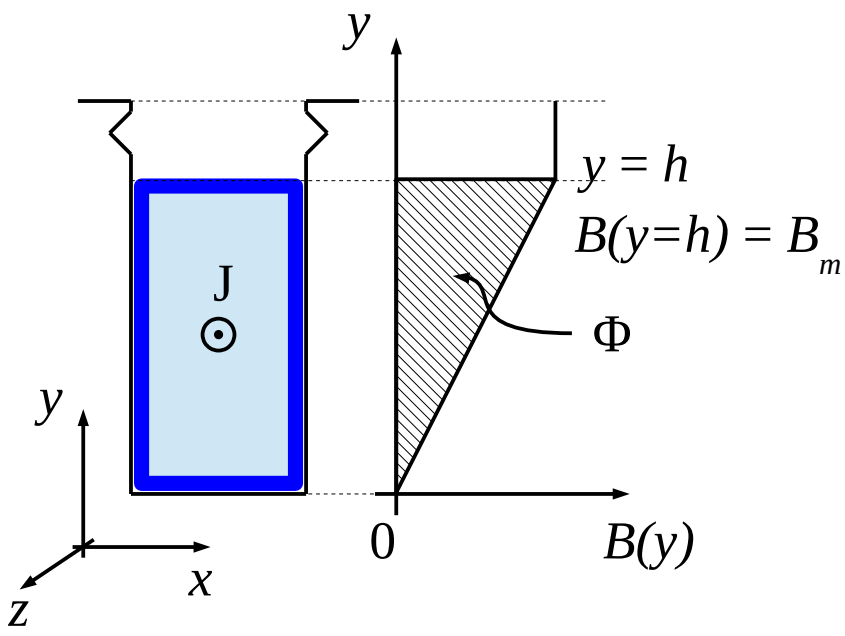
1. Triangular ascending field:



Coefficients of Partial Interaction

1. Triangular ascending field:
   - Let's integrate the amount of coil turns according to magnetic flux:

$$dU_i(y) \sim N(y)\, d\Phi$$

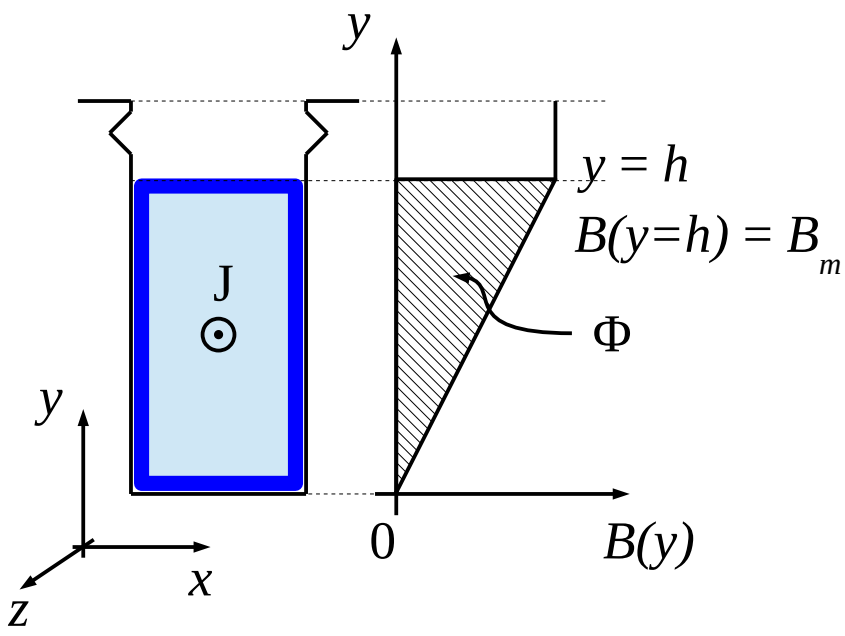


Coefficients of Partial Interaction

1. Triangular ascending field:
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Coefficients of Partial Interaction

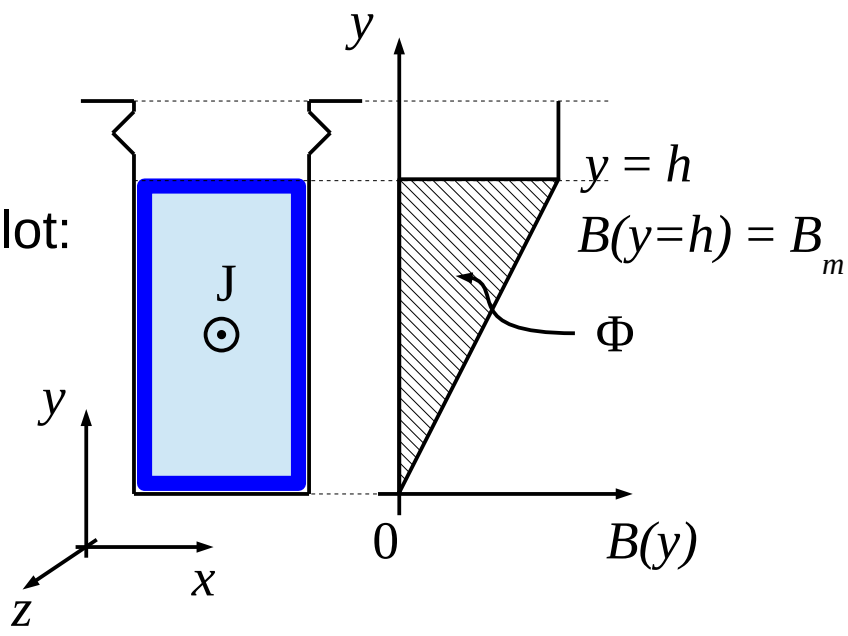
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$$N(y) = N_c \frac{y}{h}$$



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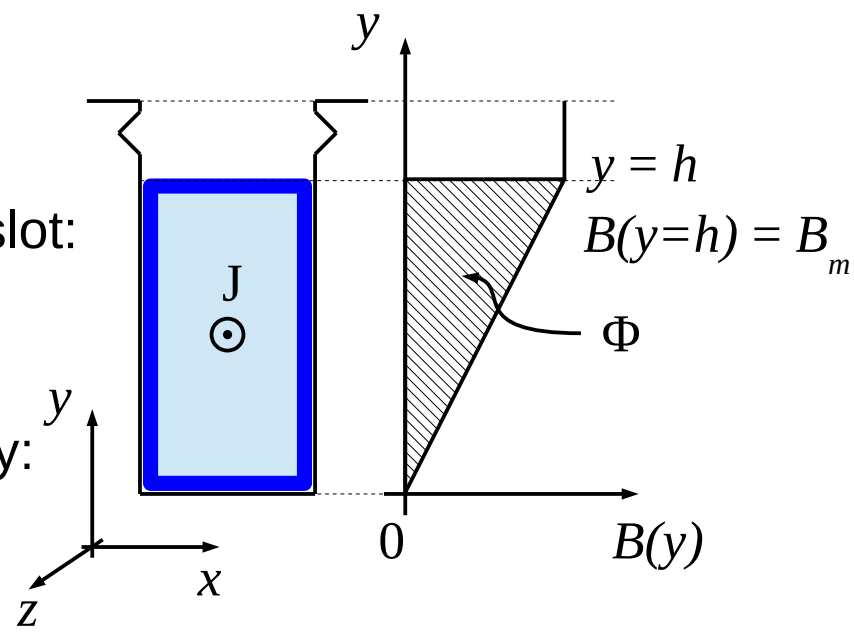
$$U_i \sim \int_0^{\Phi} N(y)\, d\Phi$$

   - It rises linearly from the bottom of the slot:

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   - Similarly rises the magnetic flux density:

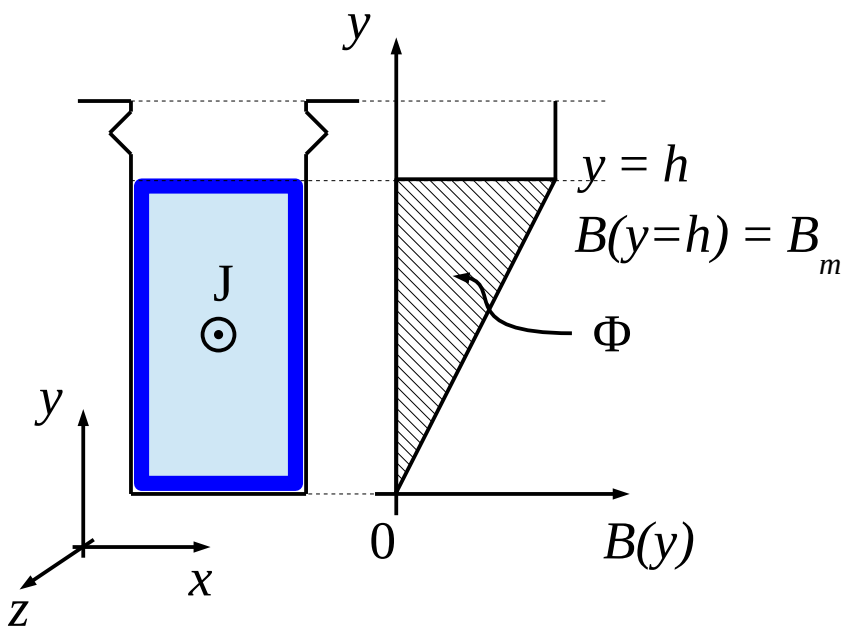
$$d\Phi = B(y)\, dS = B_m \frac{y}{h} l\, dy$$



Coefficients of Partial Interaction

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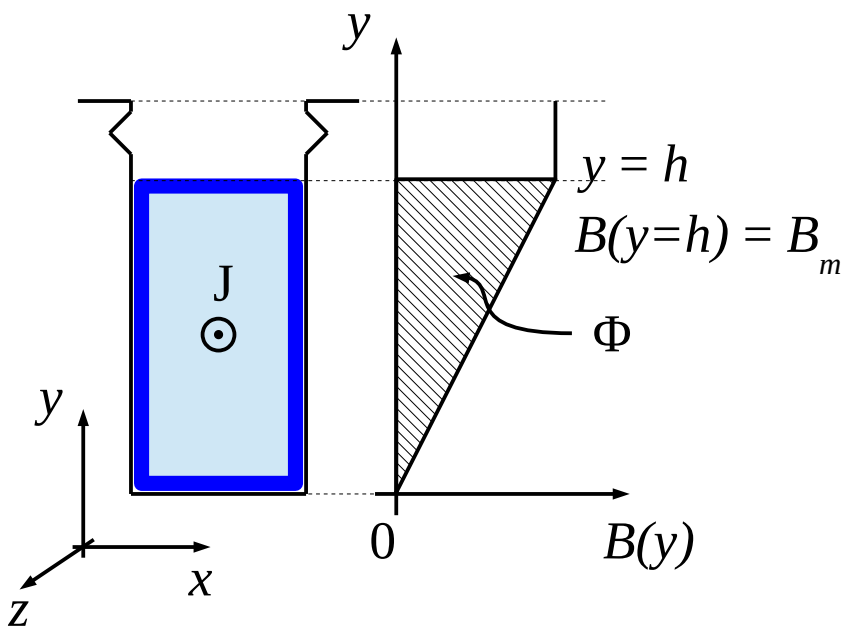


Coefficients of Partial Interaction

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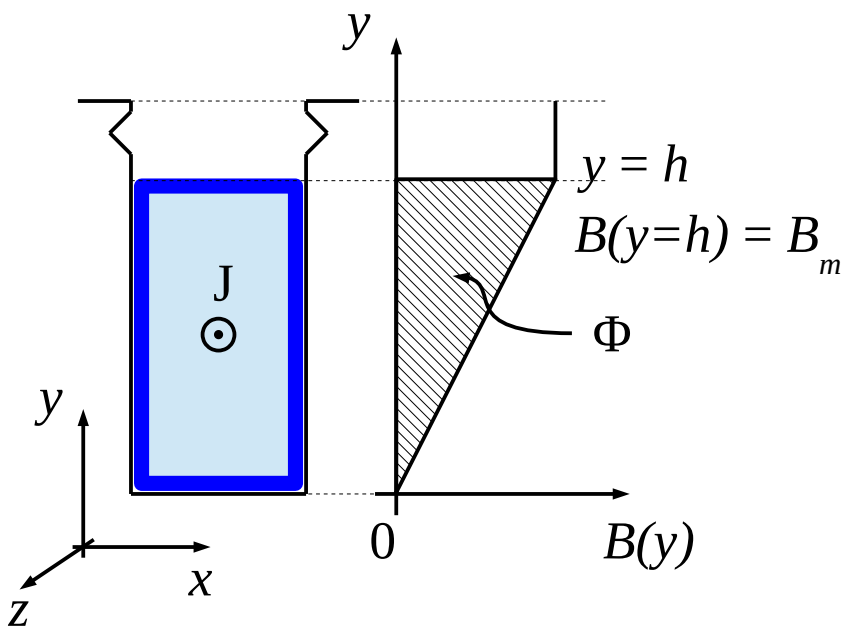
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Coefficients of Partial Interaction

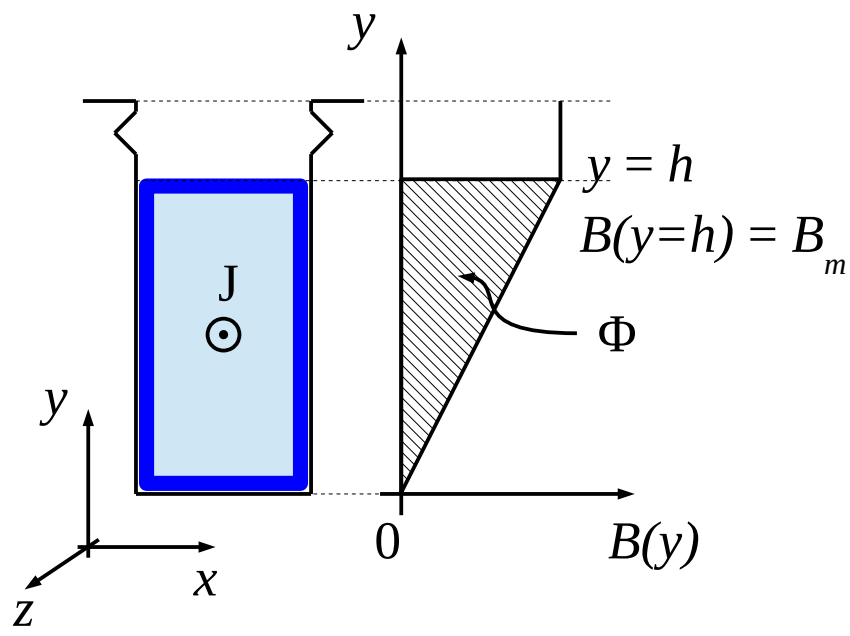
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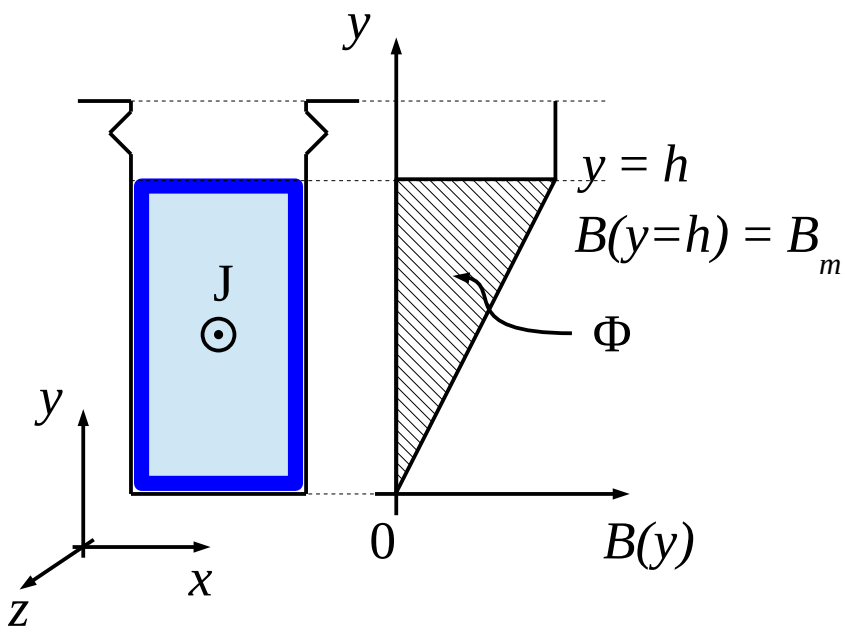
$$U_i \sim N \frac{B_m h l}{3}$$



Coefficients of Partial Interaction

1. Triangular ascending field:
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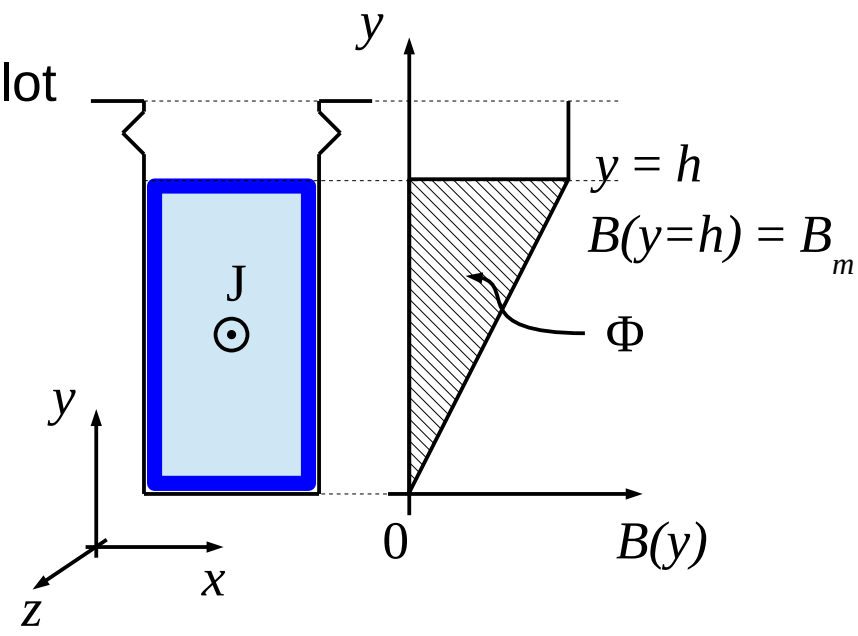
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where Φ is the magnetic flux through slot

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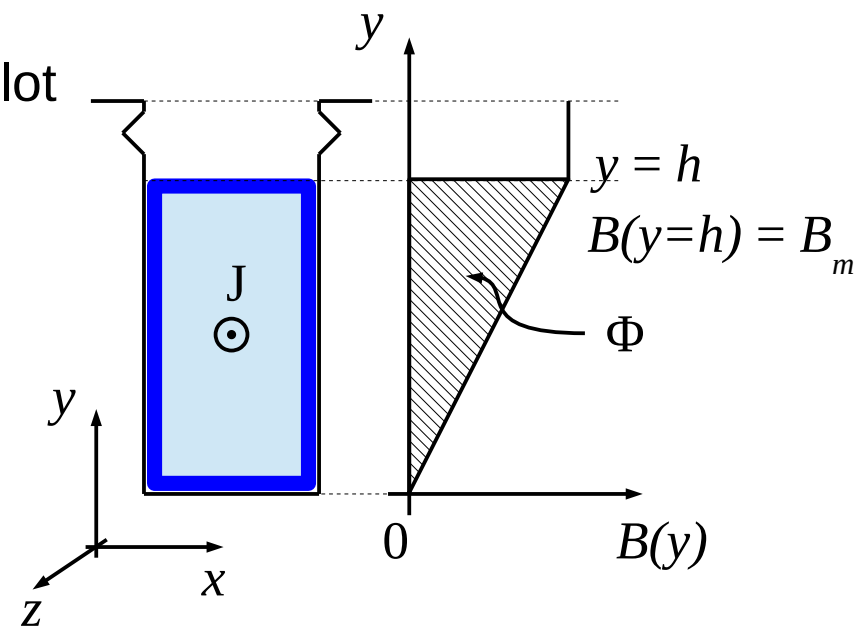
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$$\Phi = \frac{B_m h l}{2}$$

   - Thus

$$U_i \sim N \frac{B_m h l}{3} = k\, N \frac{B_m h l}{2}$$



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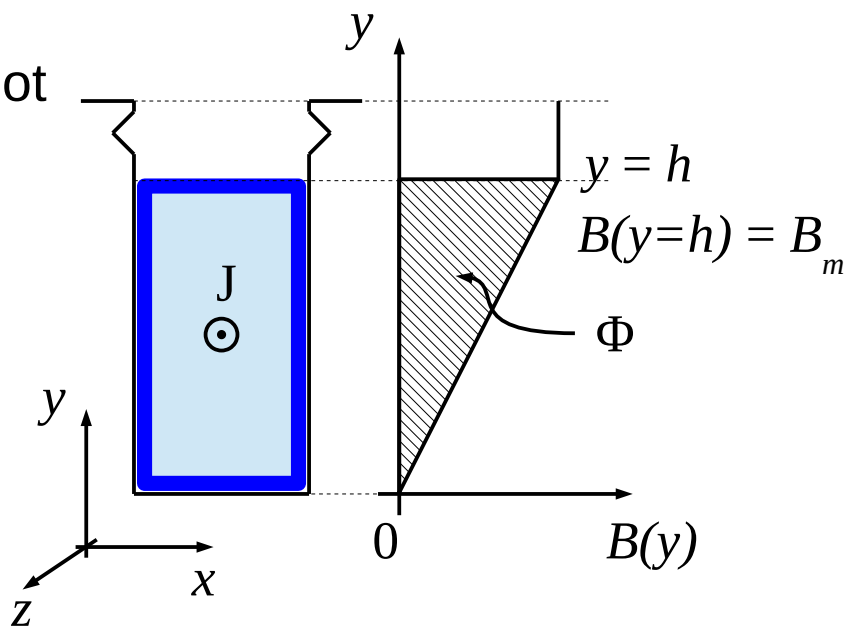
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   - Thus

     $$U_i \sim N \frac{B_m h l}{3} = k\, N \frac{B_m h l}{2}$$

   - Coefficient of partial interaction: $k = \frac{2}{3}$



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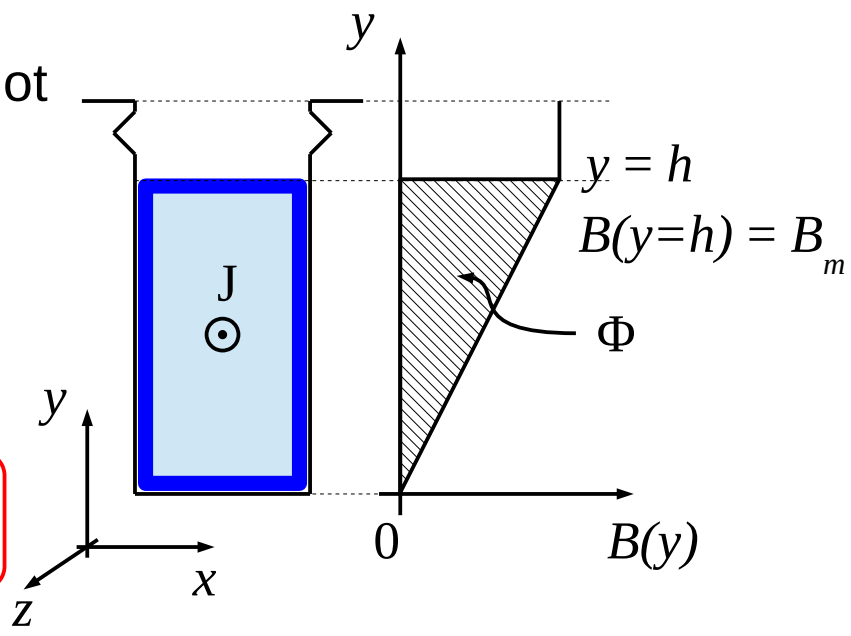
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     $$\Phi = \frac{B_m h l}{2}$$

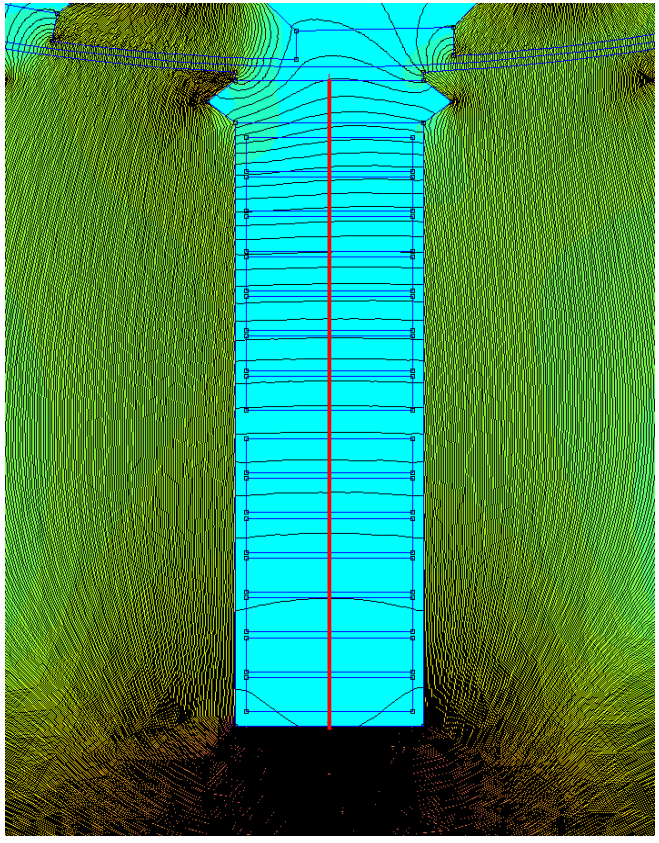
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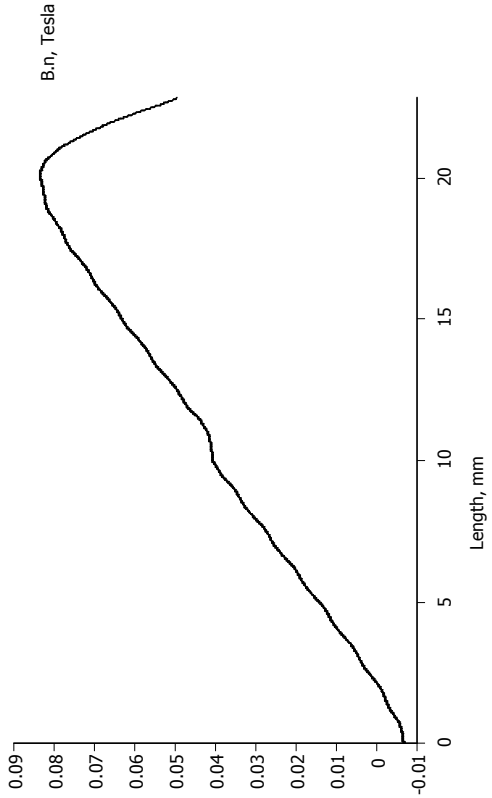
     $$U_i \sim N \frac{B_m h l}{3} = k\, N \frac{B_m h l}{2}$$

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Coefficients of Partial Interaction

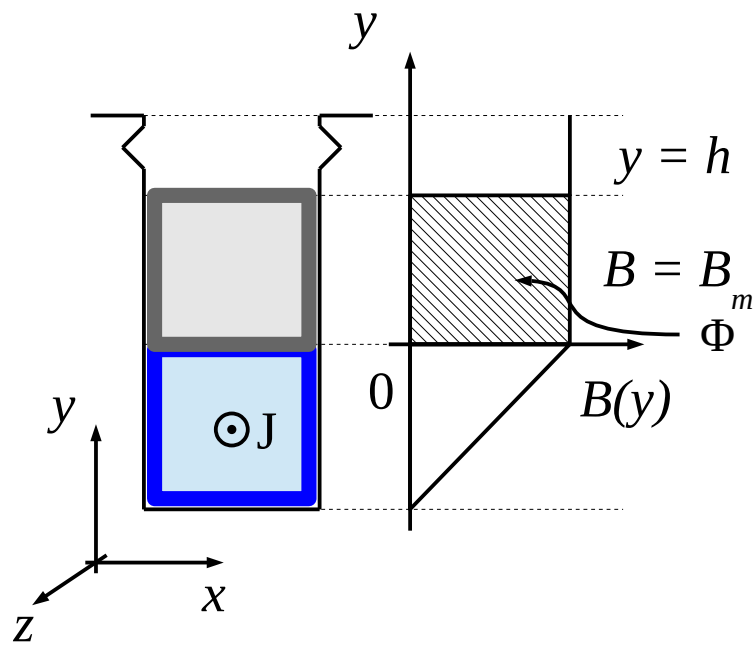




Coefficients of Partial Interaction

2. Rectangular filed:
   - Like in previous case we integrate the amount of winding turns according to flux: $d\,U_i(y) \sim N(y)\,d\,\Phi$

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Coefficients of Partial Interaction

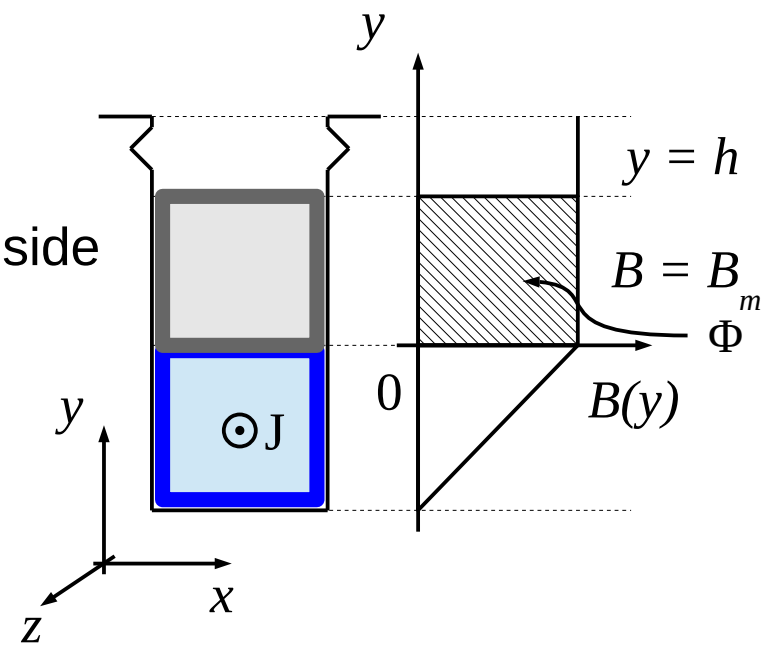
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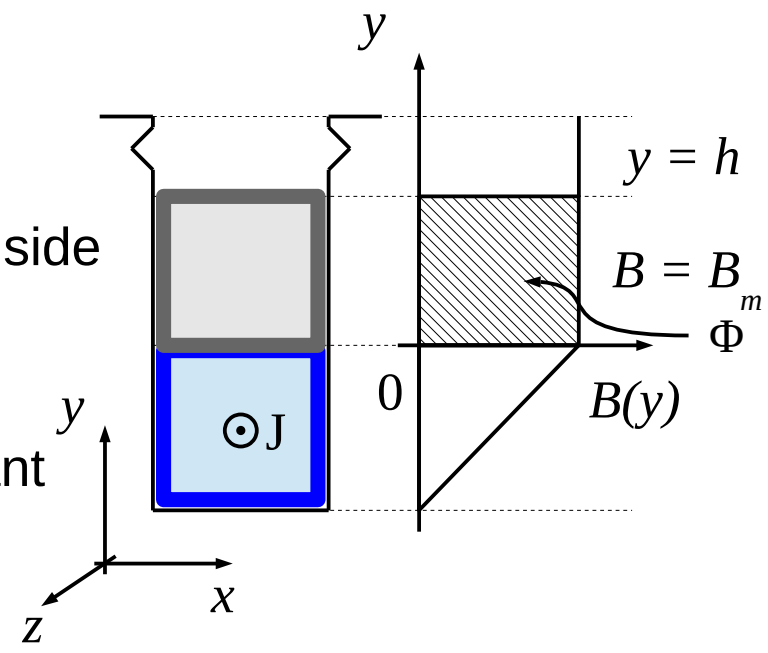
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- But the magnetic flux density is constant

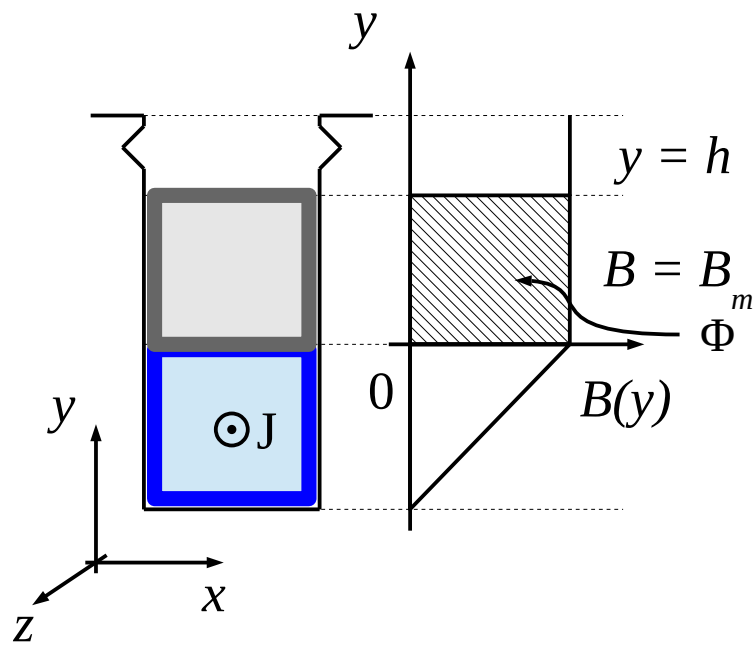
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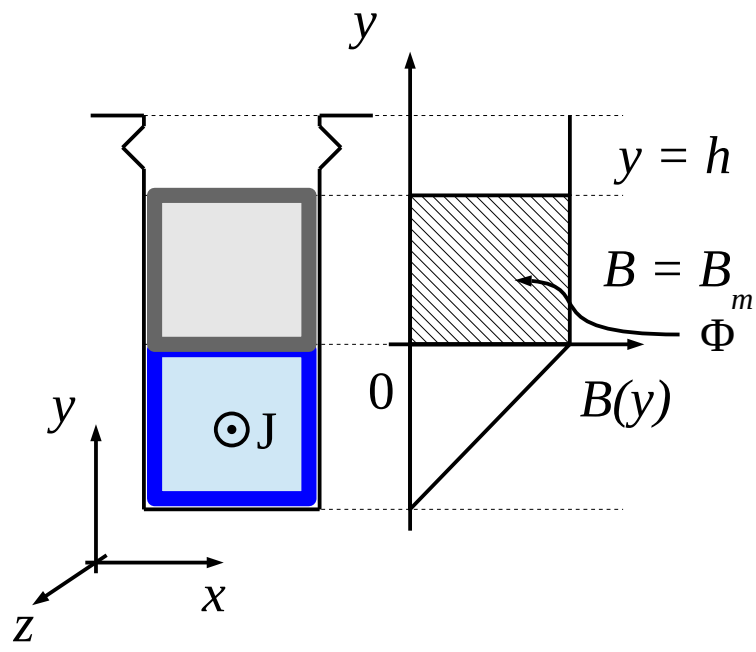


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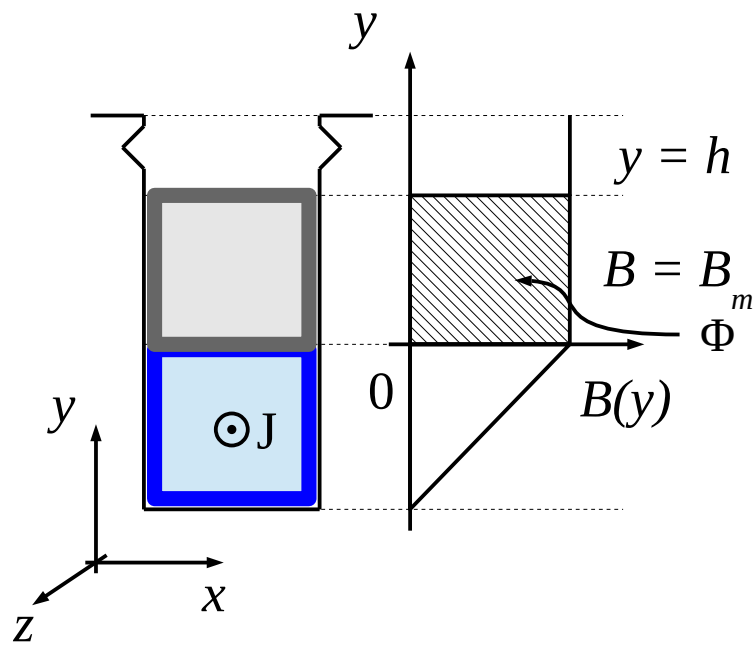
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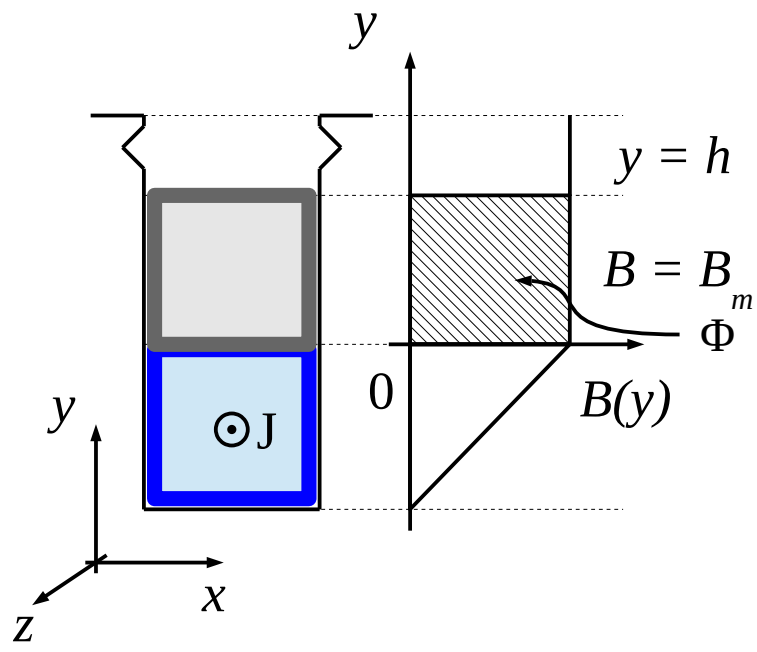
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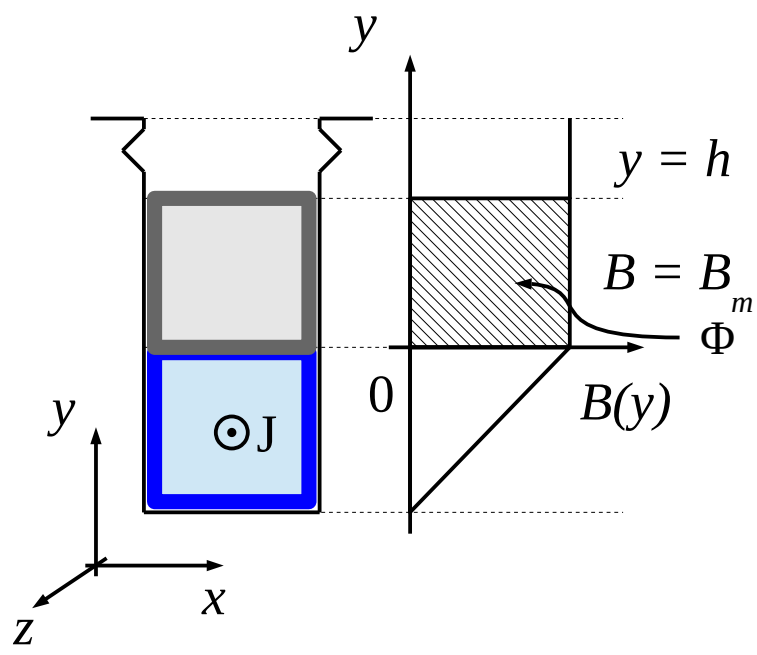
Coefficients of Partial Interaction

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is the magnetic flux in this case:

$$\Phi = B_m\, h\, l$$



Coefficients of Partial Interaction

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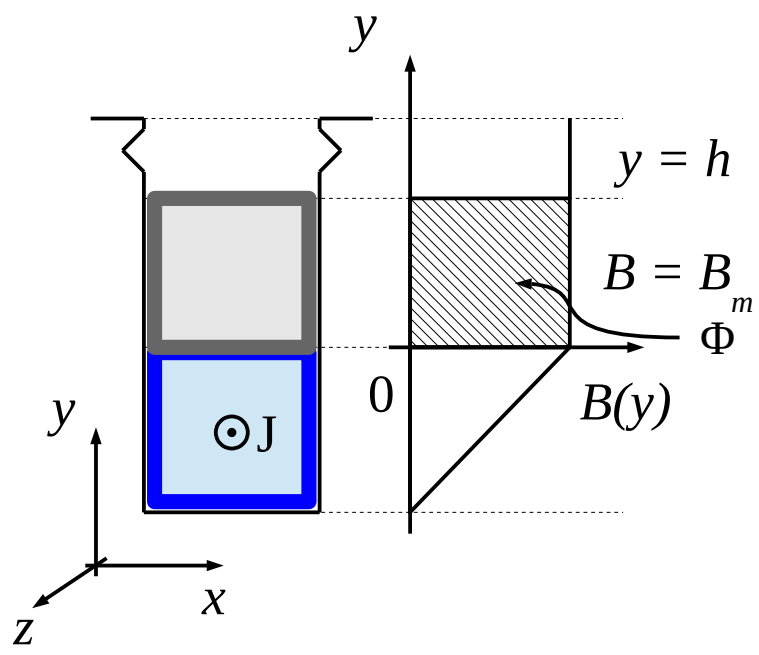
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- Comparing both results:

$$U_i \sim N\, \frac{B_m\, h\, l}{2} = k\, N\, \frac{B_m\, h\, l}{2}$$



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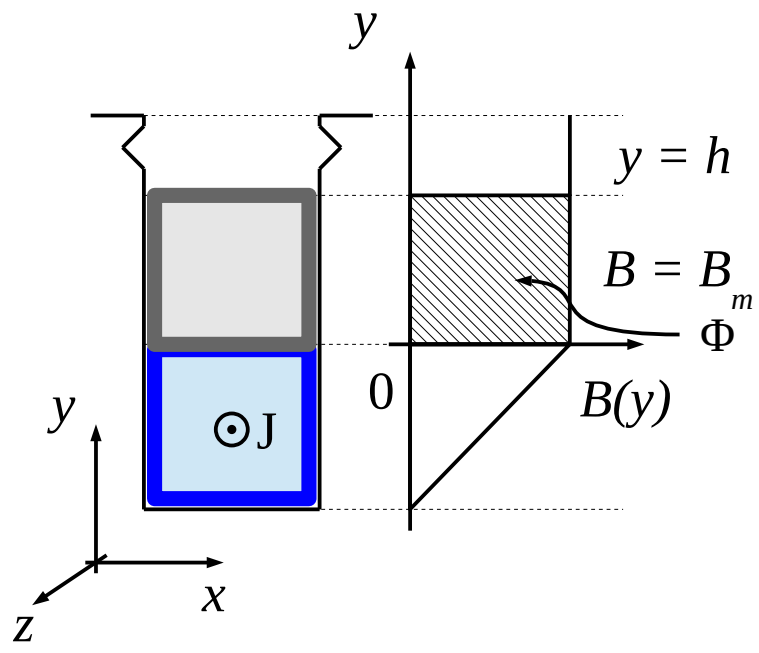
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we obtain the coefficient of partial interaction of a rectangular field:

$$k = \frac{1}{2}$$



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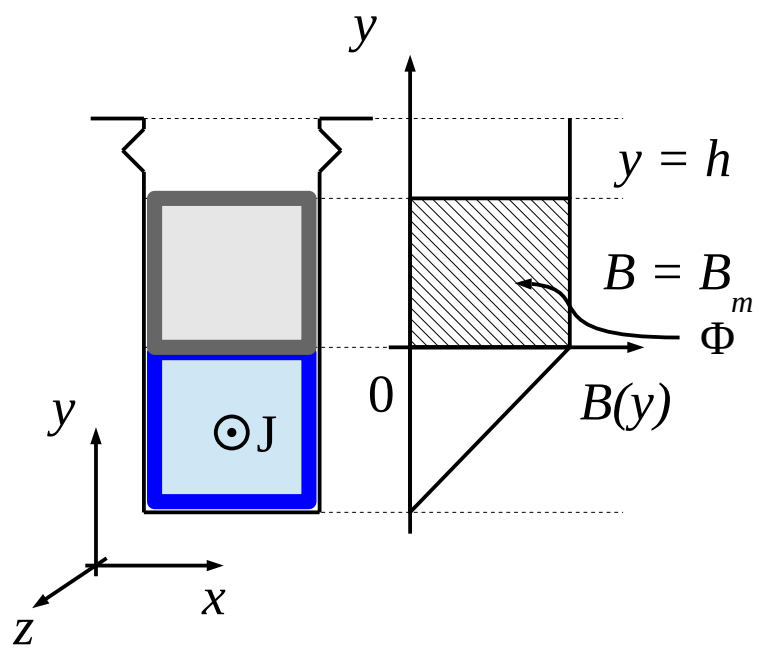
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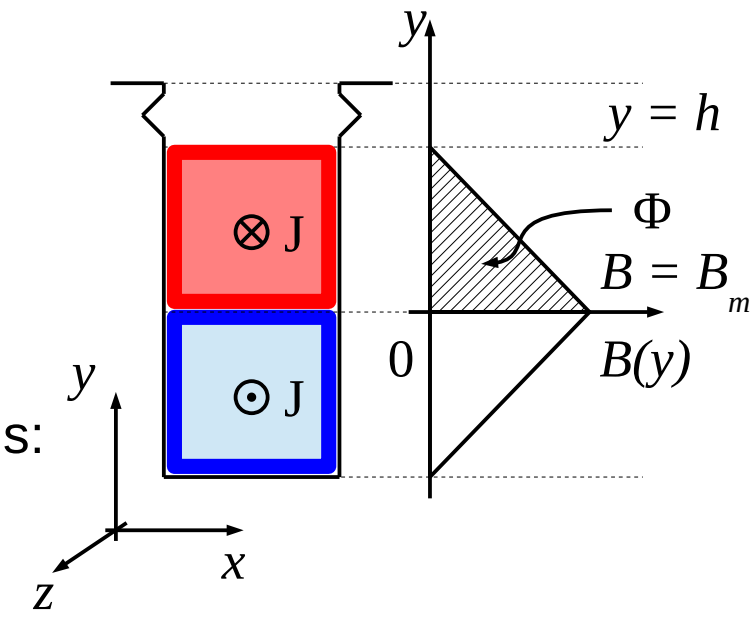
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- Which rises from the coil bottom

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- But the magnetic flux density descends:

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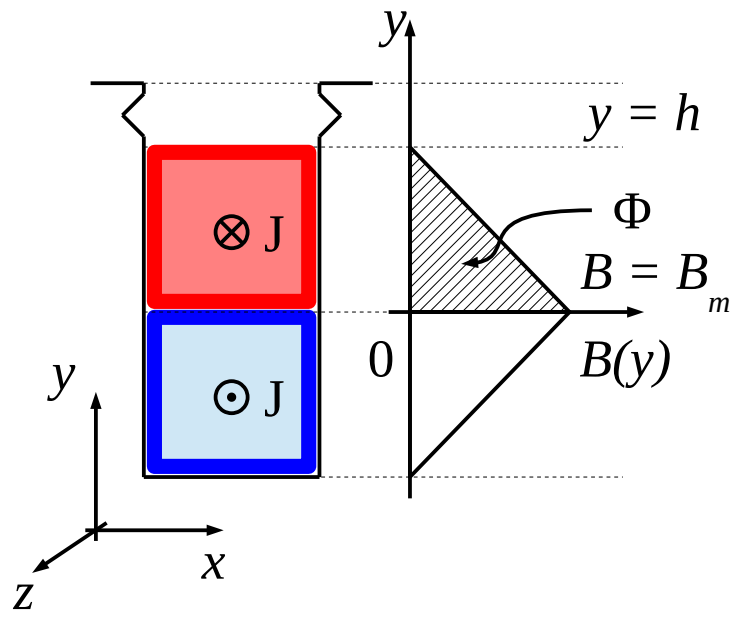


Coefficients of Partial Interaction

3. Triangular descending filed:
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$$U_i \sim \int_0^{\Phi} N(y)\, d\Phi$$

$$U_i \sim \int_0^{h} \left(N_c \frac{y}{h} \right) \left(\frac{B_m (h-y) l}{h} \right) dy$$



Coefficients of Partial Interaction

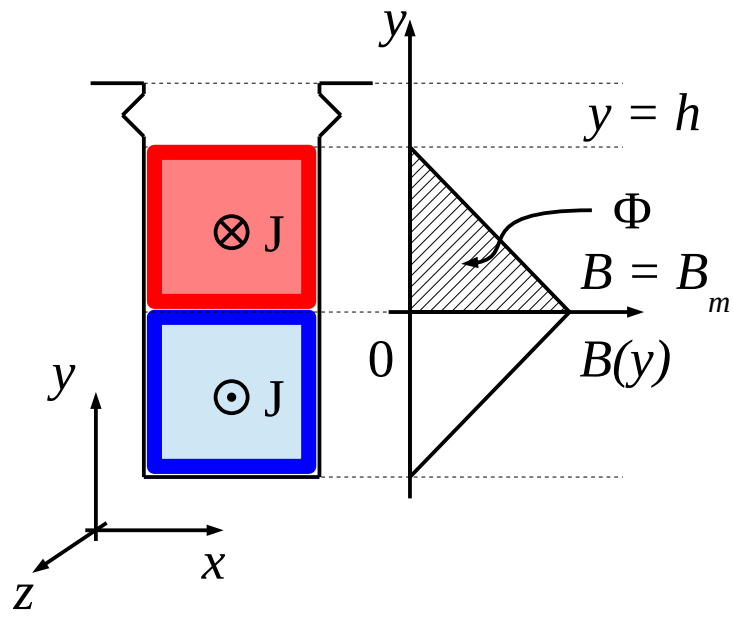
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$$= \int_0^{h} N_c \frac{B_m y l}{h} - N_c \frac{B_m y^2 l}{h^2}\, dy$$

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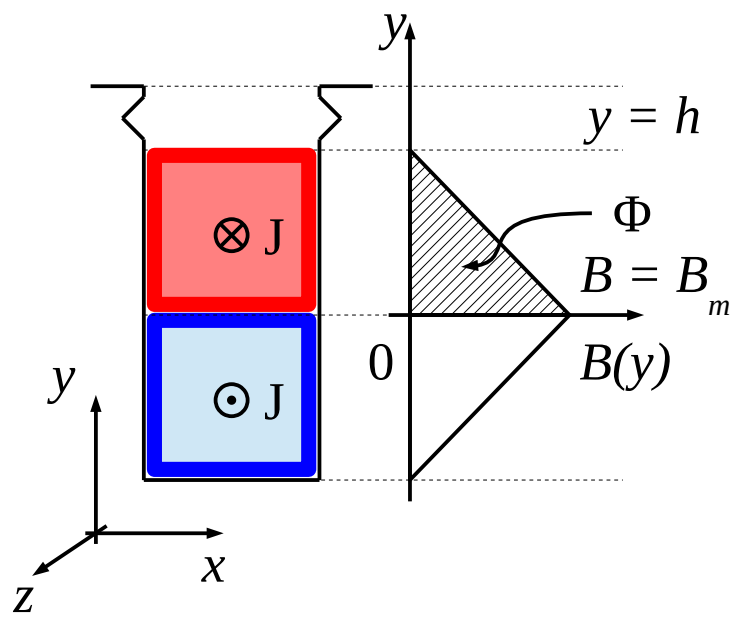
Coefficients of Partial Interaction

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$$U_i \sim N_c \frac{B_m h l}{2} - N_c \frac{B_m h l}{3}$$

$$U_i \sim N_c \frac{B_m h l}{6}$$



Coefficients of Partial Interaction

3. Triangular descending filed:

- In relation for induced voltage

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is in this case the magnetic flux

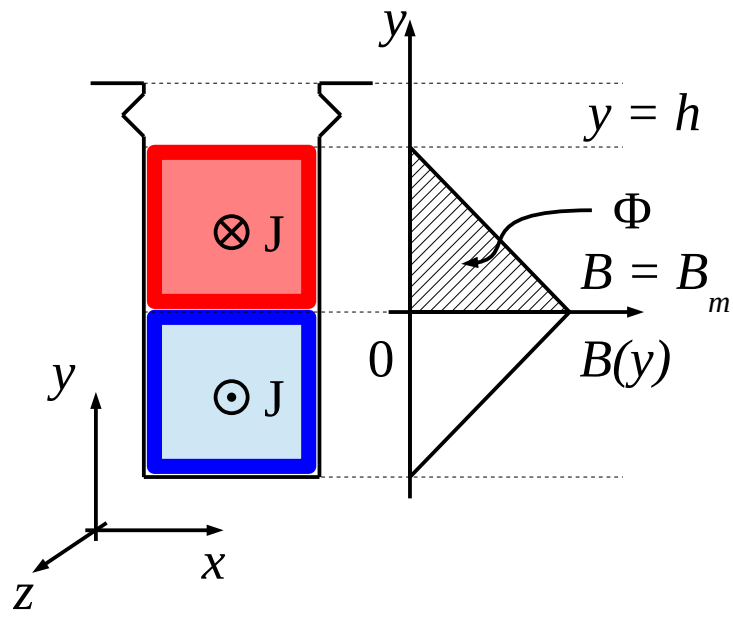
$$\Phi = \frac{B_m h l}{2}$$

- By comparison of

$$U_i \sim N_c \frac{B_m h l}{6} = k\, N_c \frac{B_m h l}{2}$$

we obtain the coefficient of partial interaction:

$$k = \frac{1}{6}$$



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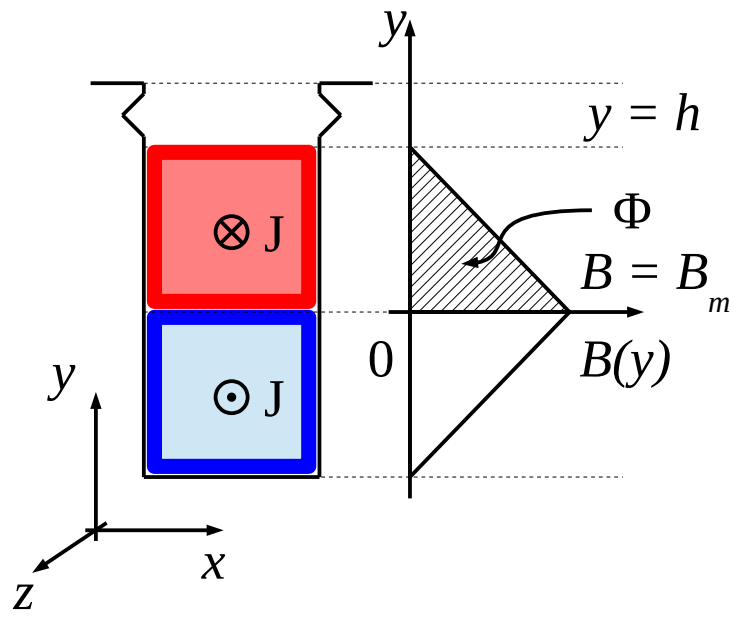
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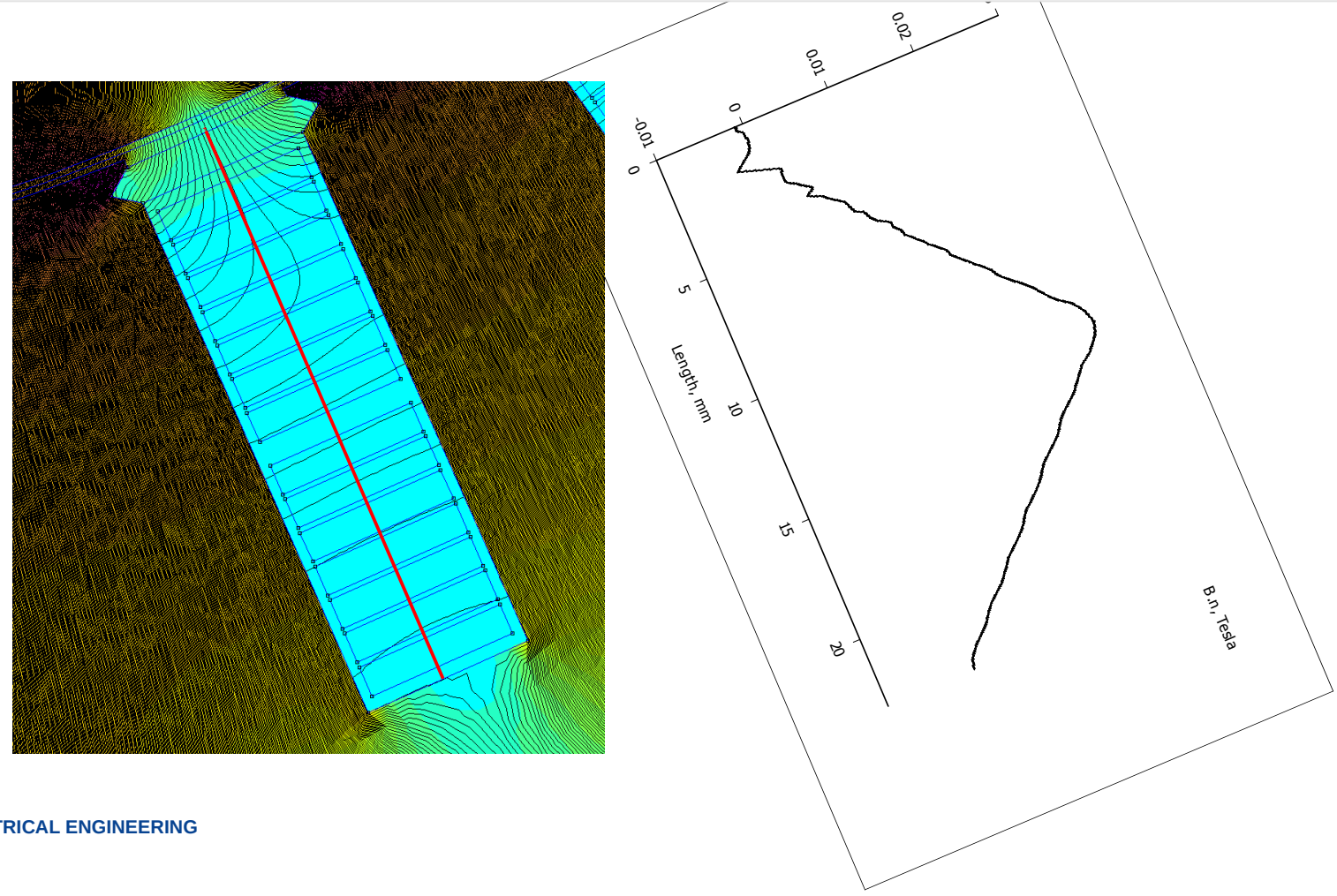
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Coefficients of Partial Interaction



FACULTY OF ELECTRICAL
ENGINEERING
UNIVERSITY OF WEST BOHEMIA

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

ESF II – project of the University of West Bohemia
reg. no. CZ.02.2.69/0.0/0.0/18_056/0013239

Transformer Leakage Inductance

Transformer Leakage Inductance

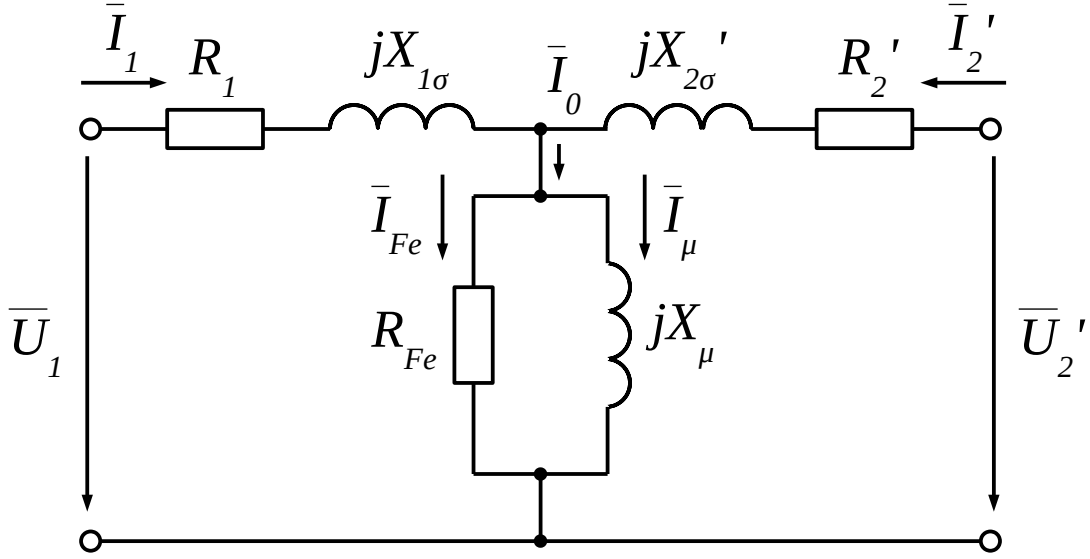
- The leakage inductance of the winding fundamentally influences the short circuit voltage of a transformer – u_k
- From the equivalent circuit of the transformer it results that if winding resistances are neglected, the transformer short circuit voltage is given by a sum of leakage inductances of both primary and secondary windings:

$$u_k = x_{1\sigma} + x_{2\sigma}$$

- But, is is possible to measure only the sum of both
- How does this situation look like from the magnetic field point of view?

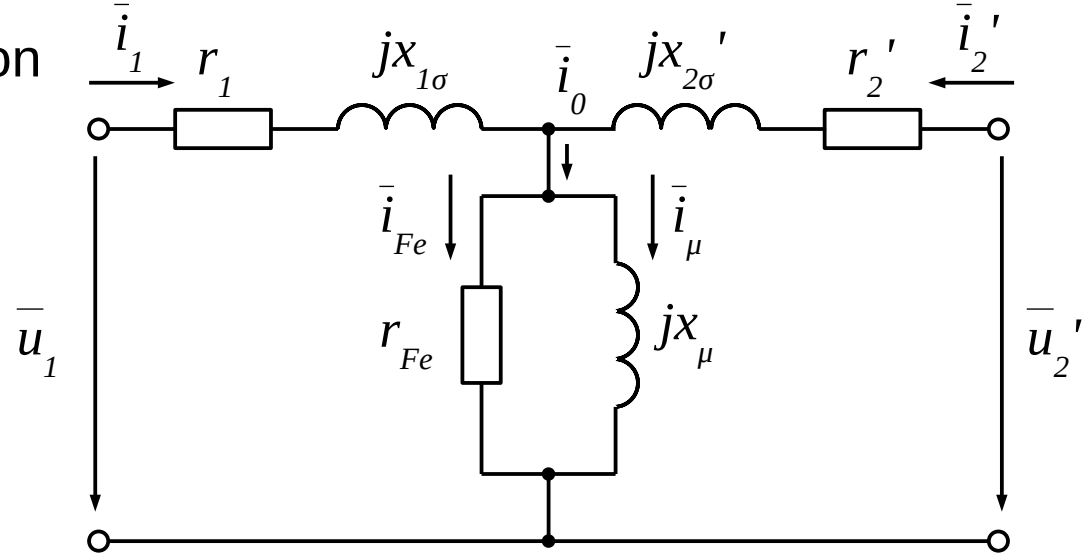
Transformer Leakage Inductance

- Presumptions:
  - we have a short circuited transformer
  - the secondary winding is power supplied
  - primary side is in short circuit



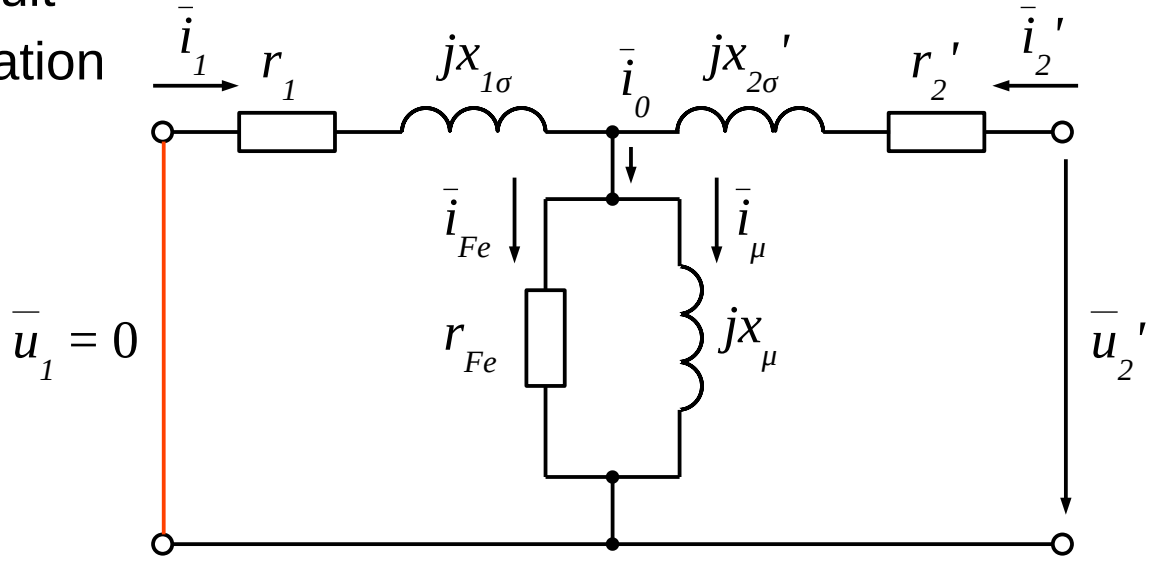
Transformer Leakage Inductance

- Presumptions:
  - we have a short circuited transformer
  - the secondary winding is power supplied
  - primary side is in short circuit
  - for the purpose of simplification we will use the system of proportional units [$p.u.$]



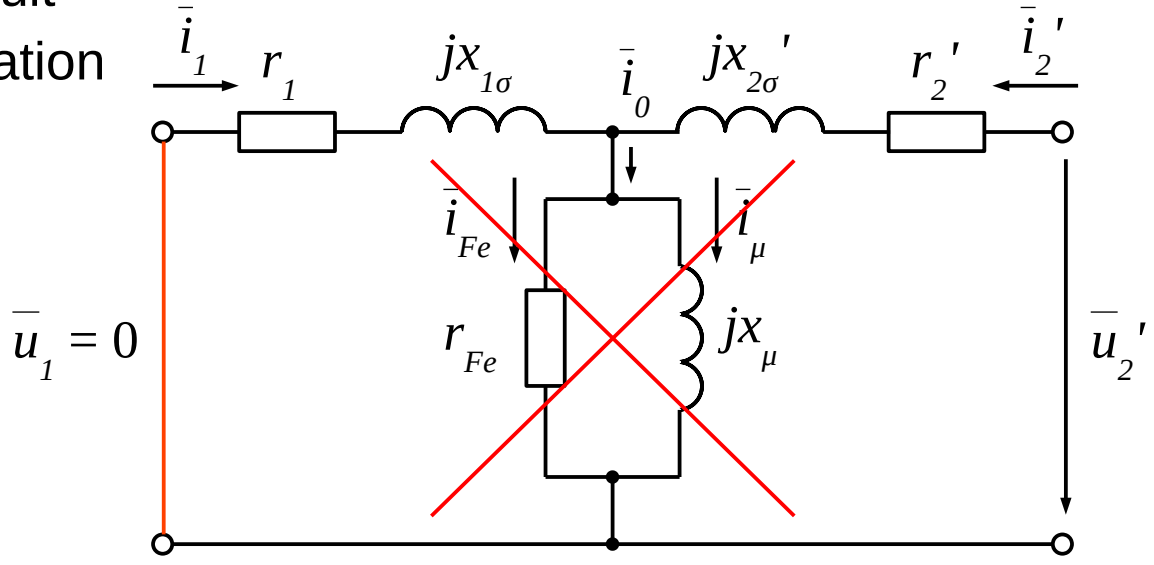
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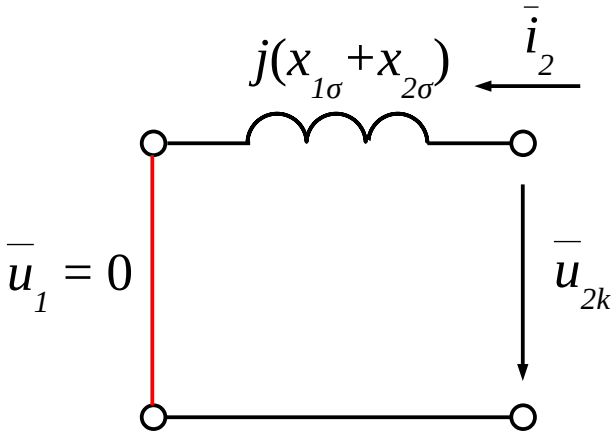
Transformer Leakage Inductance

- Presumptions:
  - we have a short circuited transformer
  - the secondary winding is power supplied
  - primary side is in short circuit
  - for the purpose of simplification we will use the system of proportional units [$p.u.$]
  - the magnetizing branch may be neglected



Transformer Leakage Inductance

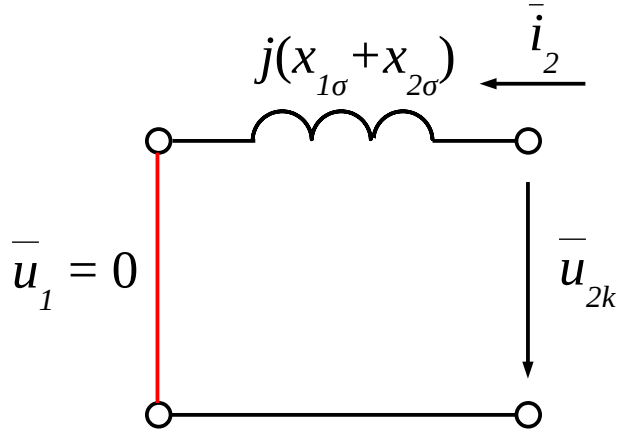
- Resulting equivalent circuit for the short-circuit state:



Transformer Leakage Inductance

- The transformer equations in per units:

$$\left(r_1 + jx_1\right)\bar{i}_1 + j\,x_\mu\,\bar{i}_2 = \bar{u}_1$$
$$j\,x_\mu\,\bar{i}_1 + \left(r_2 + jx_2\right)\bar{i}_2 = \bar{u}_2$$

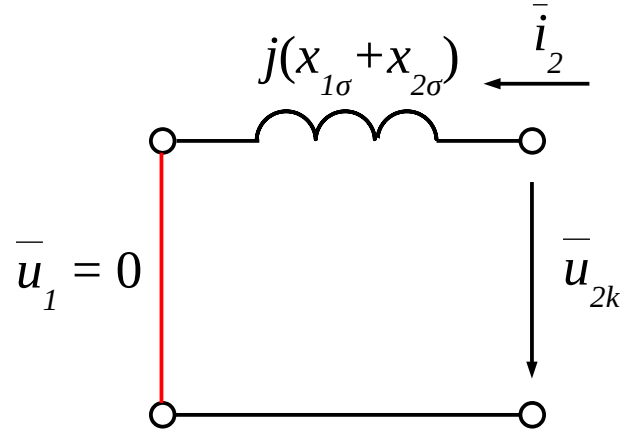


Transformer Leakage Inductance

- The transformer equations in per units:

$$\begin{aligned} \left(r_1 + jx_1\right)\bar{i}_1 + j\,x_\mu\,\bar{i}_2 &= \bar{u}_1 \\ j\,x_\mu\,\bar{i}_1 + \left(r_2 + jx_2\right)\bar{i}_2 &= \bar{u}_2 \end{aligned}$$

$$\begin{aligned} jx_1\,\bar{i}_1 + j\,x_\mu\,\bar{i}_2 &= 0 \\ j\,x_\mu\,\bar{i}_1 + jx_2\,\bar{i}_2 &= \bar{u}_{2k} \end{aligned}$$



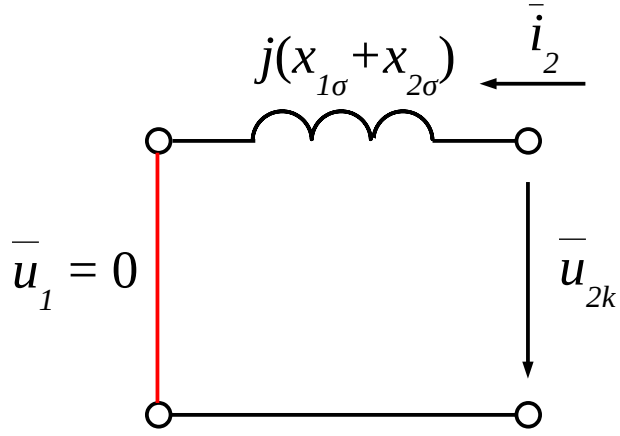
Transformer Leakage Inductance

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$$jx_1\,\bar{i}_1 + j\,x_\mu\,\bar{i}_2 = 0$$
$$j\,x_\mu\,\bar{i}_1 + jx_2\,\bar{i}_2 = \bar{u}_{2k}$$

$$j\left(x_{1\sigma} + x_\mu\right)\bar{i}_1 + j\,x_\mu\,\bar{i}_2 = 0$$
$$j\,x_\mu\,\bar{i}_1 + j\left(x_{2\sigma} + x_\mu\right)\bar{i}_2 = \bar{u}_{2k}$$



Transformer Leakage Inductance

- The transformer equations in per units:

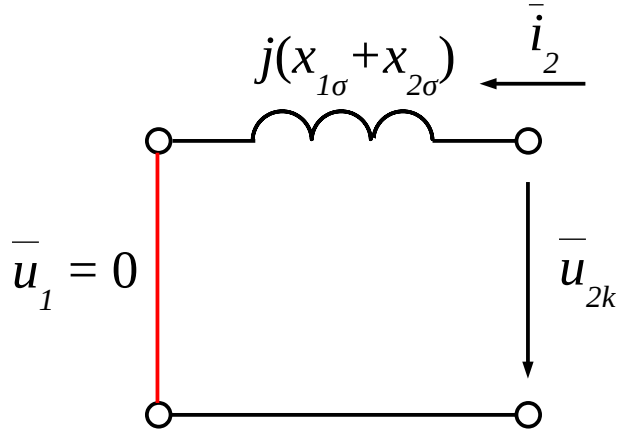
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$$j\,x_\mu\left(\bar{i}_1 + \bar{i}_2\right) + j\,x_{2\sigma}\,\bar{i}_2 = \bar{u}_{2k}$$

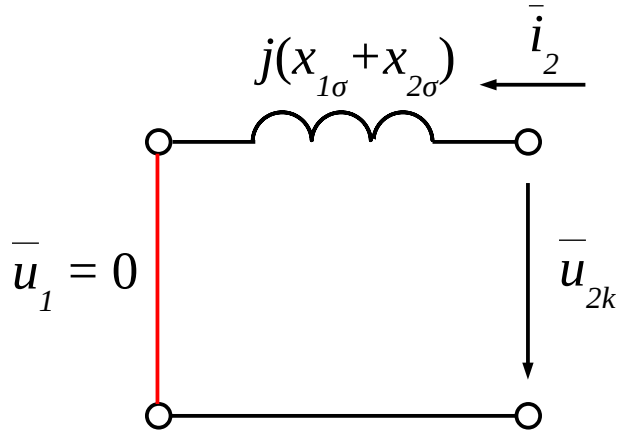
$$!\ \bar{i}_1 + \bar{i}_2 = 0\ !$$



Transformer Leakage Inductance

- The transformer equations in per units:

$$j x_{1\sigma} \bar{i}_1 = 0$$
$$j x_{2\sigma} \bar{i}_2 = \bar{u}_{2k}$$

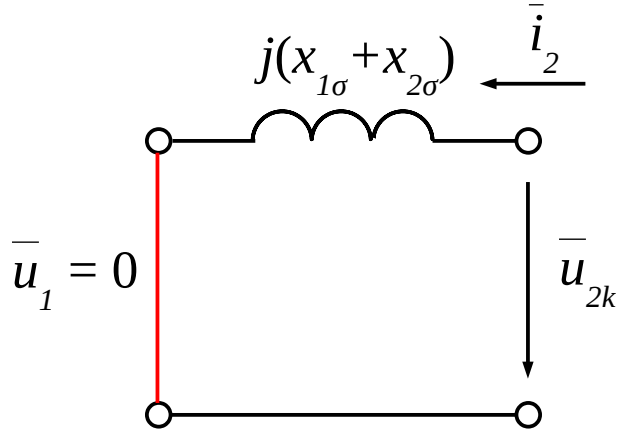


Transformer Leakage Inductance

- The transformer equations in per units:

$$j\,x_{1\sigma}\,\bar{i}_1 = 0$$
$$j\,x_{2\sigma}\,\bar{i}_2 = \bar{u}_{2k}$$

=> $x_{1\sigma} = 0$



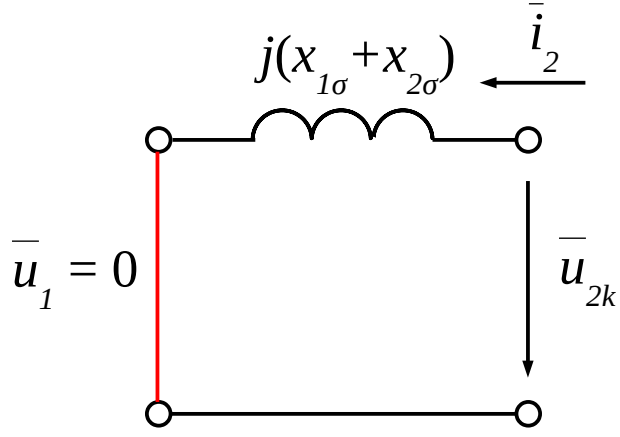
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=> $x_{1\sigma} = 0$

- The leakage inductance of the primary side results zero, thus the sum of magnetic fluxes through primary winding must be zero either.



FACULTY OF ELECTRICAL
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EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

ESF II – project of the University of West Bohemia
reg. no. CZ.02.2.69/0.0/0.0/18_056/0013239

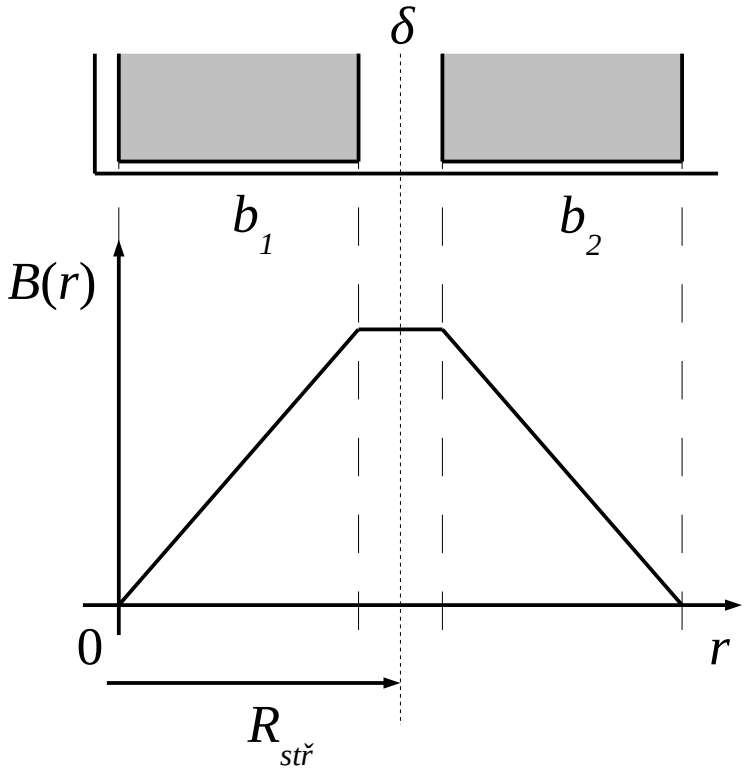
Transformer Division Plane

Transformer Division Plane

- The a short circuited transformer, the magnetic fluxes from both primary and secondary windings converge in the primary winding
- The border between both magnetic fluxes is called “division plane”
- Should the induced voltage in the primary winding be equal to zero, the sum of both magnetic fluxes must be identically equal to zero
- The determination of the position of the division plane allows calculation of transformer short-circuit voltage

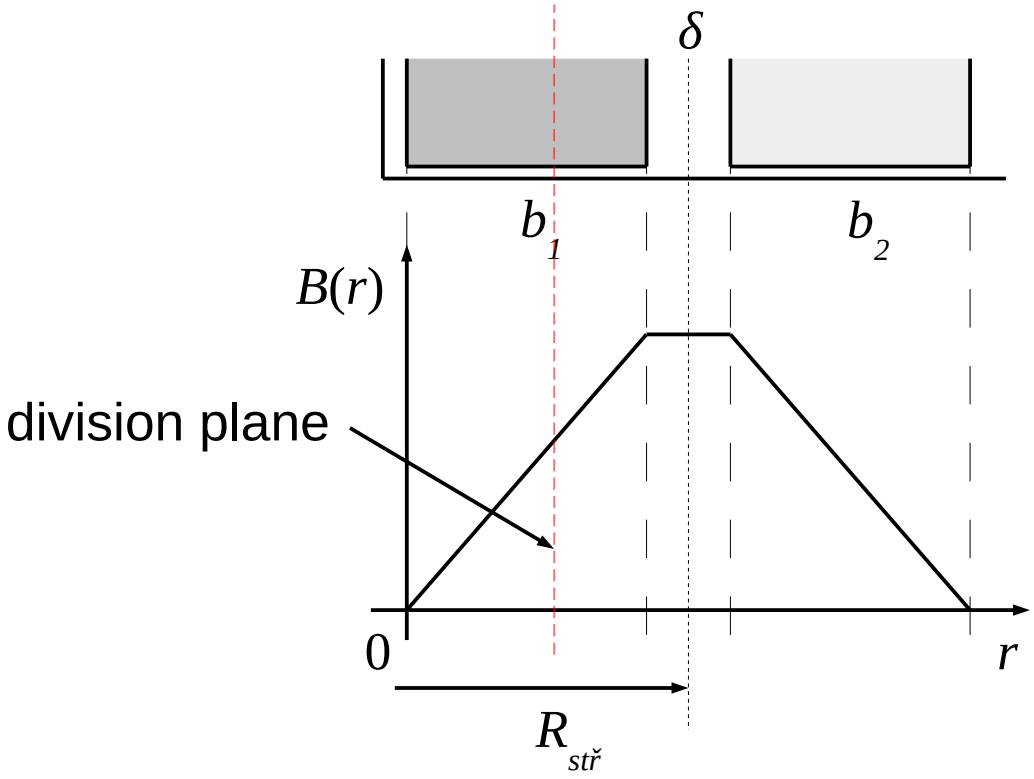
Transformer Division Plane

- Magnetic field of a transformer:



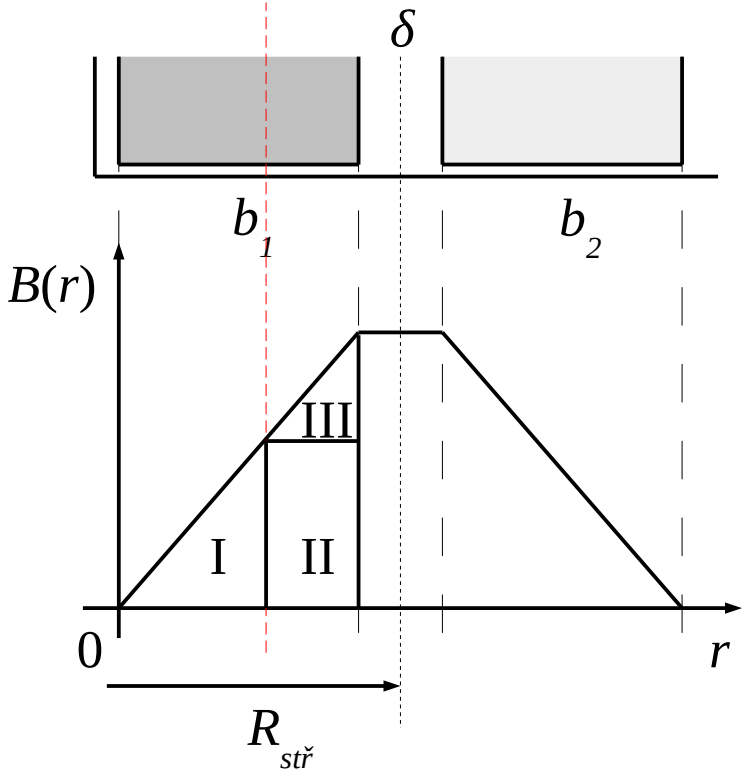
Transformer Division Plane

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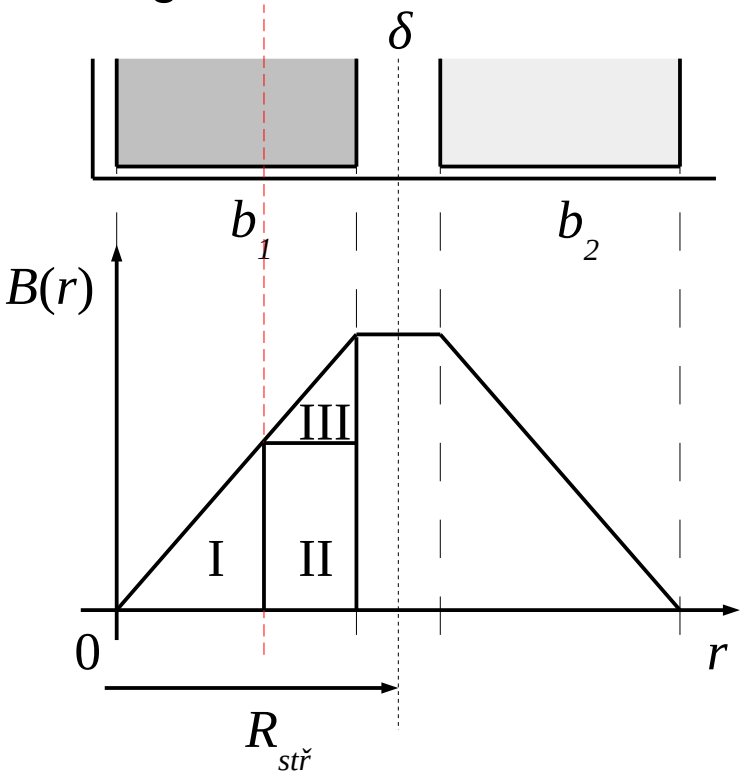
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- Magnetic field of a transformer:



Transformer Division Plane

- Magnetic field of a transformer:



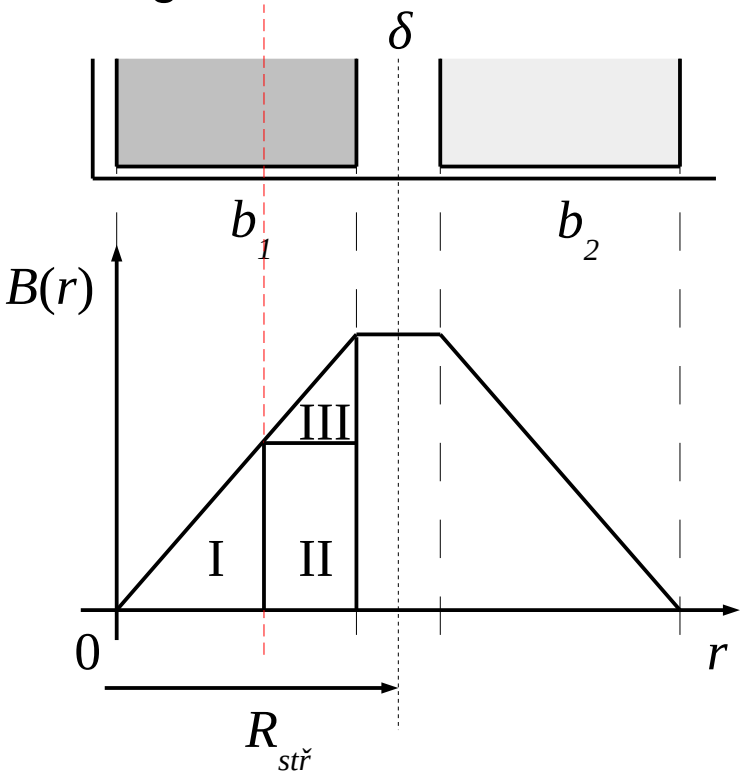
- to be able to fulfill the requirement of

$$\bar{u}_{i1} = j\, x_{1\sigma}\, \bar{i}_1 = 0$$

the sum of induced voltages on the left and on the right side from the division plane must be equal to zero.

Transformer Division Plane

- Magnetic field of a transformer:



- to be able to fulfill the requirement of

$$\bar{u}_{i1} = j\, x_{1\sigma}\, \bar{i}_1 = 0$$

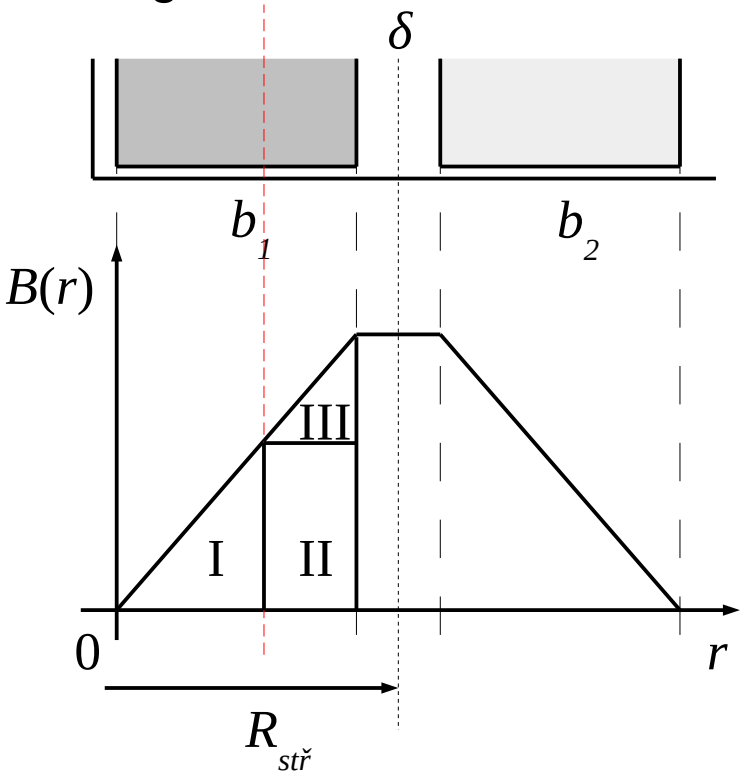
the sum of induced voltages on the left and on the right side from the division plane must be equal to zero.

=> the sum of areas of II a III must be equal to the area of I.

+ we have to respect the partial coefficient of partial interaction: $u_i \sim k\,\Phi\,N$

Transformer Division Plane

- Magnetic field of a transformer:



- k – coefficient of partial interaction
  - as per field shape
- Φ – magnetic flux

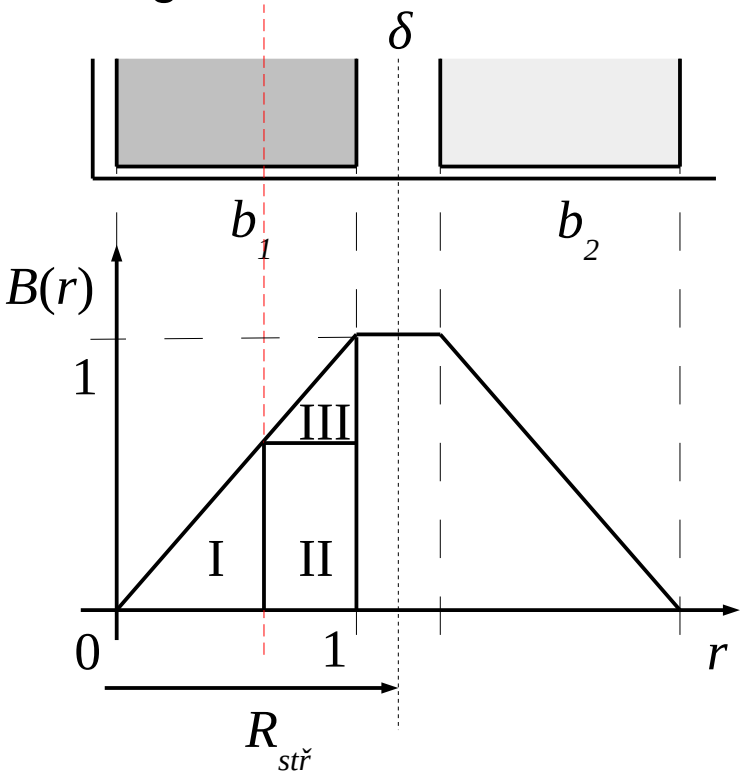
$$\Phi \sim B(r)S(r)$$

- N – amount of coil turns

$$N \sim r$$

Transformer Division Plane

- Magnetic field of a transformer:



- k – coefficient of partial interaction
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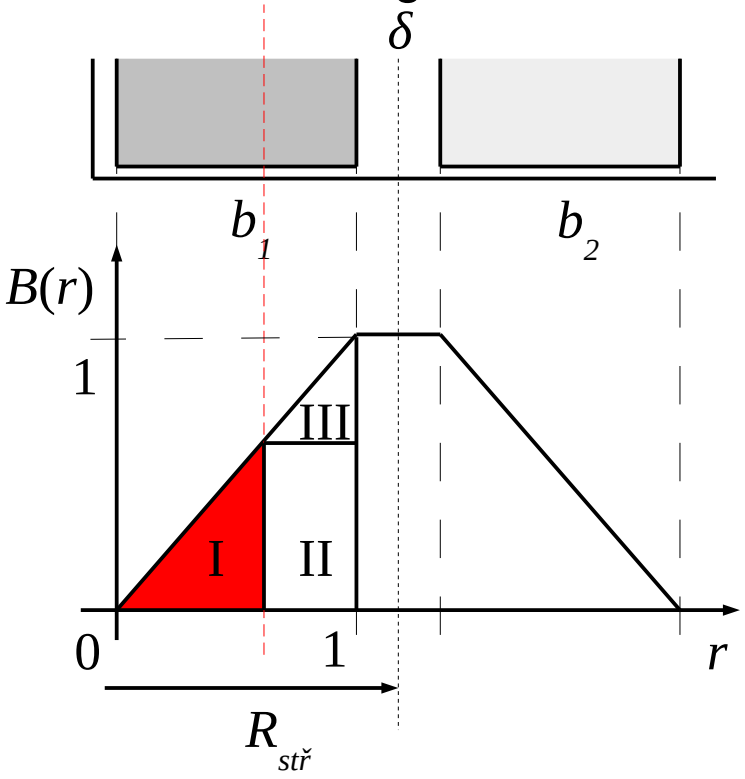
$$N \sim r$$

- furthermore let us use the system of per units

$(b_1 = 1\ p.u., B_m = 1\ p.u.)$

Transformer Division Plane

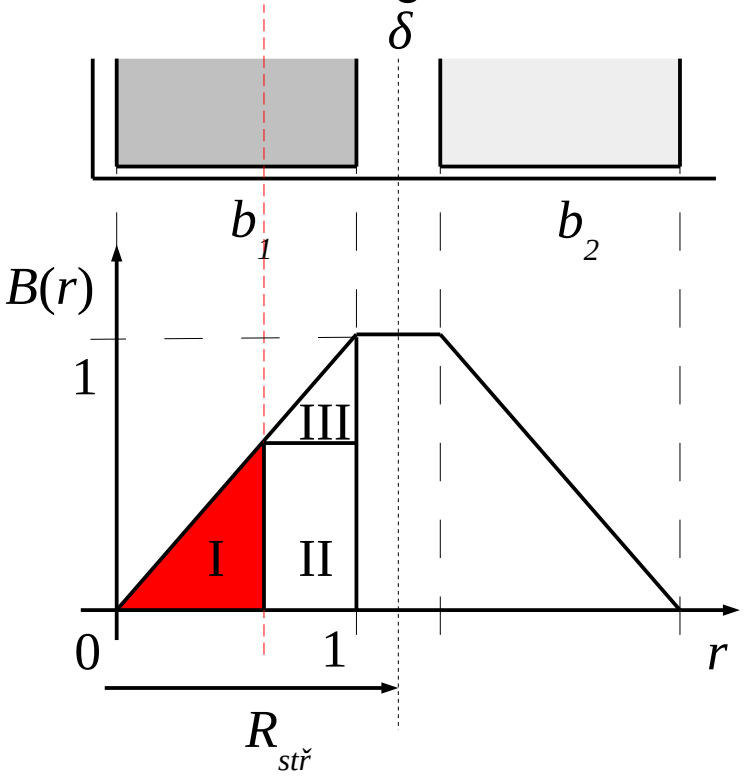
- Induced voltage in each area:



$$u_i \sim k \Phi N$$

Transformer Division Plane

- Induced voltage in each area:

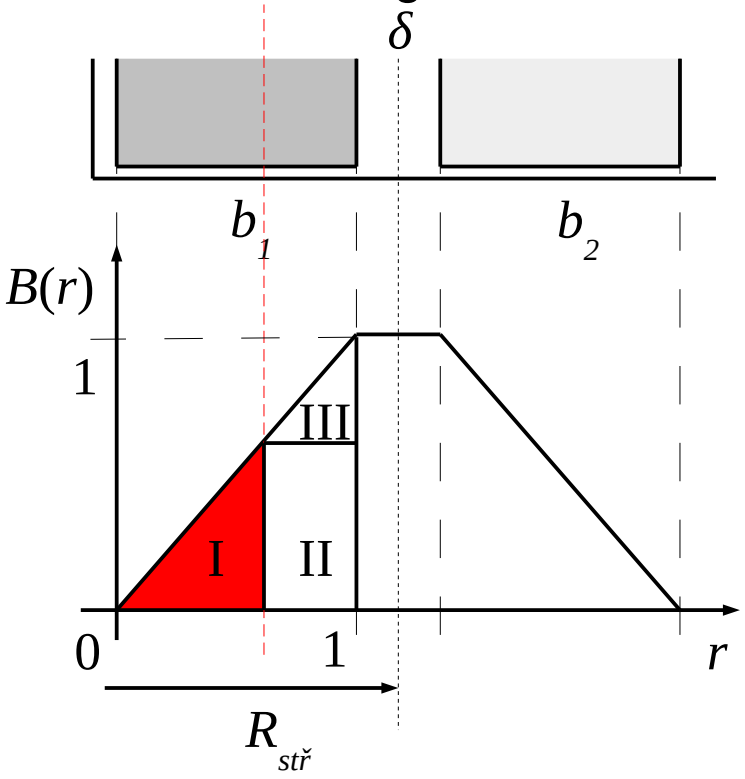


$$u_i \sim k \Phi N$$

$$k_I = \frac{2}{3}$$

Transformer Division Plane

- Induced voltage in each area:



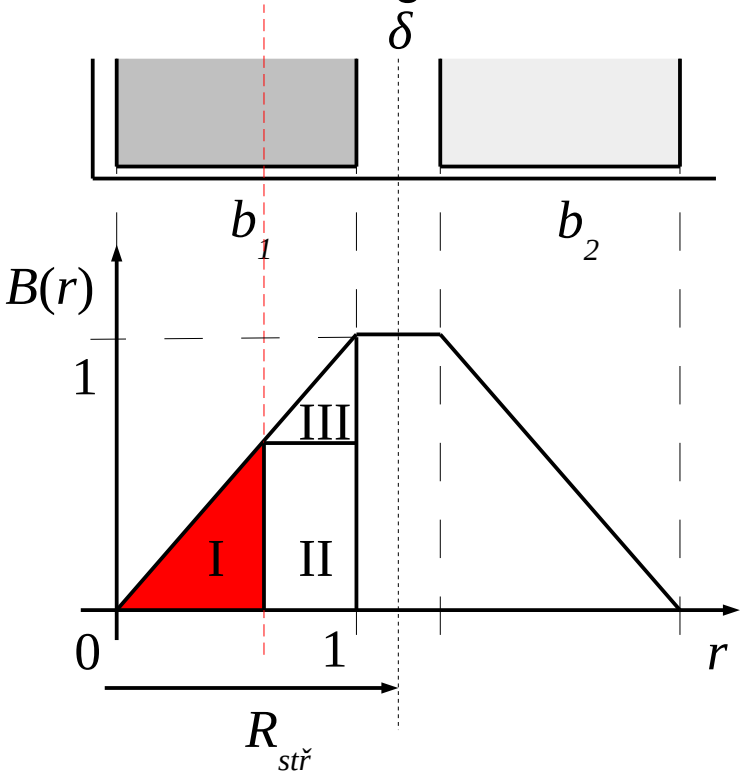
$$u_i \sim k \Phi N$$

$$k_I = \frac{2}{3}$$

$$\Phi_I \sim S_I \sim \frac{1}{2} r \cdot r = \frac{1}{2} r^2$$

Transformer Division Plane

- Induced voltage in each area:



$$u_i \sim k \Phi N$$

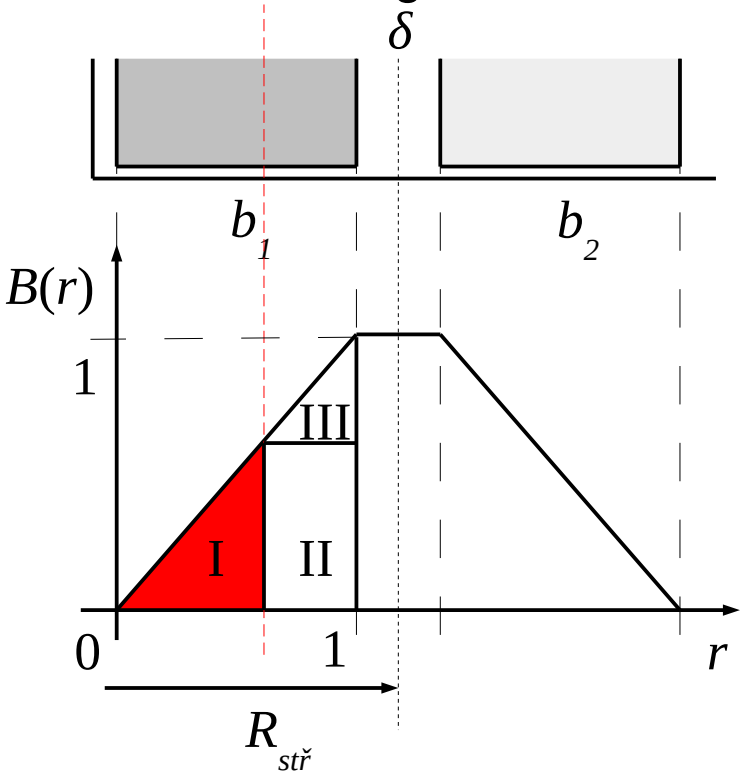
$$k_I = \frac{2}{3}$$

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$$N_I \sim r$$

Transformer Division Plane

- Induced voltage in each area:



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$$k_I = \frac{2}{3}$$

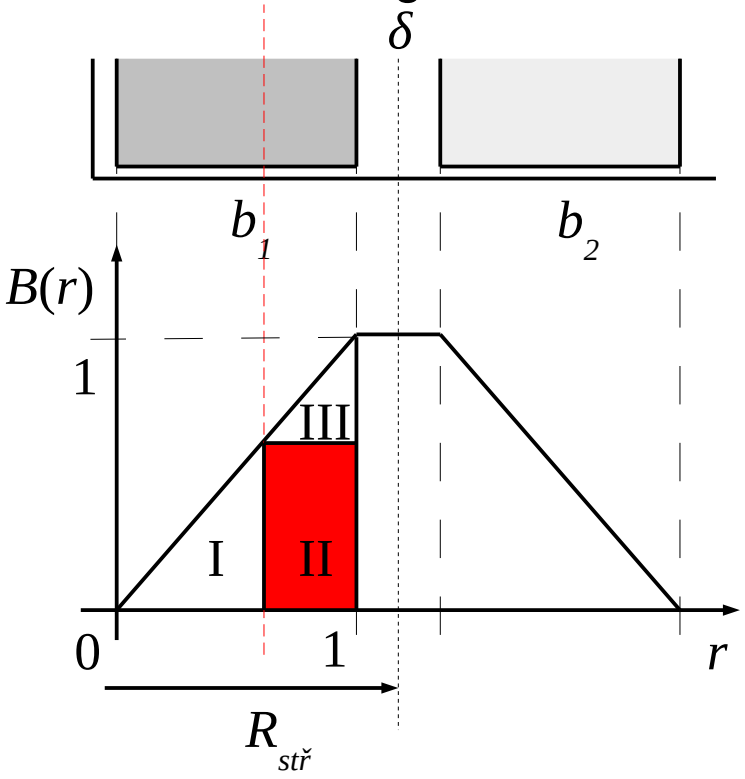
$$\Phi_I \sim S_I \sim \frac{1}{2} r \cdot r = \frac{1}{2} r^2$$

$$N_I \sim r$$

$$u_{iI} \sim k_I \Phi_I N_I \sim \frac{2}{3} \cdot \frac{r^2}{2} r \sim \frac{r^3}{3}$$

Transformer Division Plane

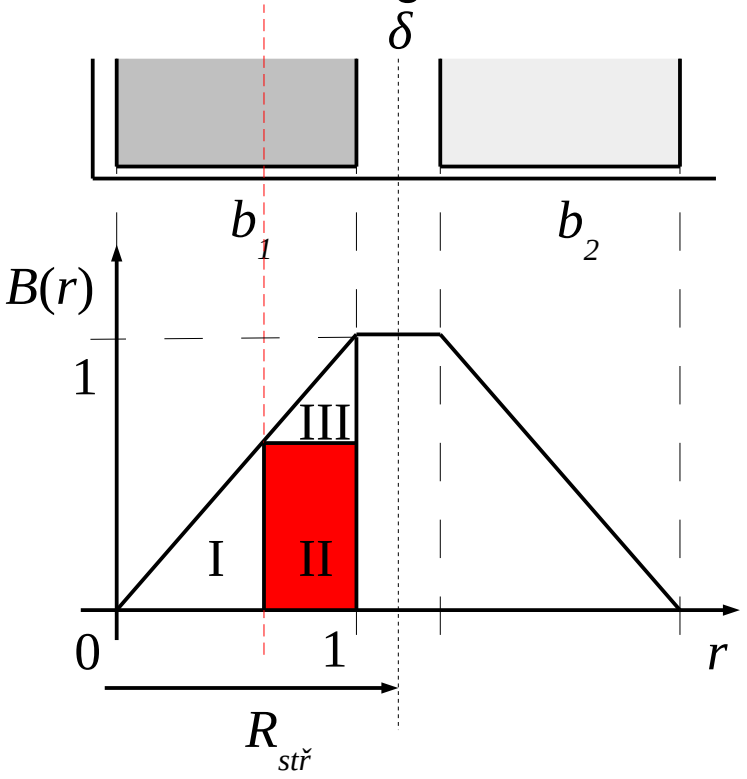
- Induced voltage in each area:



$$u_i \sim k \Phi N$$

Transformer Division Plane

- Induced voltage in each area:

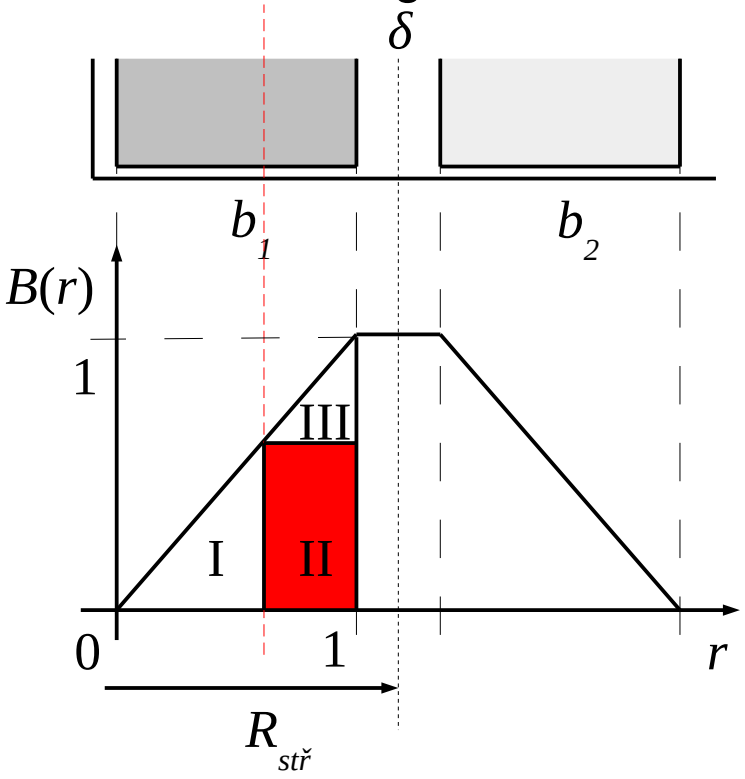


$$u_i \sim k \Phi N$$

$$k_{II} = \frac{1}{2}$$

Transformer Division Plane

- Induced voltage in each area:



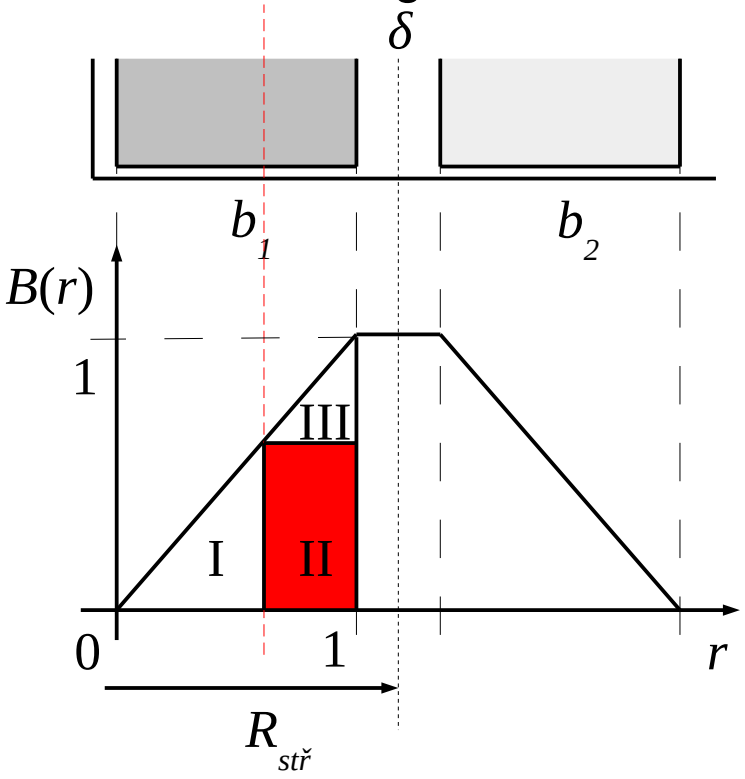
$$u_i \sim k \Phi N$$

$$k_{II} = \frac{1}{2}$$

$$\Phi_{II} \sim S_{II} \sim (1-r) \cdot r = r - r^2$$

Transformer Division Plane

- Induced voltage in each area:



$$u_i \sim k \Phi N$$

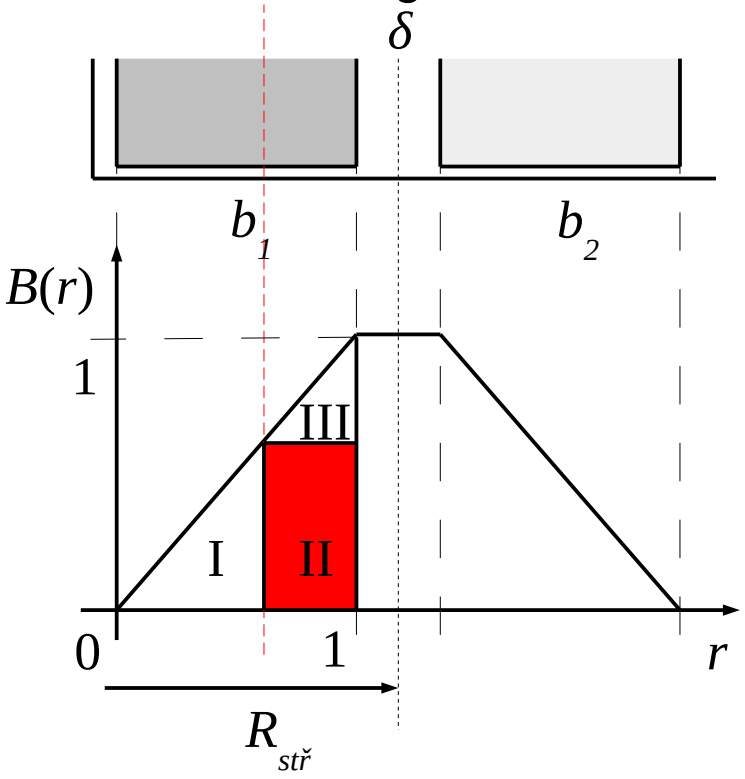
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Transformer Division Plane

- Induced voltage in each area:



$$u_i \sim k \Phi N$$

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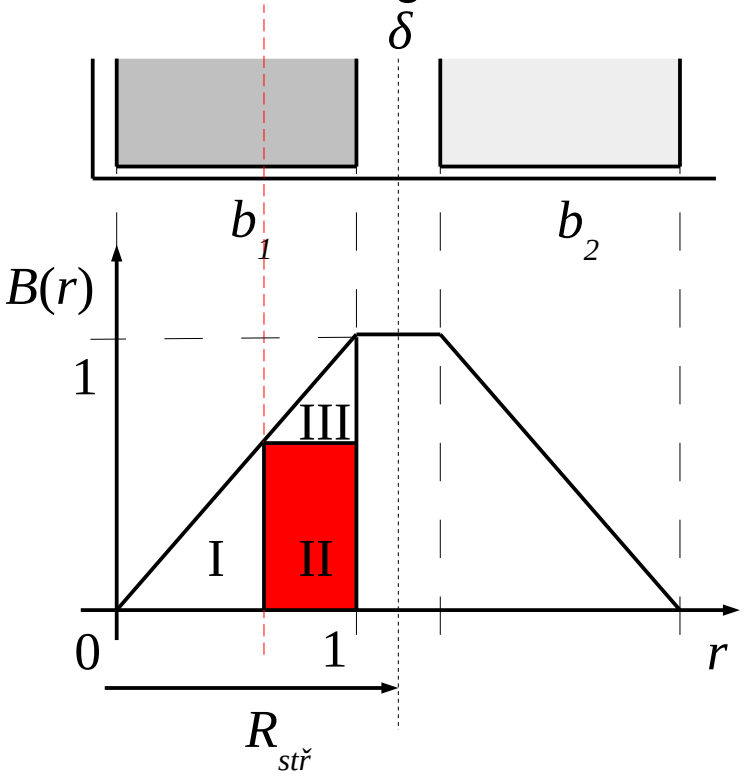
$$\Phi_{II} \sim S_{II} \sim (1-r) \cdot r = r - r^2$$

$$N_{II} \sim 1 - r$$

$$u_{iII} \sim k_{II} \Phi_{II} N_{II} \sim \frac{1}{2} \cdot (r - r^2) \cdot (1 - r)$$

Transformer Division Plane

- Induced voltage in each area:



$$u_i \sim k \Phi N$$

$$k_{II} = \frac{1}{2}$$

$$\Phi_{II} \sim S_{II} \sim (1-r) \cdot r = r - r^2$$

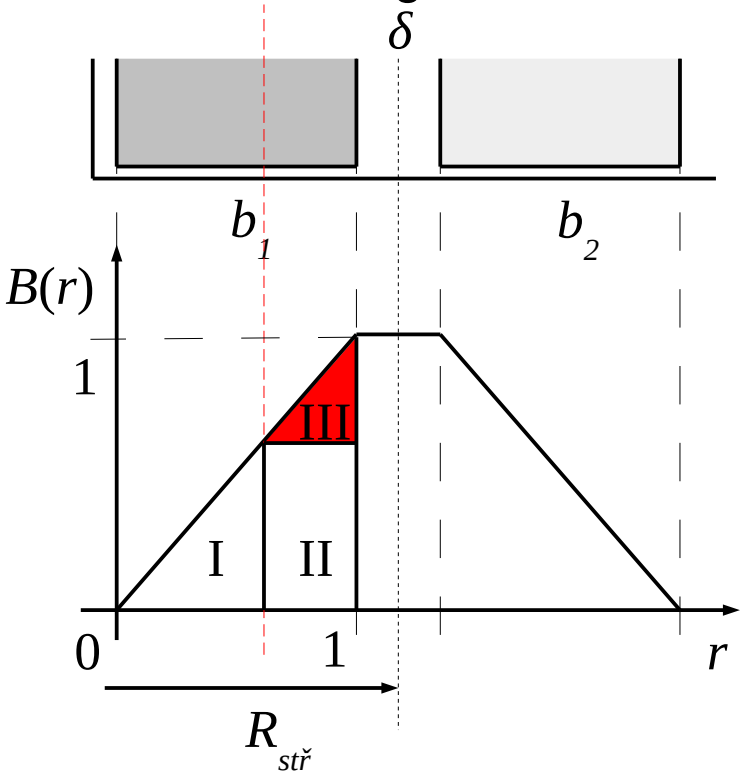
$$N_{II} \sim 1 - r$$

$$u_{iII} \sim k_{II} \Phi_{II} N_{II} \sim \frac{1}{2} \cdot (r - r^2) \cdot (1 - r)$$

$$u_{iII} \sim \frac{1}{2} \cdot (r - 2r^2 + r^3)$$

Transformer Division Plane

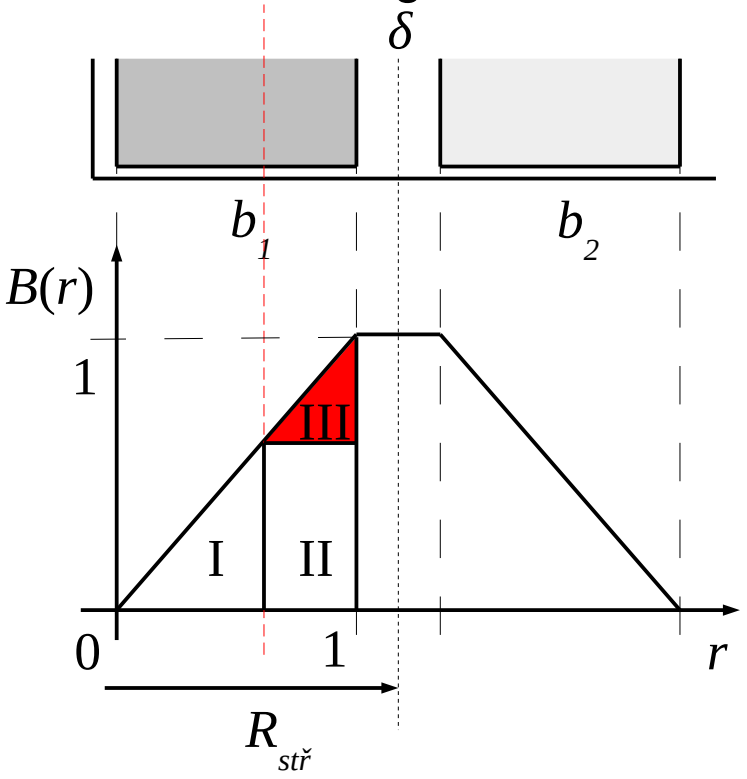
- Induced voltage in each area:



$$u_i \sim k \Phi N$$

Transformer Division Plane

- Induced voltage in each area:

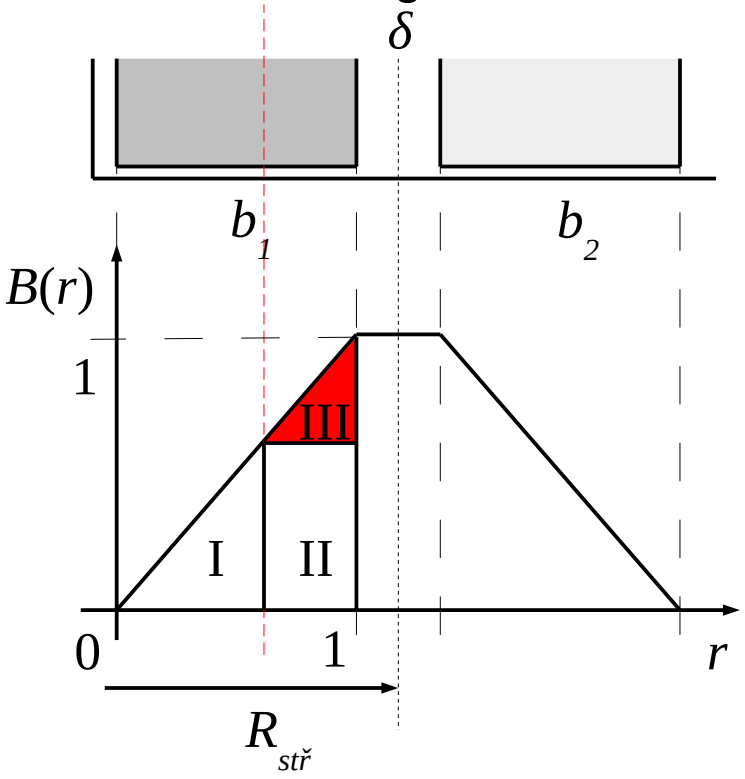


$$u_i \sim k \Phi N$$

$$k_{III} = \frac{1}{3}$$

Transformer Division Plane

- Induced voltage in each area:



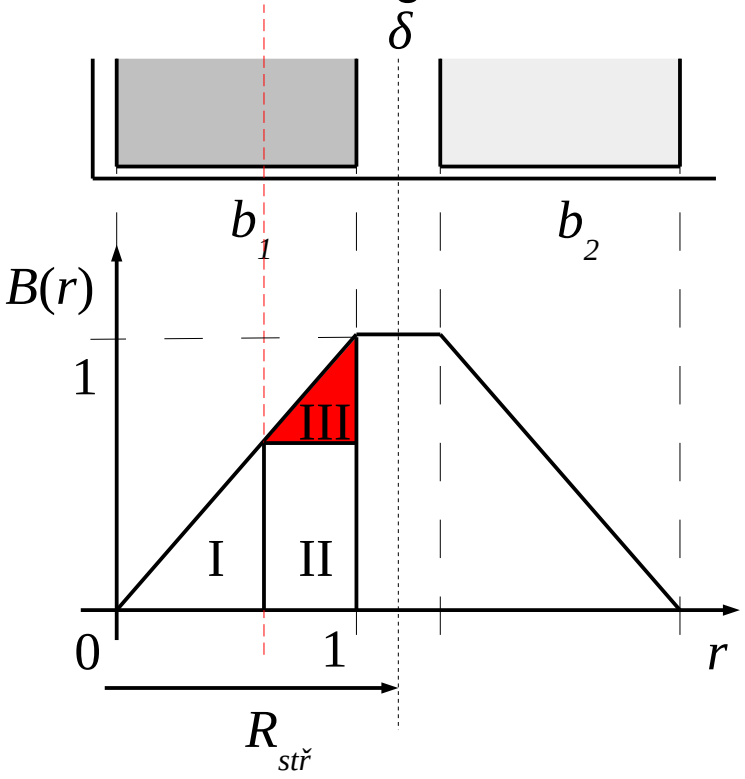
$$u_i \sim k \Phi N$$

$$k_{III} = \frac{1}{3}^{*}$$

*) From the point of view of the secondary winding it is a descending field.

Transformer Division Plane

- Induced voltage in each area:



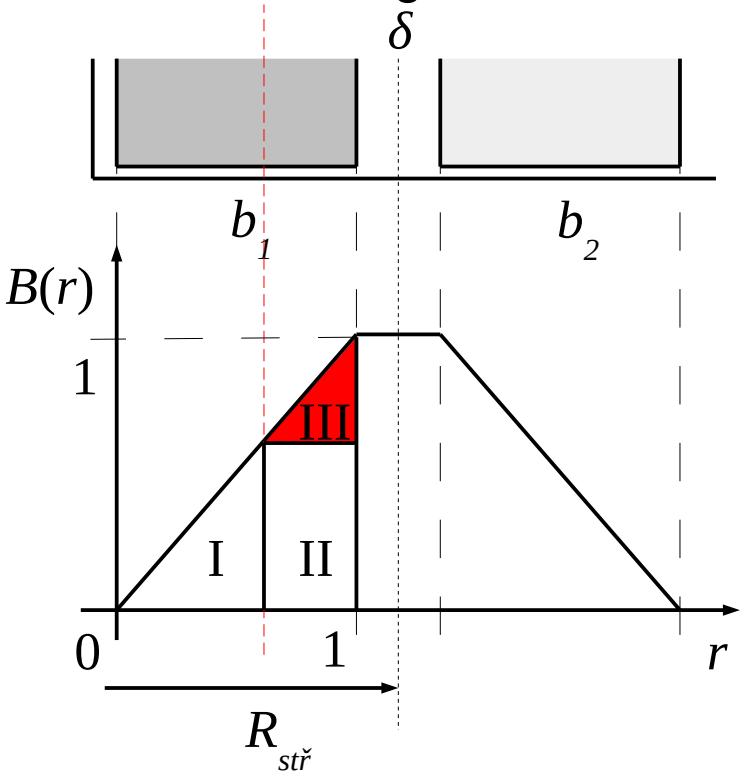
$$u_i \sim k \Phi N$$

$$k_{III} = \frac{1}{3}$$

$$\Phi_{III} \sim S_{III} \sim \frac{1}{2}(1-r)\cdot(1-r) = \frac{1}{2}(1-r)^2$$

Transformer Division Plane

- Induced voltage in each area:



$$u_i \sim k \Phi N$$

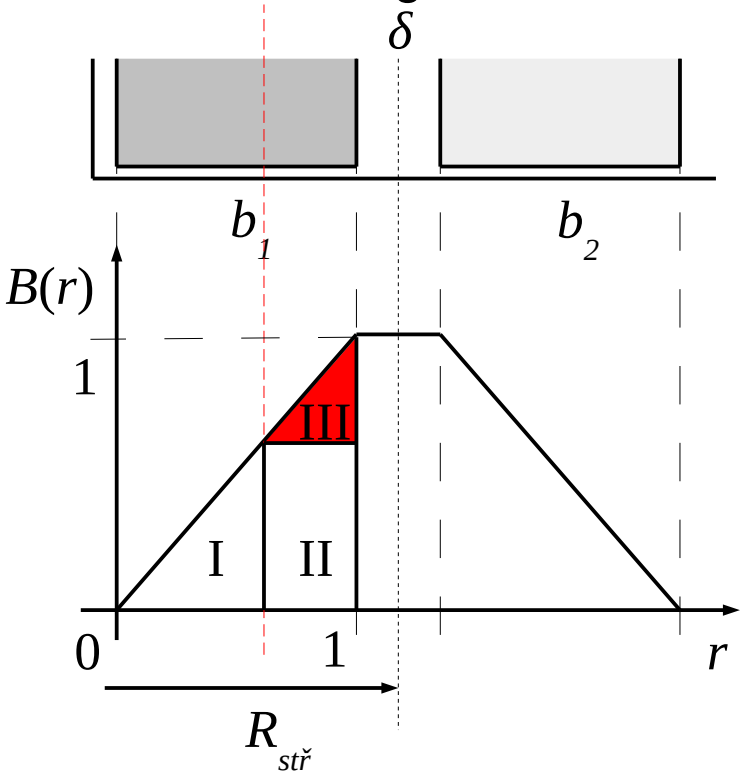
$$k_{III} = \frac{1}{3}$$

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Transformer Division Plane

- Induced voltage in each area:



$$u_i \sim k \Phi N$$

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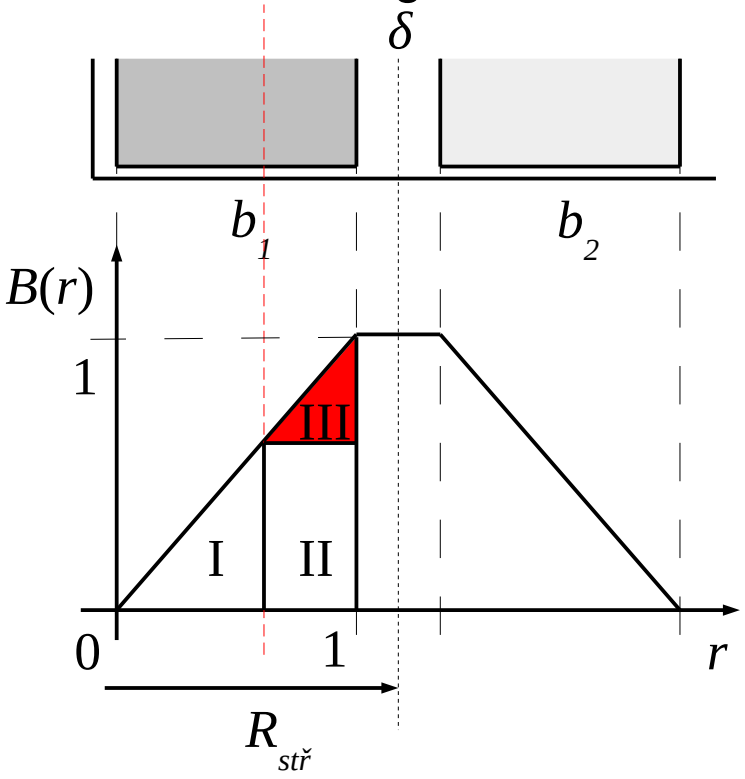
$$\Phi_{III} \sim S_{III} \sim \frac{1}{2}(1-r)\cdot(1-r) = \frac{1}{2}(1-r)^2$$

$$N_{III} \sim 1-r$$

$$u_{iIII} \sim k_{III}\Phi_{III}N_{III} \sim \frac{1}{3}\cdot\frac{(1-r)^2}{2}\cdot(1-r)$$

Transformer Division Plane

- Induced voltage in each area:



$$u_i \sim k\,\Phi\,N$$

$$k_{III} = \frac{1}{3}$$

$$\Phi_{III} \sim S_{III} \sim \frac{1}{2}(1-r)\cdot(1-r) = \frac{1}{2}(1-r)^2$$

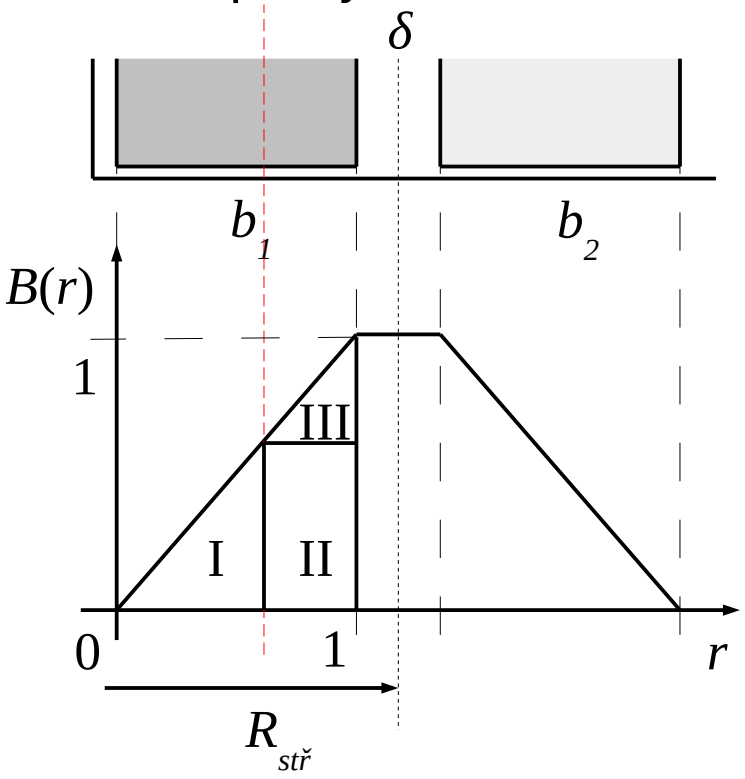
$$N_{III} \sim 1-r$$

$$u_{iIII} \sim k_{III}\,\Phi_{III}\,N_{III} \sim \frac{1}{3}\cdot\frac{(1-r)^2}{2}\cdot(1-r)$$

$$u_{iIII} \sim \frac{1}{6}\cdot\left(1-3r+3r^2-r^3\right)$$

Transformer Division Plane

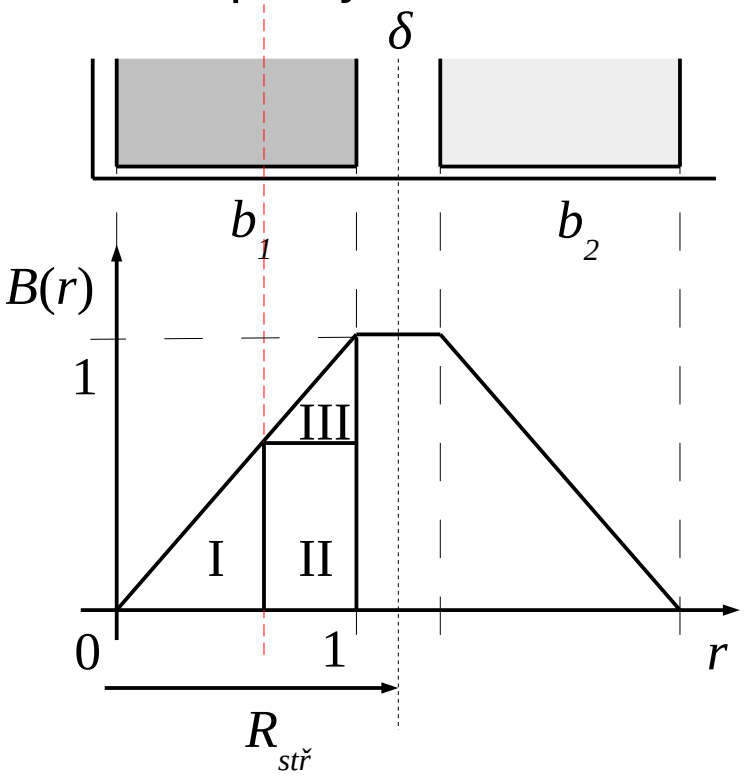
- The equality of induced voltages in each area may be then written as:



$$u_{iI} = u_{iII} + u_{iIII}$$

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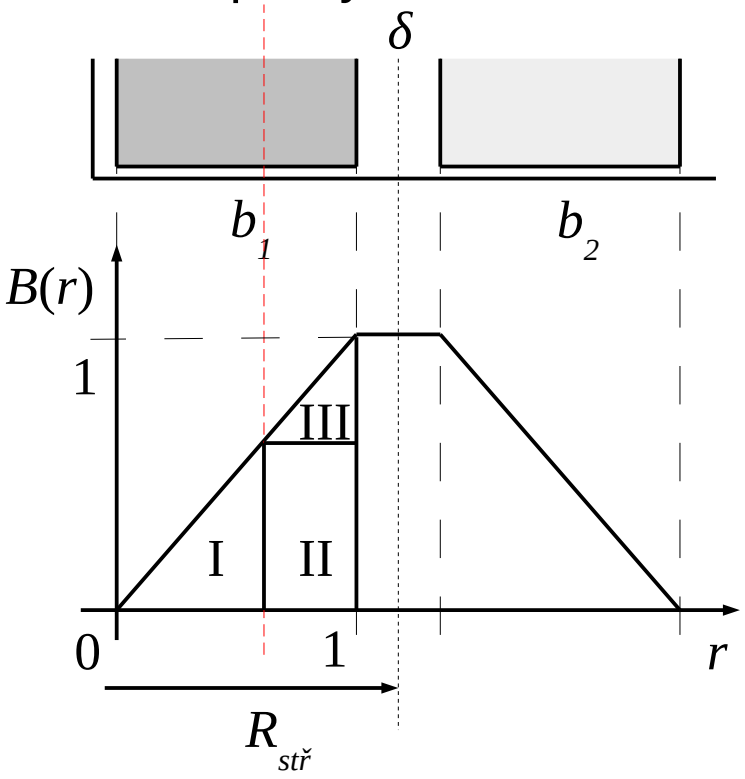
$$u_{iI} \sim \frac{r^3}{3}$$

$$u_{iII} \sim \frac{1}{2} \cdot \left(r - 2r^2 + r^3 \right)$$

$$u_{iIII} \sim \frac{1}{6} \cdot \left(1 - 3r + 3r^2 - r^3 \right)$$

Transformer Division Plane

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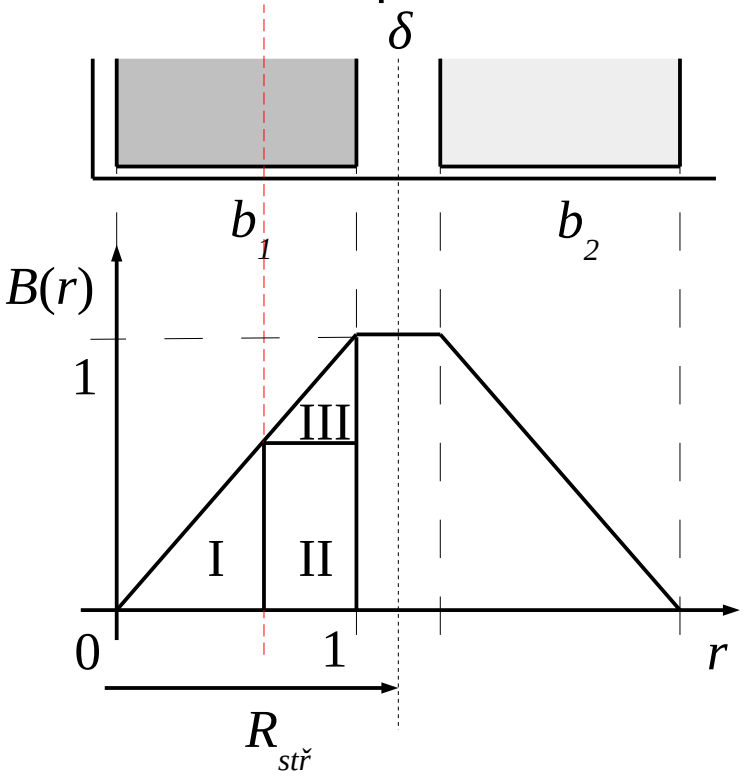
$$u_{iII} \sim \frac{1}{2} \cdot \left(r - 2r^2 + r^3 \right)$$

$$u_{iIII} \sim \frac{1}{6} \cdot \left(1 - 3r + 3r^2 - r^3 \right)$$

$$\frac{r^3}{3} = \frac{1}{2} \cdot \left(r - 2r^2 + r^3 \right) + \frac{1}{6} \cdot \left(1 - 3r + 3r^2 - r^3 \right)$$

Transformer Division Plane

- The division plane is then located at

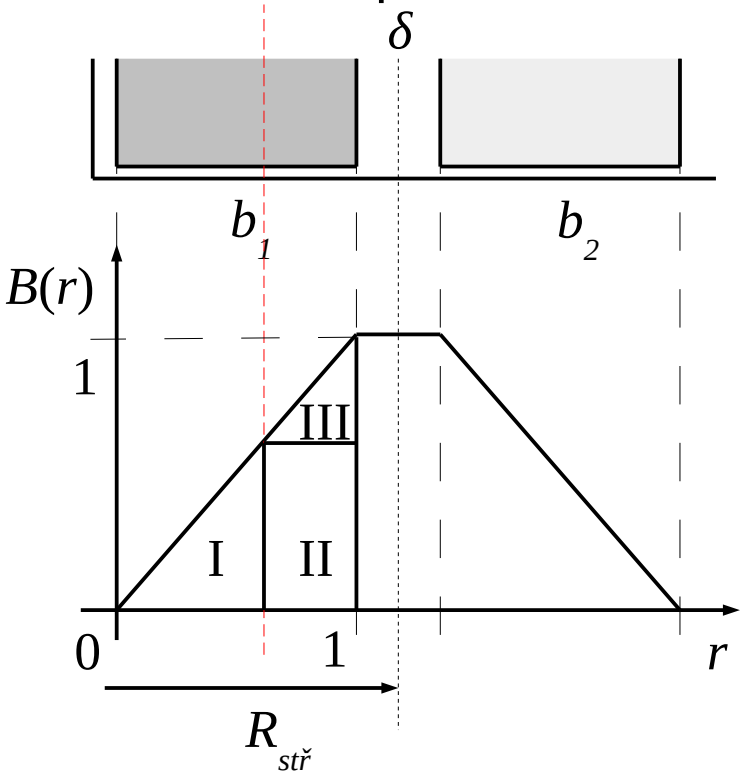


$$r = \frac{1}{\sqrt{3}}$$

… thus approximately in 57,7 per cent of the primary winding.

Transformer Division Plane

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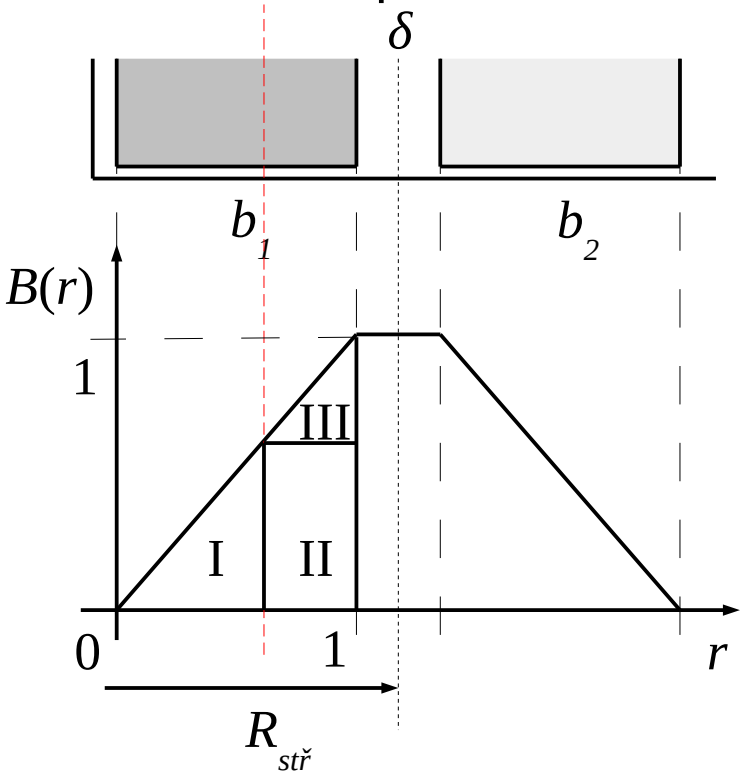
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- Between 0 a 0,577 b_1 there is only magnetic field of the primary winding.

Transformer Division Plane

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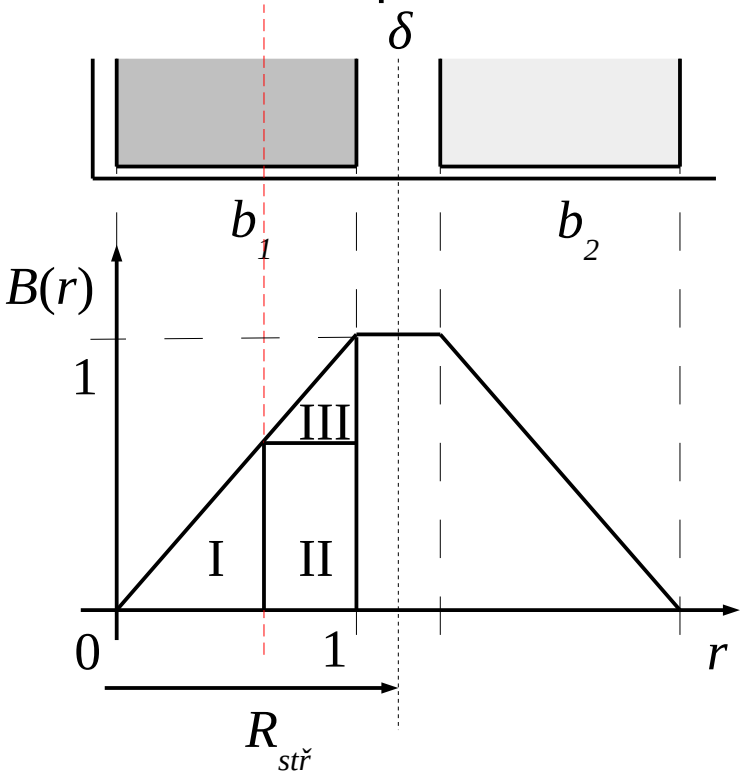
$$r = \frac{1}{\sqrt{3}}$$

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- The remaining space is filled by the field of the secondary winding.

Transformer Division Plane

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$$r = \frac{1}{\sqrt{3}}$$

… thus approximately in 57,7 per cent of the primary winding.

- Between 0 a 0,577 b_1 there is only magnetic field of the primary winding.
- The remaining space is filled by the field of the secondary winding.
- This magnetic field is bound with the transformer leakage inductance.

FACULTY OF ELECTRICAL
ENGINEERING
UNIVERSITY OF WEST BOHEMIA

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

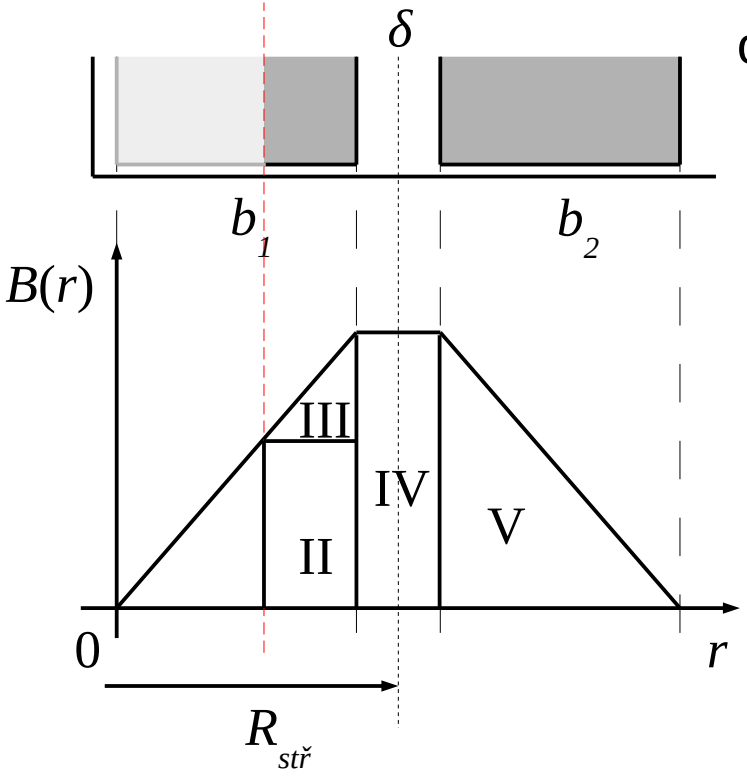
MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

ESF II – project of the University of West Bohemia
reg. no. CZ.02.2.69/0.0/0.0/18_056/0013239

The Calculation of Transformer Leakage Inductance in Short-Circuit

The Calculation of Transformer Leakage Inductance in Short-Circuit

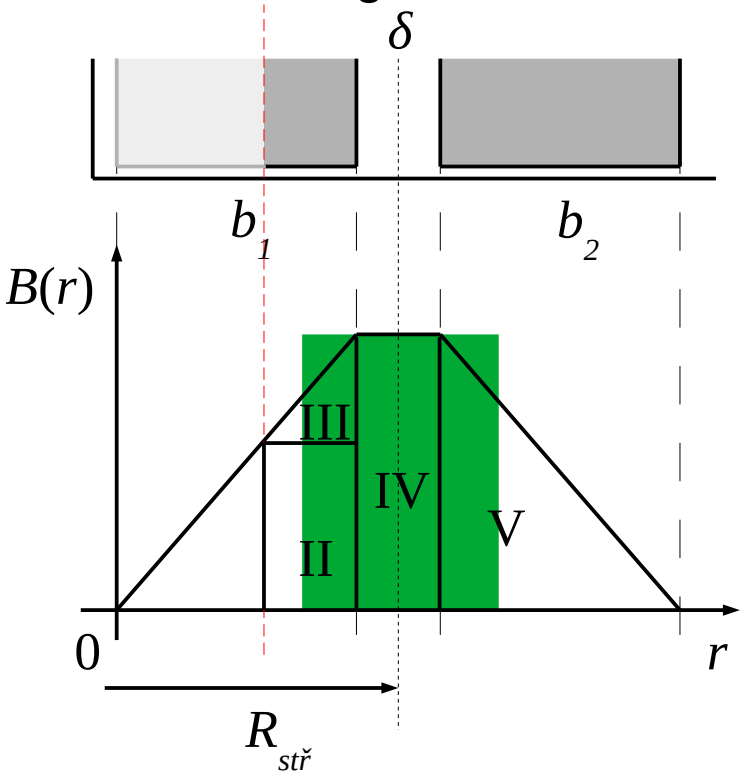
- The inductance of a transformer in short-circuit state is bound to the field of the secondary winding



  - For the purpose of simplification let us replace this magnetic field by a rectangular one
  - Substituting field:
    - rectangular shape; no winding
    - the width of the field will be δ_r
  - Let us again use the system of per units

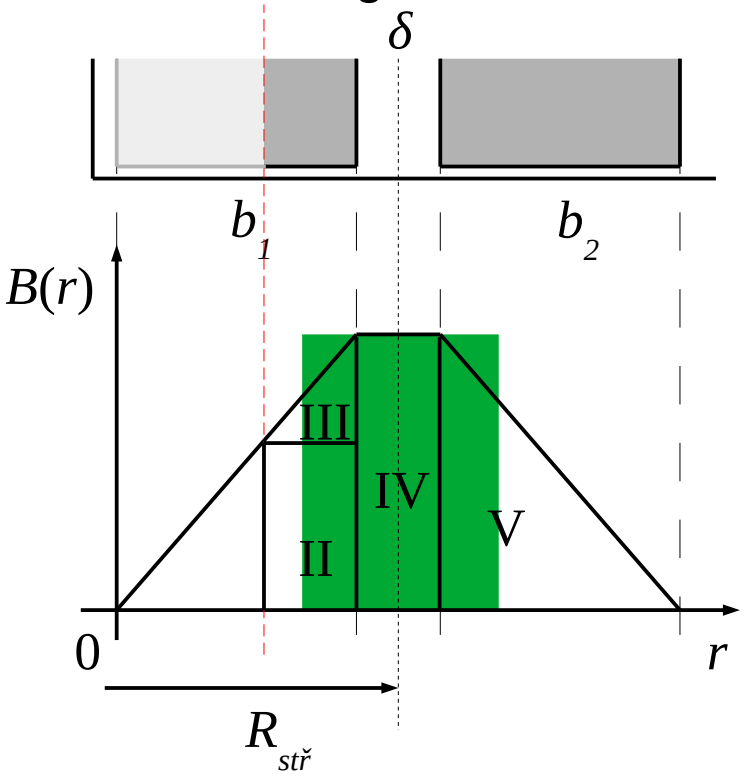
The Calculation of Transformer Leakage Inductance in Short-Circuit

- The resulting reduced field:



The Calculation of Transformer Leakage Inductance in Short-Circuit

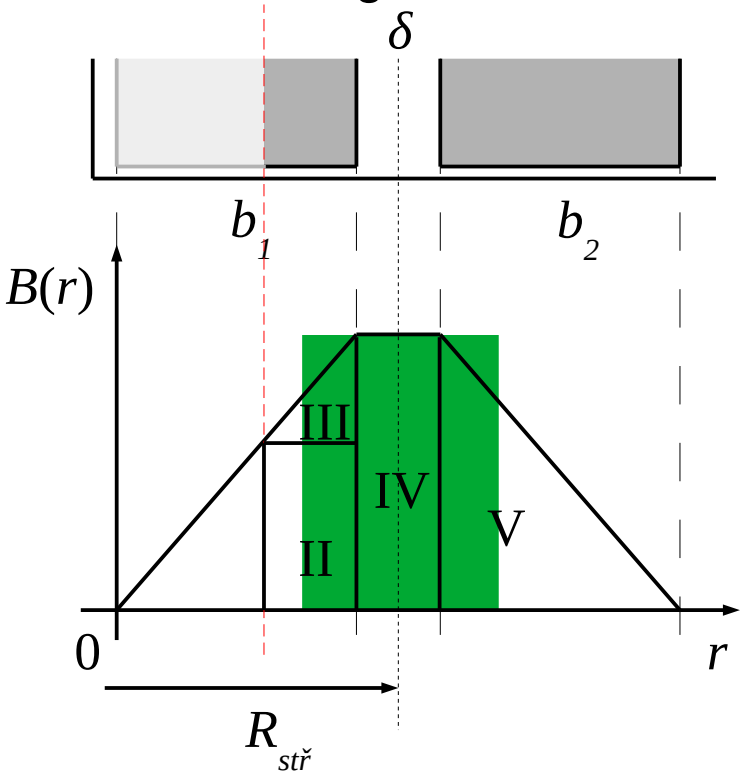
- The resulting reduced field:



$$u_{ir} \sim k_r \Phi_r N_r$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- The resulting reduced field:

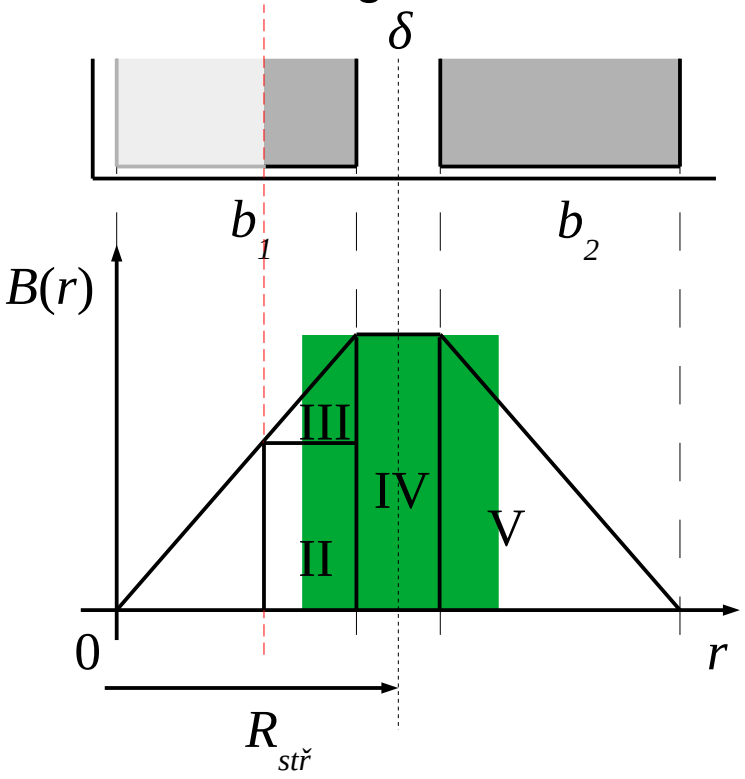


$$u_{ir} \sim k_r \Phi_r N_r$$

$$k_r = 1$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- The resulting reduced field:



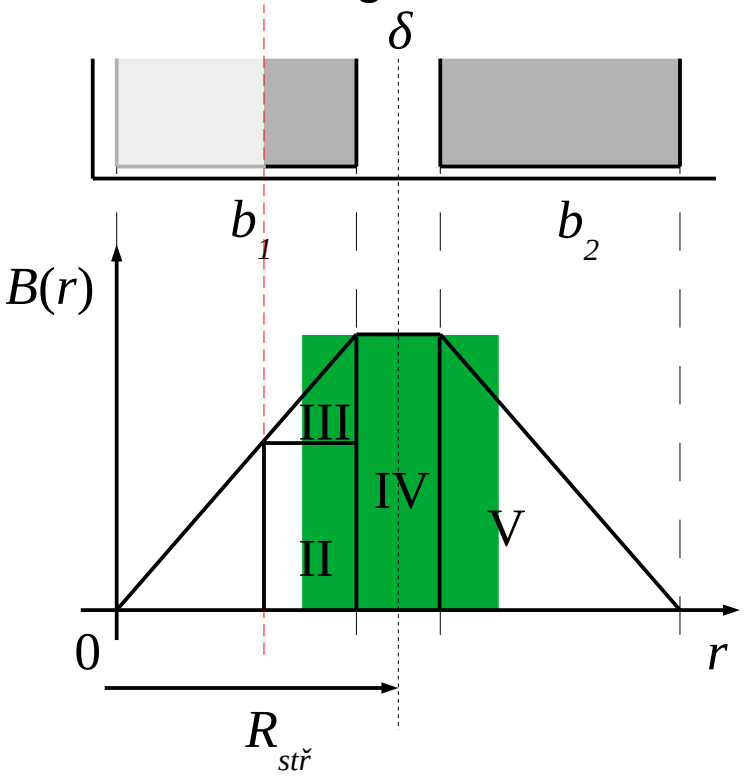
$$u_{ir} \sim k_r \Phi_r N_r$$

$$k_r = 1^*$$

*) It should be an “air” winding, thus a winding with full interaction with the magnetic field.

The Calculation of Transformer Leakage Inductance in Short-Circuit

- The resulting reduced field:



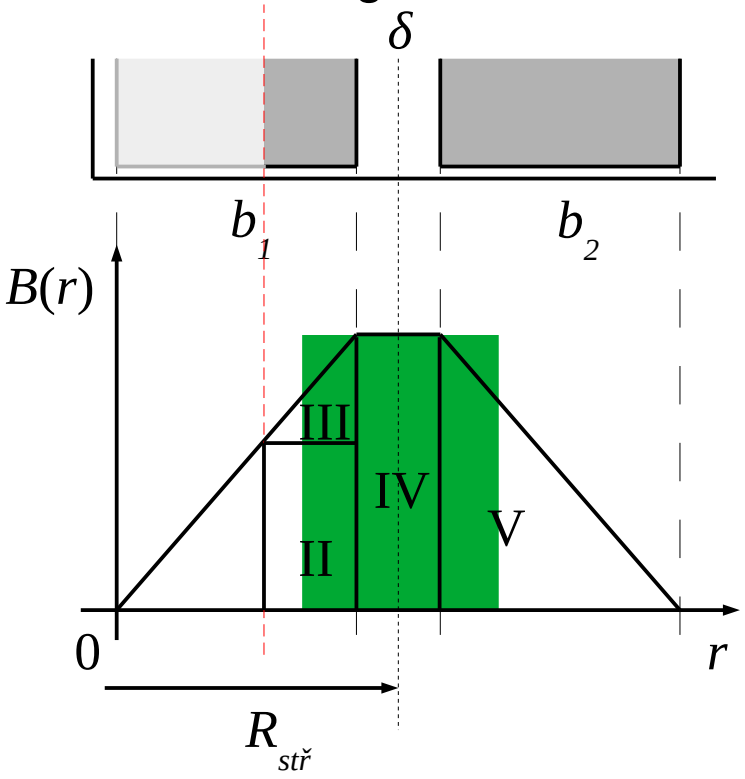
$$u_{ir} \sim k_r \Phi_r N_r$$

$$k_r = 1$$

$$\Phi_r \sim S_r \sim \delta_r \cdot 1$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- The resulting reduced field:



$$u_{ir} \sim k_r \Phi_r N_r$$

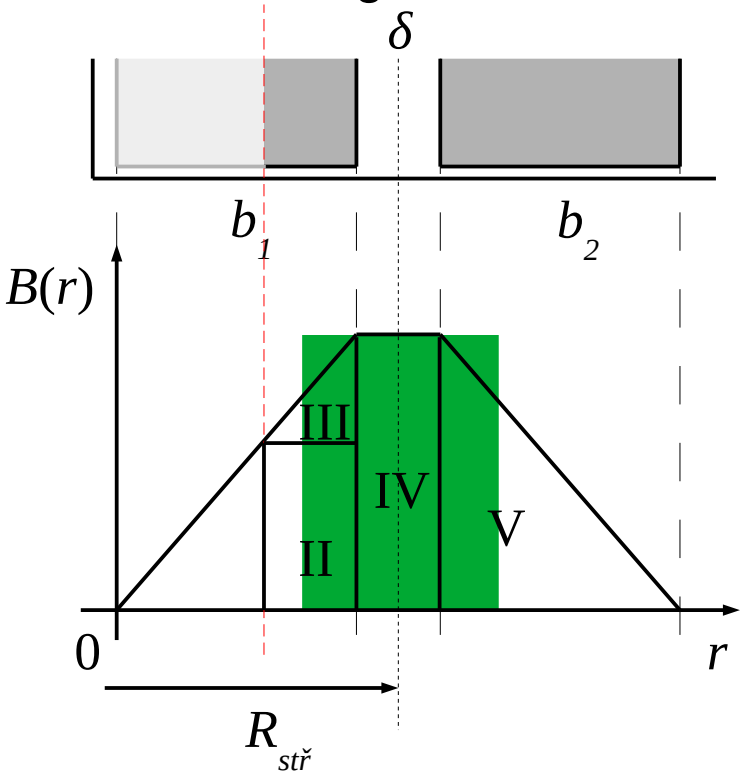
$$k_r = 1$$

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The Calculation of Transformer Leakage Inductance in Short-Circuit

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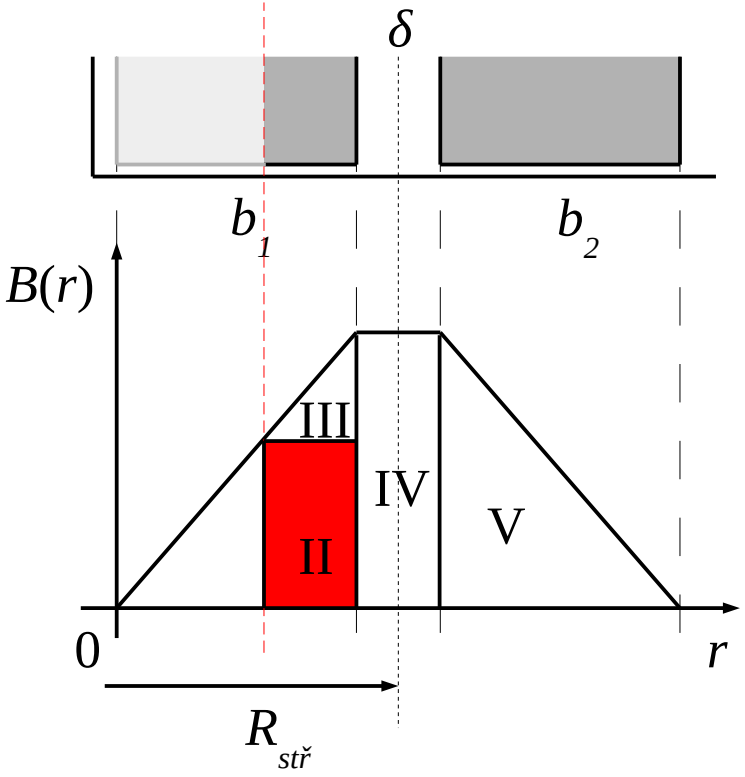
$$\Phi_r \sim S_r \sim \delta_r \cdot 1$$

$$N_r = 1$$

$$u_{ir} \sim 1 \cdot \delta_r \cdot 1 \cdot 1 \sim \delta_r$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

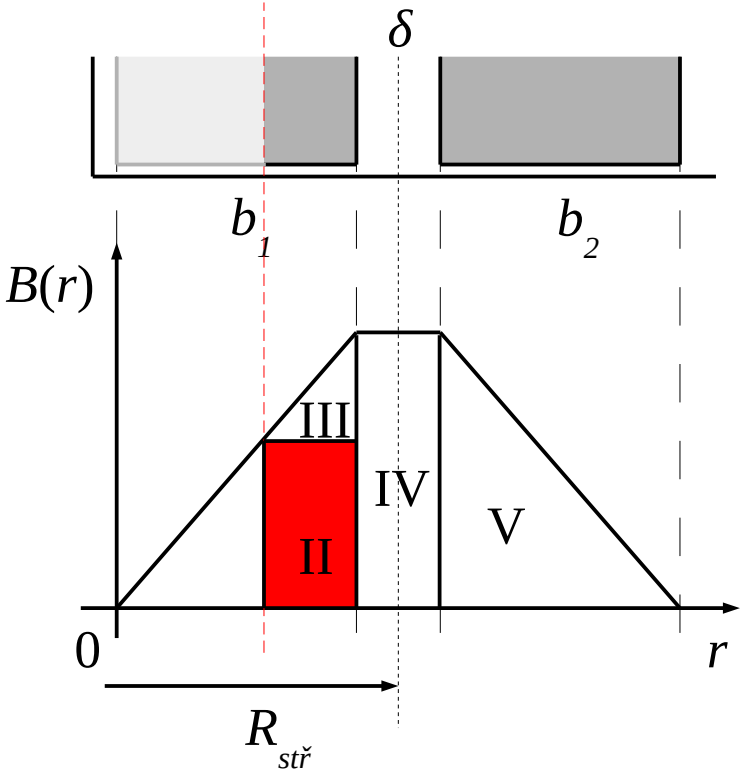
- Area II:



$$u_{iII} \sim k_{II} \Phi_{II} N_{II}$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

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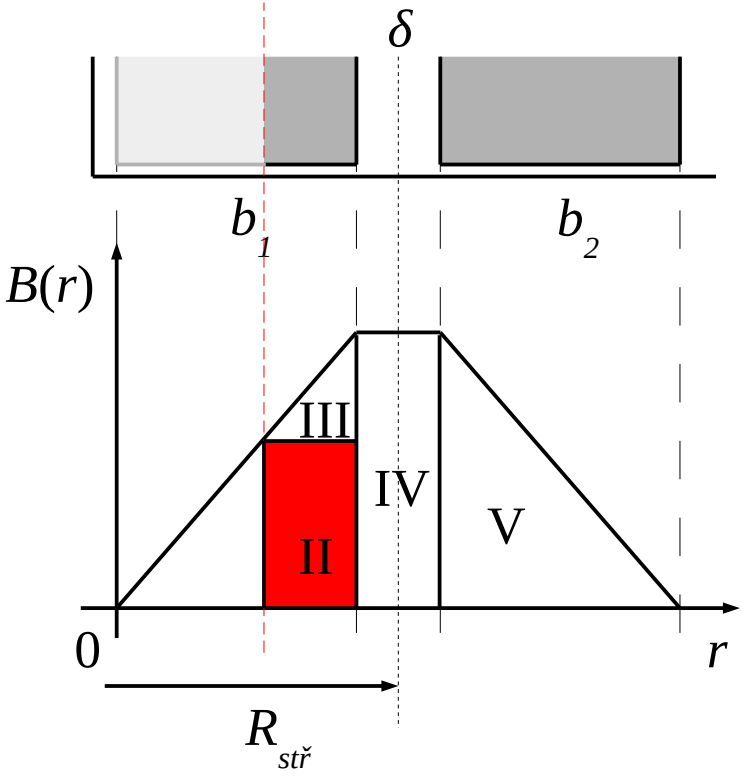


$$u_{iII} \sim k_{II} \Phi_{II} N_{II}$$

$$k_{II} = 1$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

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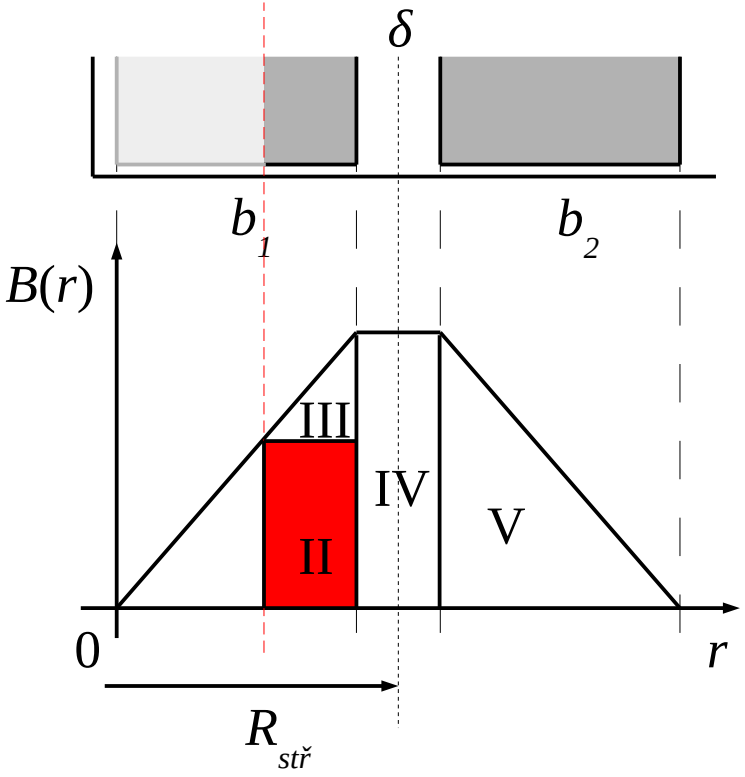
$$u_{iII} \sim k_{II} \Phi_{II} N_{II}$$

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*) The magnetic flux through area II is excited by the field of the secondary coil, thus it is in full interaction with it.

The Calculation of Transformer Leakage Inductance in Short-Circuit

- Area II:



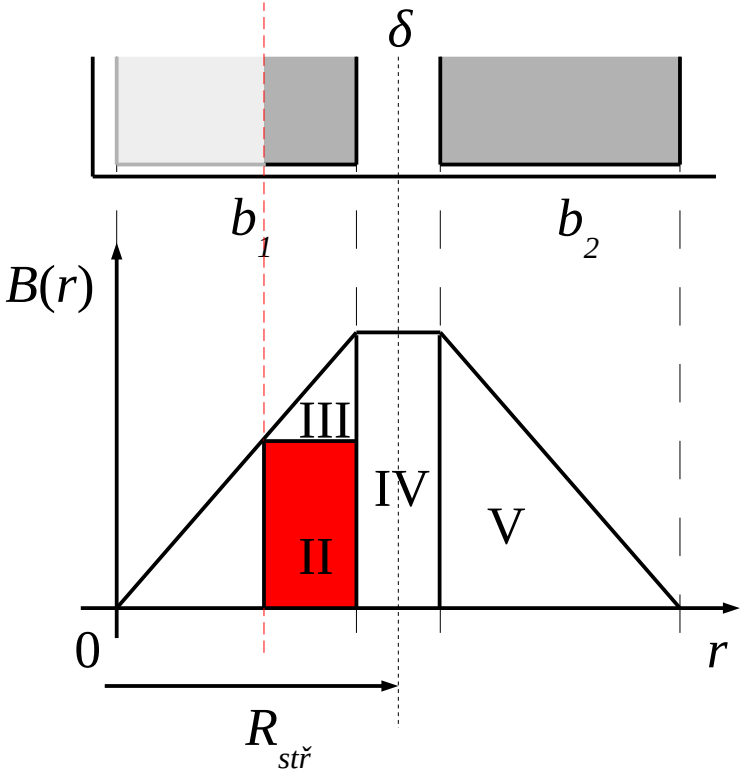
$$u_{iII} \sim k_{II} \Phi_{II} N_{II}$$

$$k_{II} = 1$$

$$\Phi_{II} \sim S_{II} \sim r \cdot b_1 \cdot (1 - r)$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- Area II:



$$u_{iII} \sim k_{II} \Phi_{II} N_{II}$$

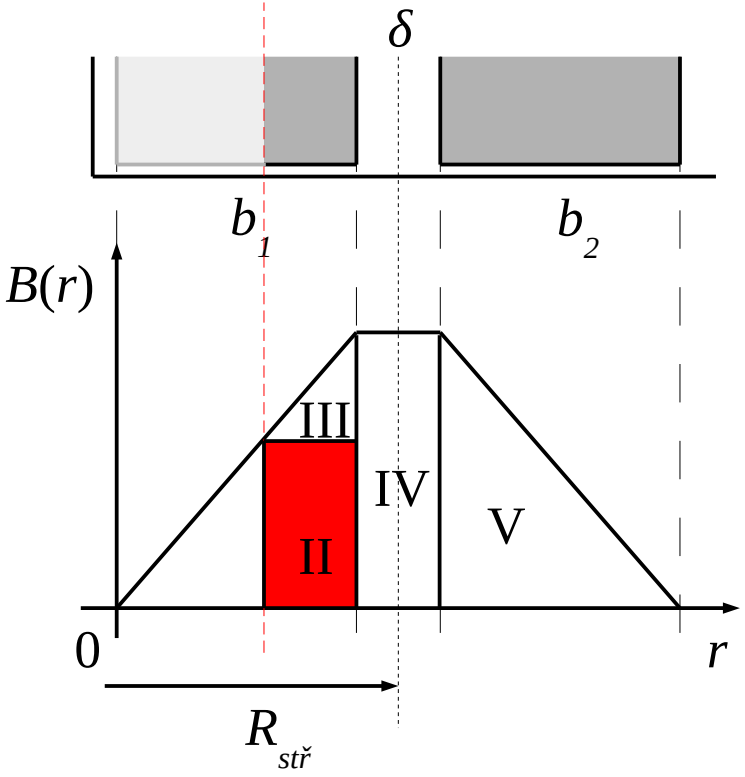
$$k_{II} = 1$$

$$\Phi_{II} \sim S_{II} \sim r \cdot b_1 \cdot (1 - r)$$

$$N_{II} = 1$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- Area II:



$$u_{iII} \sim k_{II}\,\Phi_{II}\,N_{II}$$

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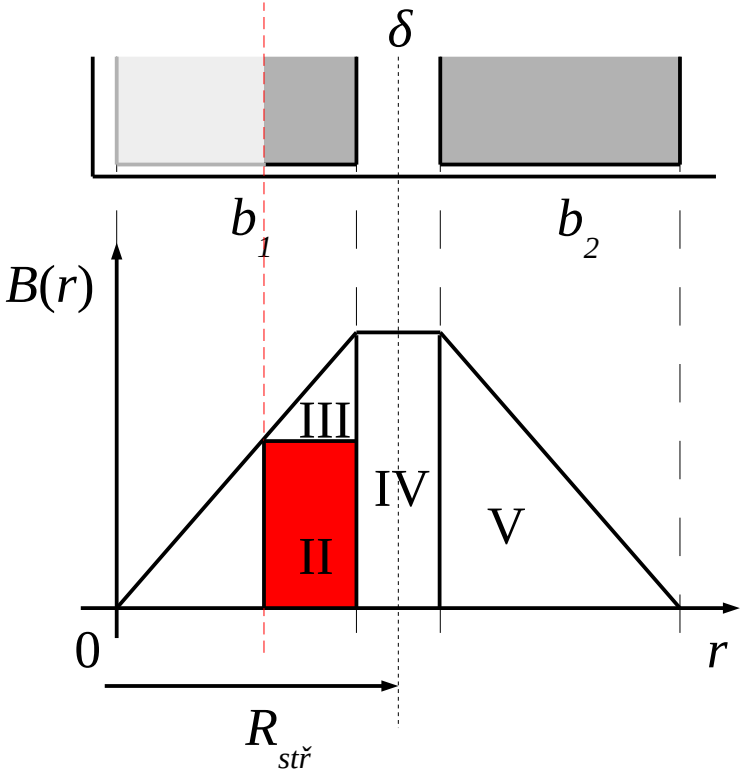
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The Calculation of Transformer Leakage Inductance in Short-Circuit

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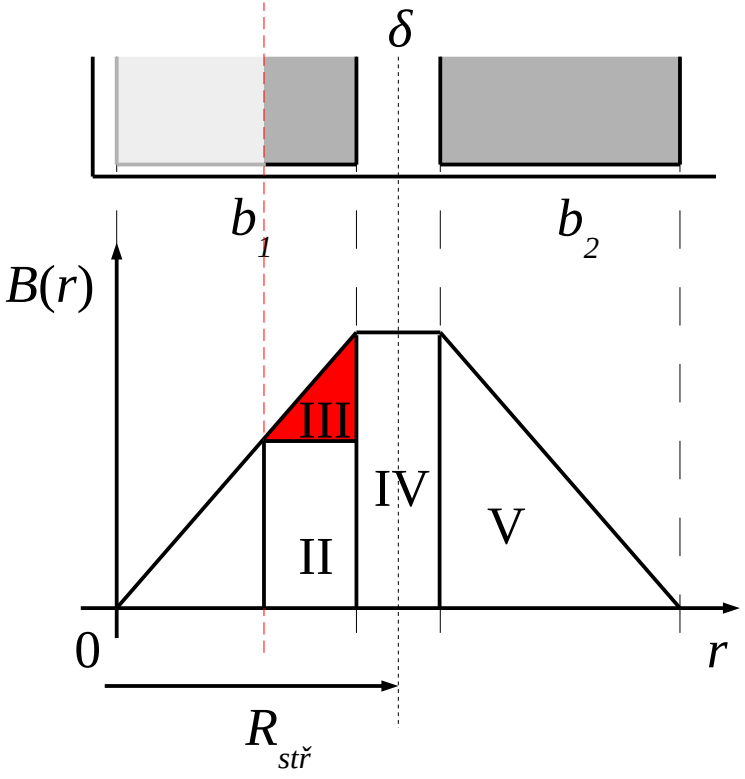
$$\Phi_{II} \sim S_{II} \sim r \cdot b_1 \cdot (1 - r)$$

$$N_{II} = 1$$

$$u_{iII} \sim 1 \cdot r \cdot b_1 \cdot (1 - r) \cdot 1$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

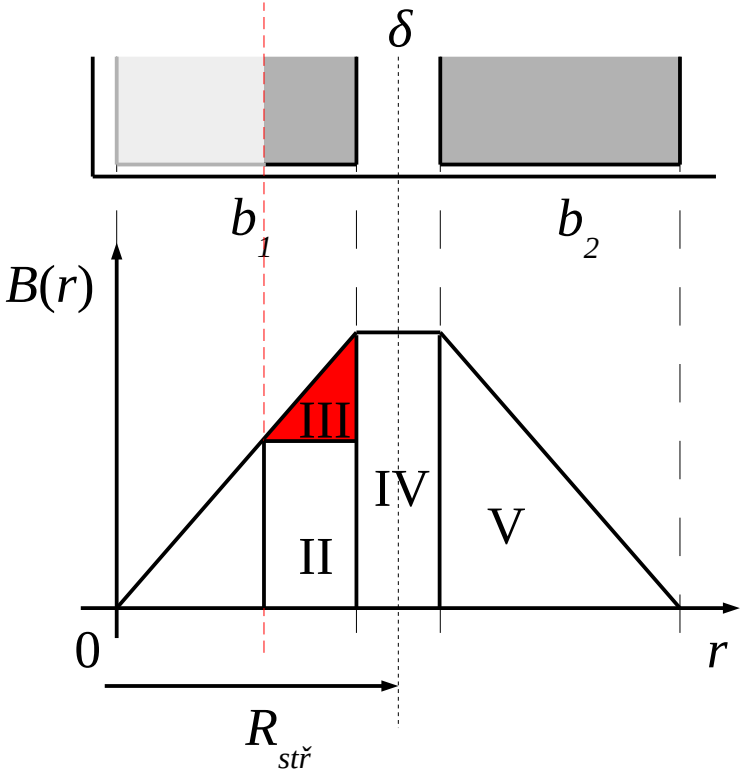
- Area III:



$$u_{iIII} \sim k_{III} \Phi_{III} N_{III}$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- Area III:

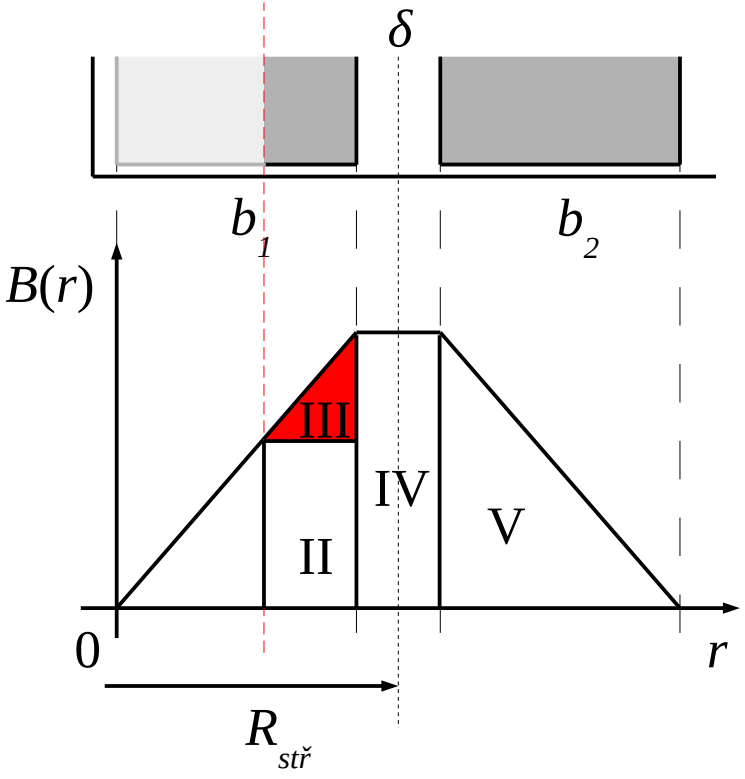


$$u_{iIII} \sim k_{III} \Phi_{III} N_{III}$$

$$k_{III} = 1$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- Area III:



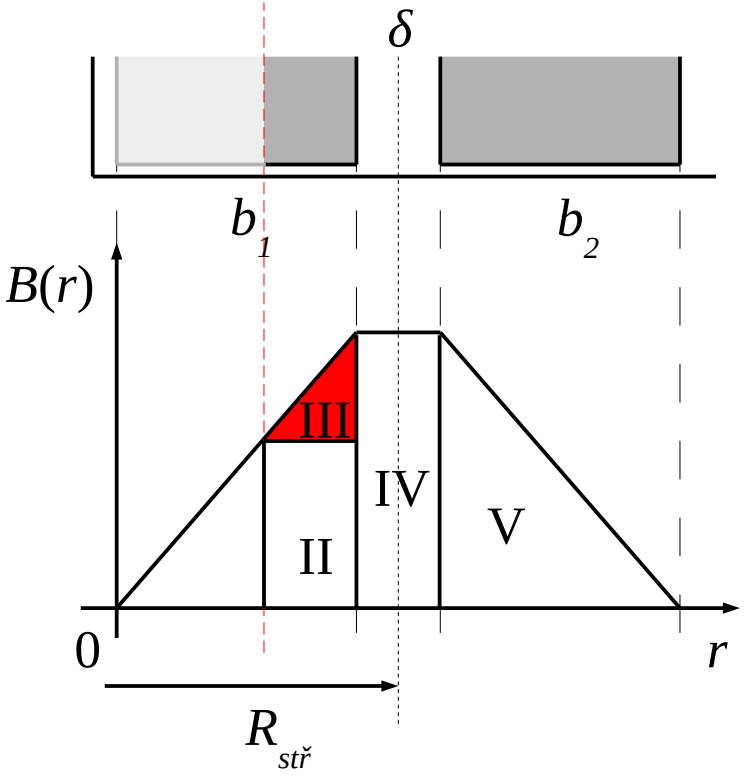
$$u_{iIII} \sim k_{III} \Phi_{III} N_{III}$$

$$k_{III} = 1$$

$$\Phi_{III} \sim S_{III} \sim \frac{1}{2} \cdot (1-r) \cdot b_1 \cdot (1-r)$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- Area III:



$$u_{iIII} \sim k_{III} \Phi_{III} N_{III}$$

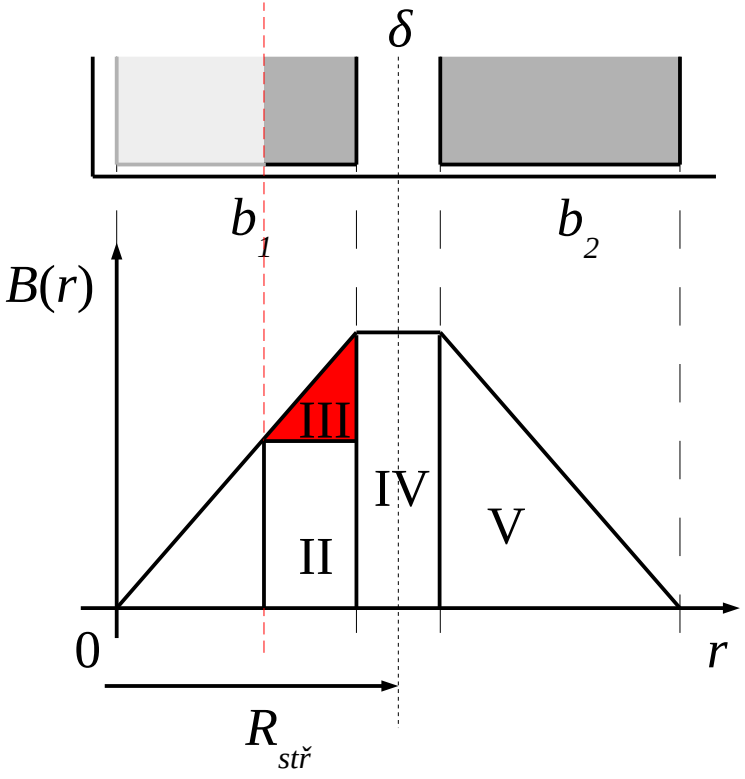
$$k_{III} = 1$$

$$\Phi_{III} \sim S_{III} \sim \frac{1}{2} \cdot (1-r) \cdot b_1 \cdot (1-r)$$

$$N_{III} = 1$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- Area III:



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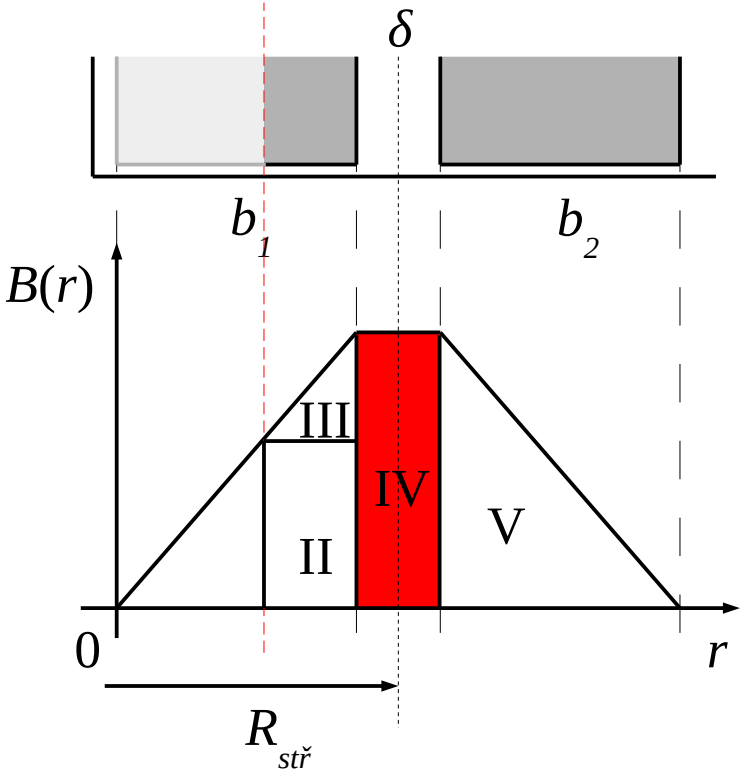
$$\Phi_{III} \sim S_{III} \sim \frac{1}{2} \cdot (1-r) \cdot b_1 \cdot (1-r)$$

$$N_{III} = 1$$

$$u_{iIII} \sim 1 \cdot \frac{1}{2} \cdot (1-r) \cdot b_1 \cdot (1-r) \cdot 1$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

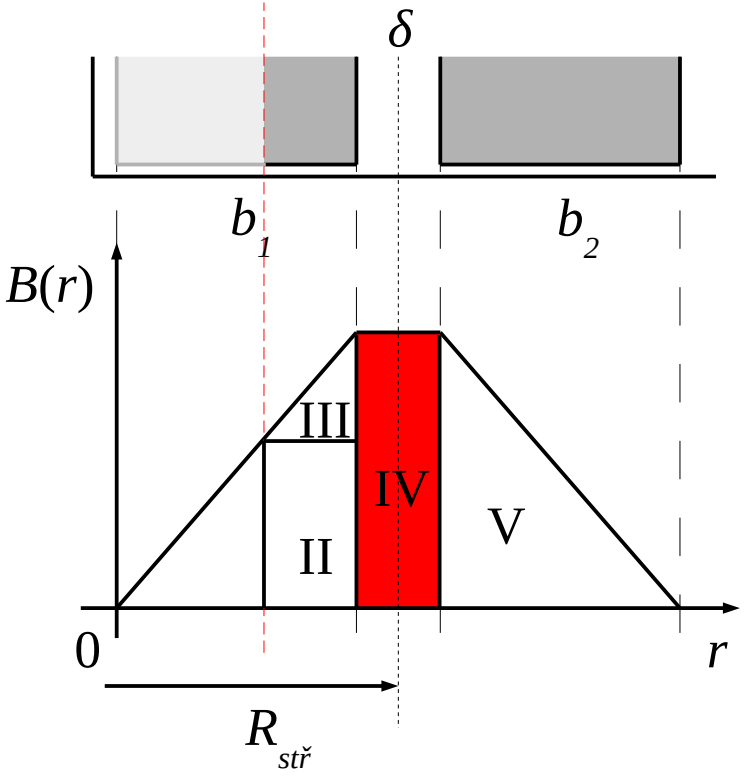
- Area IV:



$$u_{iIV} \sim k_{IV} \Phi_{IV} N_{IV}$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- Area IV:

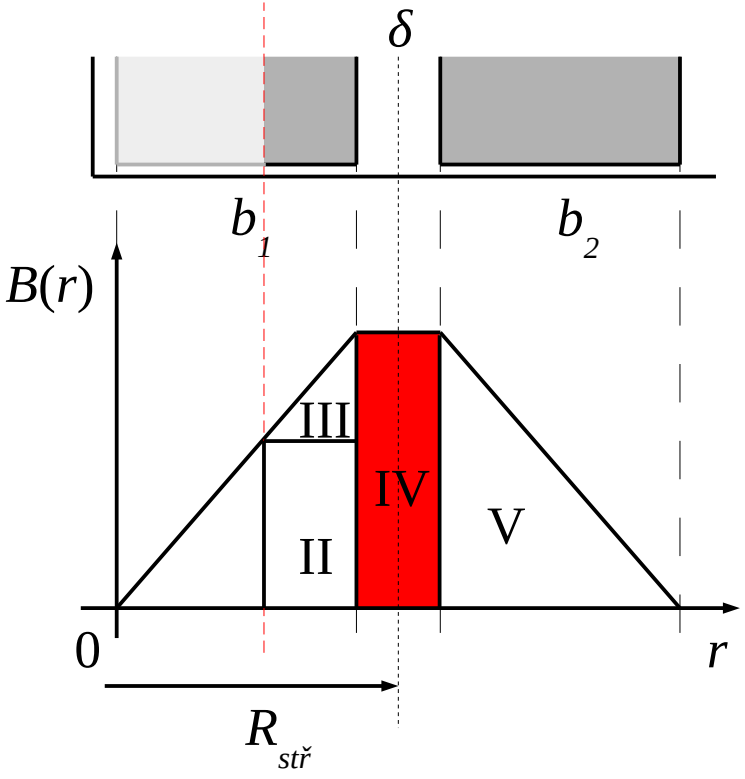


$$u_{iIV} \sim k_{IV} \Phi_{IV} N_{IV}$$

$$k_{IV} = 1$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- Area IV:



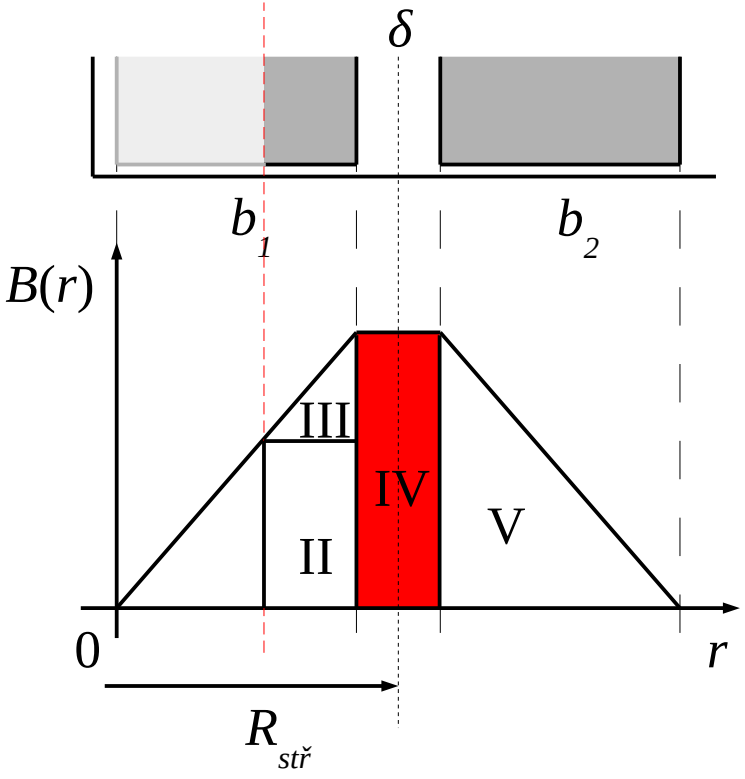
$$u_{iIV} \sim k_{IV} \Phi_{IV} N_{IV}$$

$$k_{IV} = 1$$

$$\Phi_{IV} \sim S_{IV} \sim 1 \cdot \delta$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- Area IV:



$$u_{iIV} \sim k_{IV} \Phi_{IV} N_{IV}$$

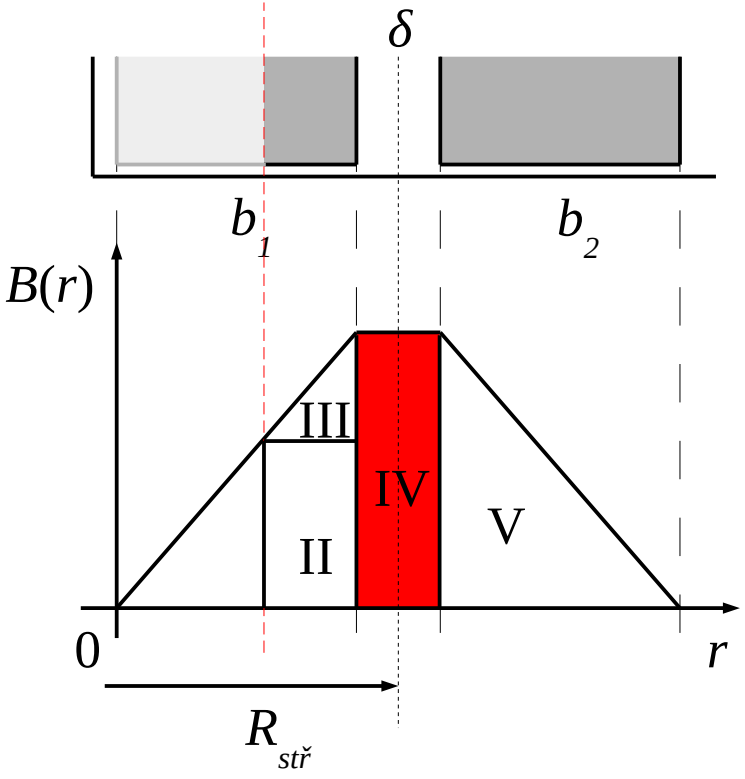
$$k_{IV} = 1$$

$$\Phi_{IV} \sim S_{IV} \sim 1 \cdot \delta$$

$$N_{IV} = 1$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- Area IV:



$$u_{iIV} \sim k_{IV} \Phi_{IV} N_{IV}$$

$$k_{IV} = 1$$

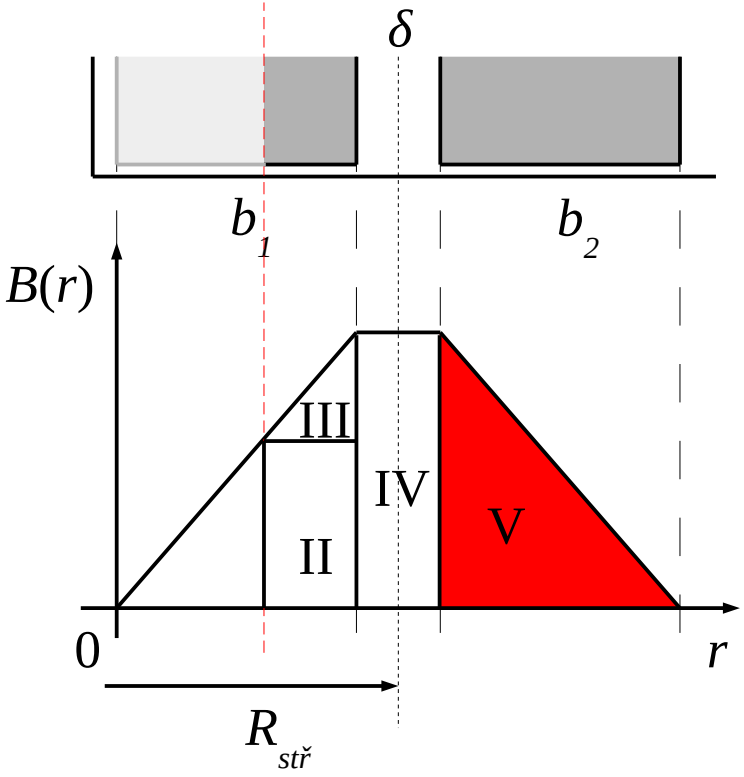
$$\Phi_{IV} \sim S_{IV} \sim 1 \cdot \delta$$

$$N_{IV} = 1$$

$$u_{iIV} \sim 1 \cdot 1 \cdot \delta \cdot 1 \sim \delta$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

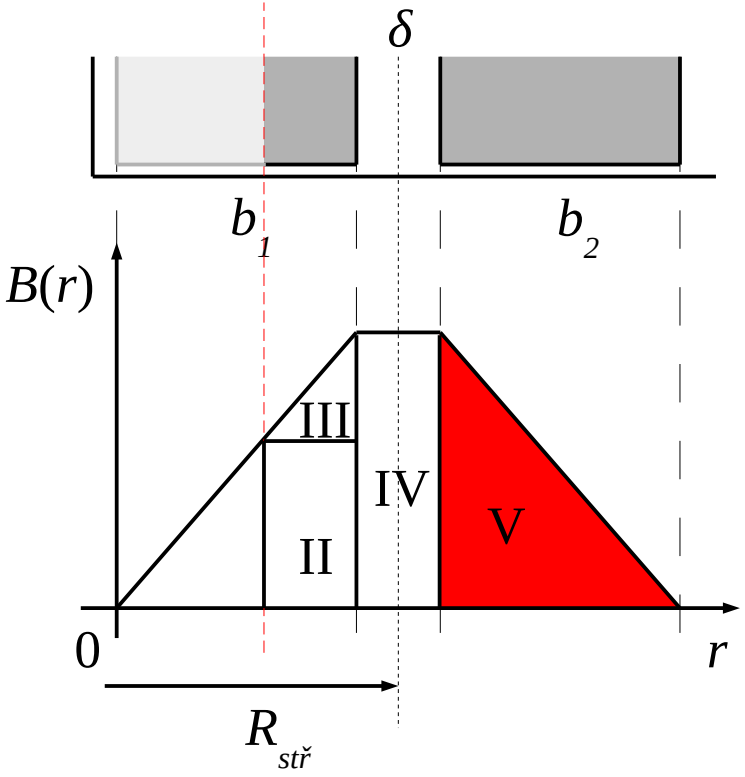
- Area V:



$$u_{iV} \sim k_V \Phi_V N_V$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- Area V:

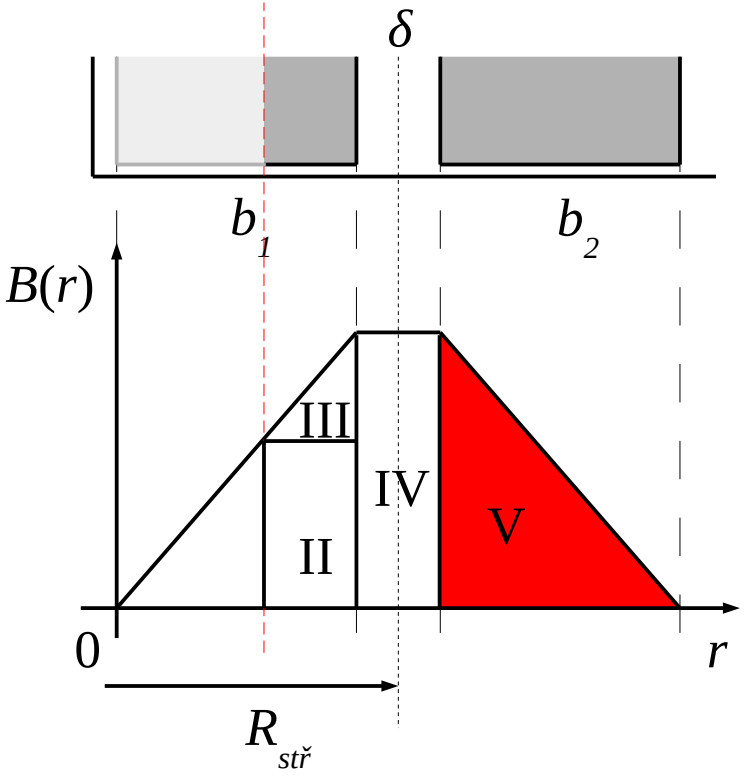


$$u_{iV} \sim k_V \Phi_V N_V$$

$$k_V = \frac{2}{3}$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- Area V:



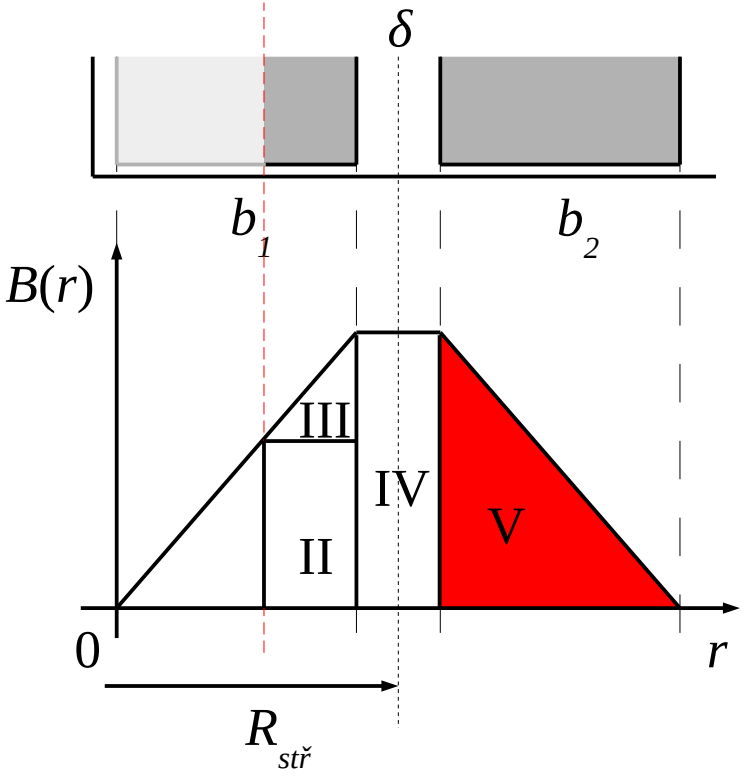
$$u_{iV} \sim k_V \Phi_V N_V$$

$$k_V = \frac{2}{3}$$

$$\Phi_V \sim S_V \sim \frac{1}{2} \cdot b_2 \cdot 1$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- Area V:



$$u_{iV} \sim k_V \Phi_V N_V$$

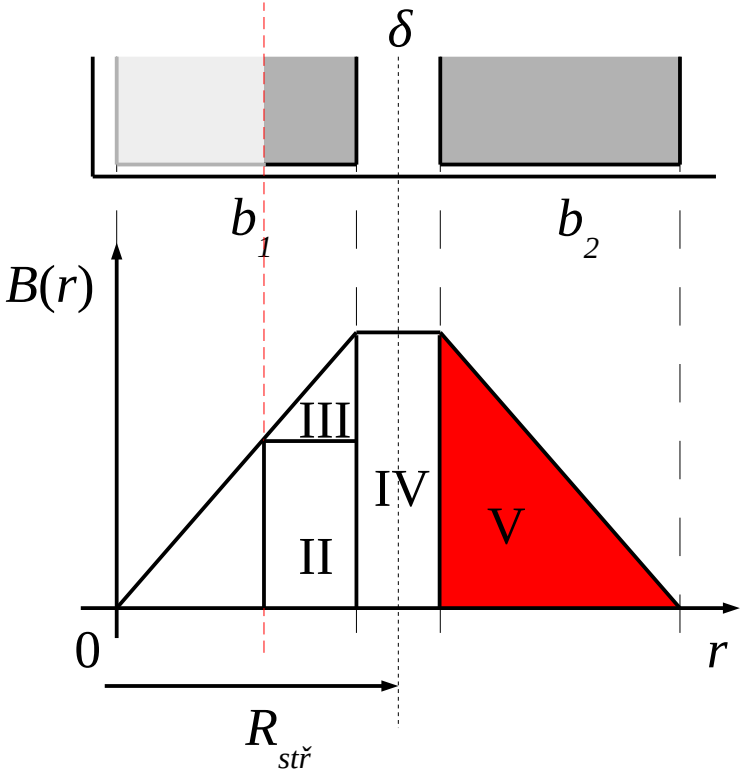
$$k_V = \frac{2}{3}$$

$$\Phi_V \sim S_V \sim \frac{1}{2} \cdot b_2 \cdot 1$$

$$N_V = 1$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- Area V:



$$u_{iV} \sim k_V \Phi_V N_V$$

$$k_V = \frac{2}{3}$$

$$\Phi_V \sim S_V \sim \frac{1}{2} \cdot b_2 \cdot 1$$

$$N_V = 1$$

$$u_{iV} \sim \frac{1}{3} \cdot b_2 \cdot 1 \cdot 1$$

The Calculation of Transformer Leakage Inductance in Short-Circuit

- The induced voltage to the reduced area must be equal to the sum of voltages from areas II – V:

$$u_{ir} = u_{iII} + u_{iIII} + u_{iIV} + u_{iV}$$

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$$u_{ir} \sim 1 \cdot \delta_r \cdot 1 \cdot 1 \sim \delta_r$$

$$u_{iII} \sim 1 \cdot r \cdot b_1 \cdot (1 - r) \cdot 1$$

$$u_{iIII} \sim 1 \cdot \frac{1}{2} \cdot (1 - r) \cdot b_1 \cdot (1 - r) \cdot 1$$

$$u_{iIV} \sim 1 \cdot 1 \cdot \delta \cdot 1 \sim \delta$$

$$u_{iV} \sim \frac{1}{3} \cdot b_2 \cdot 1 \cdot 1$$

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- The division plane is at $r = \frac{1}{\sqrt{3}}$

- The magnetic field excited by the secondary winding may be substituted by a simple rectangular field with the same magnitude and width of:

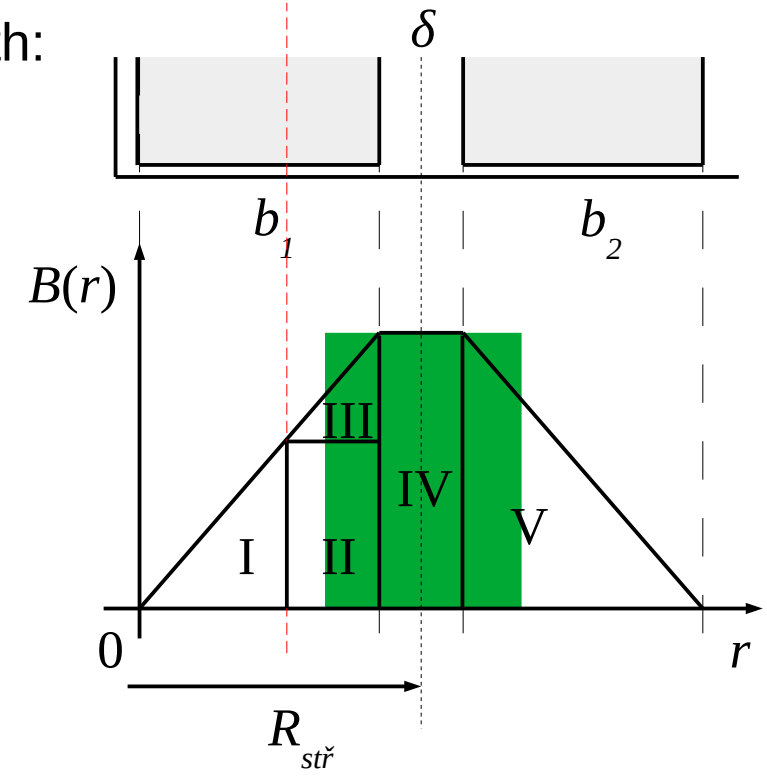
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The Calculation of Transformer Leakage Inductance in Short-Circuit

- Now it is possible to calculate the leakage inductance of the transformer from the magnetic reluctance of the leakage flux path:

$$L_\sigma = \Lambda N^2$$

$$\Lambda = \mu_0 \frac{S}{l}$$

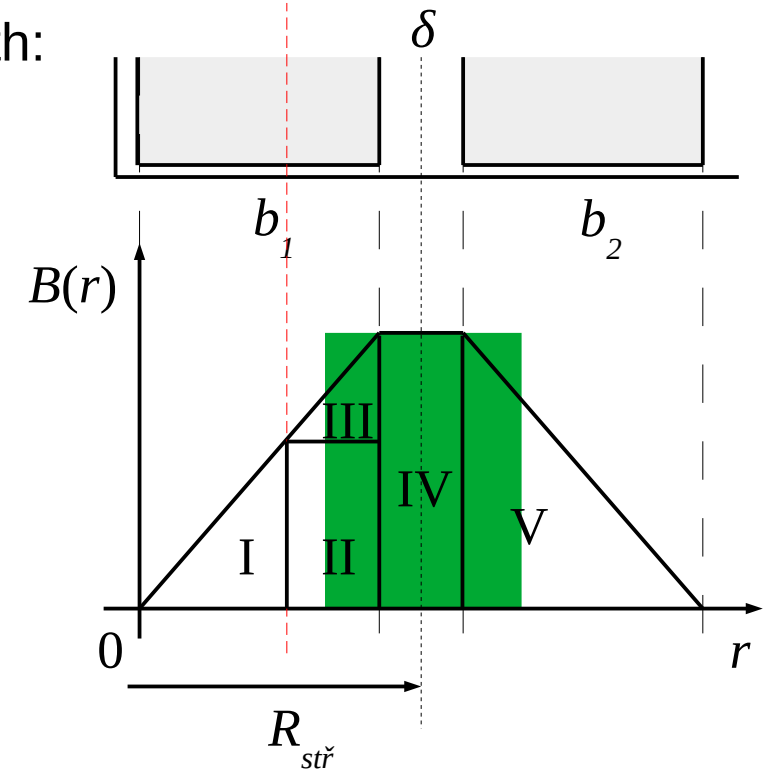


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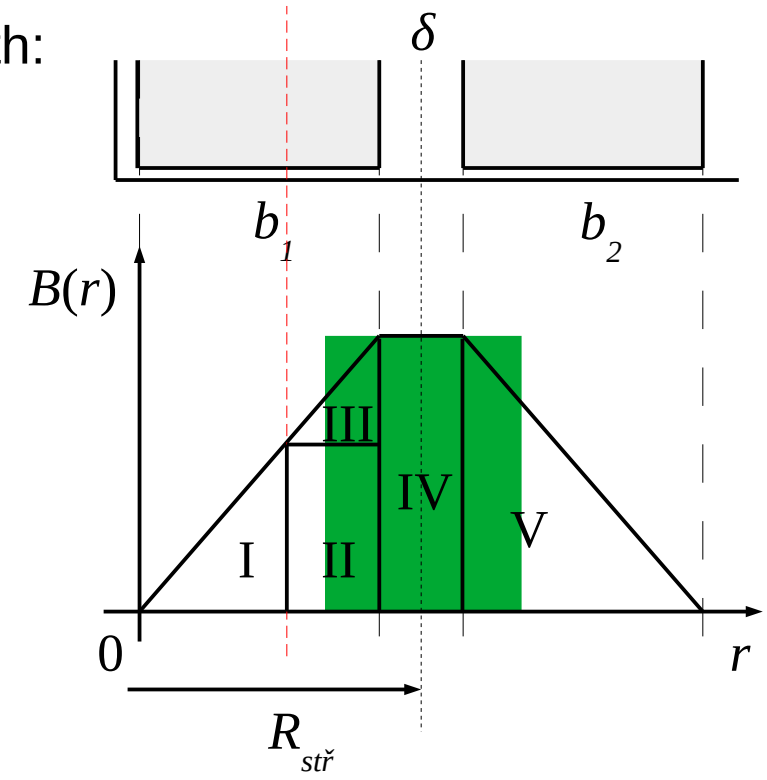
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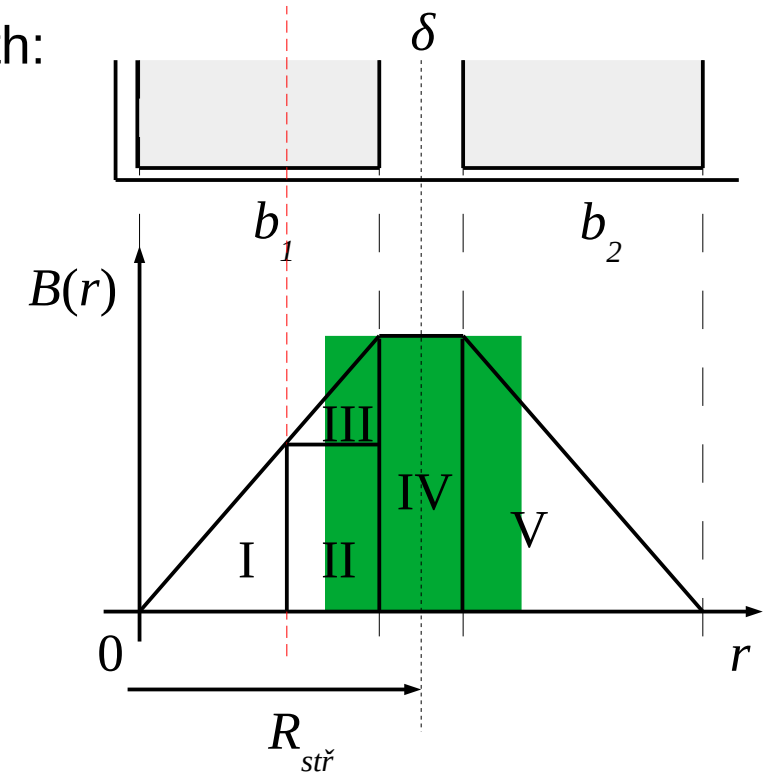
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$$\delta_r = \frac{b_1}{3} + \delta + \frac{b_2}{3}$$

$$L_\sigma = 2\mu_0 \pi R_{stř} \frac{N^2}{l} \left(\frac{b_1}{3} + \delta + \frac{b_2}{3} \right)$$



The Calculation of Transformer Leakage Inductance in Short-Circuit

- Thus the per unit value of the leakage reactance as well as the short circuit voltage is:

$$x_{\sigma} \simeq x_{k} \simeq u_{k} \simeq \frac{\omega L_{\sigma}}{Z_{n}}$$

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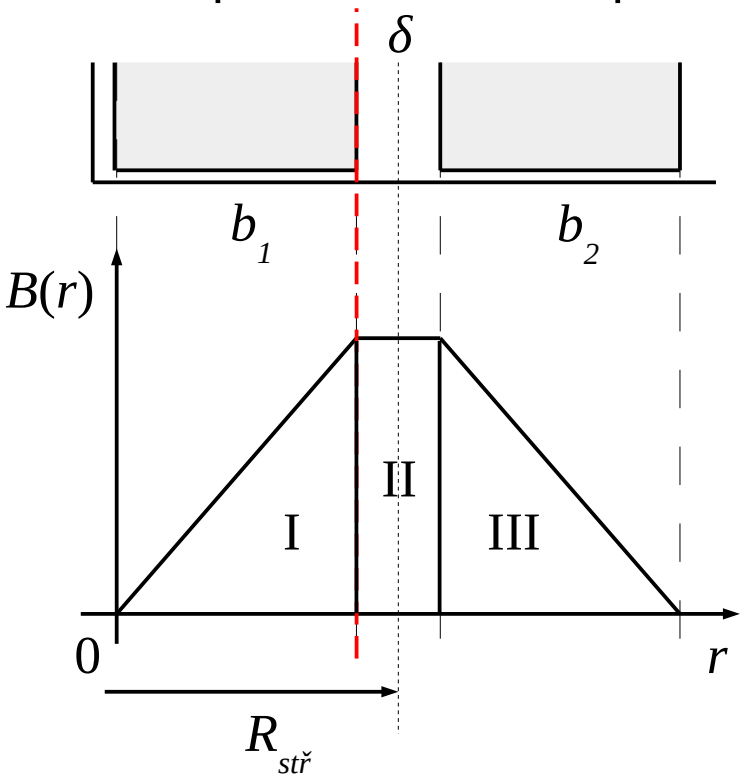
The Location of the Division Plane in a Real Transformer

The Location of the Division Plane in a Real Transformer

- The position of the division plane changes through time in normal duty of the transformer
- Presuming winding resistances r_1, $r_2 = 0$ and a real duty with $i_\mu \neq 0$, then currents i_1 and i_2 are not equal and the division plane moves to the centre of the coil
- In case when r_1, $r_2 \neq 0$, the ratio i_1/i_2 moves from the set of real numbers to the set of complex numbers and the division plane “travels” between windings
- The total value of the leakage inductance remains constant

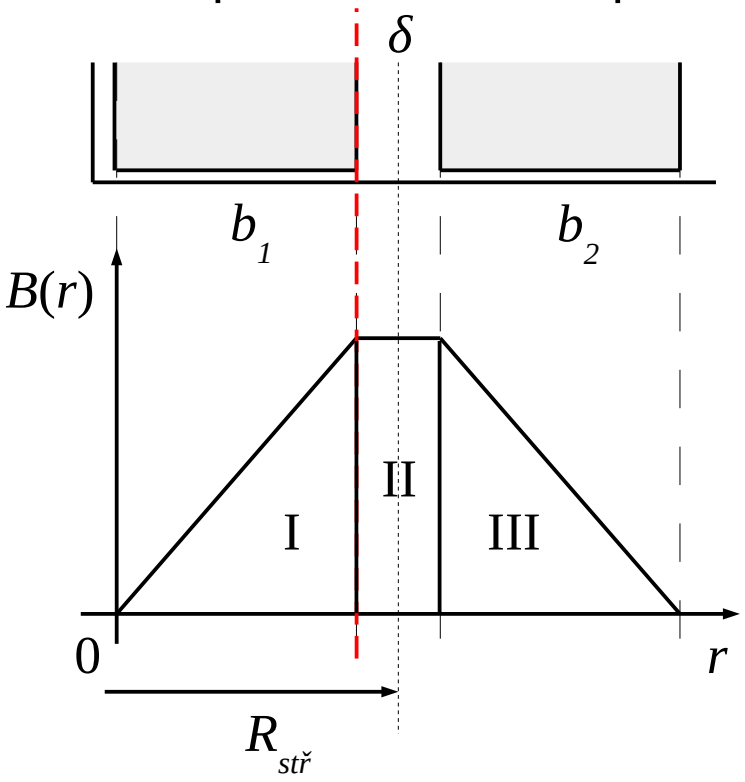
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- Example: The division plane is located on the surface of the primary winding



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$$u_{ir} \sim 1 \cdot \delta_r \cdot 1 \cdot 1$$

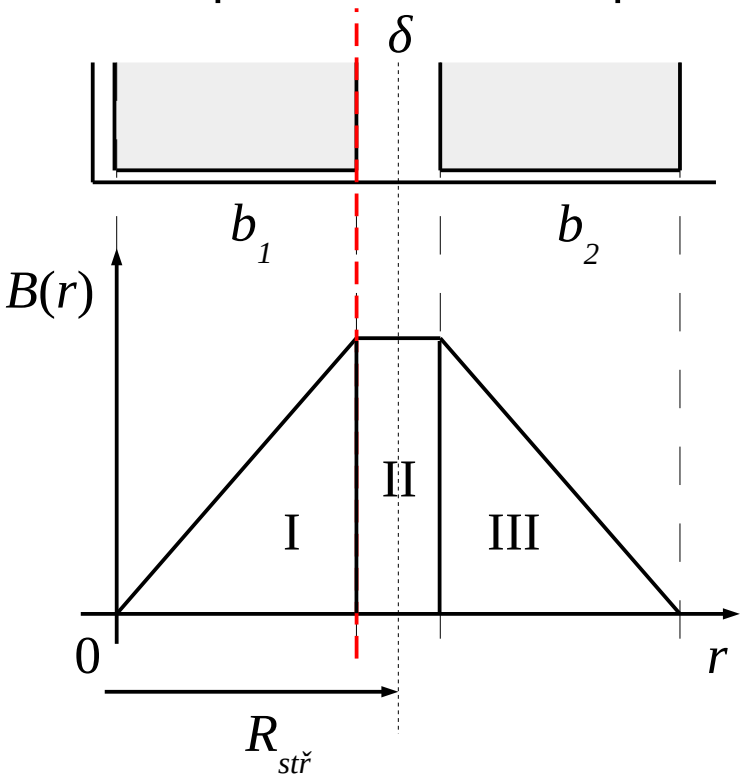
$$u_{iI} \sim \frac{2}{3} \cdot \frac{b_1}{2} \cdot 1 \cdot 1$$

$$u_{iII} \sim 1 \cdot \delta \cdot 1 \cdot 1$$

$$u_{iIII} \sim \frac{2}{3} \cdot \frac{b_2}{2} \cdot 1 \cdot 1$$

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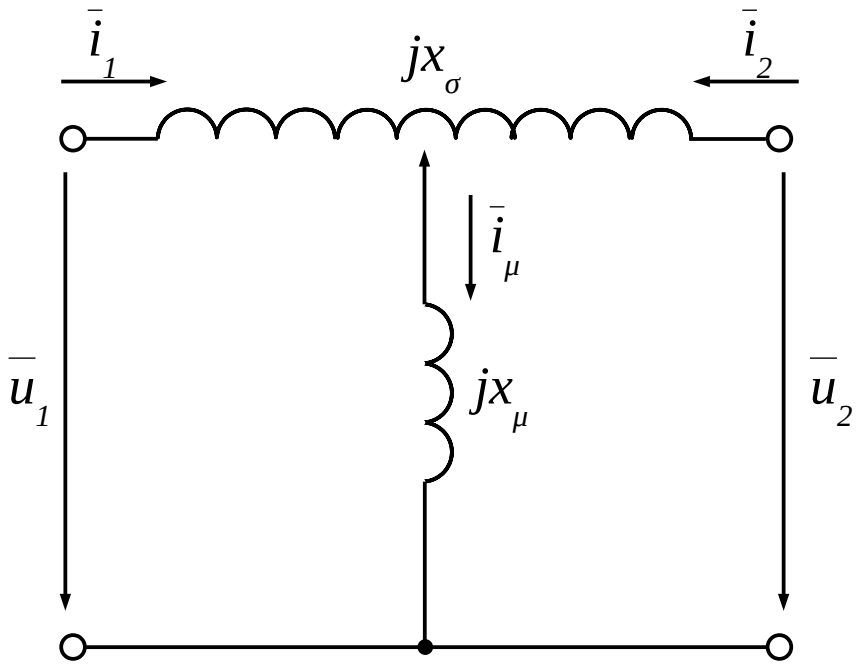
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$$u_{ir} = u_{iI} + u_{iII} + u_{iIII}$$

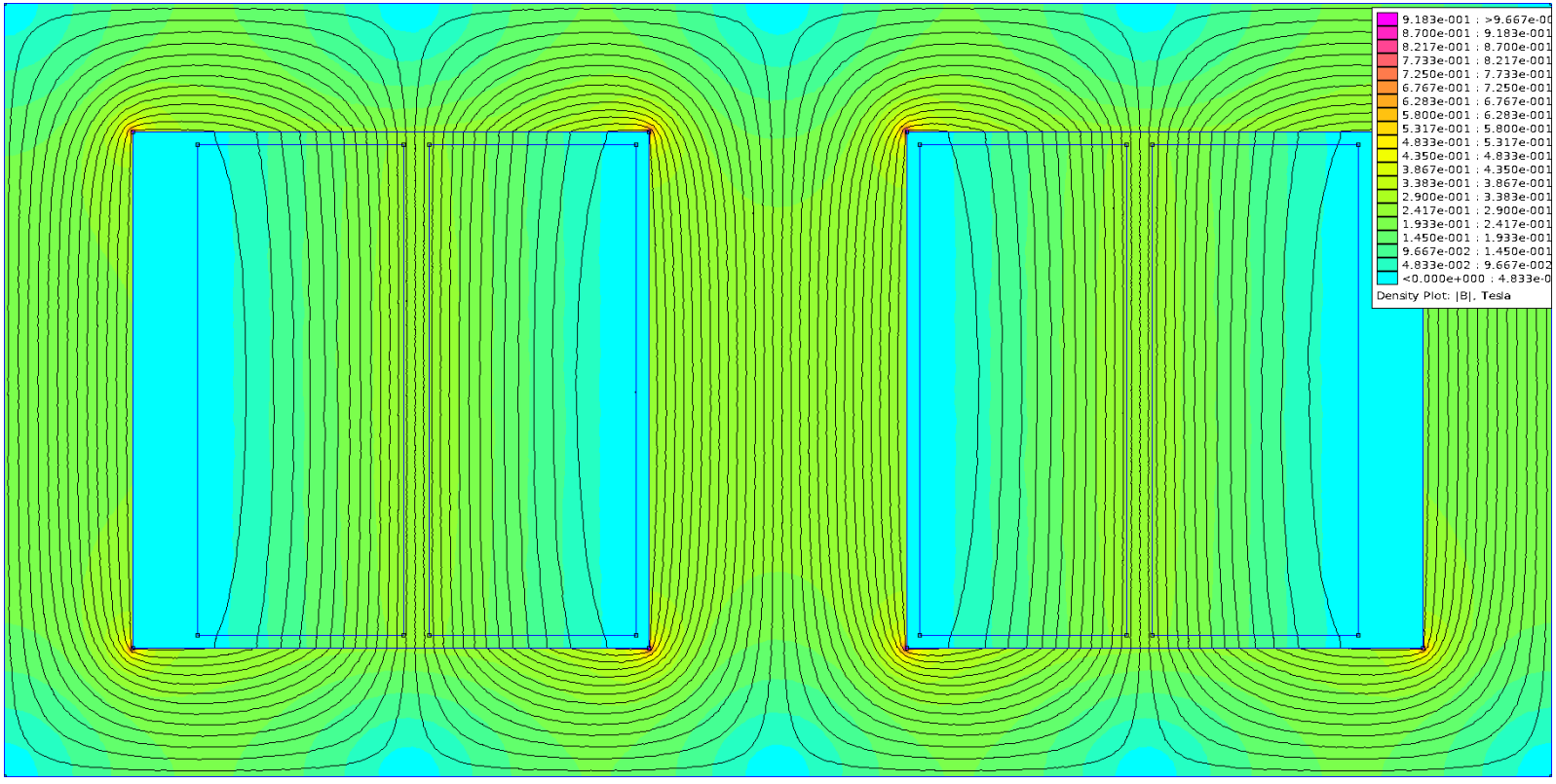
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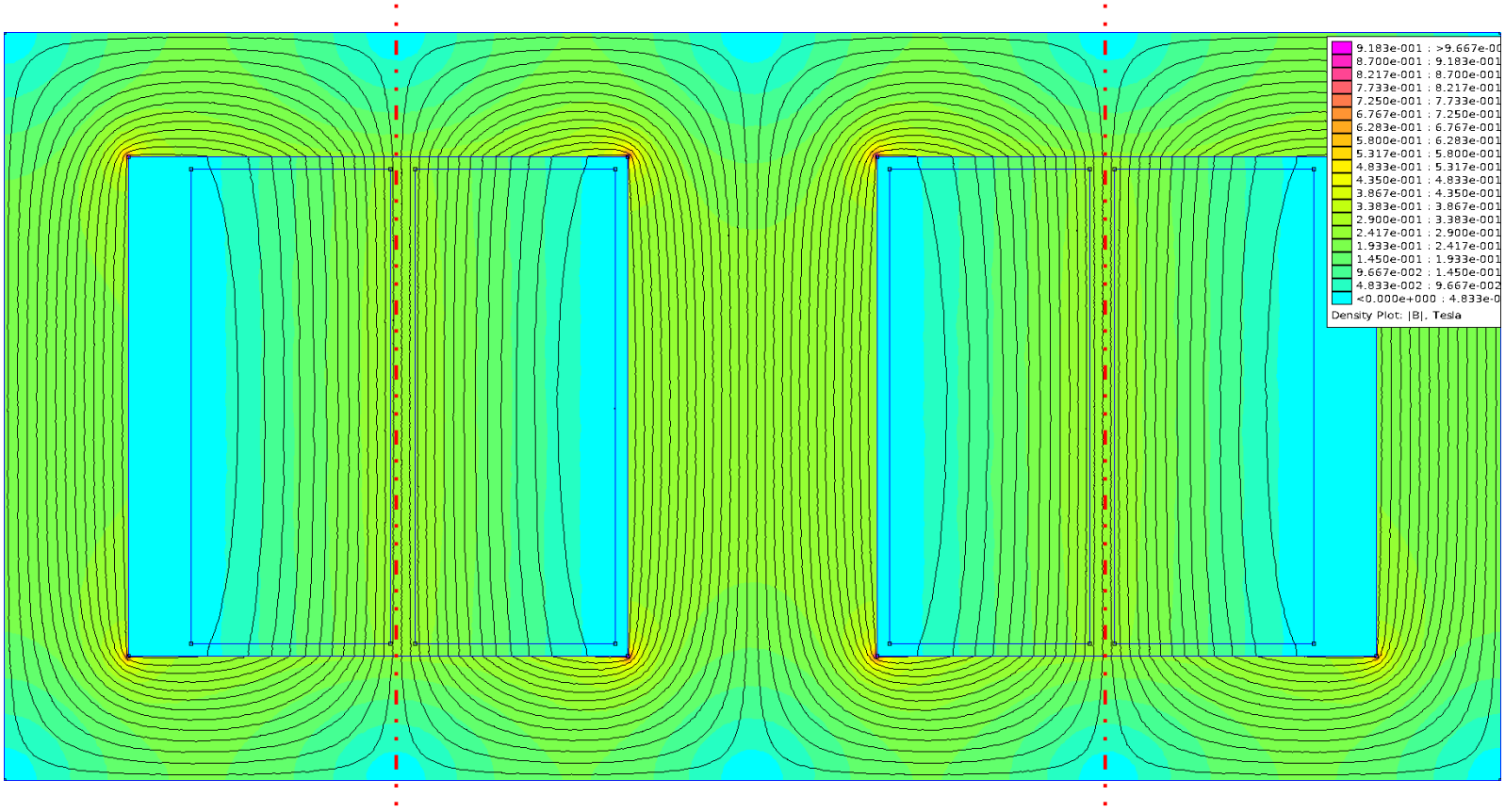
- The total leakage inductance remains identical, what is changing is the ratio between $x_{1\sigma}/x_{2\sigma}$



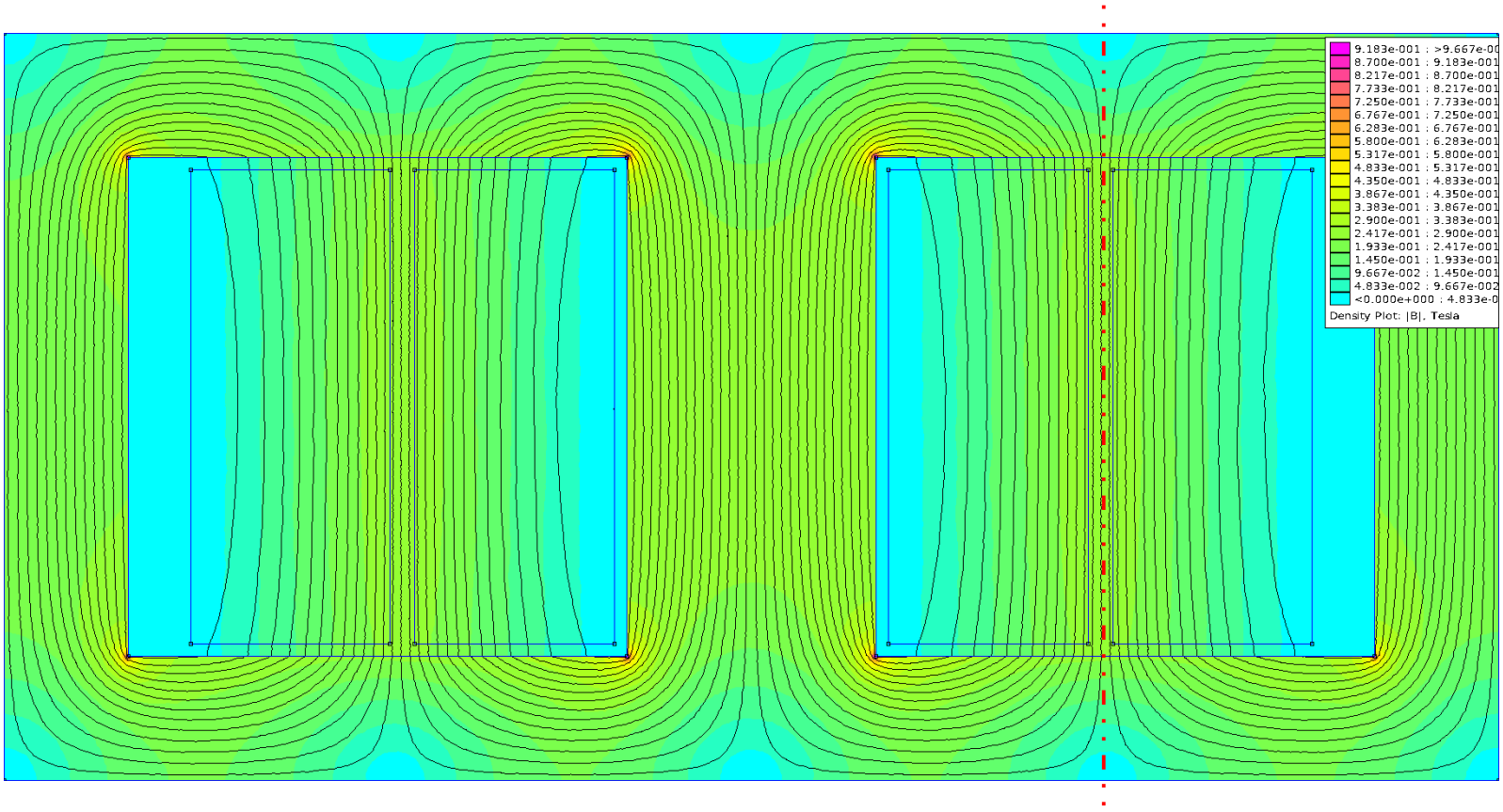
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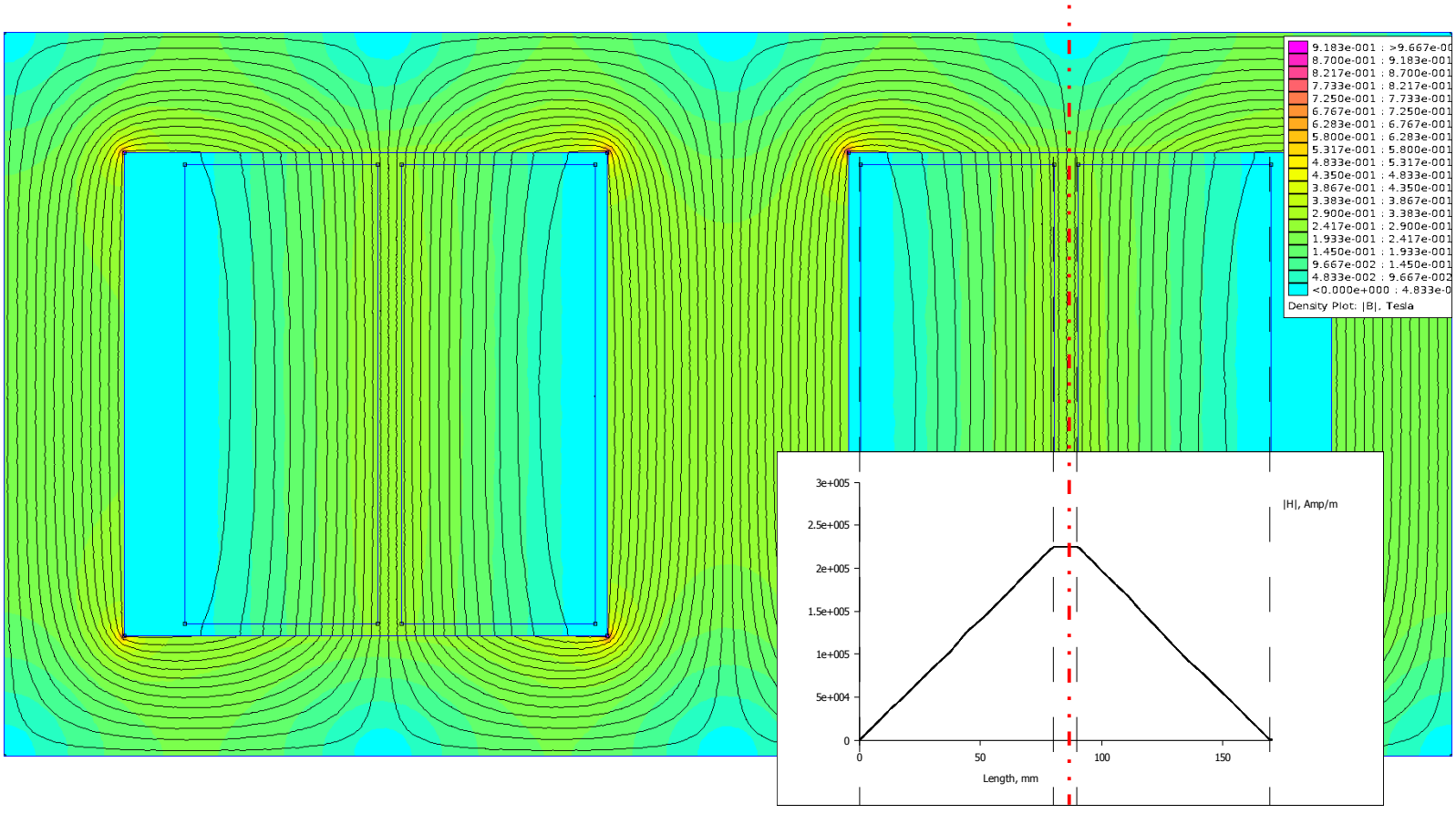
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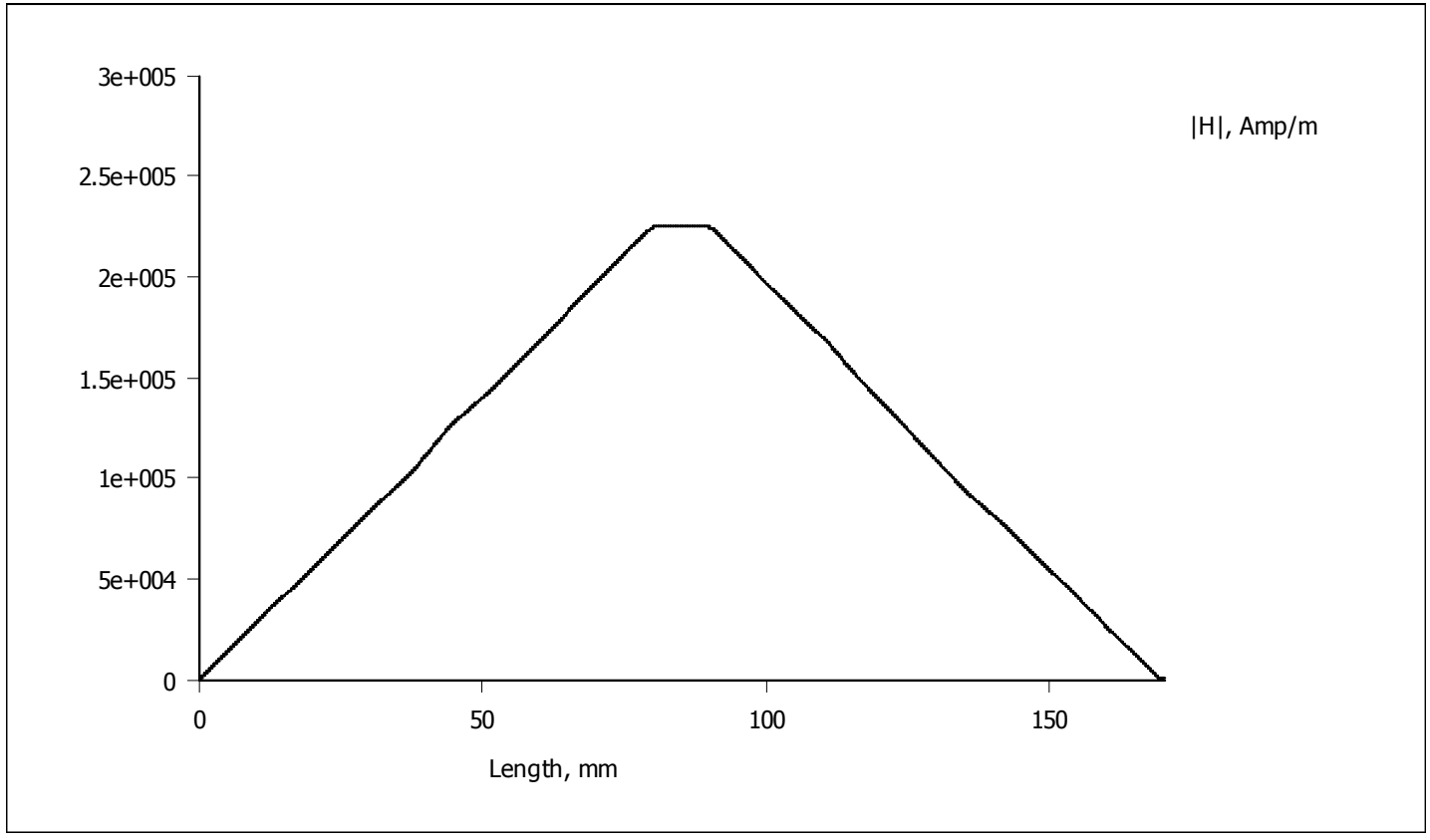
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Thank You for Your Attention!

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