FACULTY OF ELECTRICAL
ENGINEERING
UNIVERSITY OF WEST BOHEMIA

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

ESF II – project of the University of West Bohemia
reg. no. CZ.02.2.69/0.0/0.0/18_056/0013239

# Theory of Electric Machines 2

## KEV/TS2

**Karel Hruska** / r20201117

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# Circle Diagram of an Induction Machine

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- The circle diagram of an induction machine is a graphical representation of rotor current dependency on slip in complex plane
- Considering dependency of rotor parameters on slip, the circle is only one of possible cases of this dependency (i.e. current diagram)
- With satisfying accuracy it is valid only for machines, whose parameters are slip independent

  => wound-rotor induction machines

# Circle Diagram of an Induction Machine

- In case of squirrel-cage machines it is valid only with certain accuracy and in area of low slips
- Two possible constructions:
  - Based on measurement results (see KEV/ES1, KEV/TS1)
  - Based on equivalent circuit parameters
- Both constructions should lead to identical result

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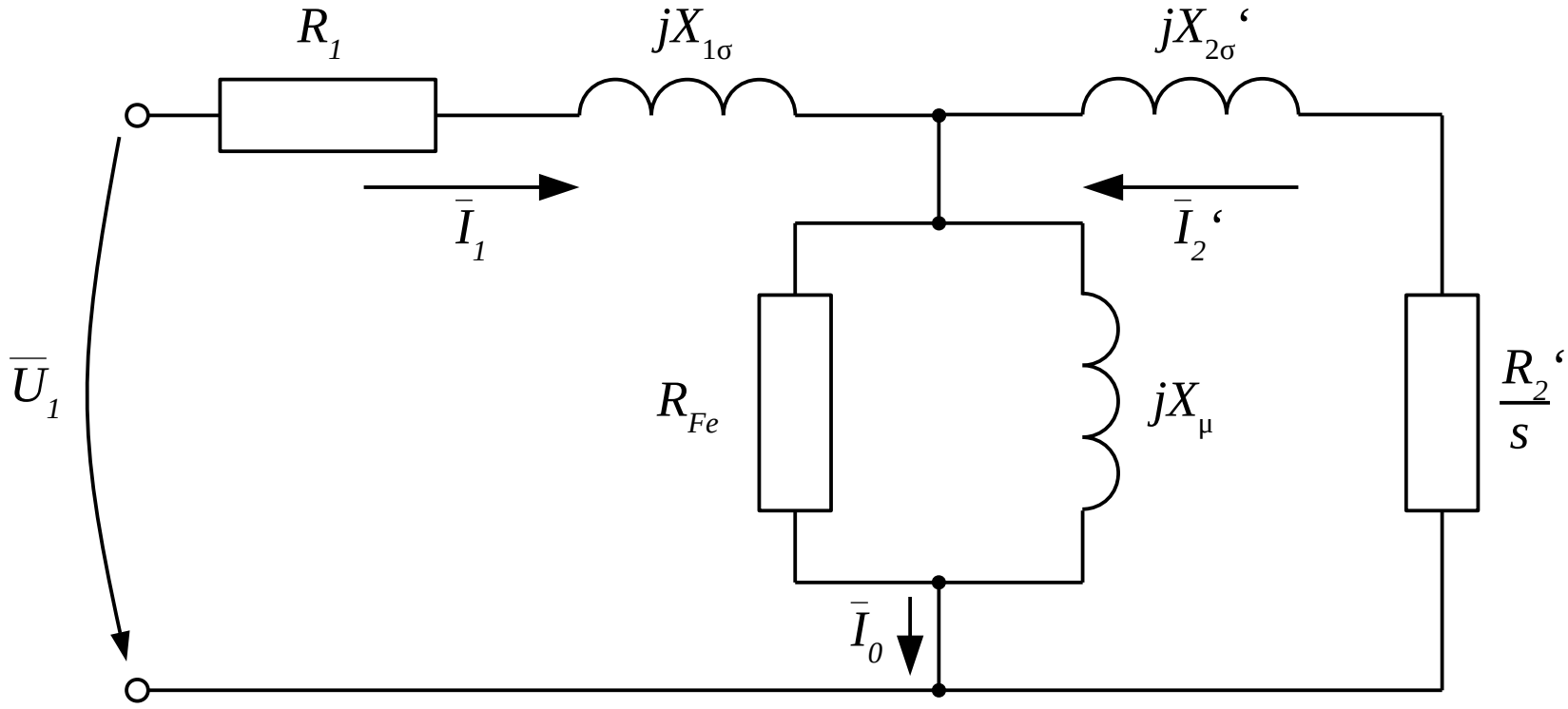
# Induction Machine Equivalent Circuit

# Induction Machine Equivalent Circuit

- The equivalent circuit forms is a circuit-form of illustration of function of an induction machine
- Commonly a T-shape equivalent circuit is used, but it does not contain a direct relation between stator voltage and rotor current
  - Necessary for determination of torque
- This relation is obtained by transformation of the T-shape equivalent circuit to its Γ-shape

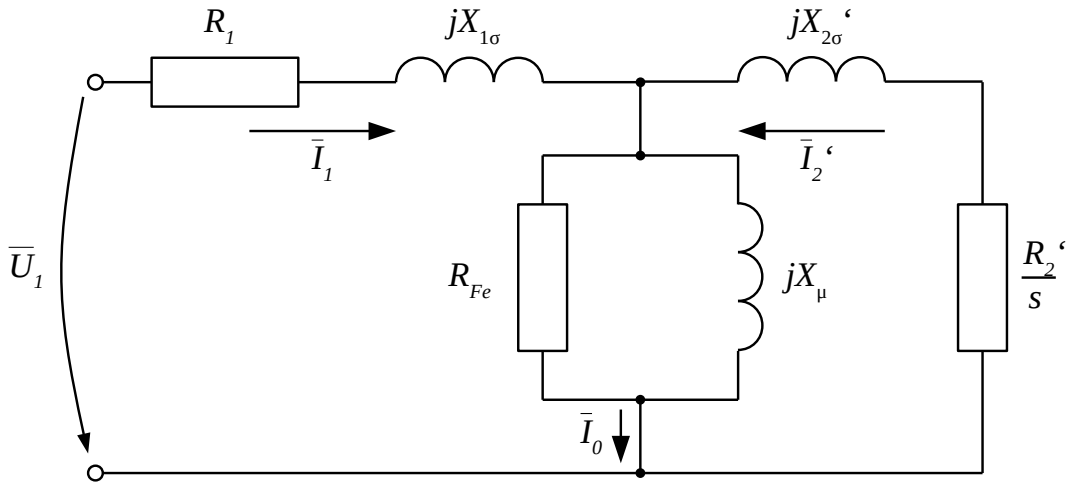
# Induction Machine Equivalent Circuit

- T-shape of the equivalent circuit:



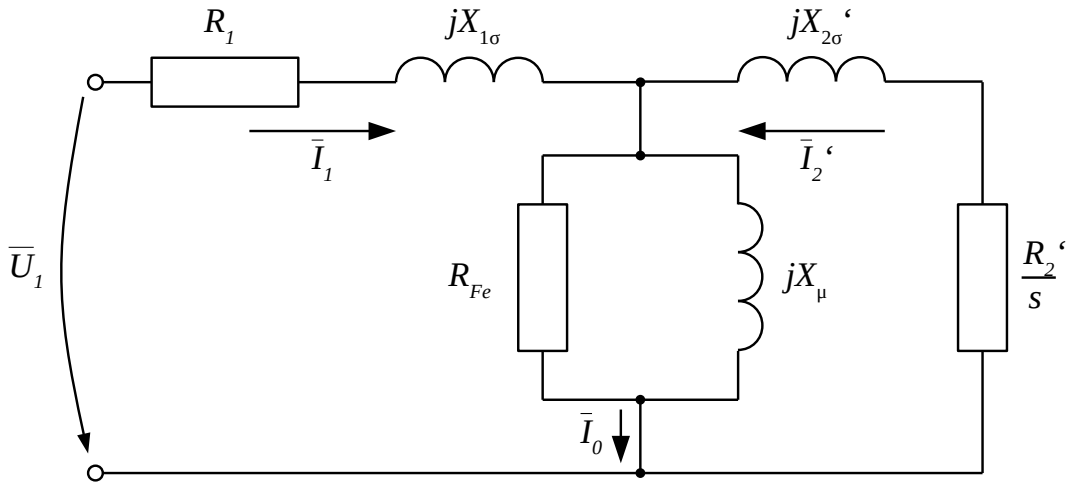
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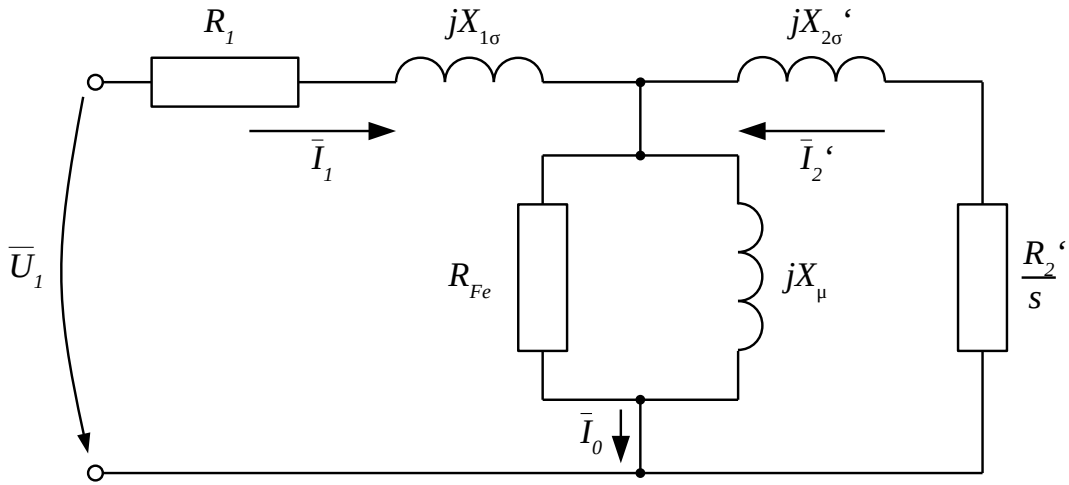
- T-shape of the equivalent circuit:
  - Voltage equations:



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- T-shape of the equivalent circuit:
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$$\bar{U}_1 = \left( R_1 + jX_{1\sigma} + \bar{Z}_{1h} \right) \bar{I}_1 + \bar{Z}_{1h} \bar{I}_2'$$

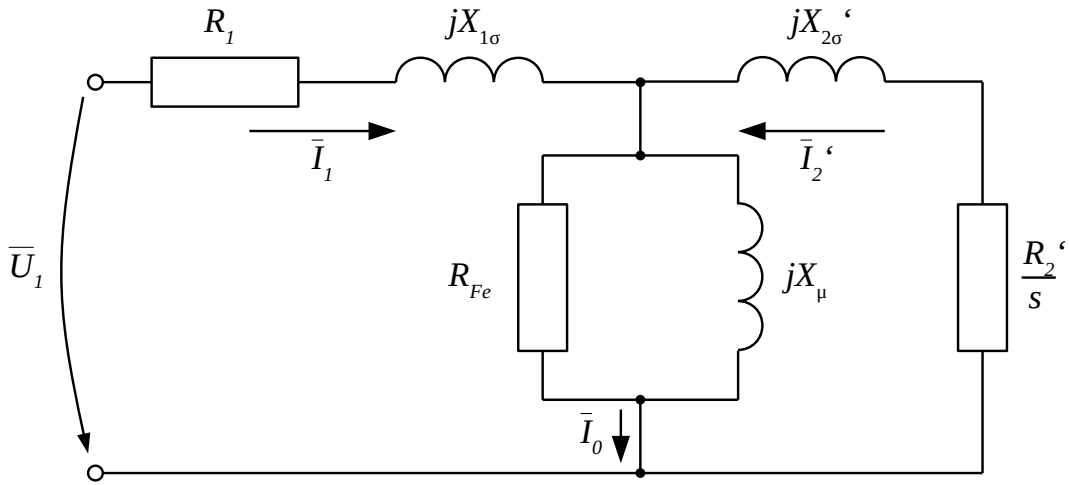


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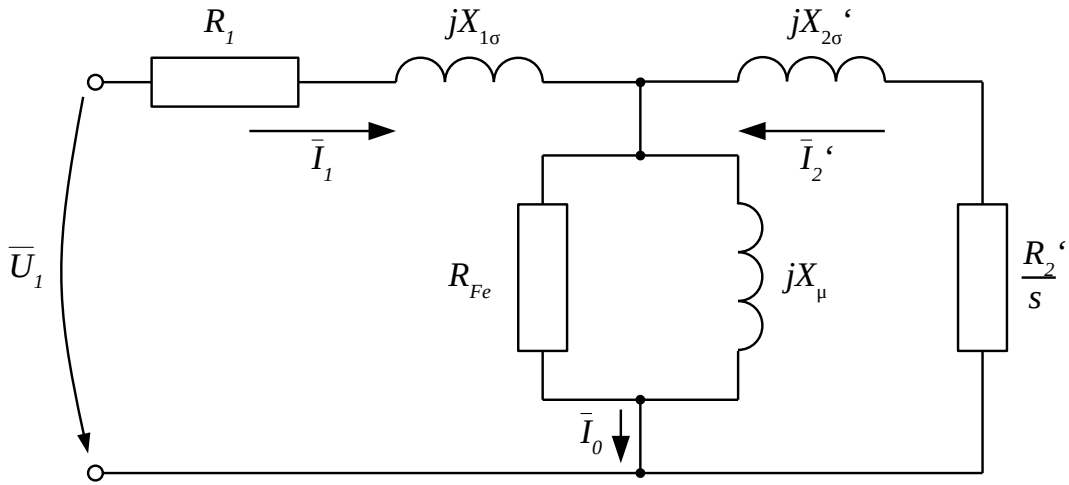
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where

$$\bar{Z}_{1h} = \frac{j R_{Fe} X_{\mu}}{R_{Fe} + jX_{\mu}}$$



# Induction Machine Equivalent Circuit

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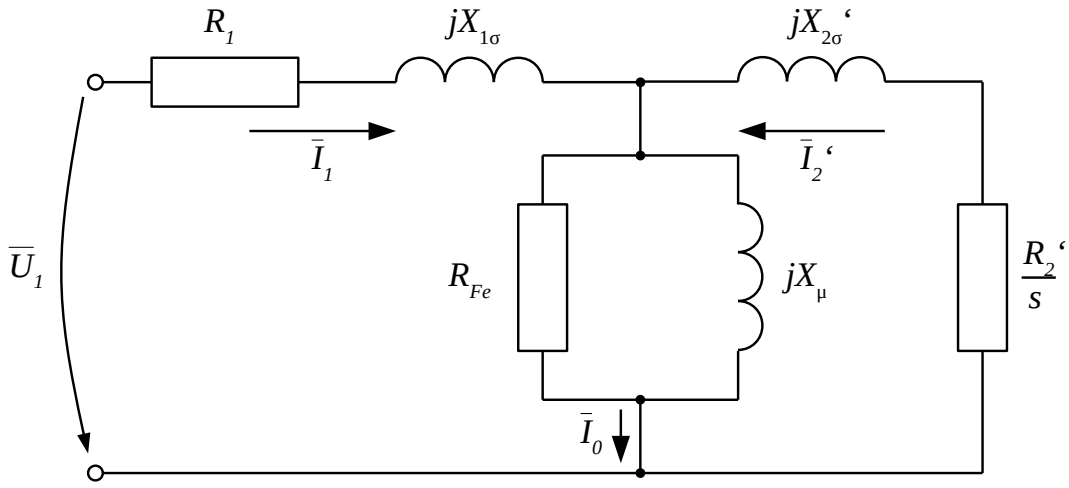
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let us assign

$$\bar{Z}_1 = R_1 + jX_{1\sigma} + \bar{Z}_{1h}$$

$$\bar{Z}_2' = \frac{R_2'}{s} + jX_{2\sigma}' + \bar{Z}_{1h}$$



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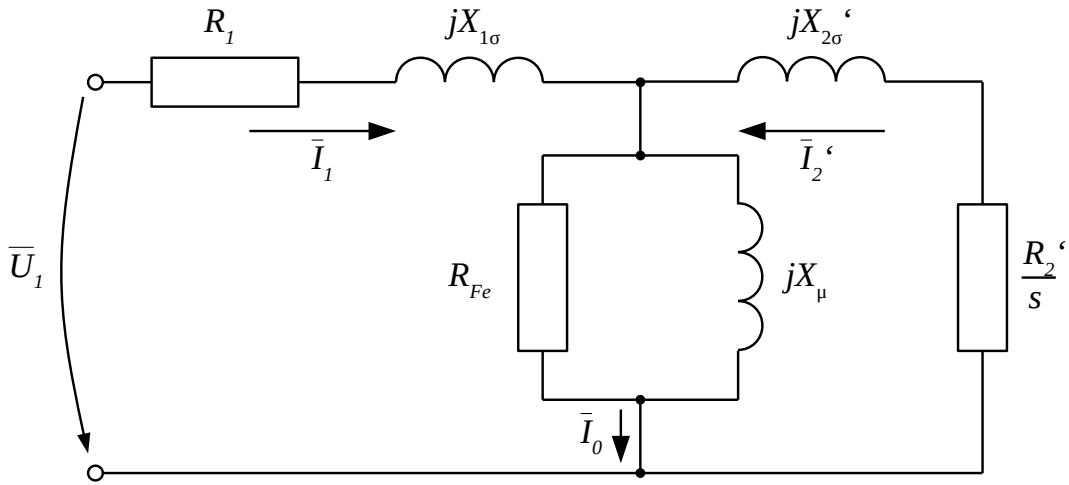
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then we obtain

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# Induction Machine Equivalent Circuit

- For transformation of the equivalent circuit into the Γ-shape, it is necessary to obtain stator current

$$\bar{I}_1 = \frac{\bar{U}_1}{\bar{Z}_1} - \frac{\bar{Z}_{1h}}{\bar{Z}_1}\bar{I}_2'$$

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- The rotor current is then

$$\bar{I}_2' = \frac{-\bar{U}_1\bar{Z}_{1h}}{\bar{Z}_1\bar{Z}_2' - \bar{Z}_{1h}^2}$$

# Induction Machine Equivalent Circuit

- Now it is possible to introduce a complex variable

$$\bar{c}_1 = \frac{\bar{Z}_1}{\bar{Z}_{1h}} = \frac{\bar{Z}_{1h} + R_1 + jX_{1\sigma}}{\bar{Z}_{1h}} = 1 + \frac{R_1 + jX_{1\sigma}}{\bar{Z}_{1h}}$$

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- Presuming rotor current $\bar{I}_2$' equal to zero, then the stator current is equal to the no-load current:

$$\bar{I}_0 = \frac{\bar{U}_1}{\bar{Z}_1}$$

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# Induction Machine Equivalent Circuit

- Now let us determine from the rotor equation the meaning of ratio $\bar{I}_2'/\bar{c}_1$...

$$\frac{\bar{I}_2'}{\bar{c}_1} = \frac{1}{\bar{c}_1} \cdot \frac{-\bar{U}_1 \bar{Z}_{1h}}{\bar{Z}_1 \bar{Z}_2' - \bar{Z}_{1h}^2}$$

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which corresponds with the rotor current in the Γ-shape equivalent circuit and the stator current is then

$$\bar{I}_1=\frac{\bar{U}_1}{\bar{Z}_1}-\frac{\bar{I}_2'}{\bar{c}_1}=\frac{\bar{U}_1}{\bar{Z}_1}+\frac{\bar{U}_1}{\bar{c}_1^2\left(\frac{R_2'}{s}+jX_{2\sigma}\right)+\bar{c}_1\left(R_1+jX_{1\sigma}\right)}$$

# Induction Machine Equivalent Circuit

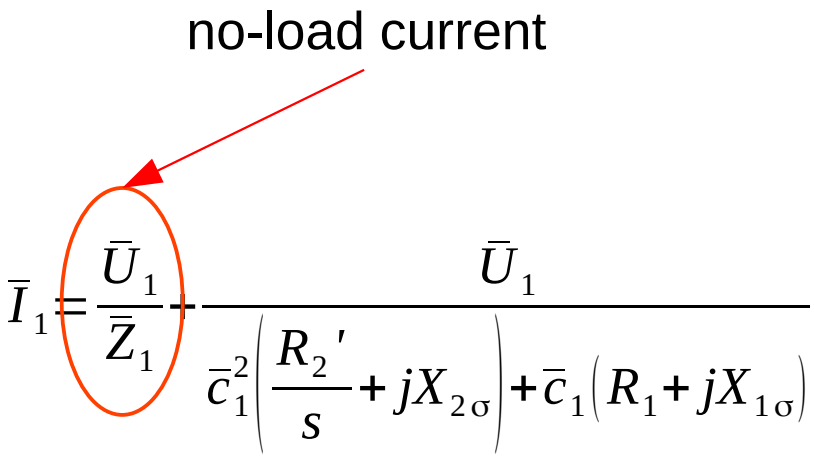
$$\bar{I}_1 = \frac{\bar{U}_1}{\bar{Z}_1} + \frac{\bar{U}_1}{\bar{c}_1^2\left(\frac{R_2'}{s} + jX_{2\sigma}\right) + \bar{c}_1\left(R_1 + jX_{1\sigma}\right)}$$

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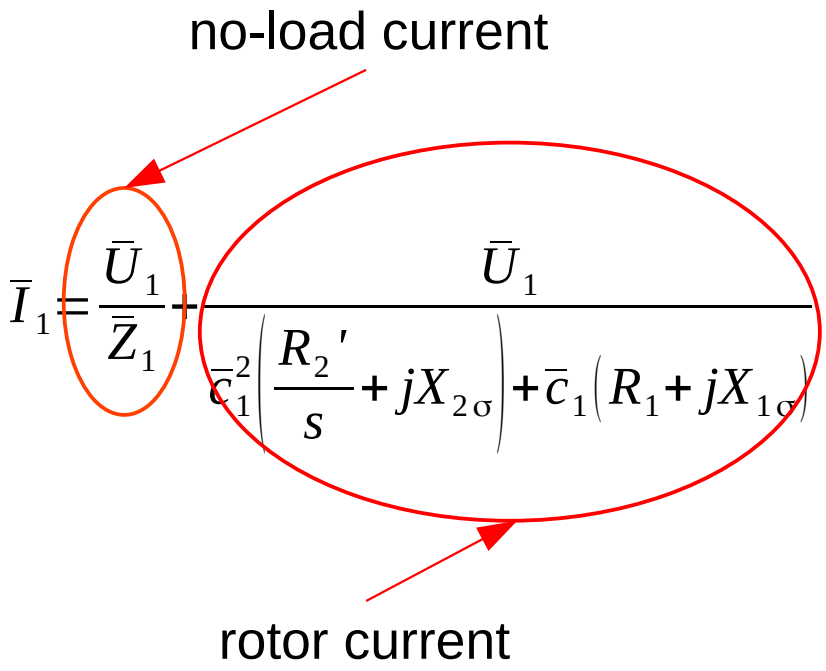
common terminal voltage

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# Induction Machine Equivalent Circuit



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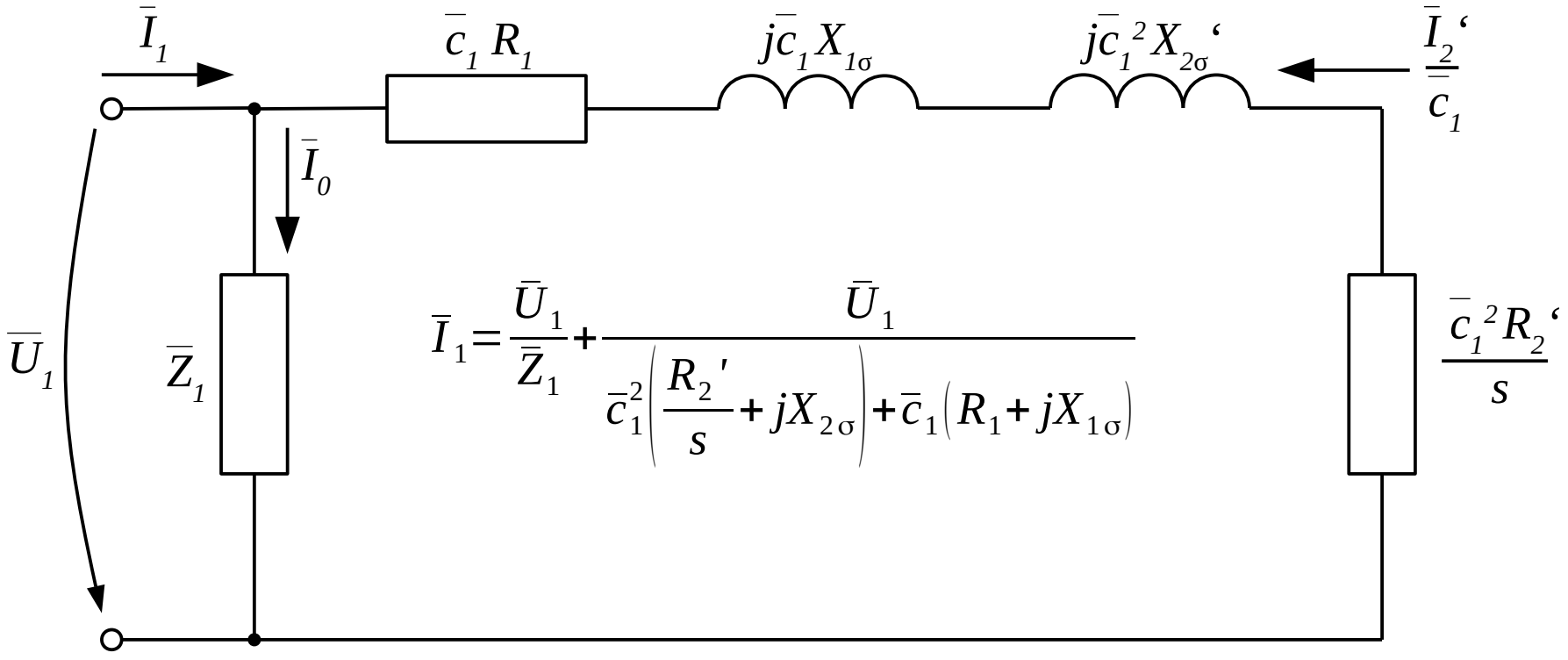


# Induction Machine Equivalent Circuit

$$\bar{I}_1 = \frac{\bar{U}_1}{\bar{Z}_1} + \underbrace{\frac{\bar{U}_1}{\bar{c}_1^2\left(\frac{R_2'}{s} + jX_{2\sigma}\right) + \bar{c}_1\left(R_1 + jX_{1\sigma}\right)}}$$

parallel combination

# Induction Machine Equivalent Circuit



# Induction Machine Equivalent Circuit

- The complex variable $\bar{c}_1$ may be expressed as

$$\bar{c}_1 = |\bar{c}_1| e^{j\delta}$$

  where $|\bar{c}_1|$ is close to one a $\delta$ is a small negative angle

- The impedance of horizontal branch may be then written as

$$\bar{Z} = \bar{c}_1 R_1 + \bar{c}_1 jX_{1\sigma} + \bar{c}_1^2 jX_{2\sigma}' + \bar{c}_1^2 \frac{R_2'}{s}$$

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… which is, in fact, a function of slip in a complex plane, i.e.

$$\bar{Z}(s) = \bar{c}_1 R_1 + \bar{c}_1 jX_{1\sigma} + \bar{c}_1^2 jX_{2\sigma}' + \bar{c}_1^2 \frac{R_2'}{s}$$

# Induction Machine Equivalent Circuit

slip independent
(complex constant)

$$\bar{Z}(s) = \bar{c}_1 R_1 + \bar{c}_1 jX_{1\sigma} + \bar{c}_1^2 jX_{2\sigma}' + \bar{c}_1^2 \frac{R_2'}{s}$$

# Induction Machine Equivalent Circuit

slip dependent
(complex variable)

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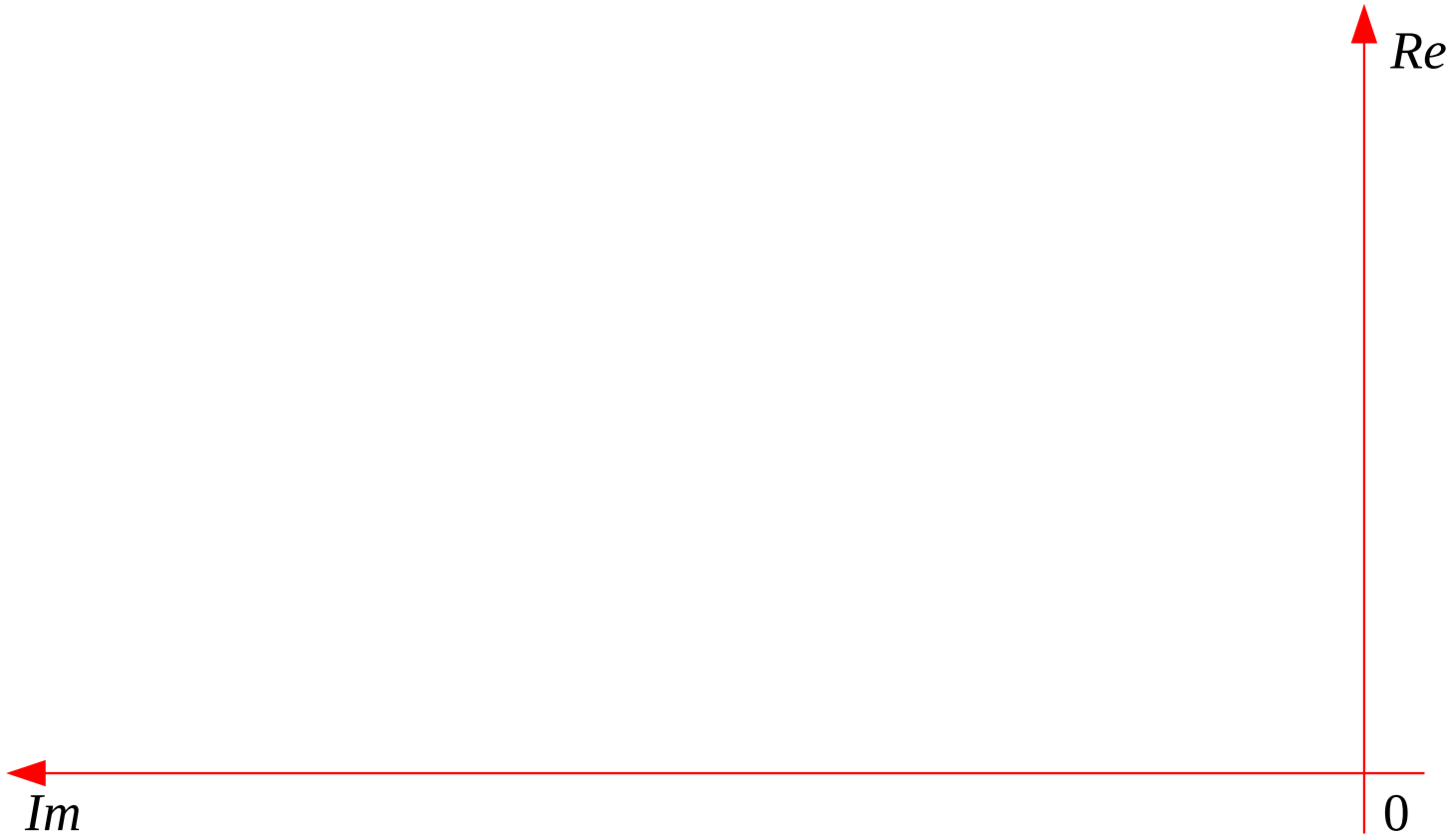
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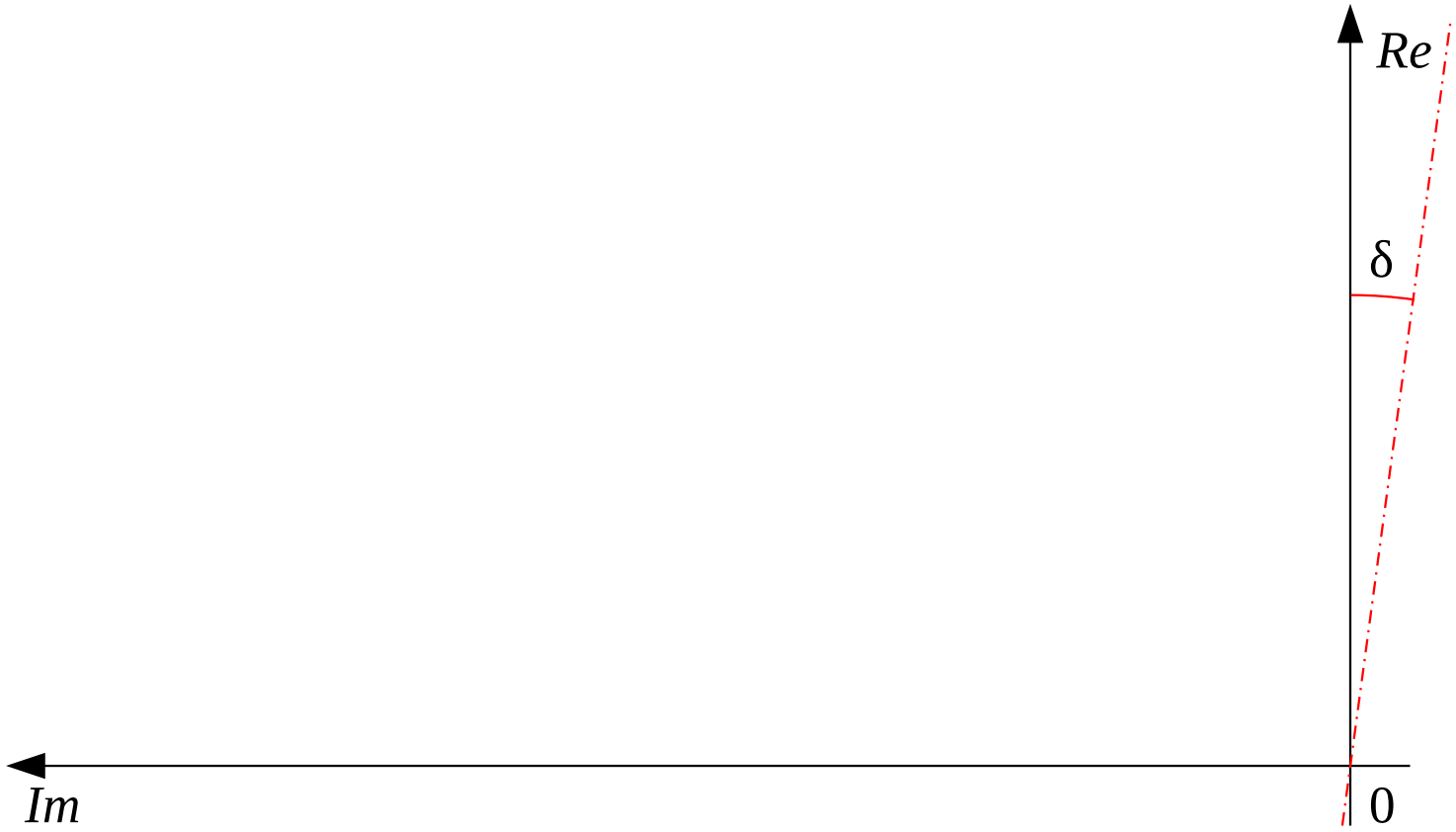
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… and the result will be a straight line in complex plane ...

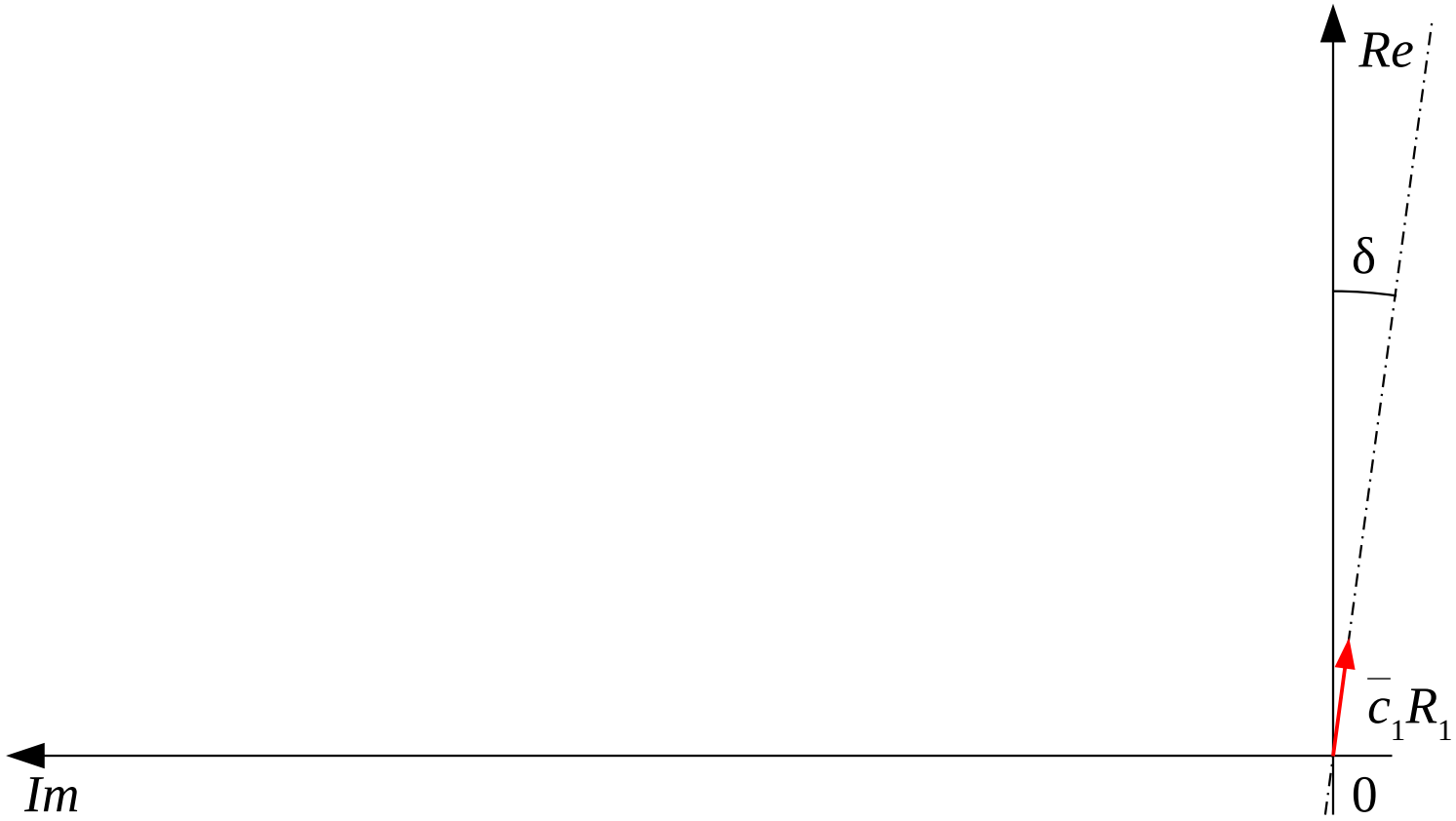
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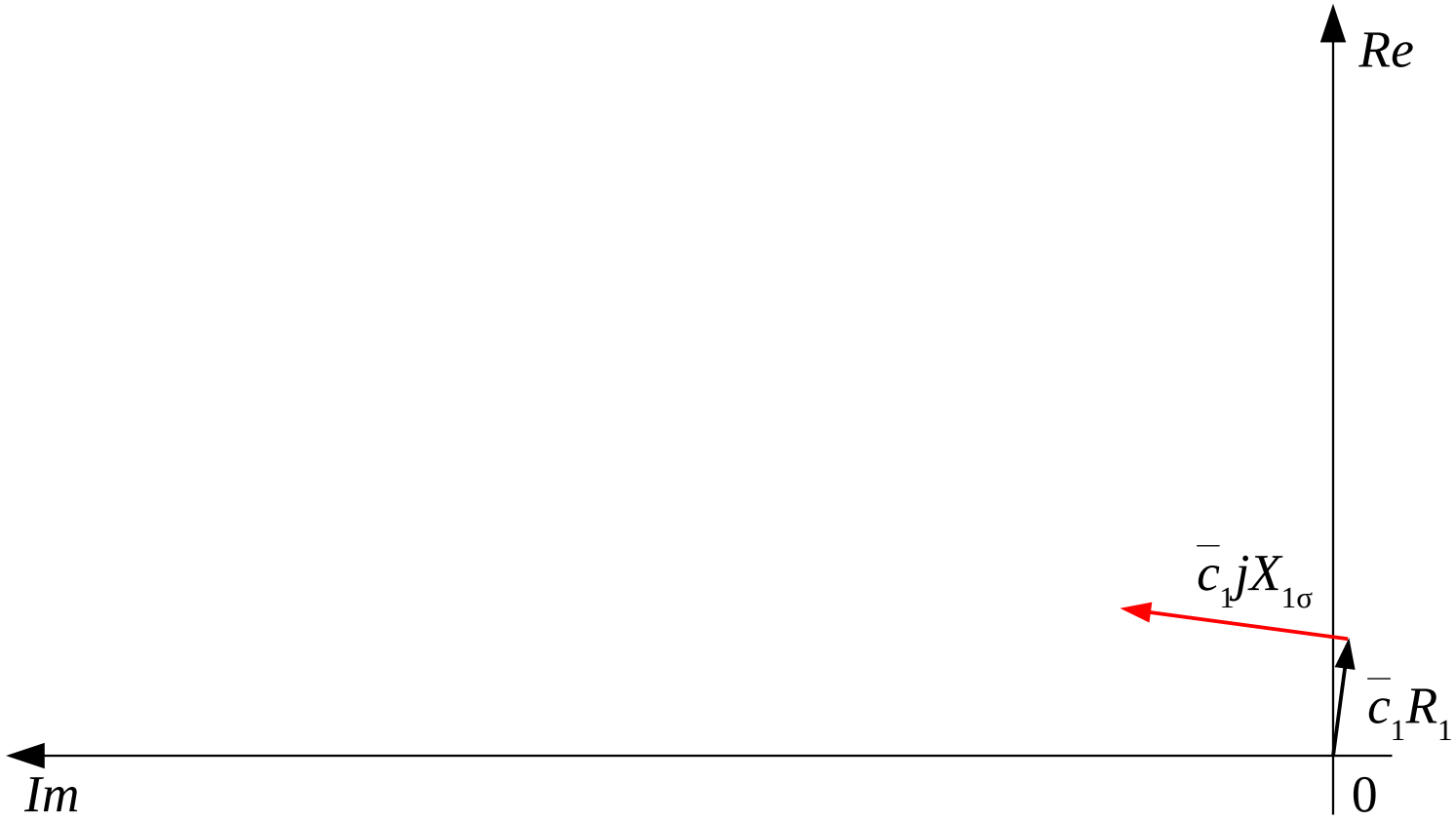
# Induction Machine Equivalent Circuit



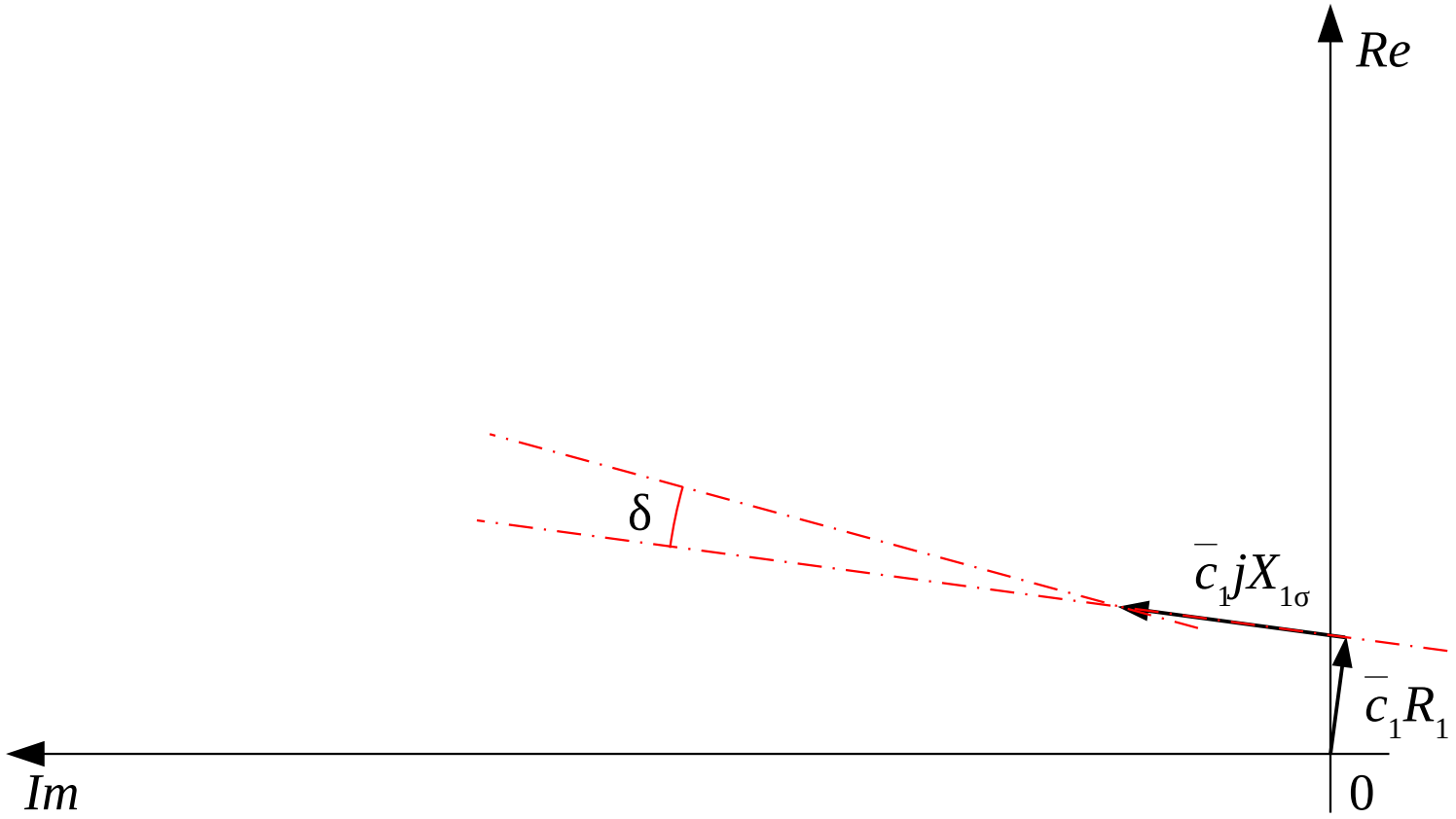
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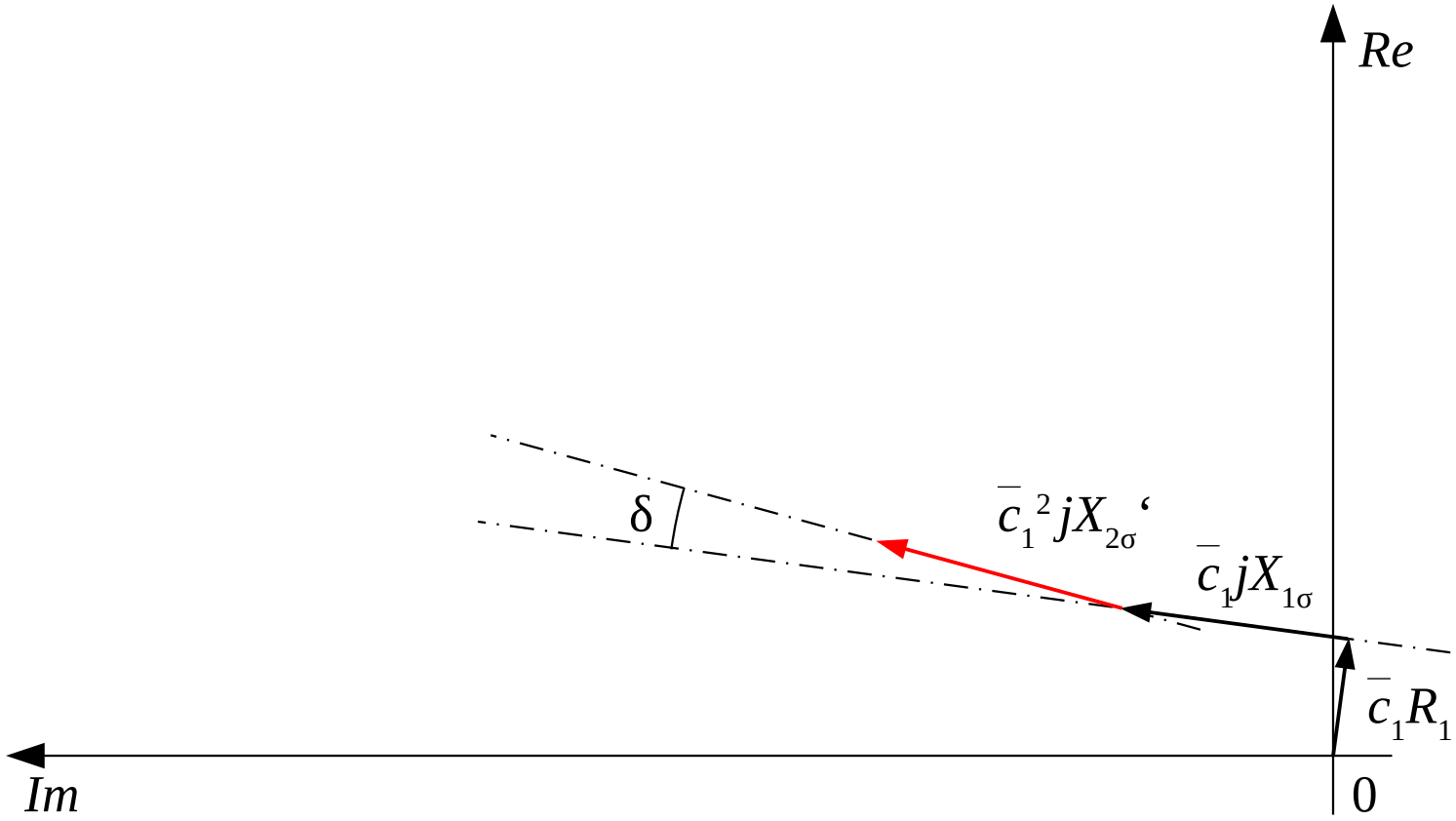
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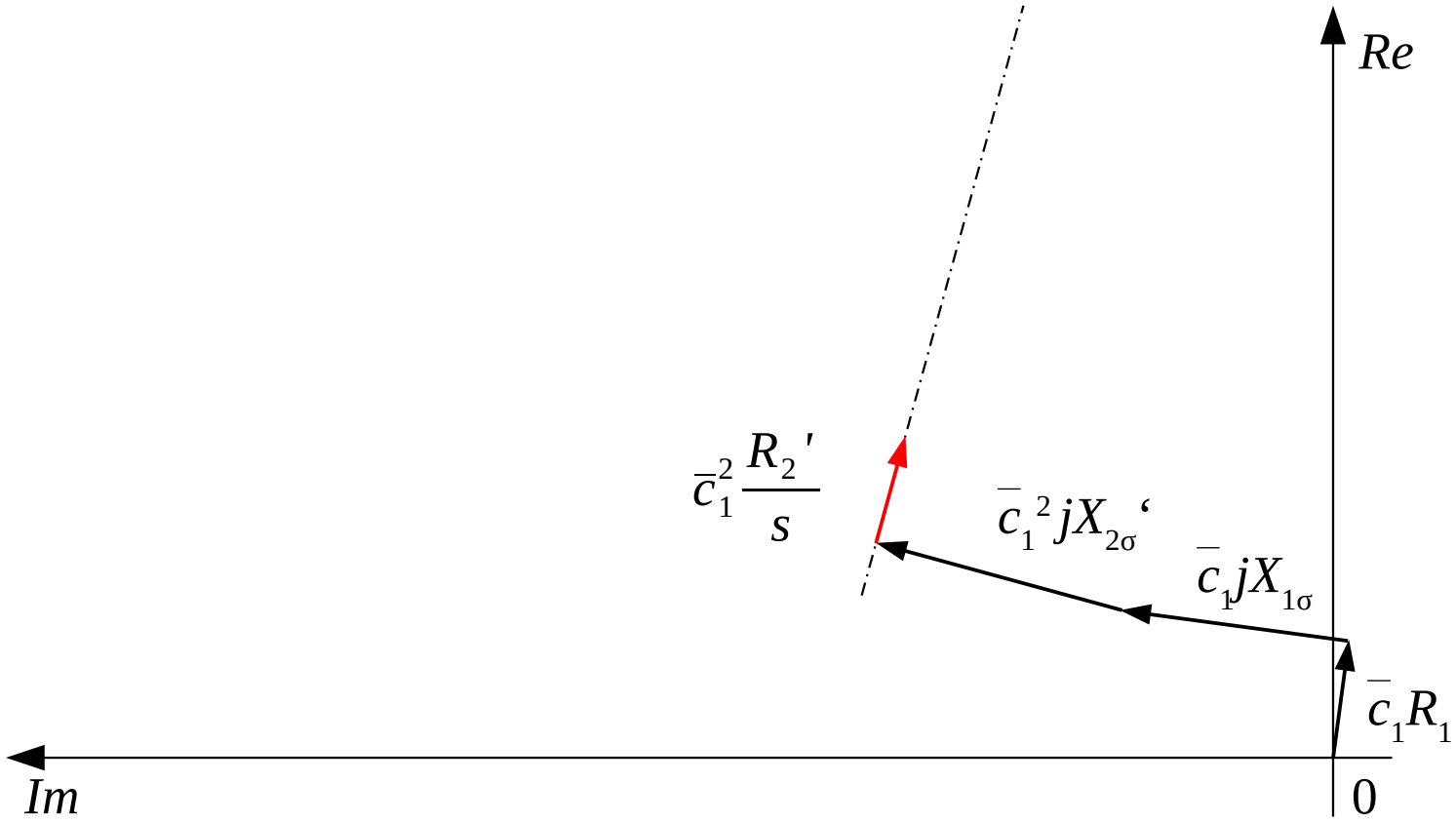
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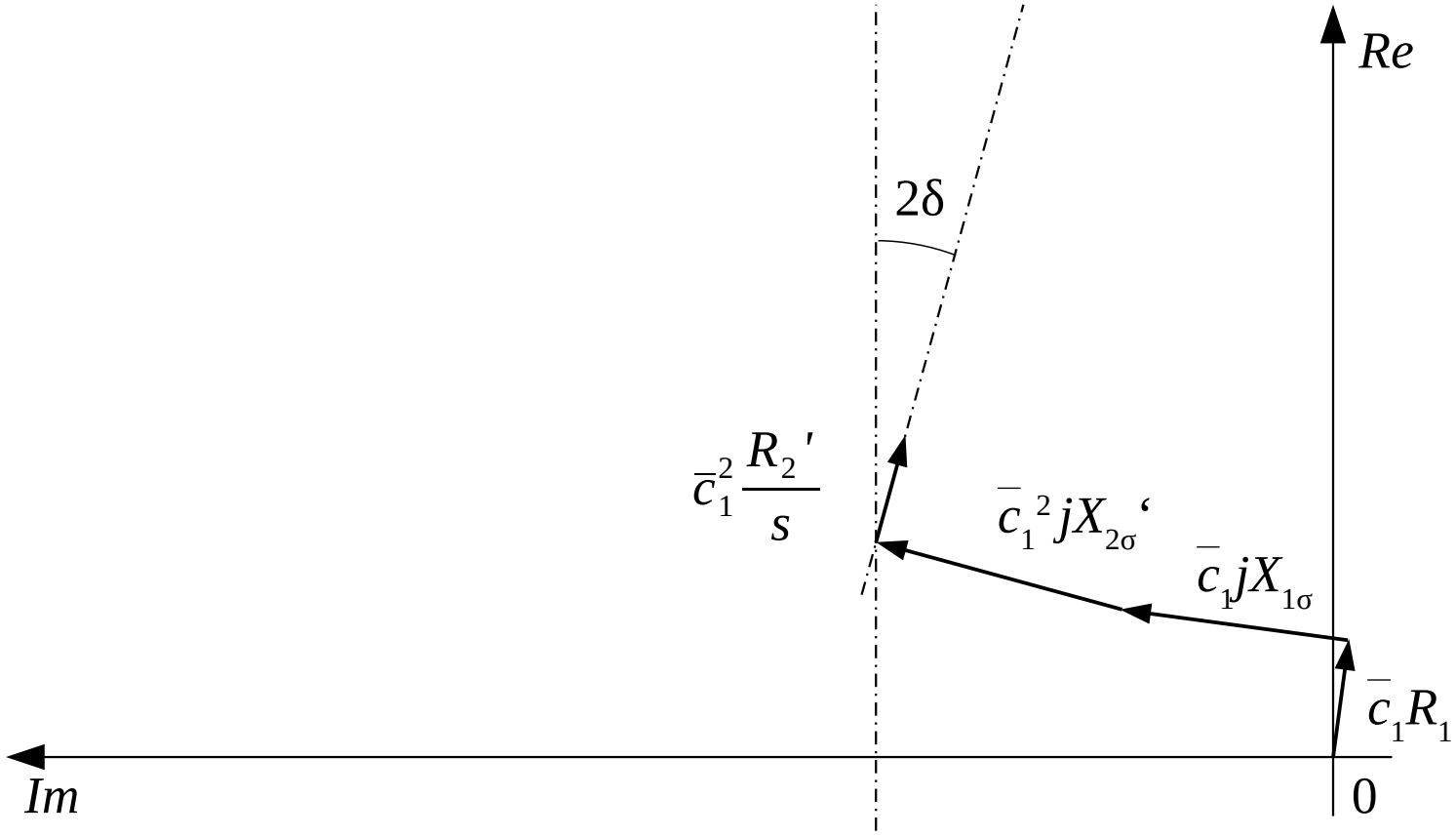
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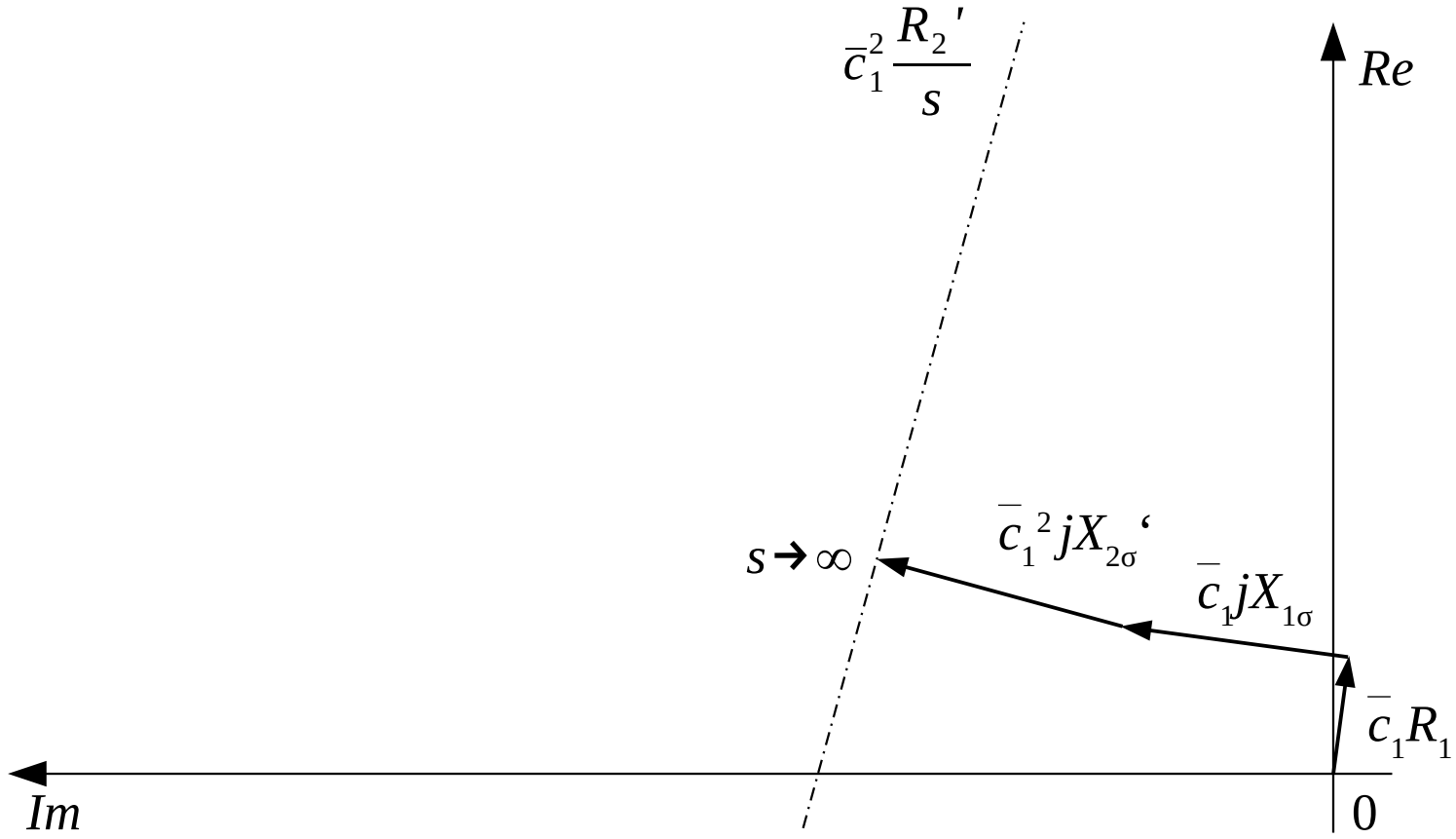
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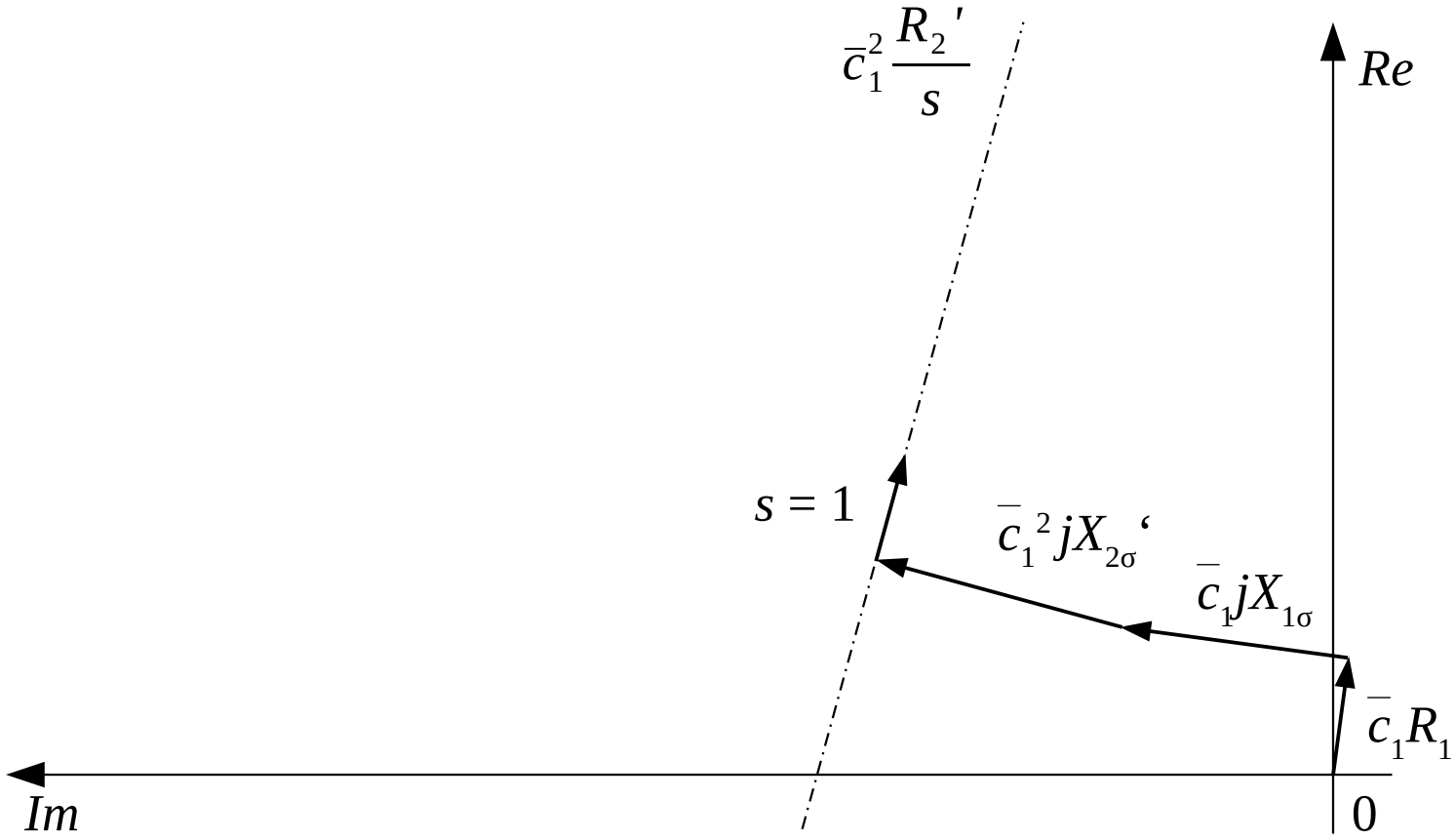
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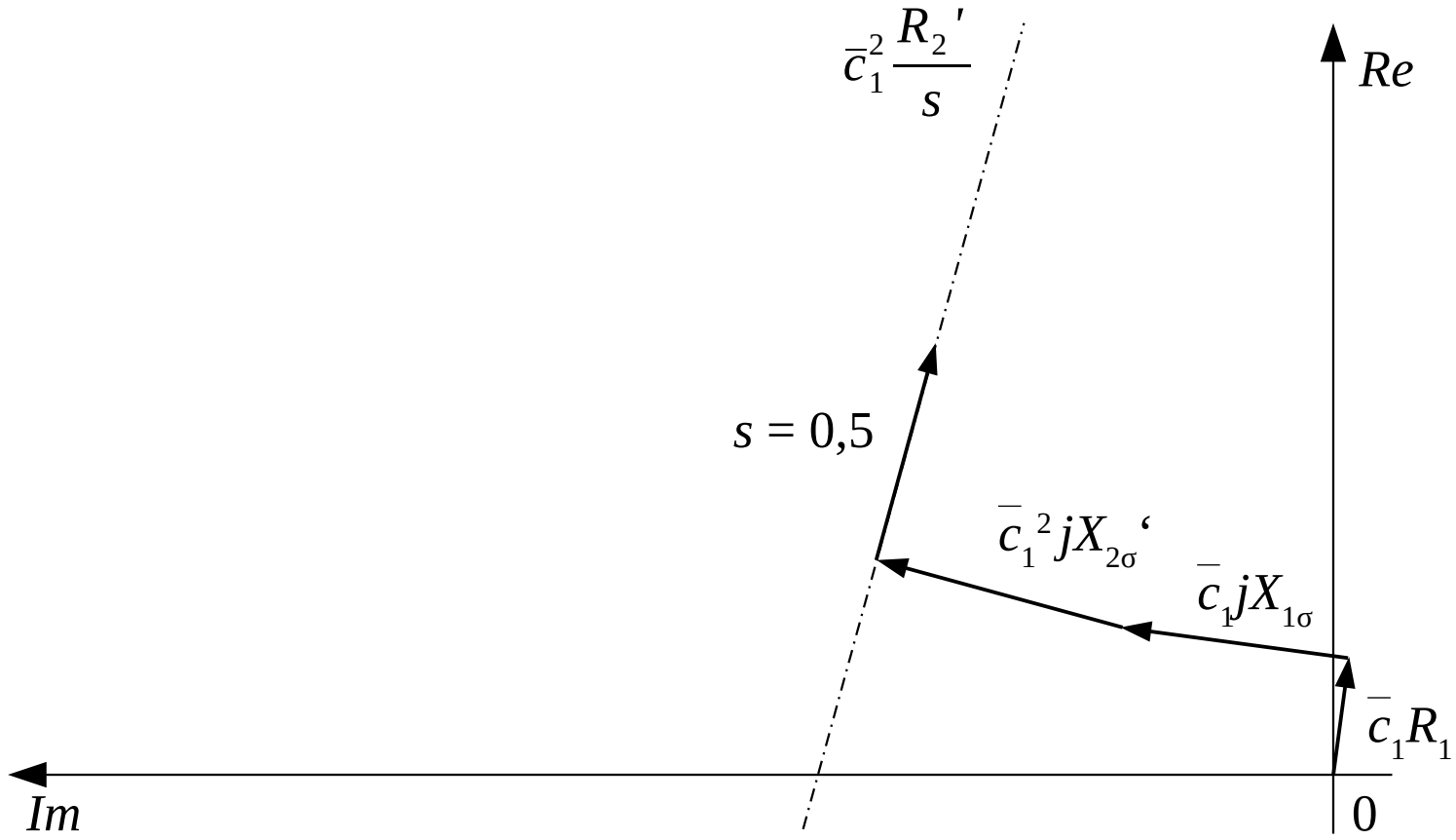
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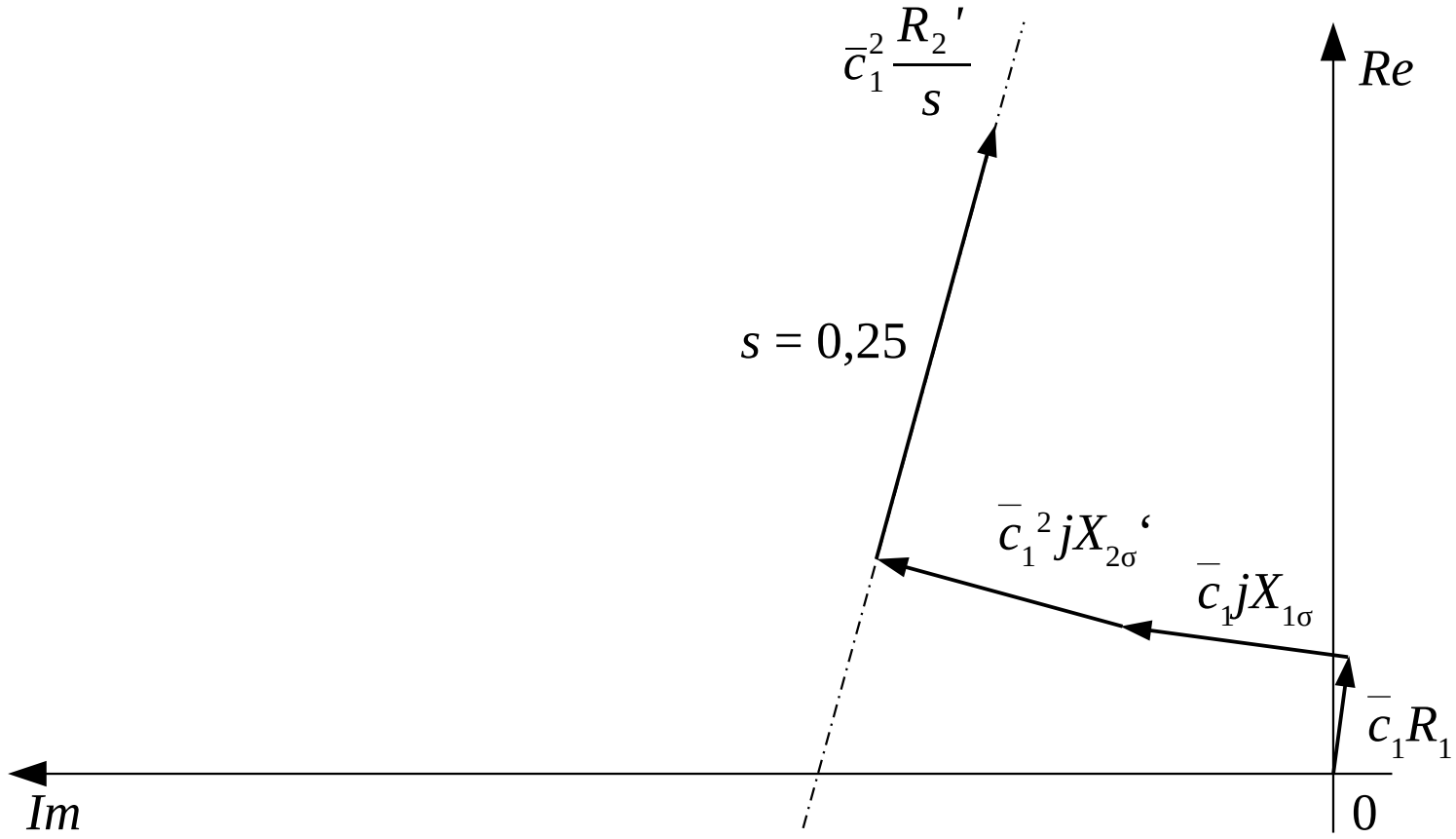
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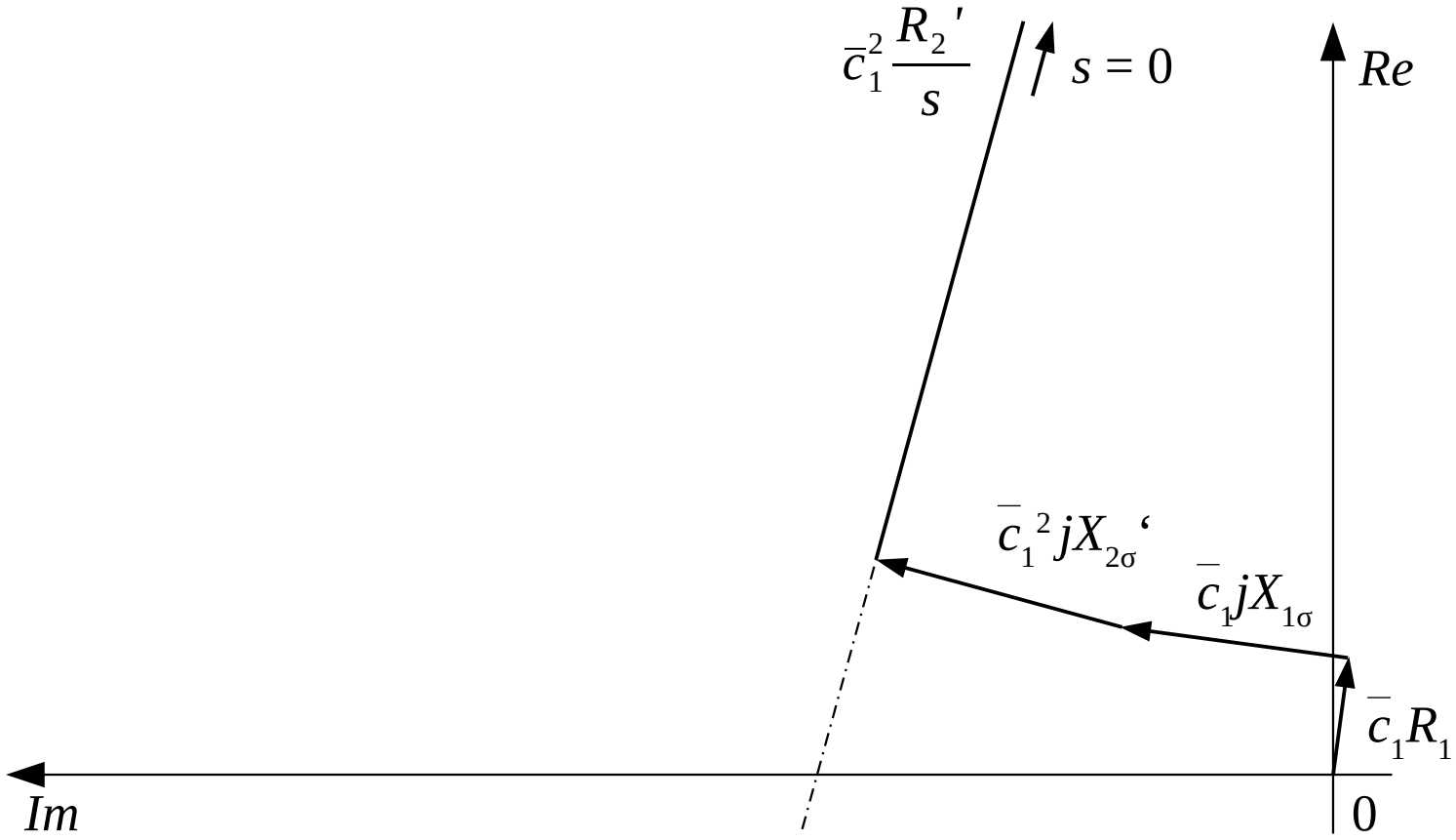
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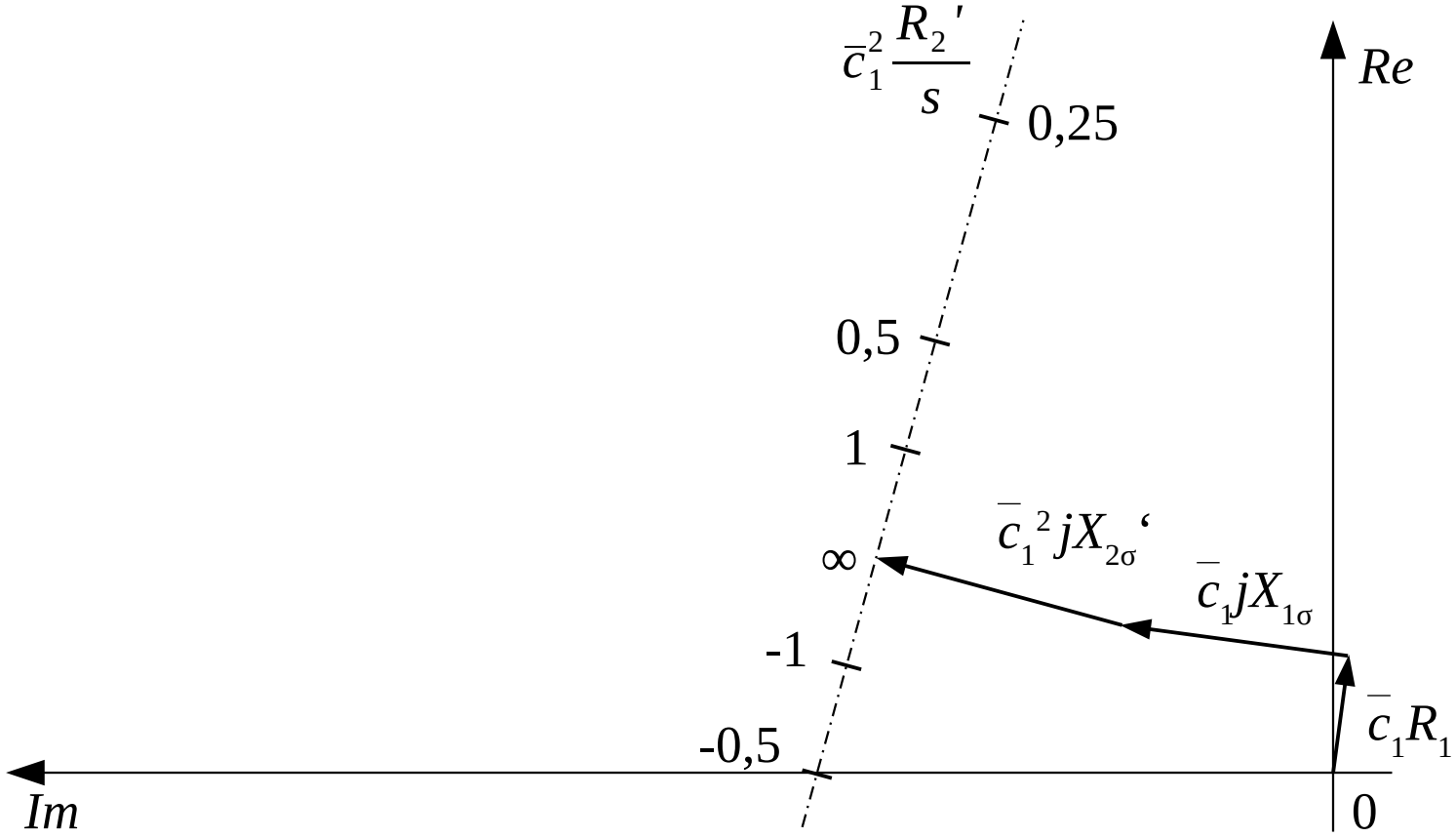
# Induction Machine Equivalent Circuit



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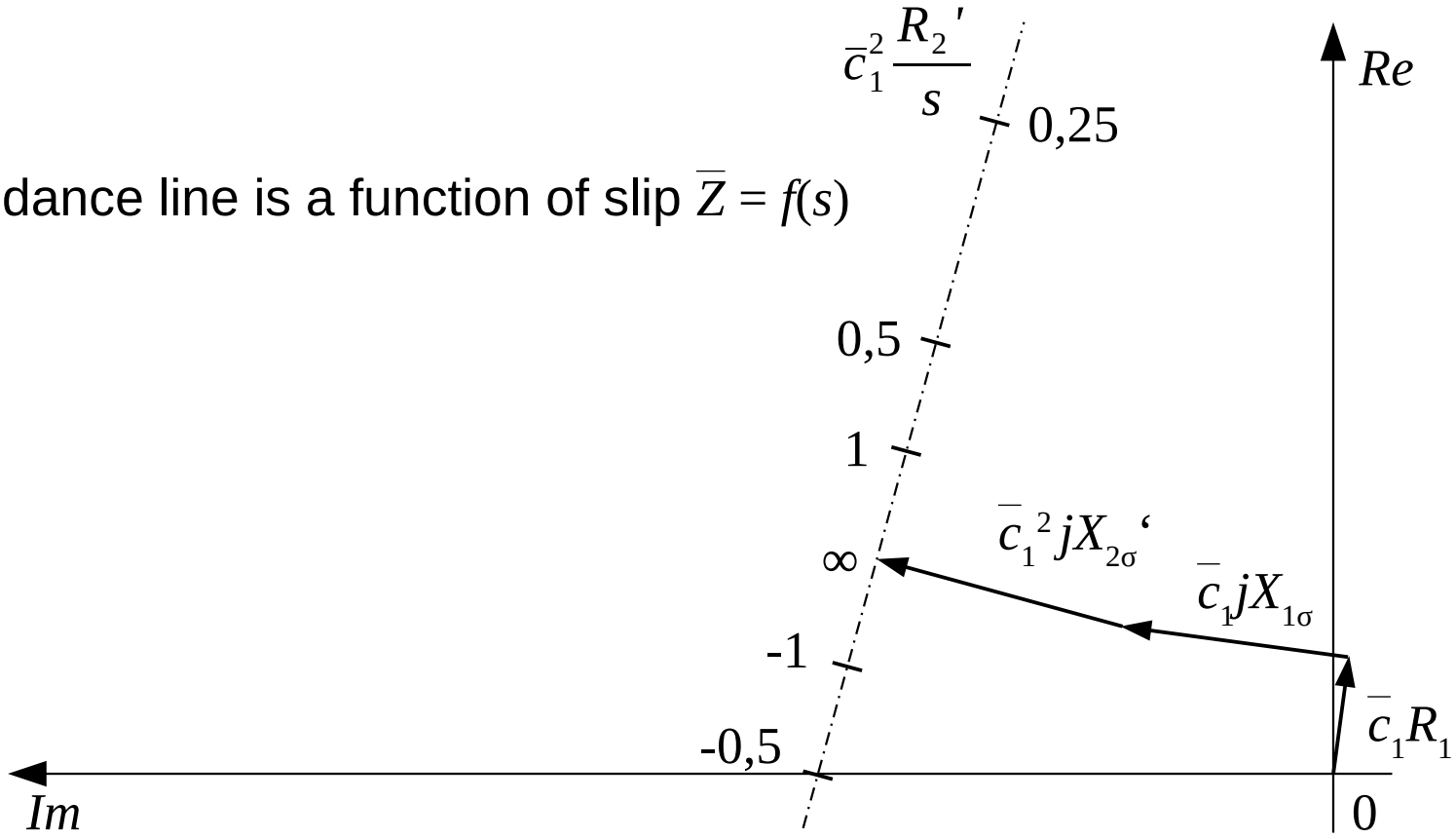


# Induction Machine Equivalent Circuit



# Induction Machine Equivalent Circuit

Impedance line is a function of slip $\underline{Z} = f(s)$



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MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

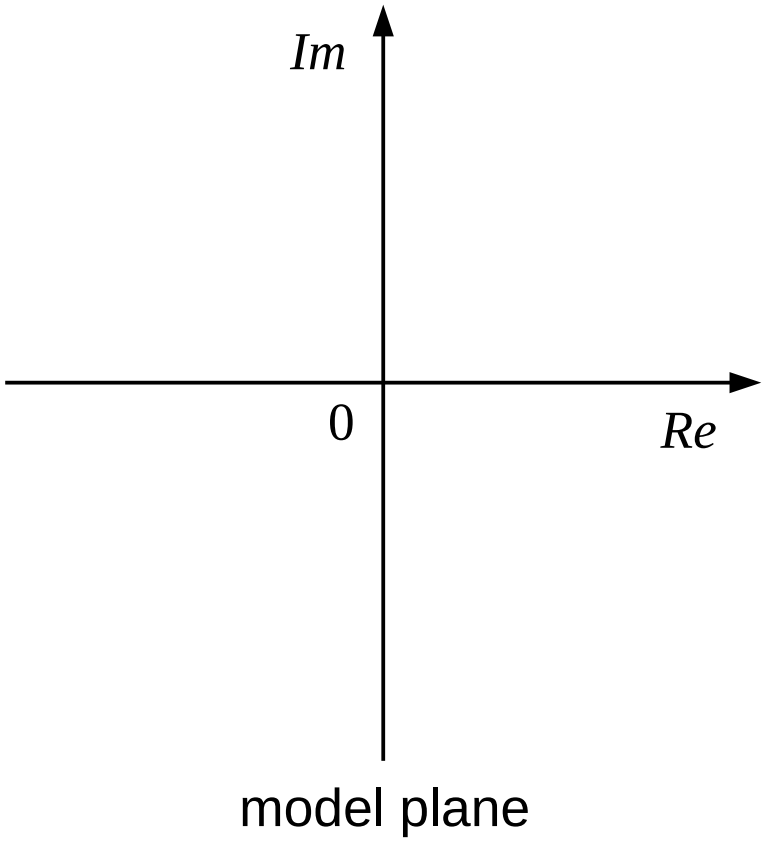
ESF II – project of the University of West Bohemia
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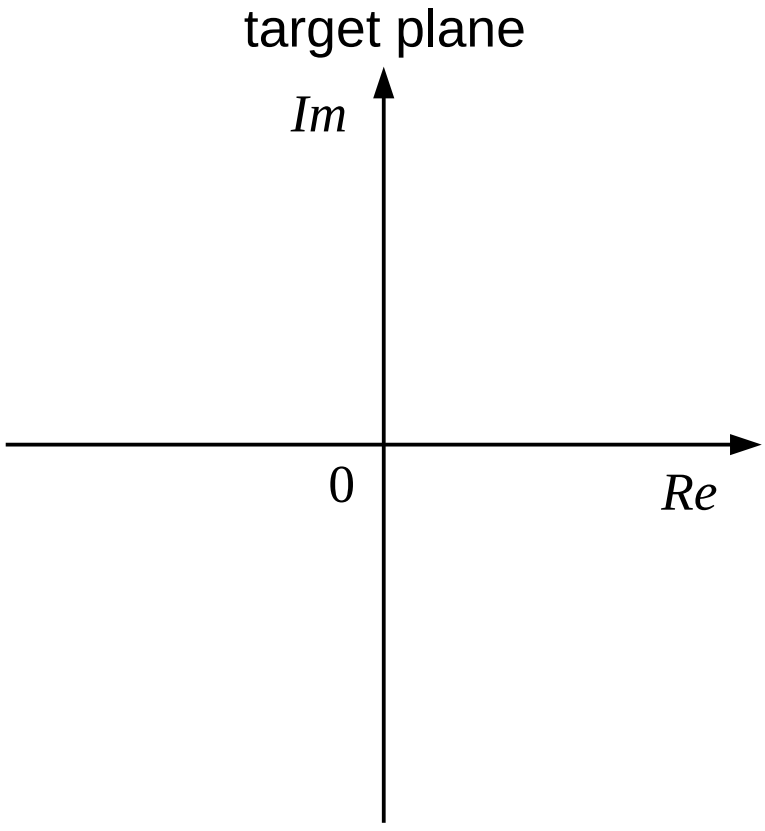
# Inversive Geometry

# Inversive Geometry

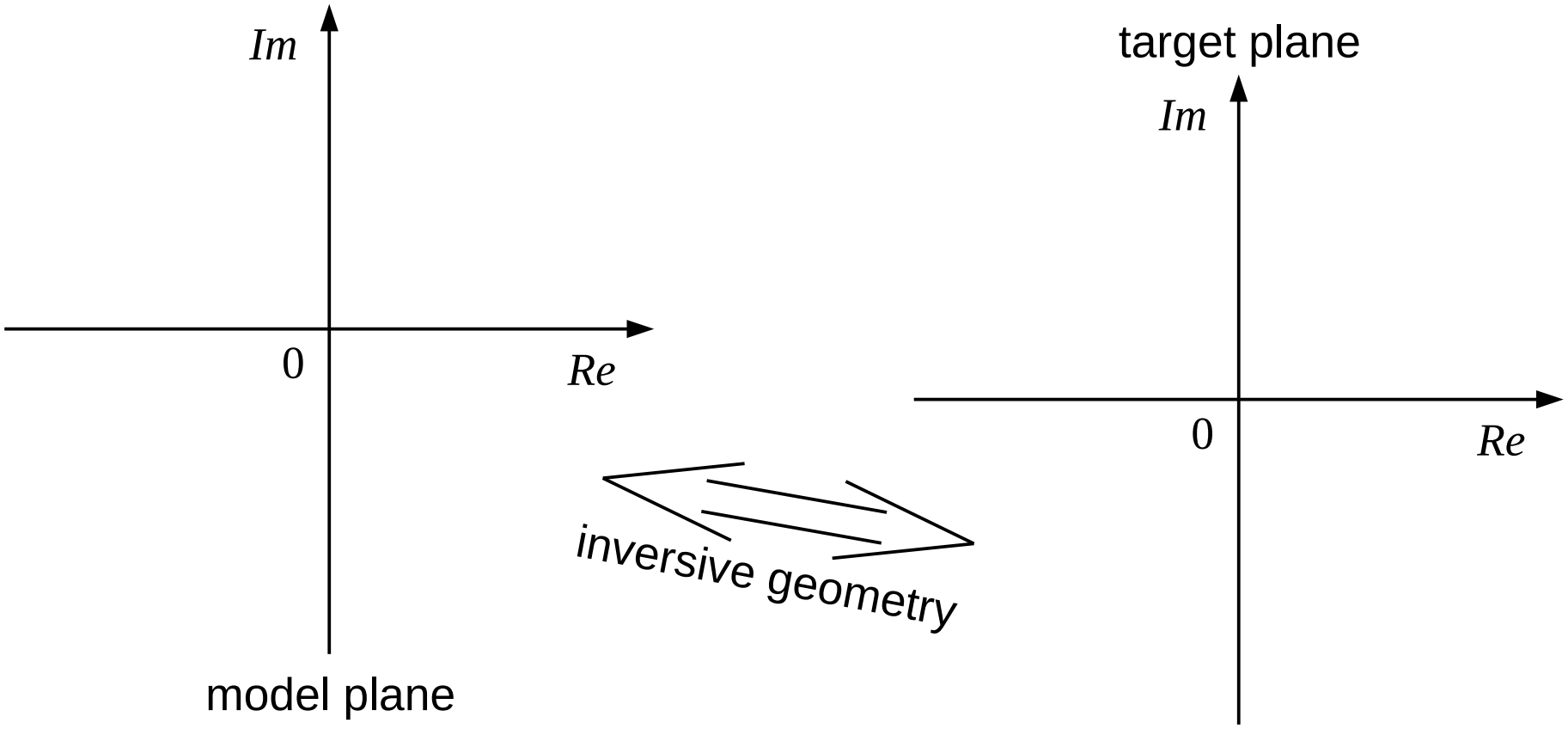
- For the purpose of projection we use an inversive geometry in complex plane
- This projection expresses a relation between model complex plane and target complex plane
- For the purpose of projection of point $X$ from the model plane to point $X'$ in the target plane the following rules apply:
  - The angular position of the point $X$ is identical
  - The distance of the point $X$ from the zero point in the model plane is $|0X|$ while in target plane it changes to $1/|0X|$

# Inversive Geometry

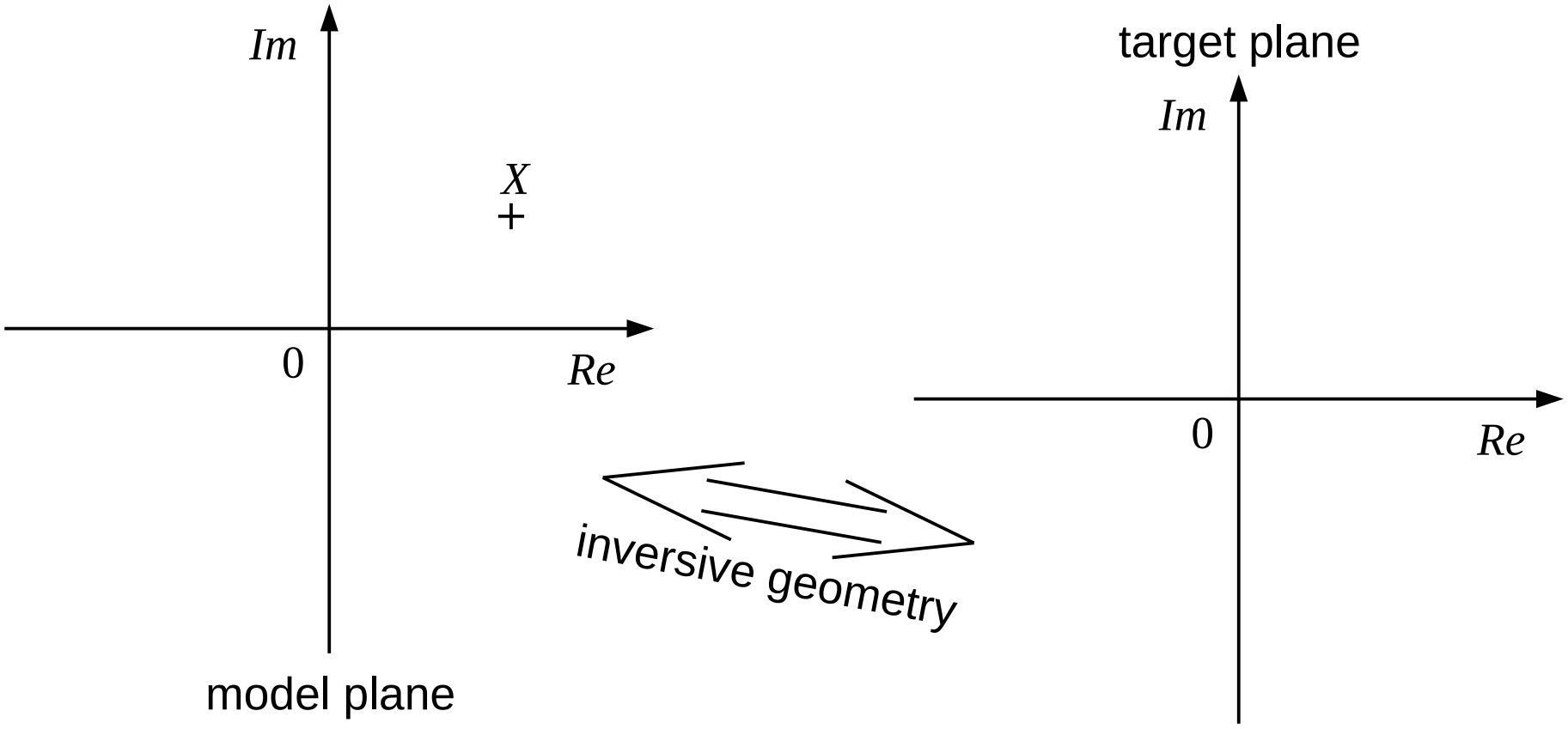


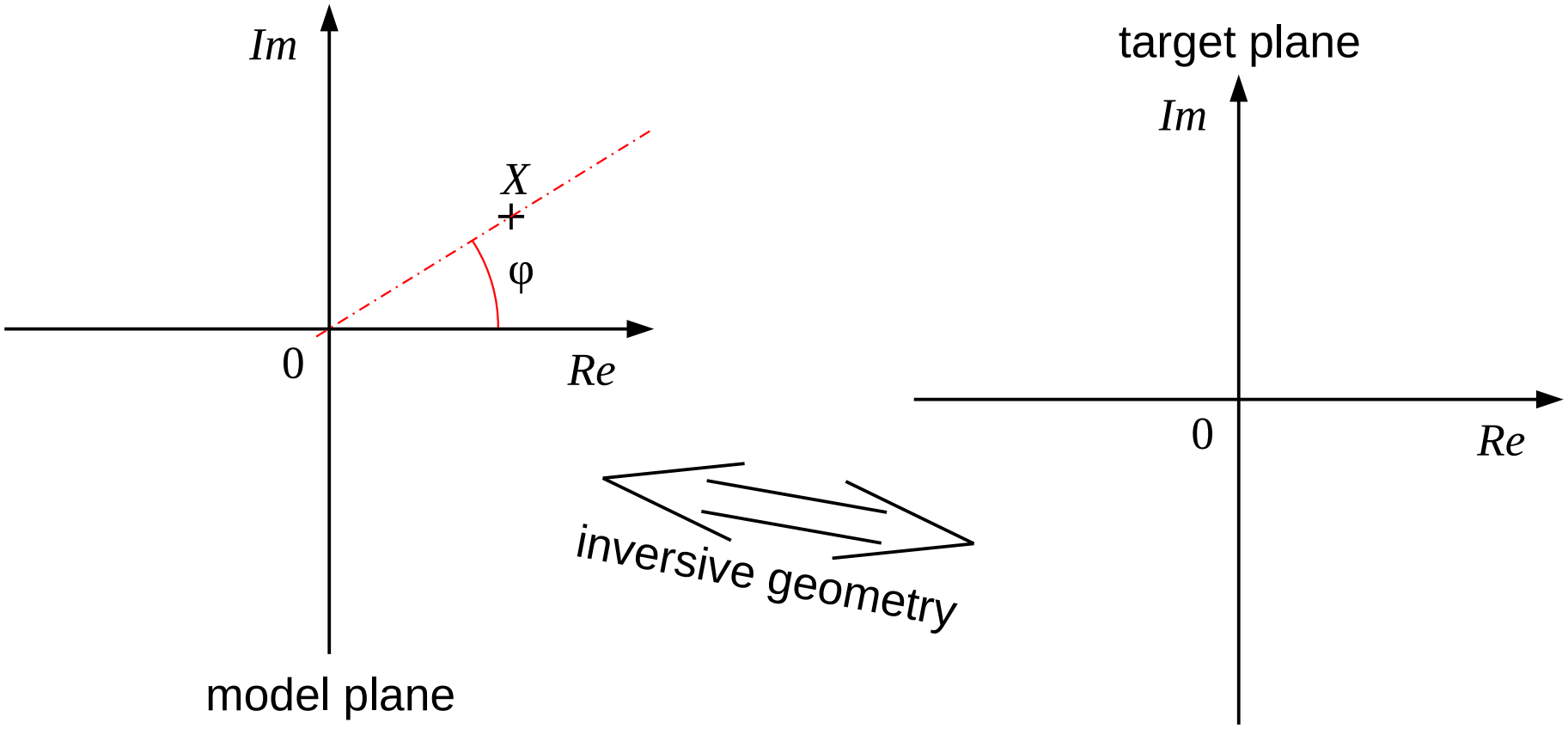
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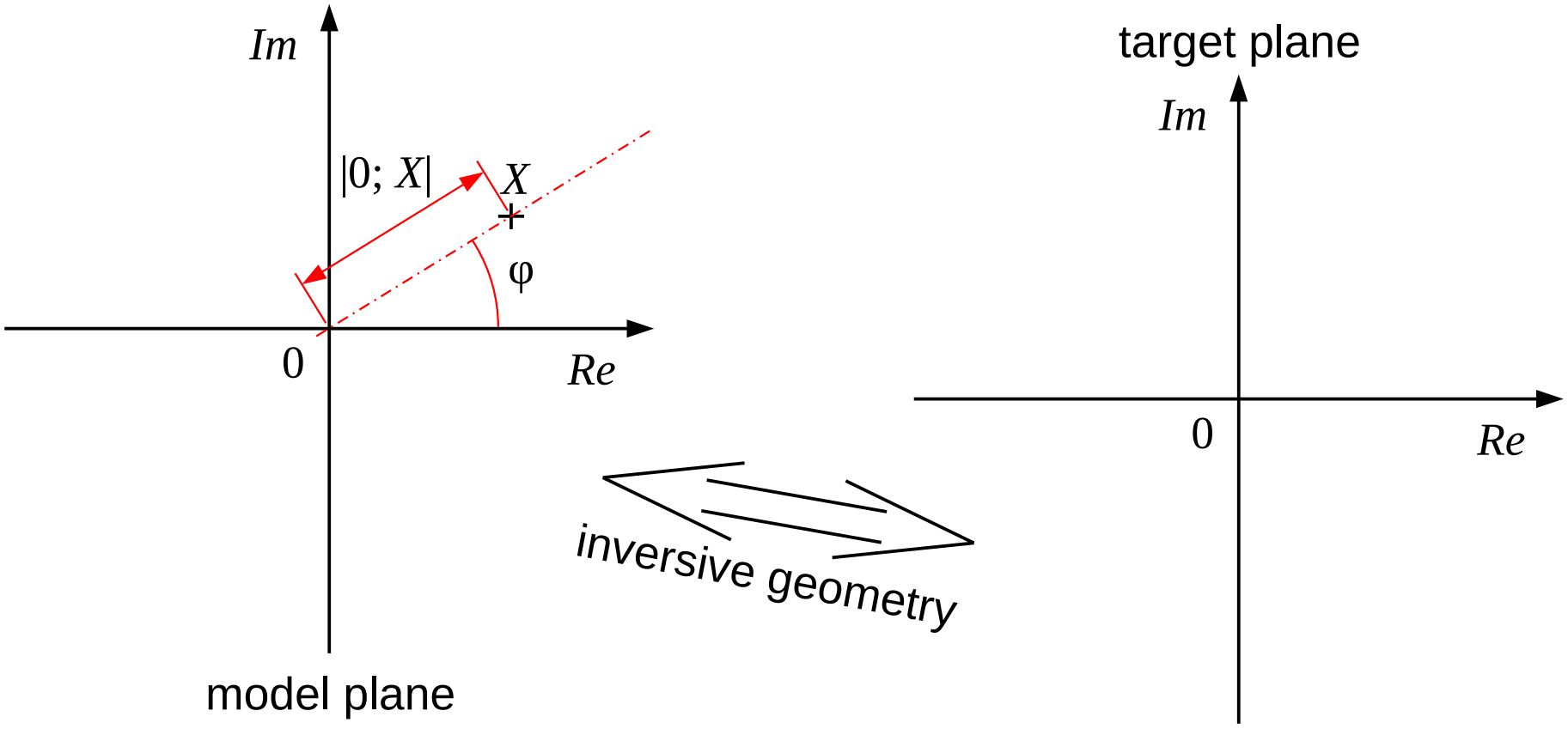
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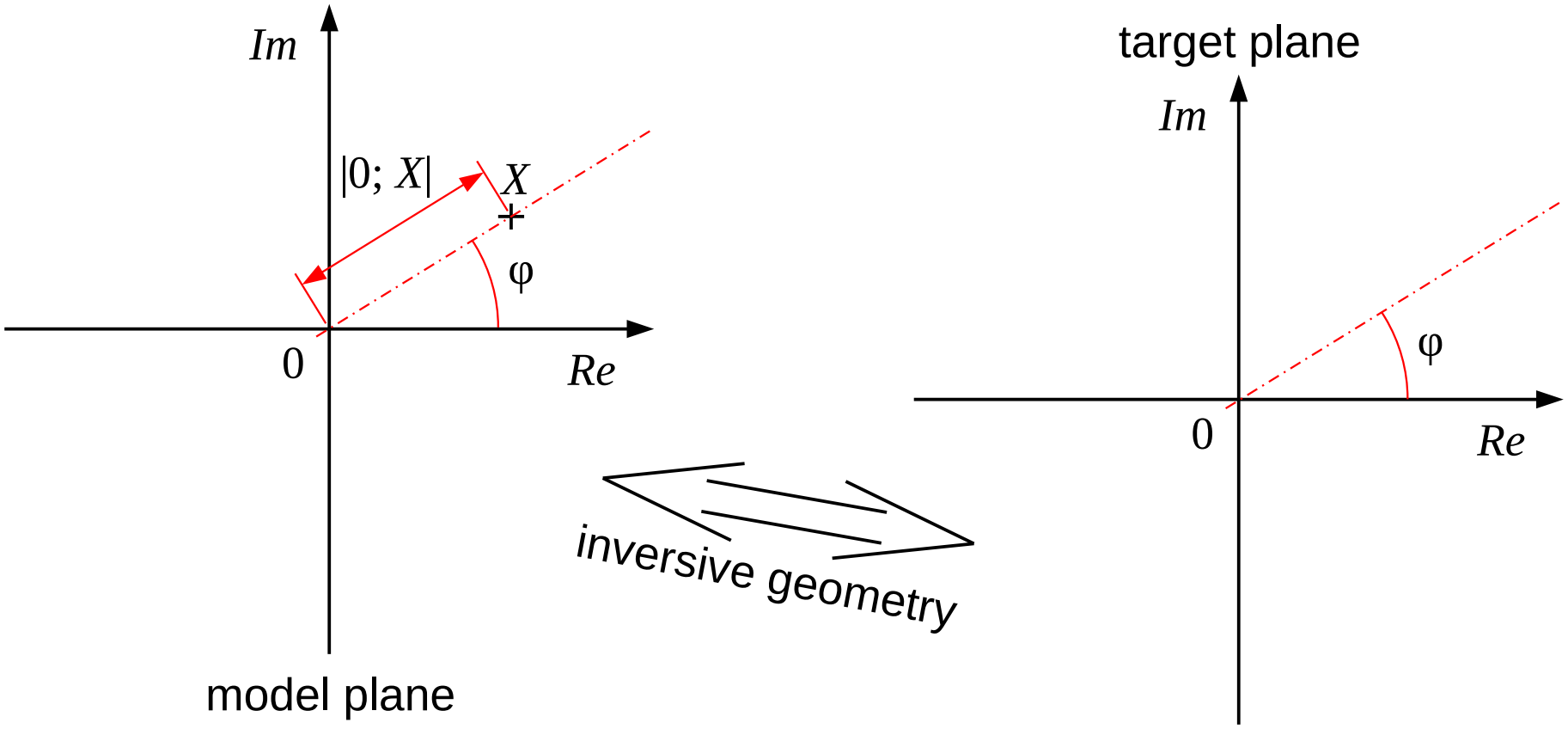
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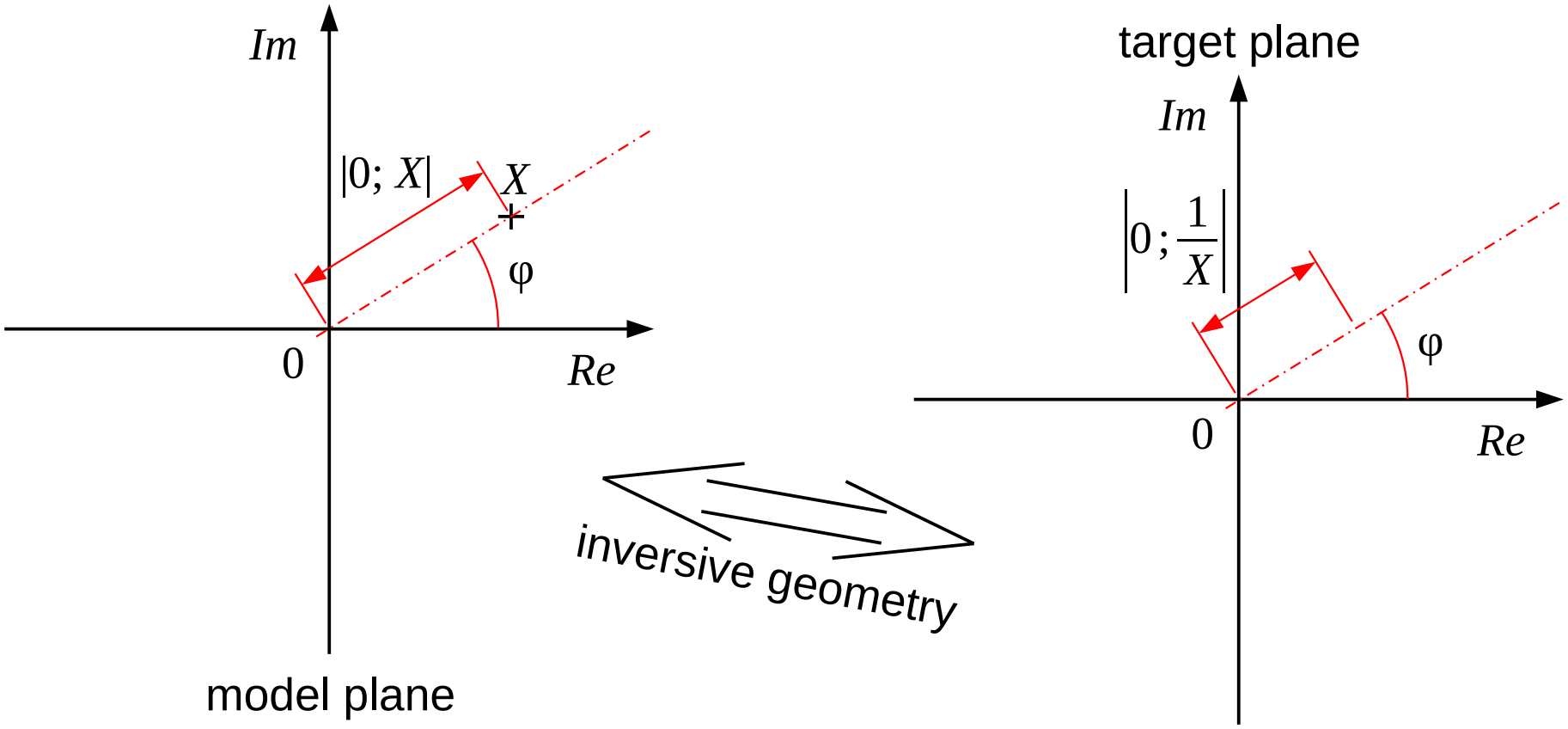
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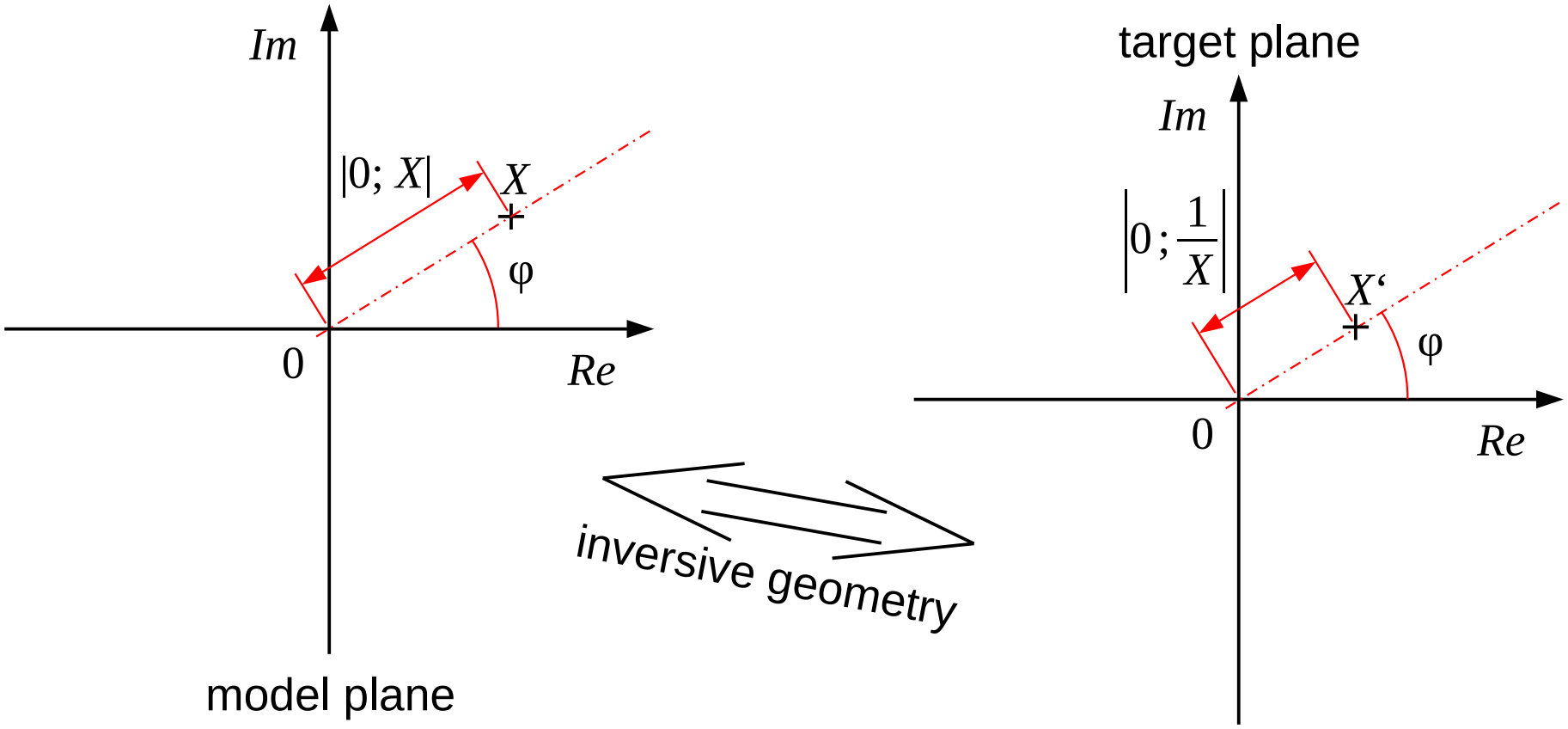
# Inversive Geometry



# Inversive Geometry



# Inversive Geometry



# Inversive Geometry

- The transformation of different typed of curces leads to thrier deformation
  - A line through zero point id projected as a line with reverse orientation
  - A line not going through zero point will project as a circle...

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# Circle Diagram

# Circle Diagram

- A circle diagram of an induction machine is a projection of impedance line using inversive geometry
- By projection of impedance line through inversive geometry we obtain admitance circle:

$$\bar{Z}(s) \Rightarrow \frac{1}{\bar{Z}(s)} = \bar{Y}(s)$$

- The current through the stator of an induction machine is according to the equivalent circuit in Γ – shape:

$$\bar{I}_2 = \frac{\bar{U}_1}{\bar{Z}} = \bar{U}_1 \bar{Y}$$

# Circle Diagram

- Considering the fact, that the horizontal branch of the equivalent circuit is slip dependent, the relation

$$\bar{I}_2 = \frac{\bar{U}_1}{\bar{Z}} = \bar{U}_1 \bar{Y}$$

verges to:

$$\bar{I}_2(s) = \frac{\bar{U}_1}{\bar{Z}(s)} = \bar{U}_1 \bar{Y}(s)$$

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- Considering projection of the voltage into real axis ($\bar{U}_1$ has no imaginary component) we have:

$$\bar{I}_2(s) = \frac{U_1}{\bar{Z}(s)} = U_1 \bar{Y}(s)$$

# Circle Diagram

- The resulting form is similar to the result of inversive geometry:

$$\bar{I}_2(s) = \frac{U_1}{\bar{Z}(s)} = U_1 \bar{Y}(s) \qquad \bar{Z}(s) = \frac{1}{\bar{Z}(s)} = \bar{Y}(s)$$

# Circle Diagram

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$$\bar{I}_2(s) = \frac{U_1}{\bar{Z}(s)} = U_1 \bar{Y}(s) \qquad \bar{Z}(s) = \frac{1}{\bar{Z}(s)} = \bar{Y}(s)$$

- The only difference is the multiplication of the result of the inversive geometry by real constant $U_1$

  => scale of current $m_i$

$$m_i = U_1 m_y \qquad [A/mm]$$

# Circle Diagram

- The fundamental scale for construction of the circle diagram using inversive geometry is the scale of admittance $m_y\ [S/mm]$
- The result of the inversive geometry of the impedance line is, after multiplication by stator voltage, a circle of rotor currents
- The circle of stator currents may be obtained by addition of current $\bar{I}_0$ to the resulting circle

  => results in movement of the coordinate system by the size and direction of the vector $\bar{I}_0$

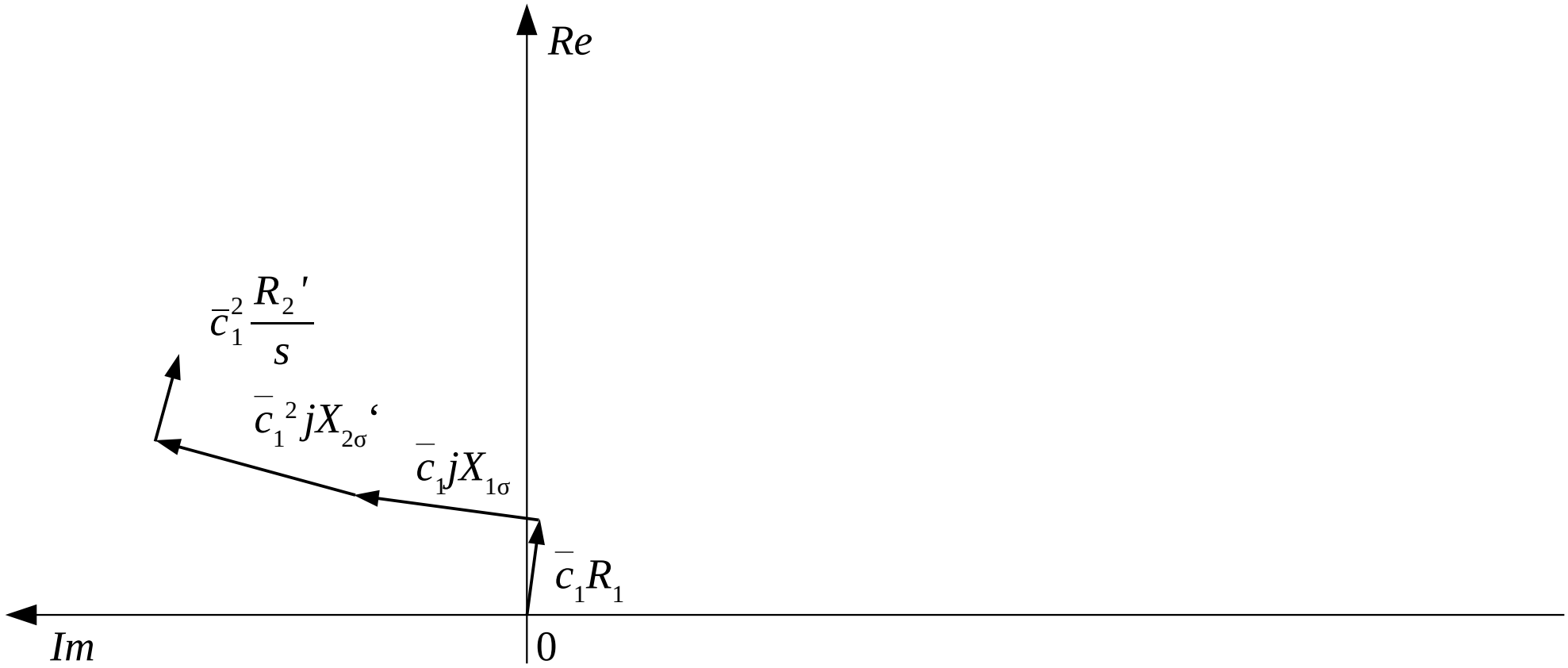
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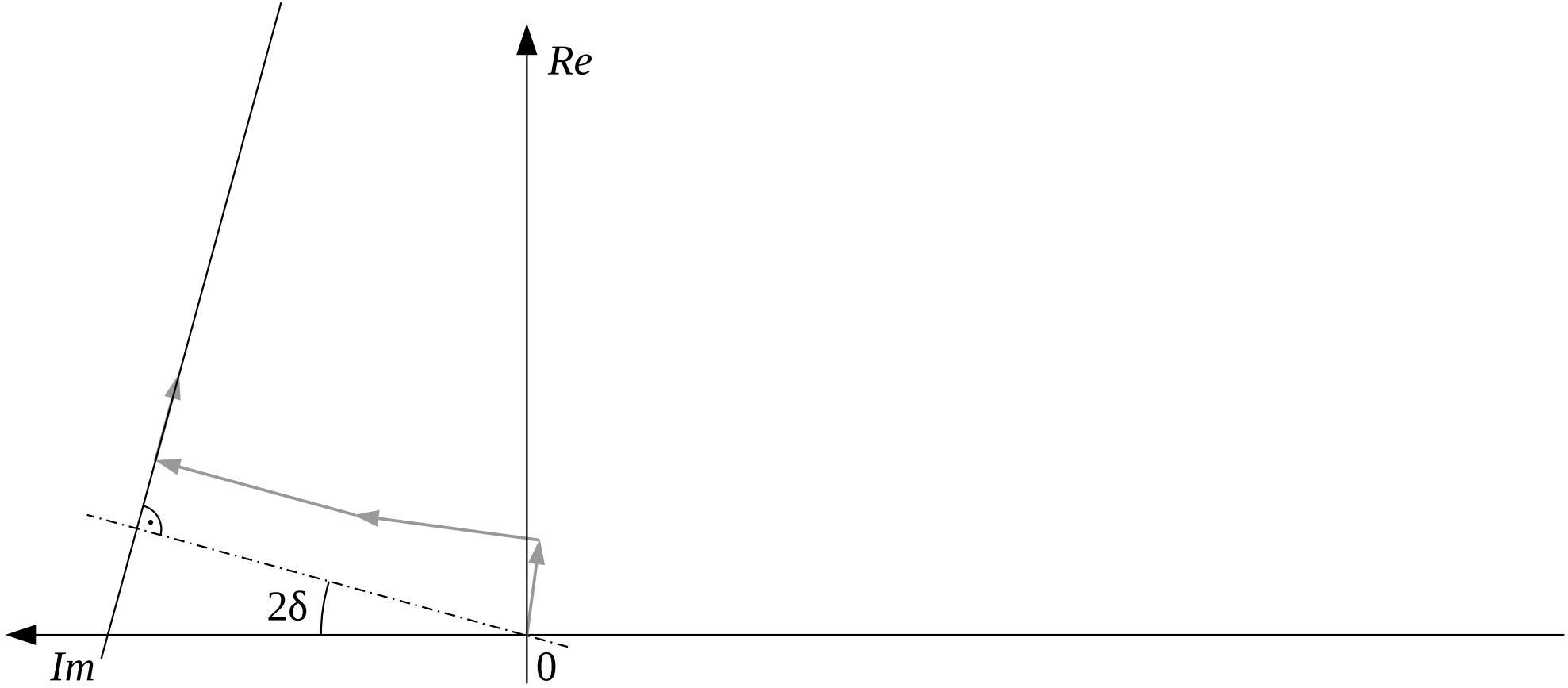
  => results in movement of the coordinate system by the size and direction of the vector $\bar{I}_0$

$$\bar{I}_1 = \frac{\bar{U}_1}{\bar{Z}_1} + \frac{\bar{U}_1}{\bar{c}_1^2\left(\frac{R_2'}{s} + jX_{2\sigma}\right) + \bar{c}_1\left(R_1 + jX_{1\sigma}\right)}$$

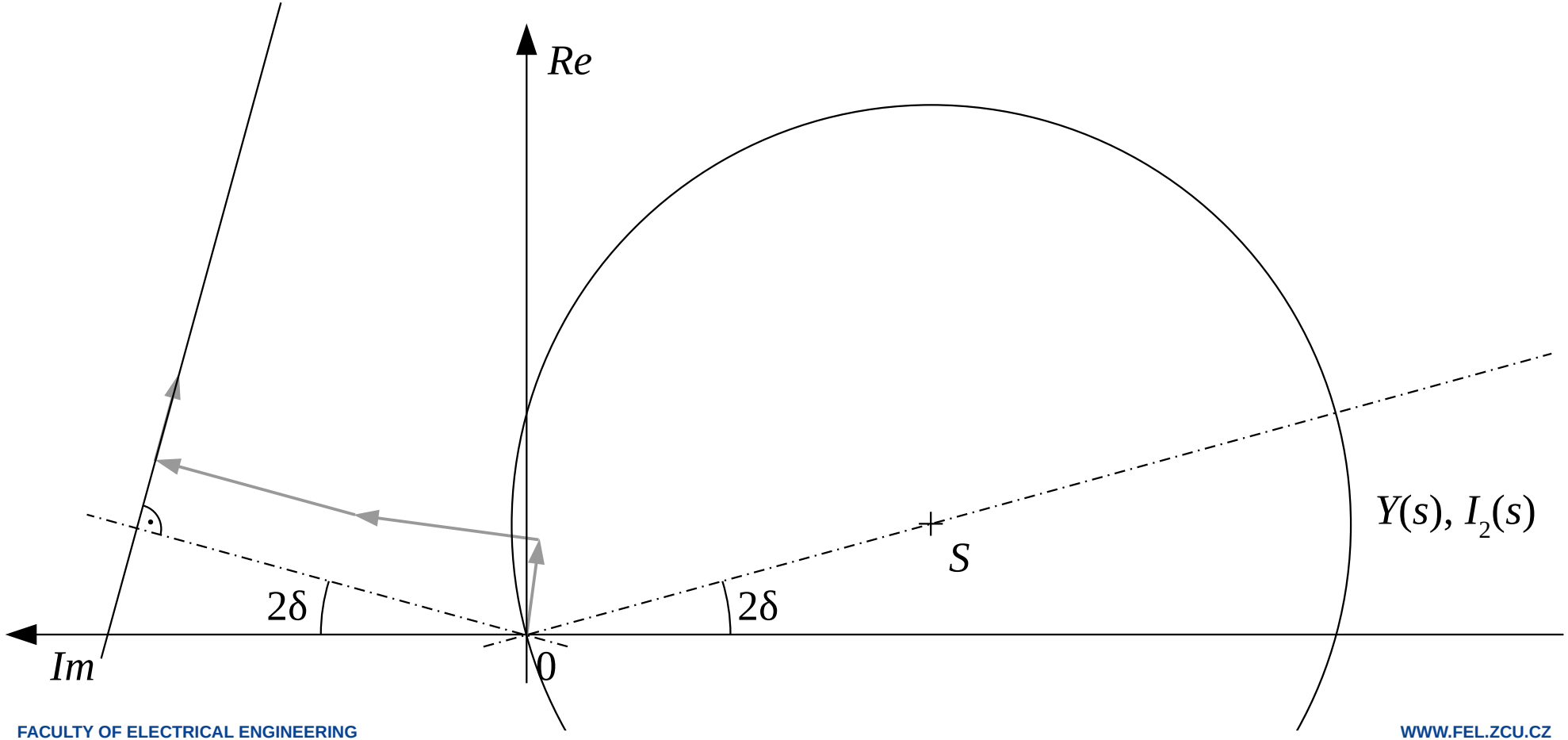
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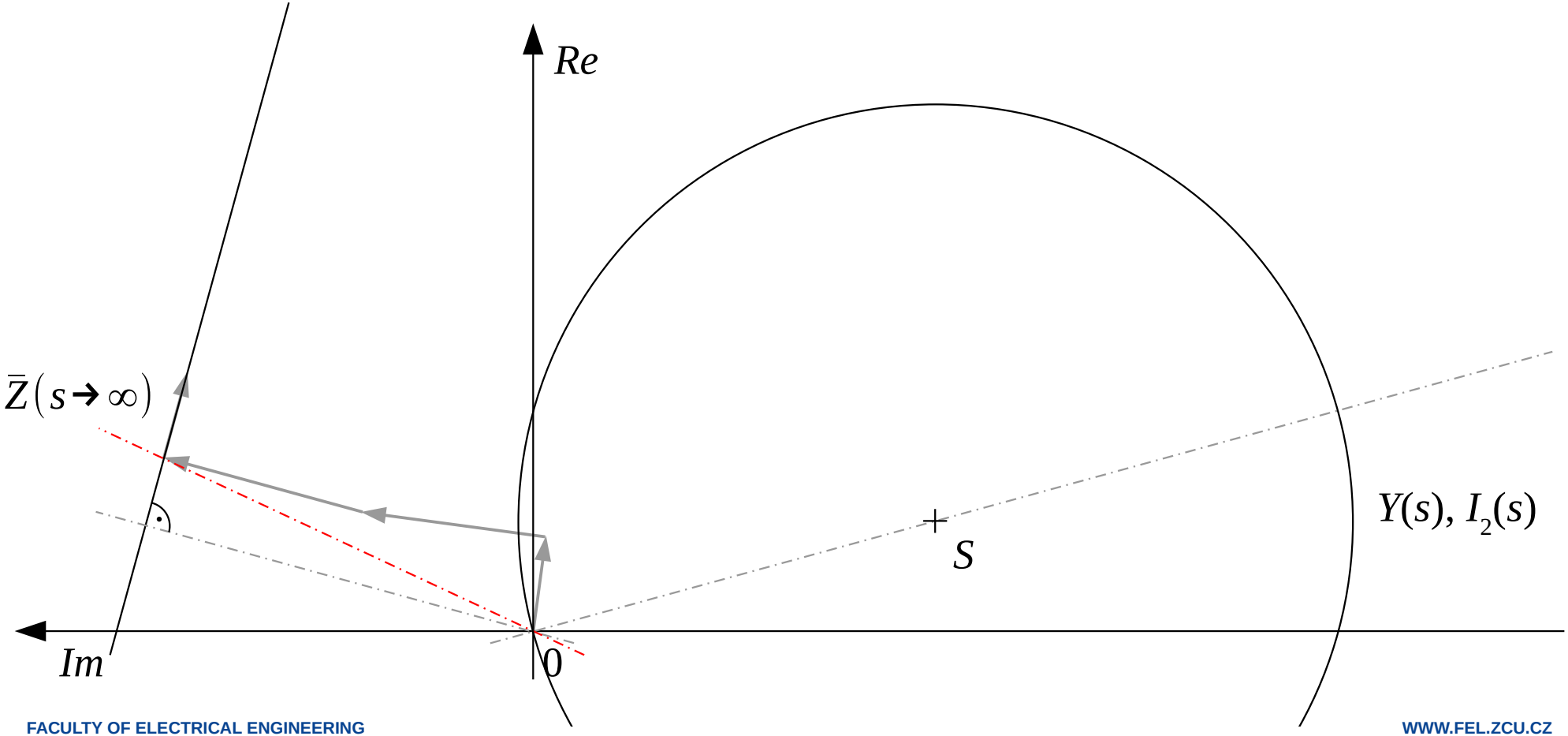
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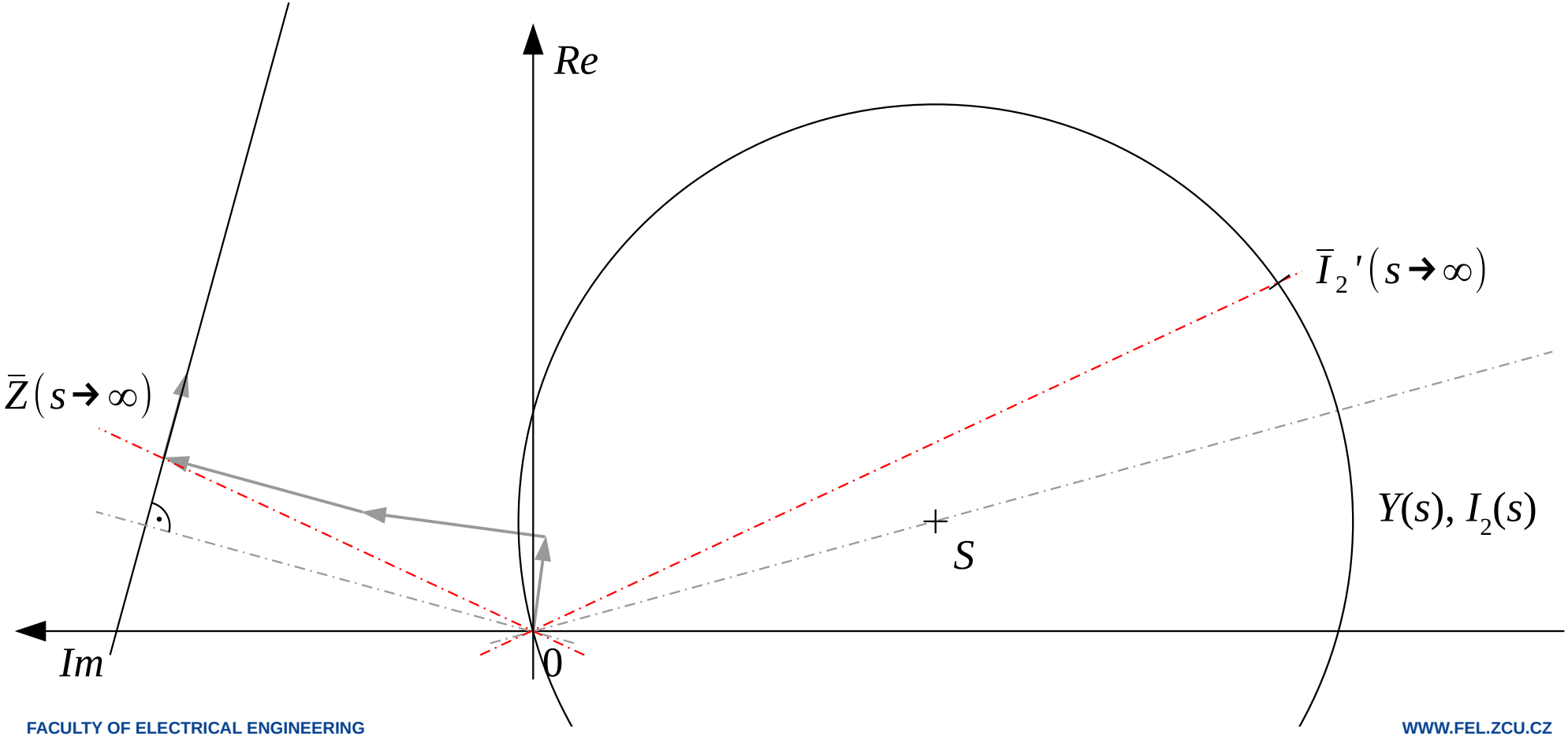
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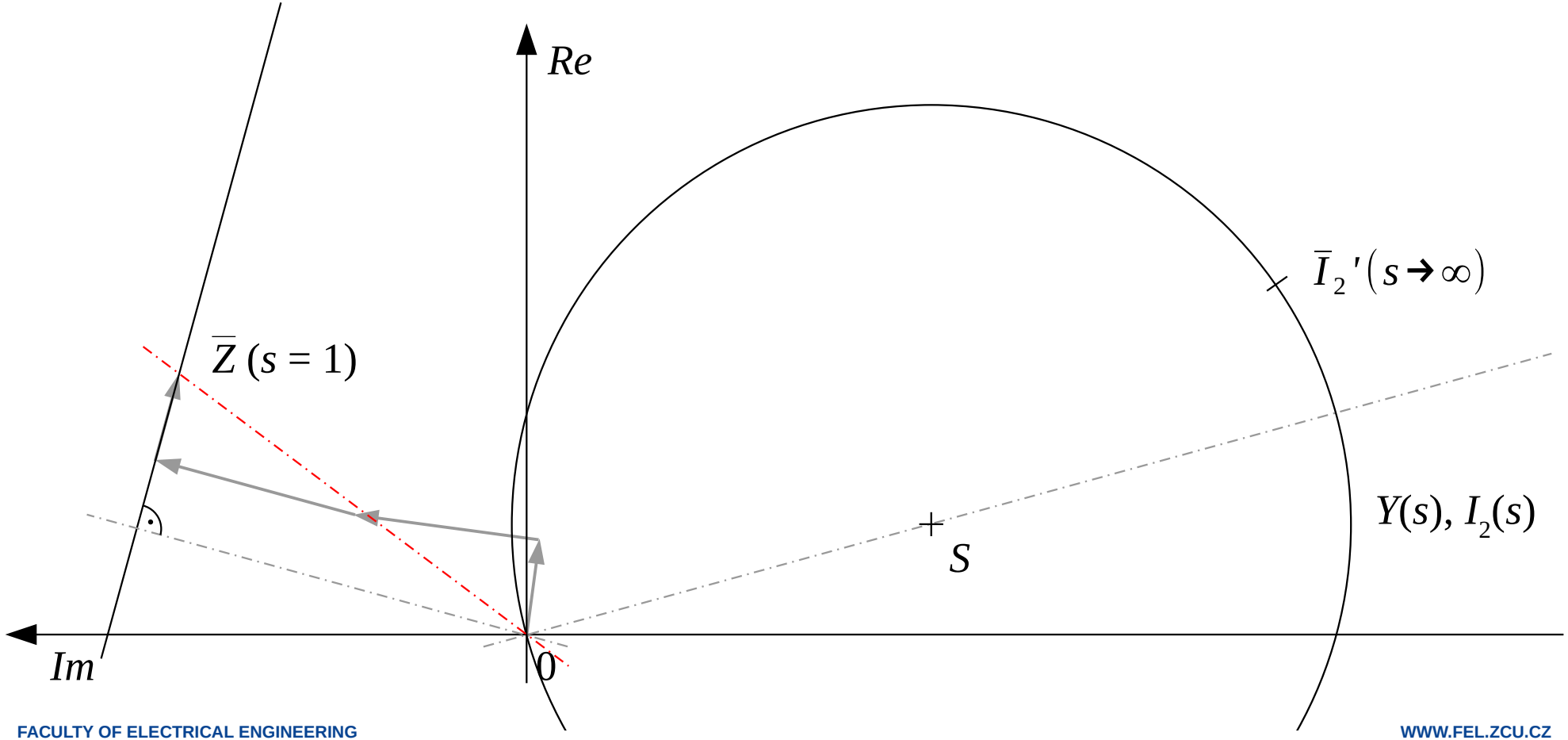
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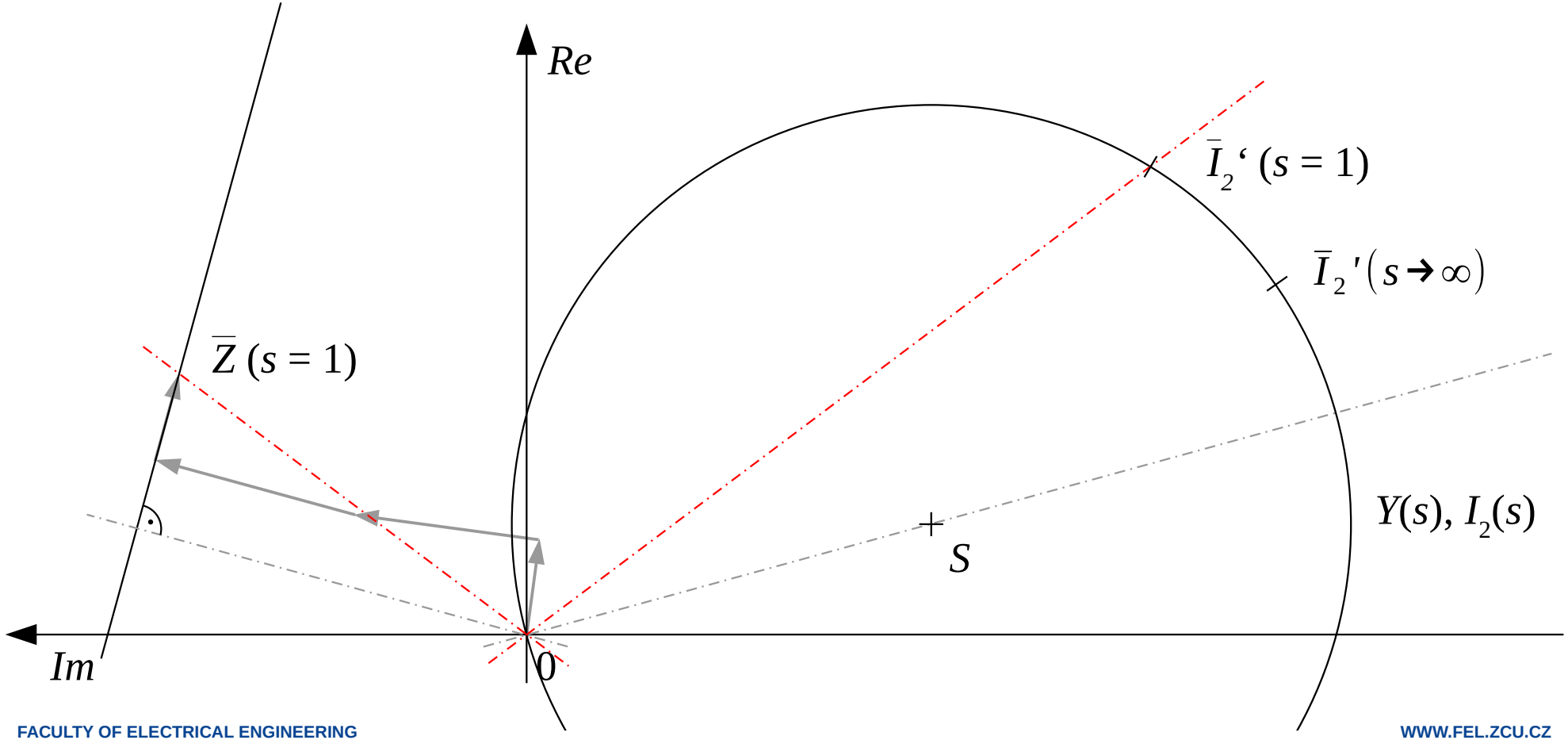
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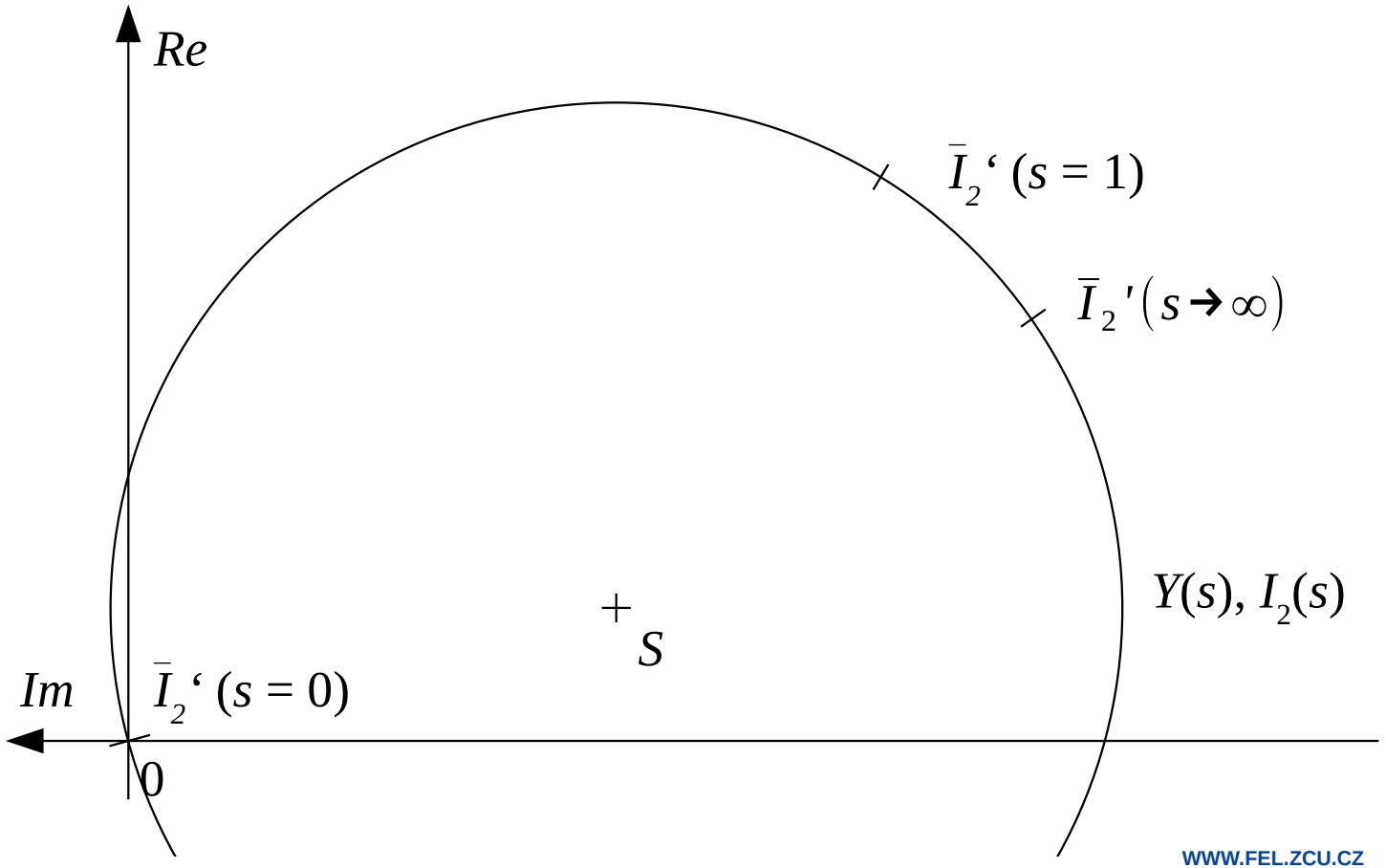
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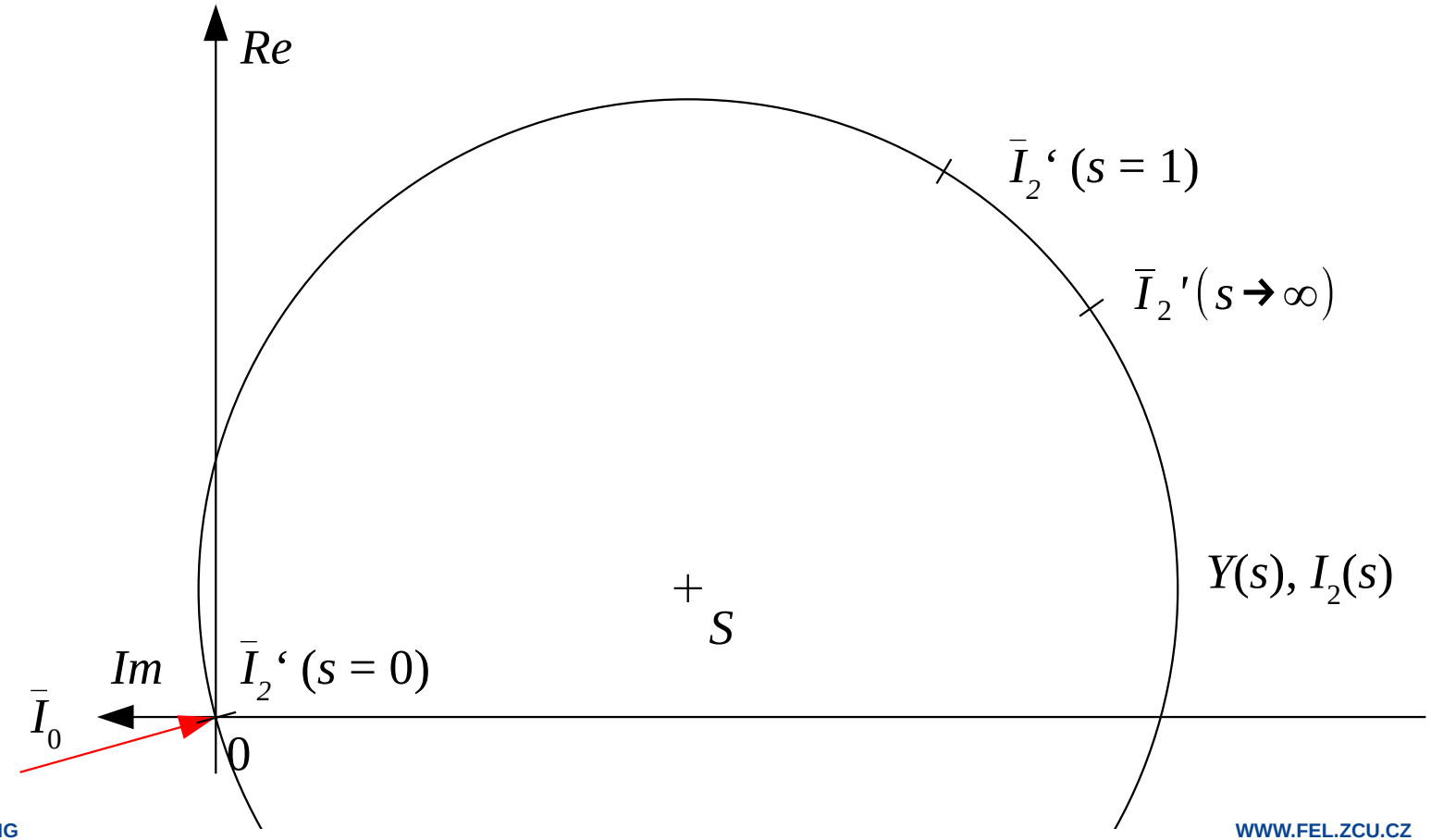
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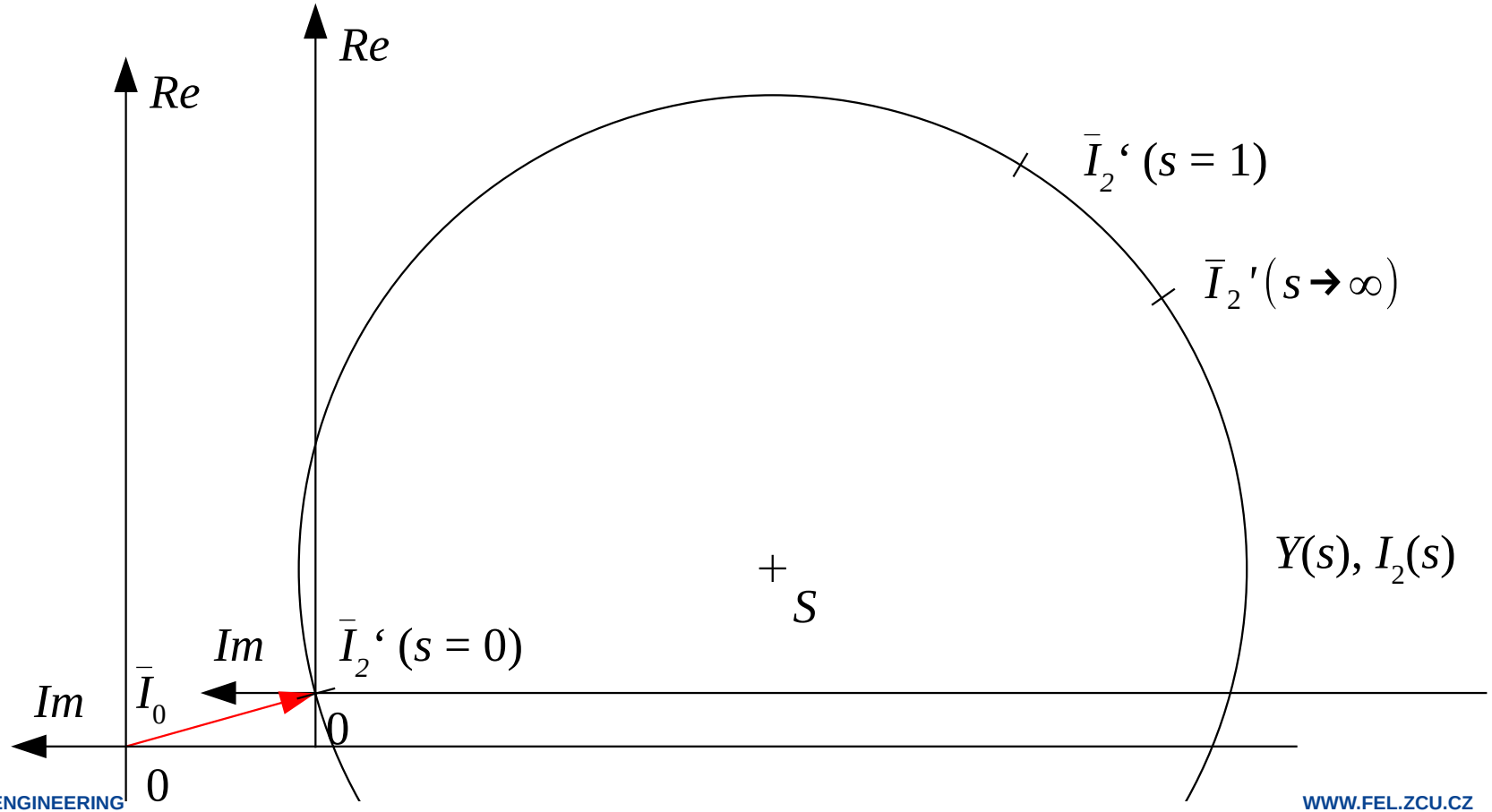
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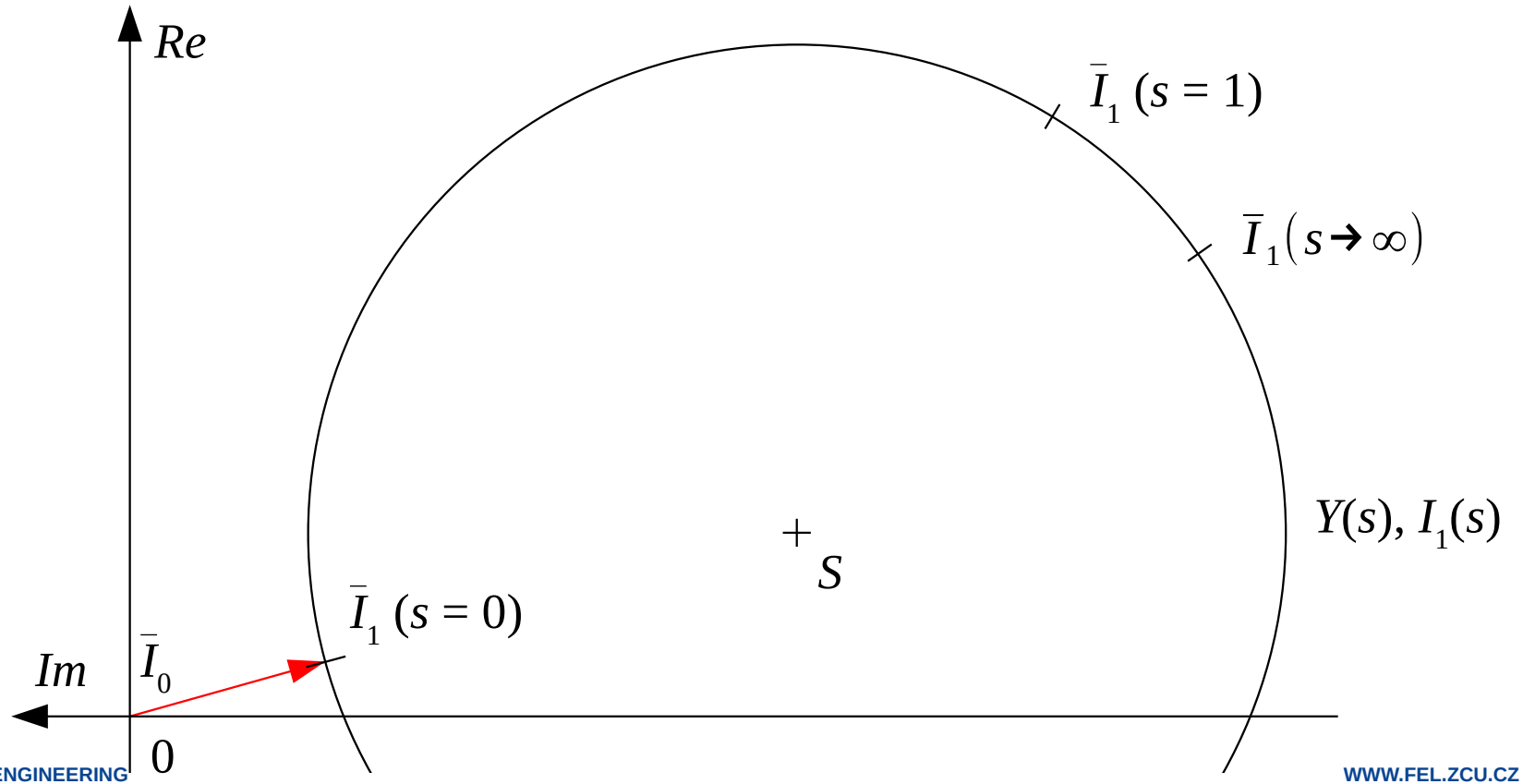
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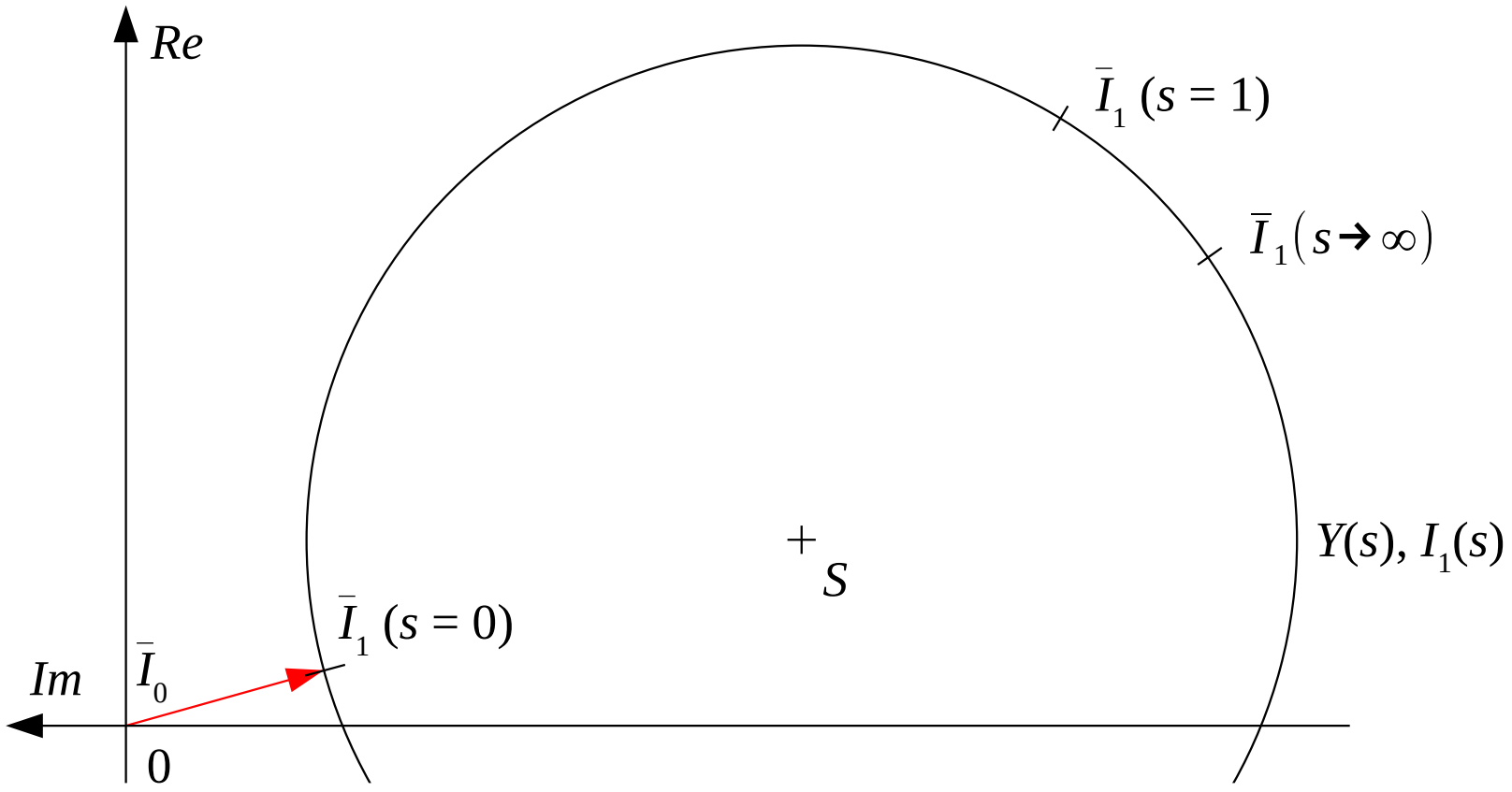
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# Circle Diagram



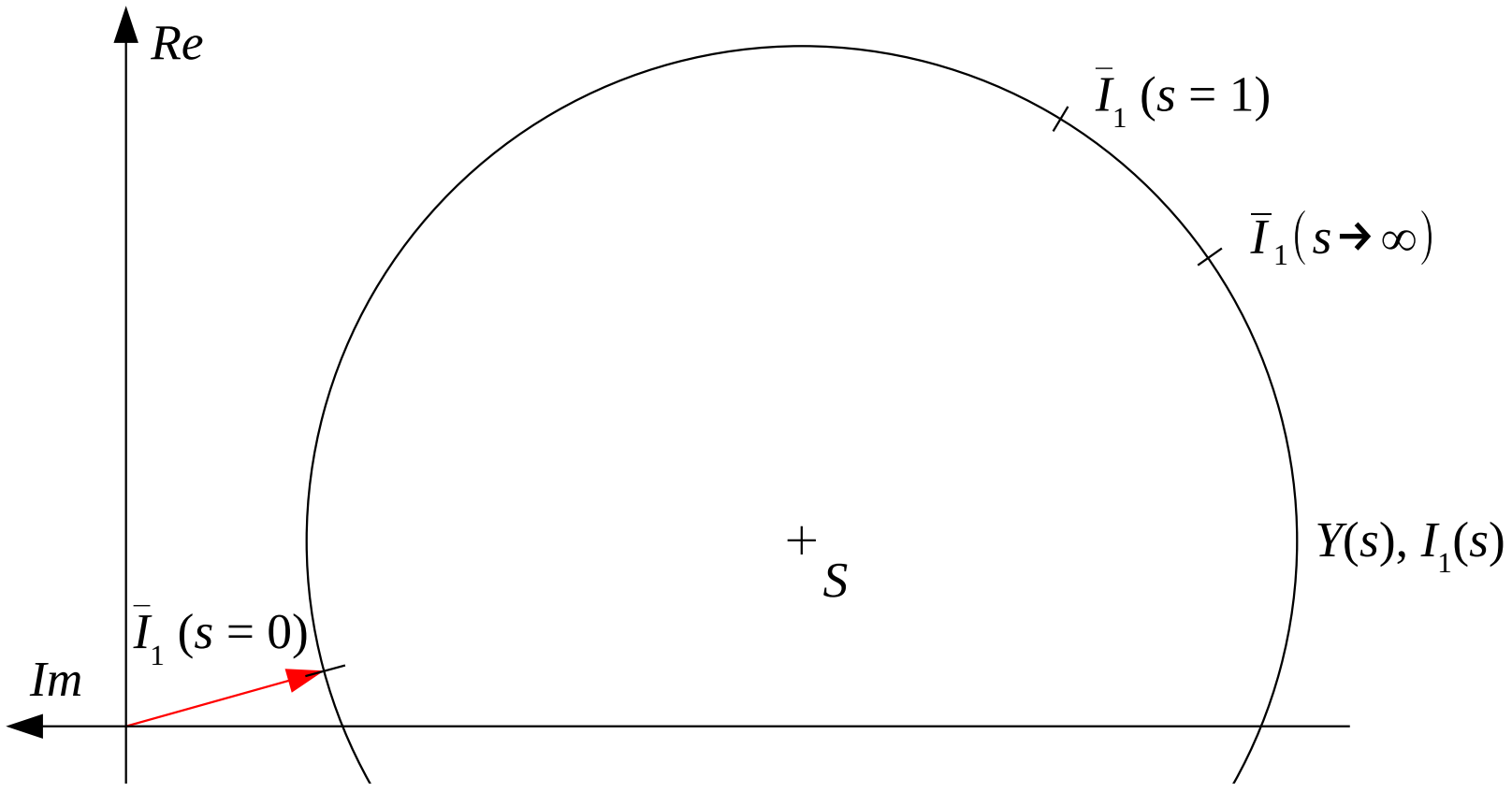
# Circle Diagram

- The slip scale may be then obtained in two ways:
  - Directly during construction of the circle diagram on the drawn circle
    - Leads to non-linear scale with extreme density in are of low slips
  - From any point $L$ as a line parallel to a line segment between point $L$ and point $Y(\infty)$
    - Results in linear scale

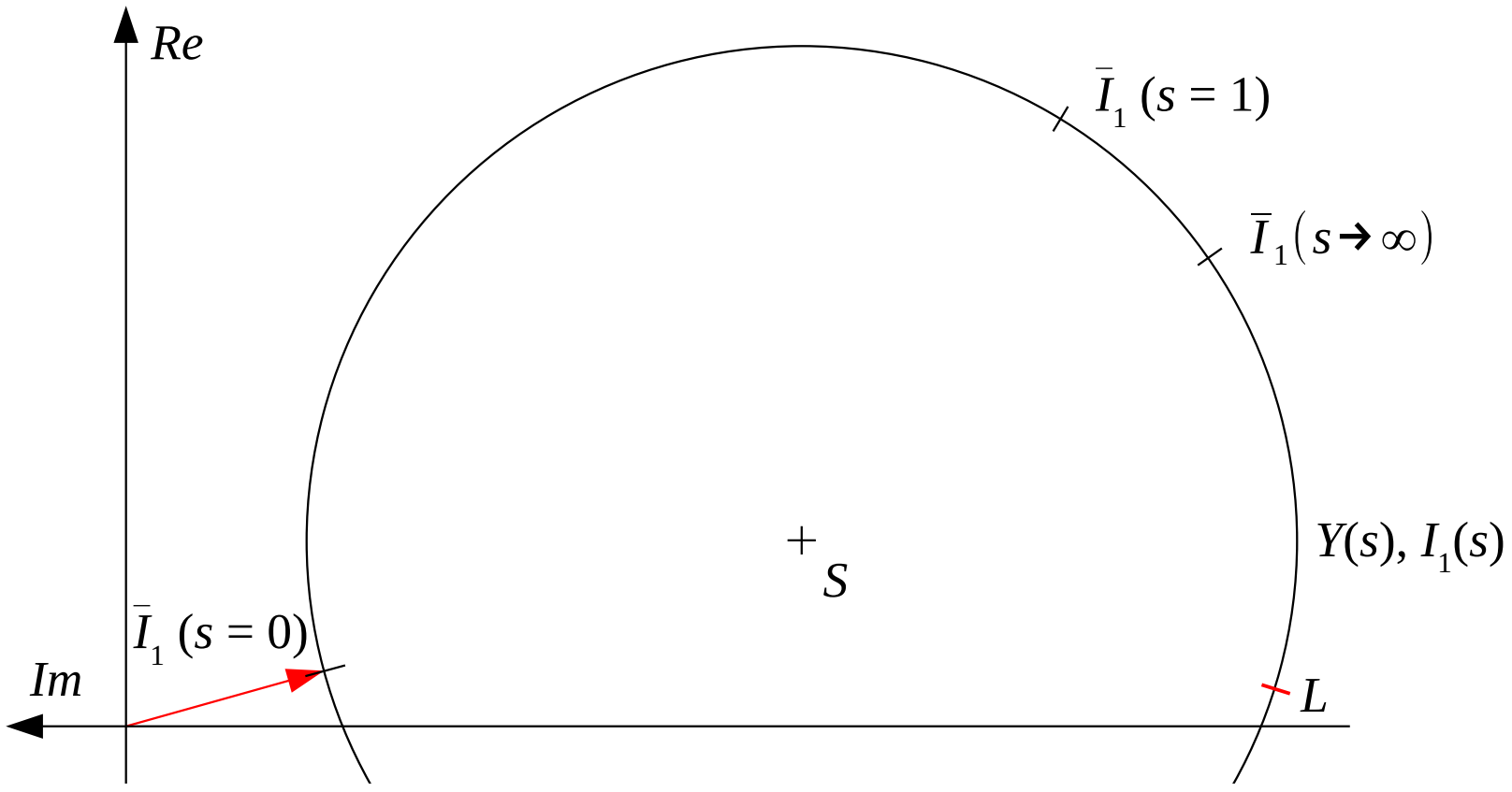
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- Now let us choose the second variant

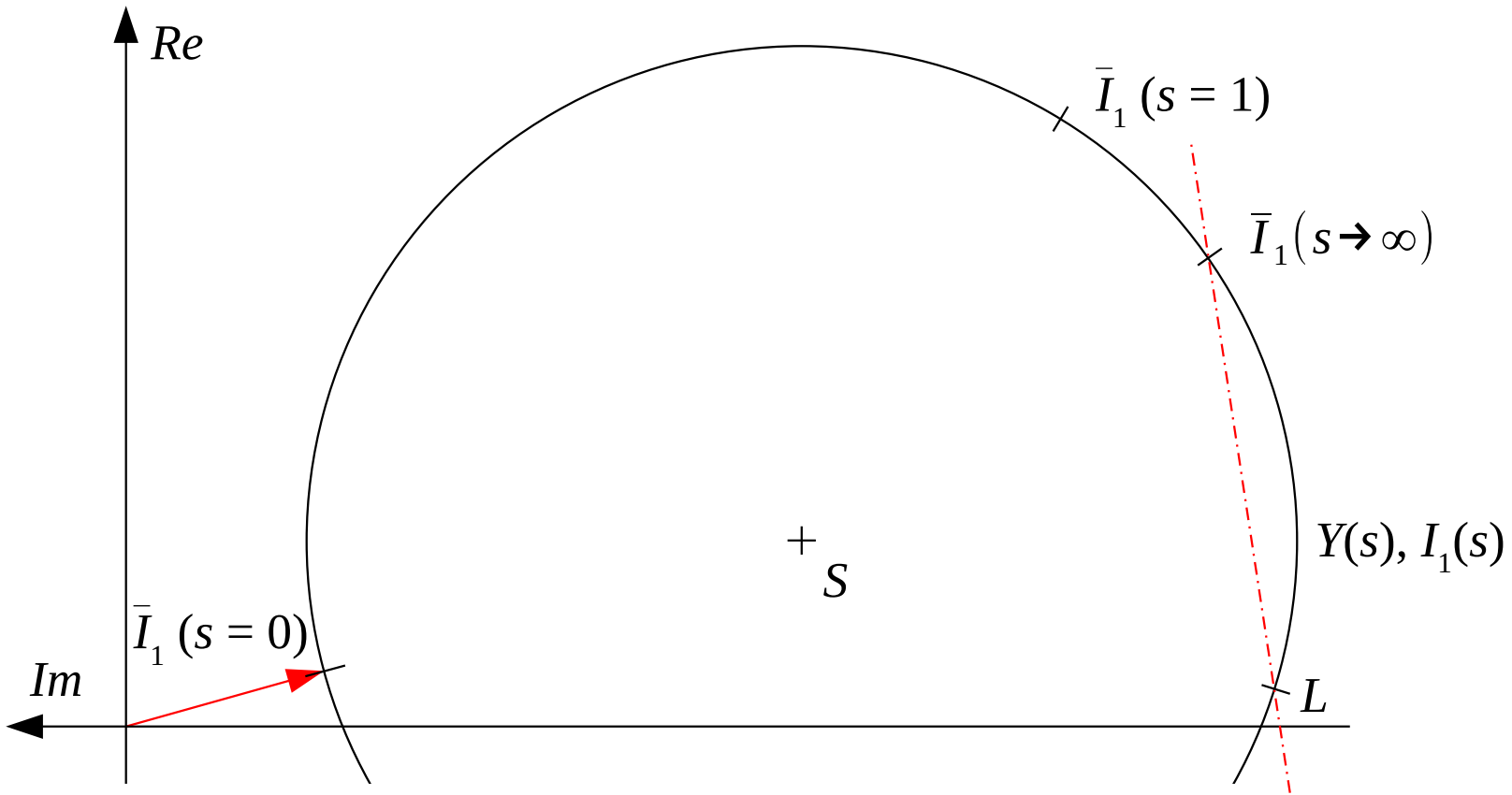
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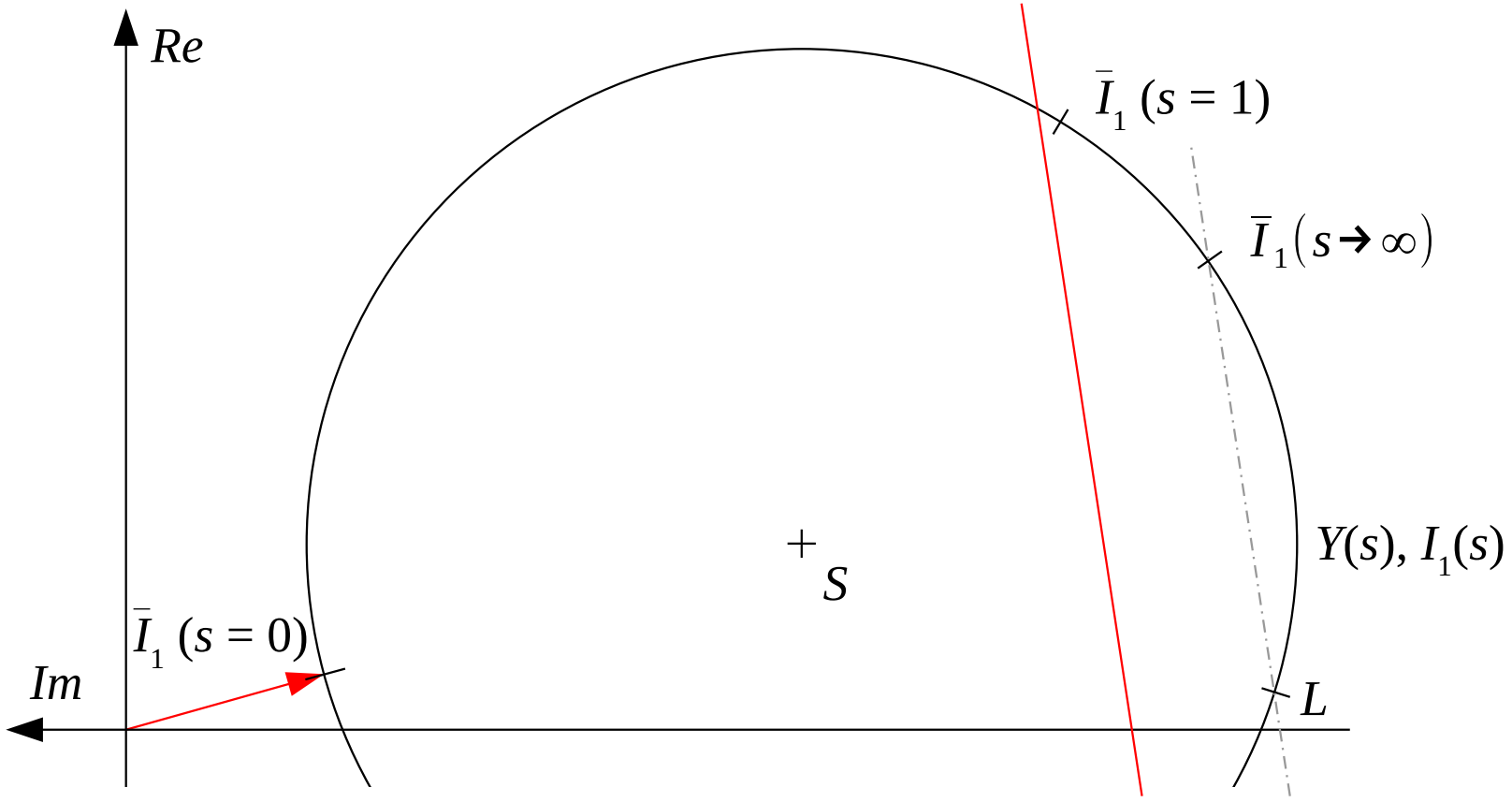
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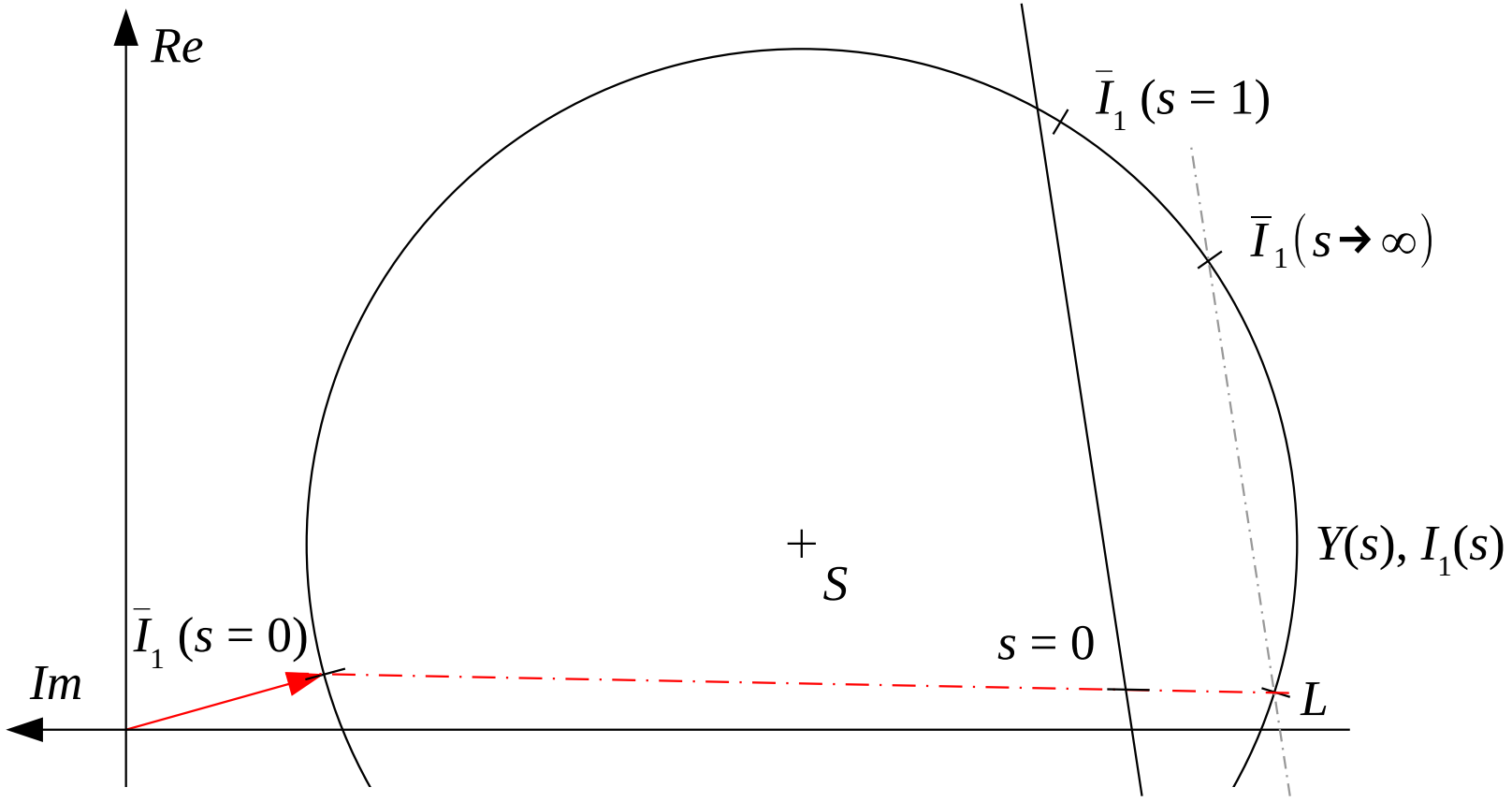
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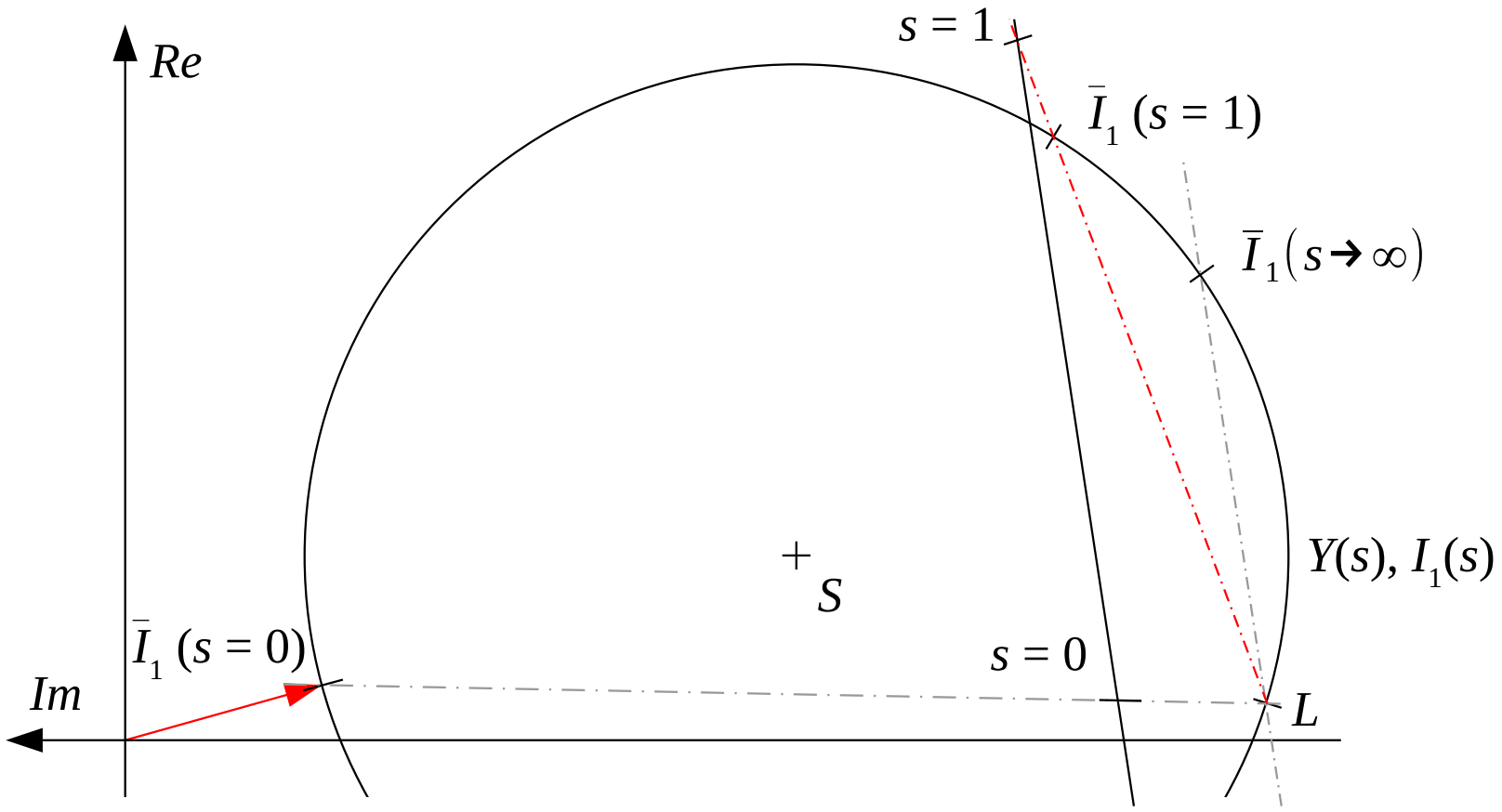
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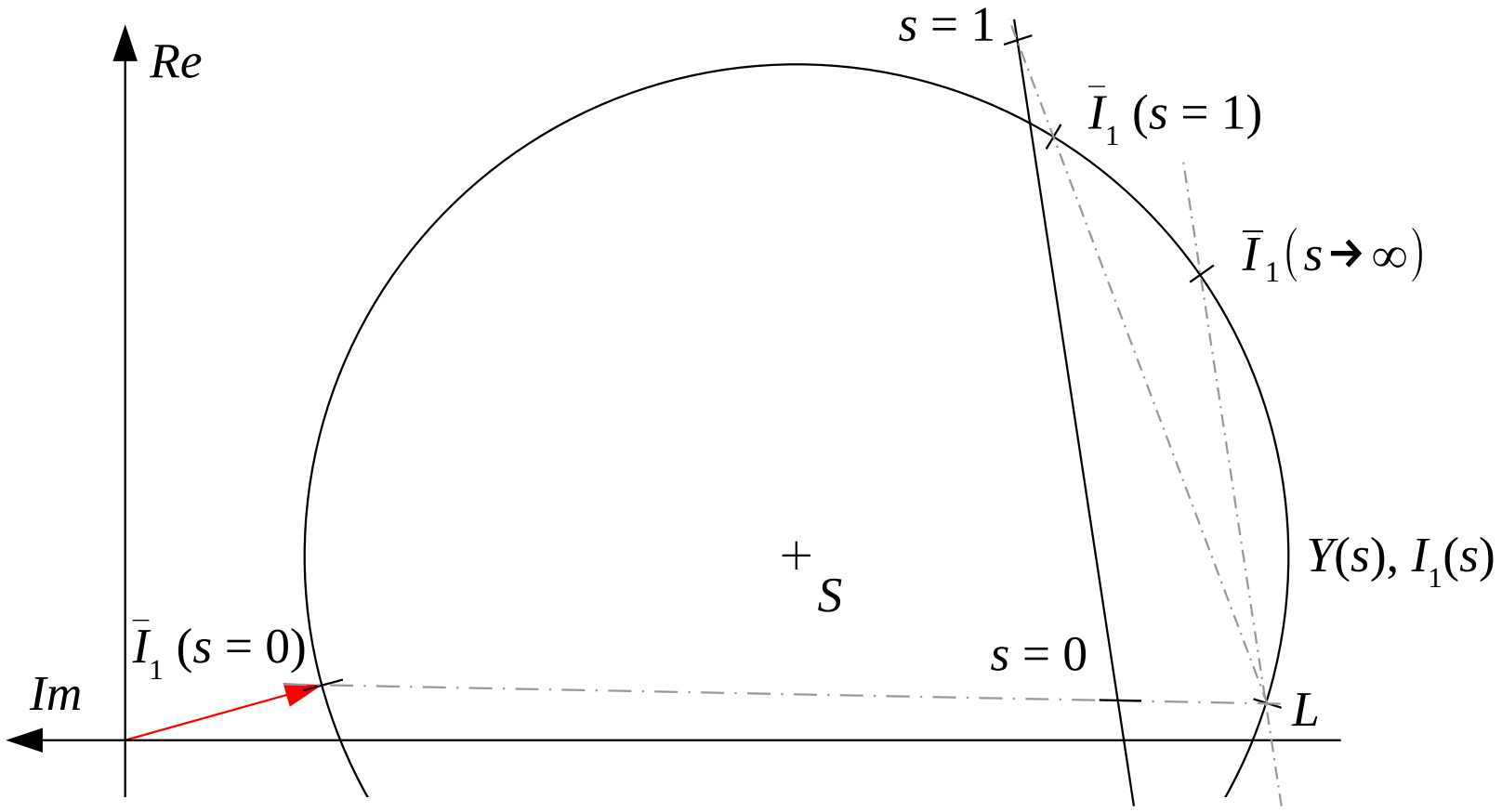
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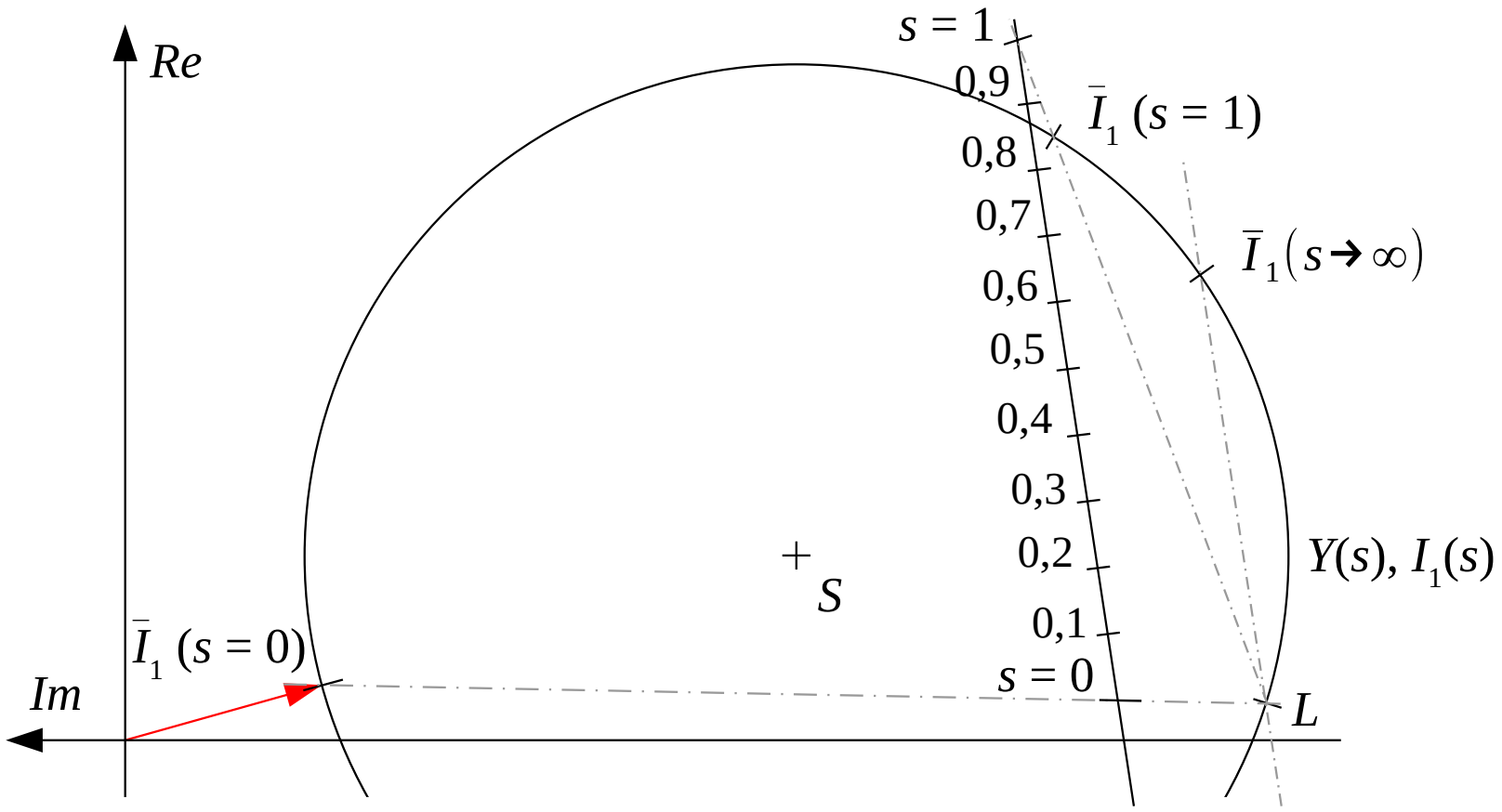
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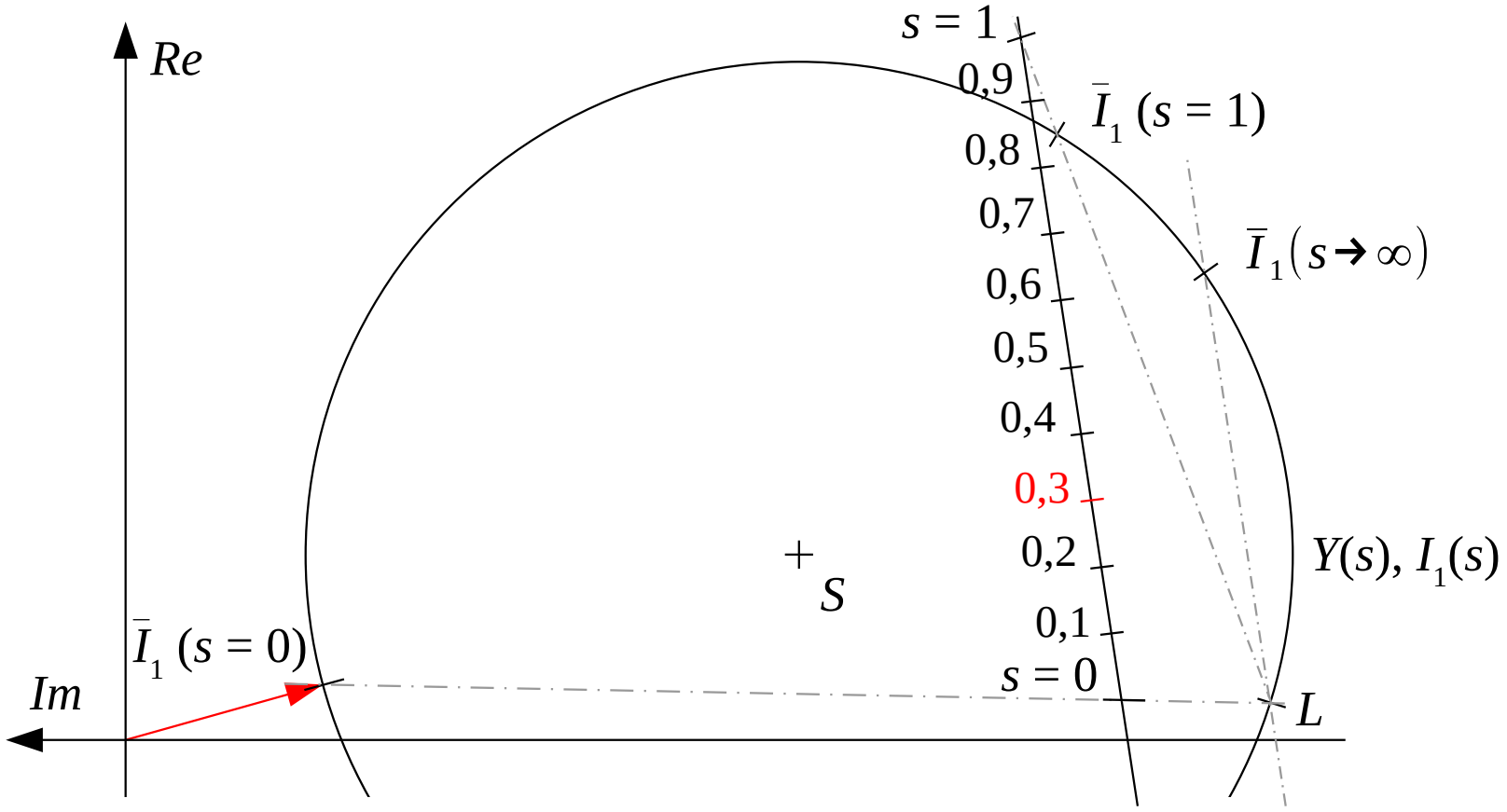
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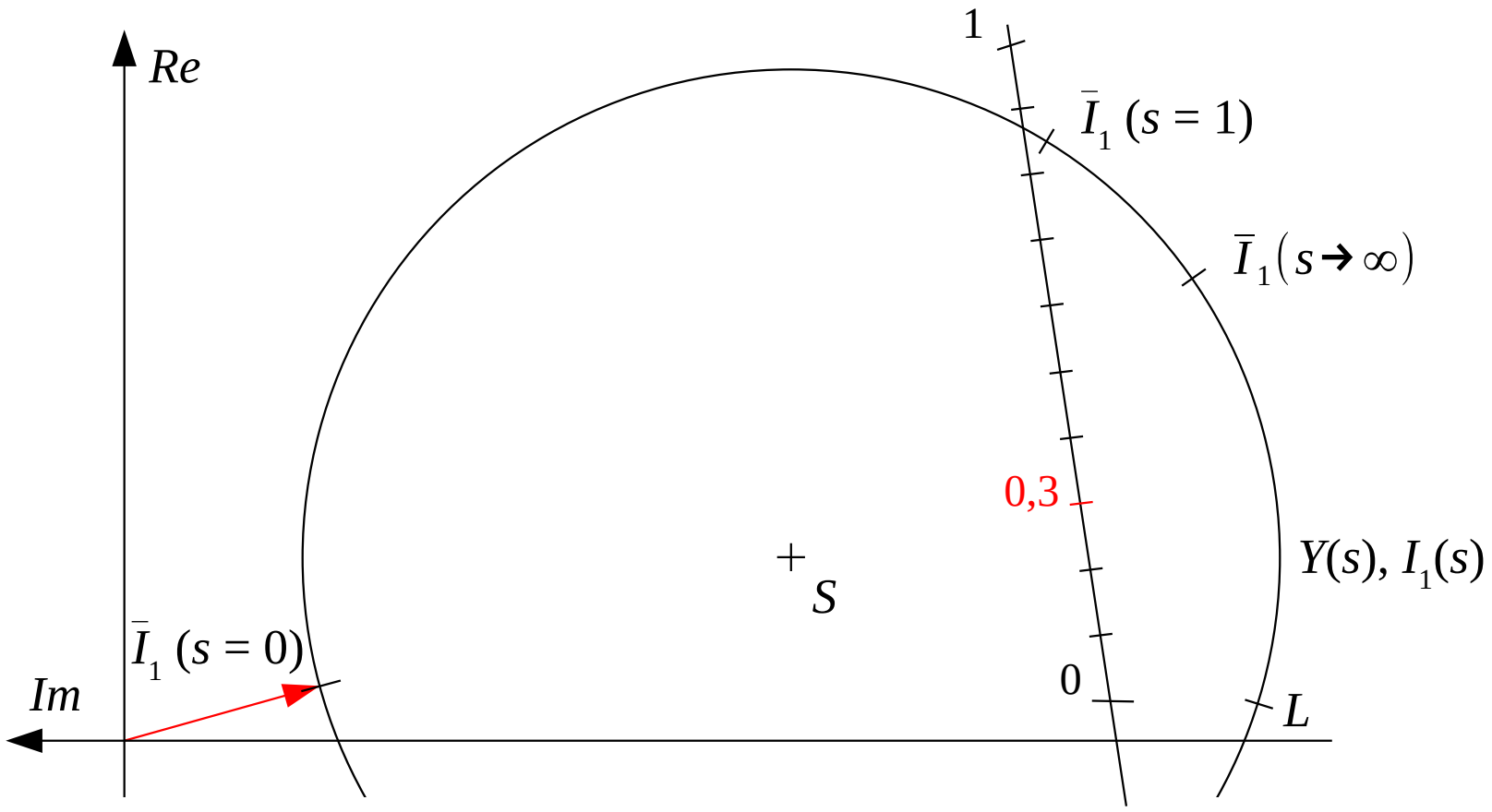
# Circle Diagram

- Reading variables from the circle diagram
  - Currents:
    - May be read directly, it is necessary to know the scale of currents $m_i$
    - The distance between point $Y(s)$ and the centre of the coordinate system is equal to the value of stator current $I_1$
    - The distance between point $Y(s)$ and the point $I_0$ is equal to the value of the rotor current $I_2$
    - The angle between line segments given by these points and the real axis is equal to the phase shift between the voltage and the current

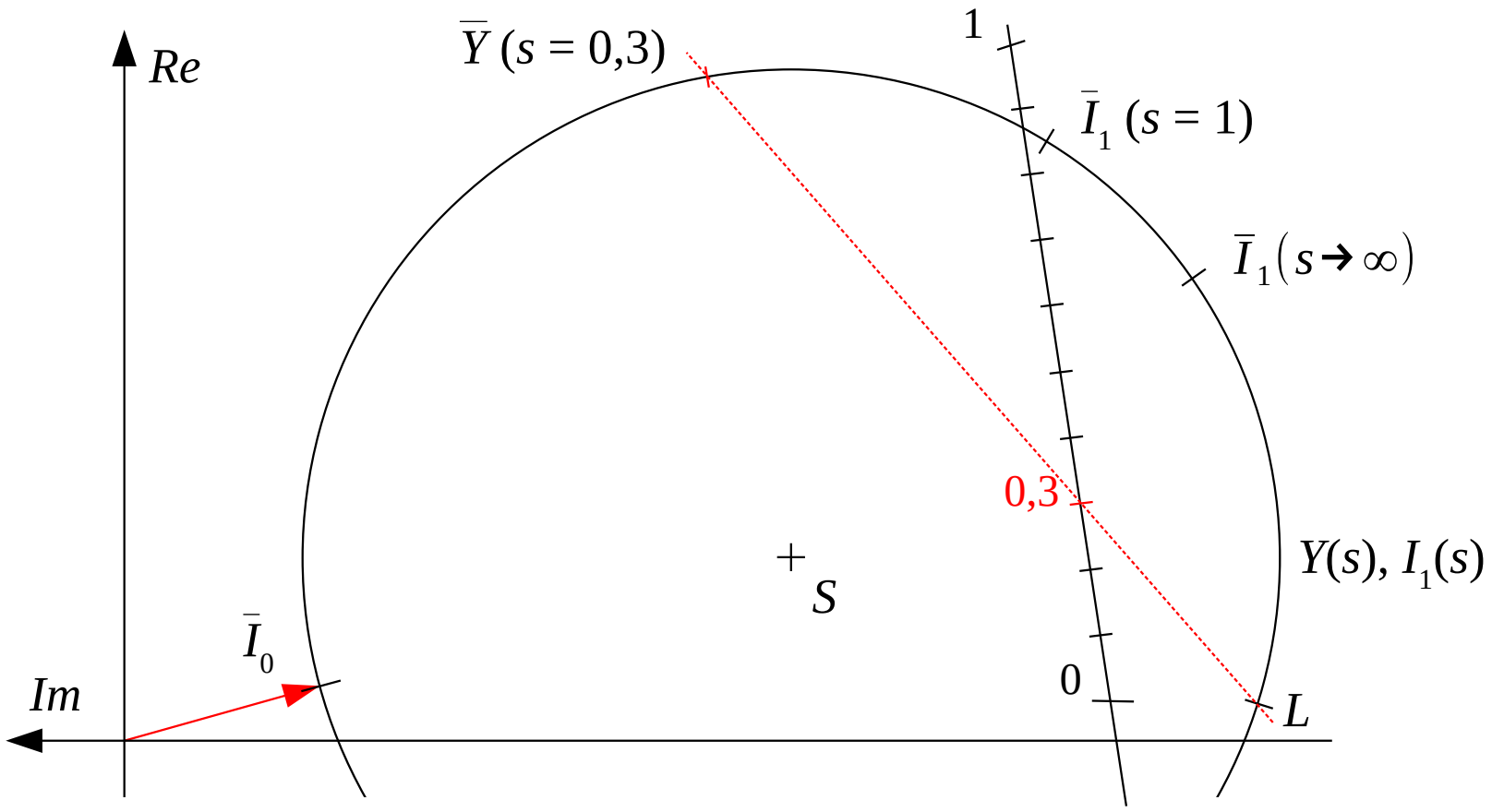
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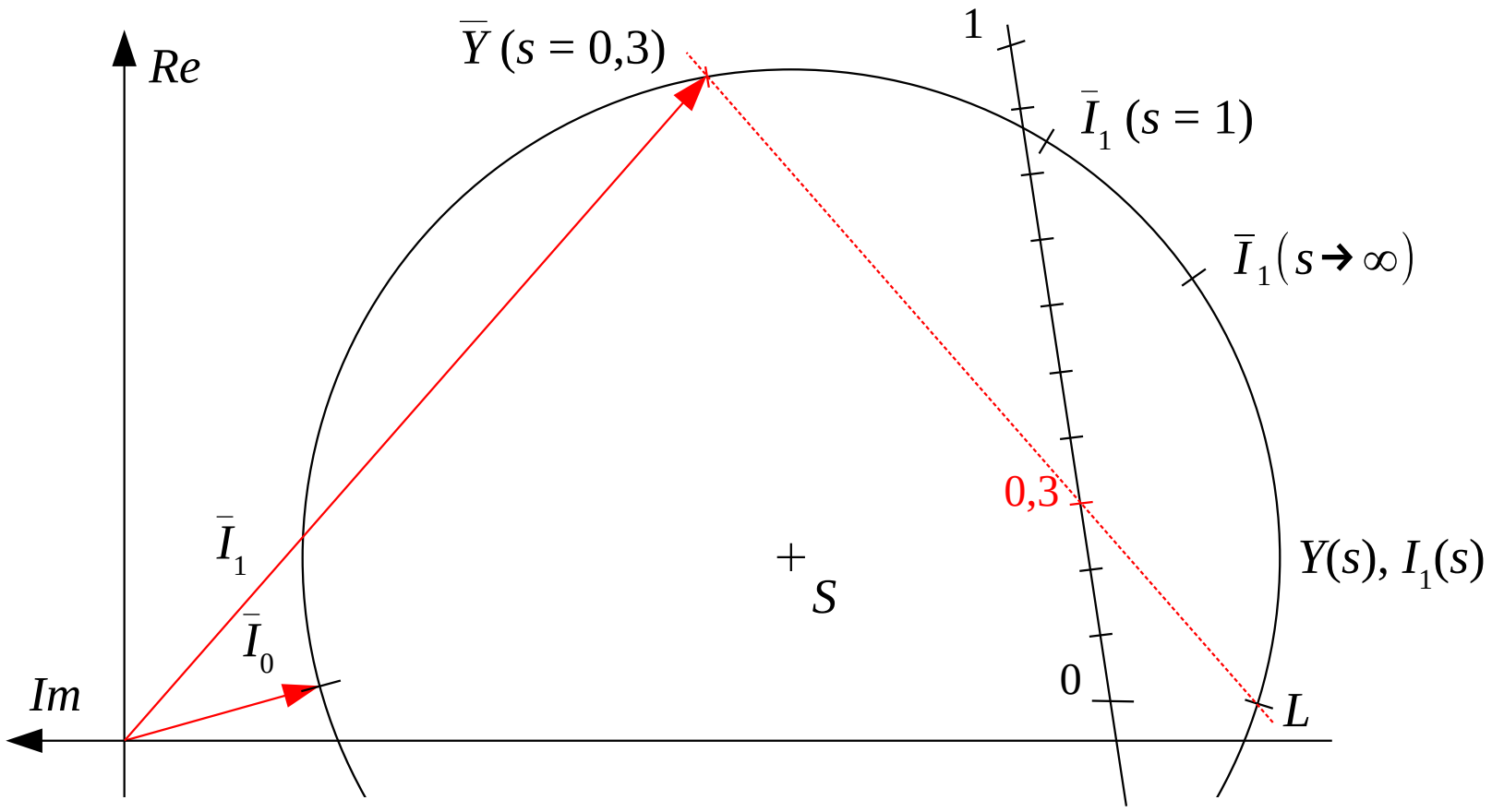
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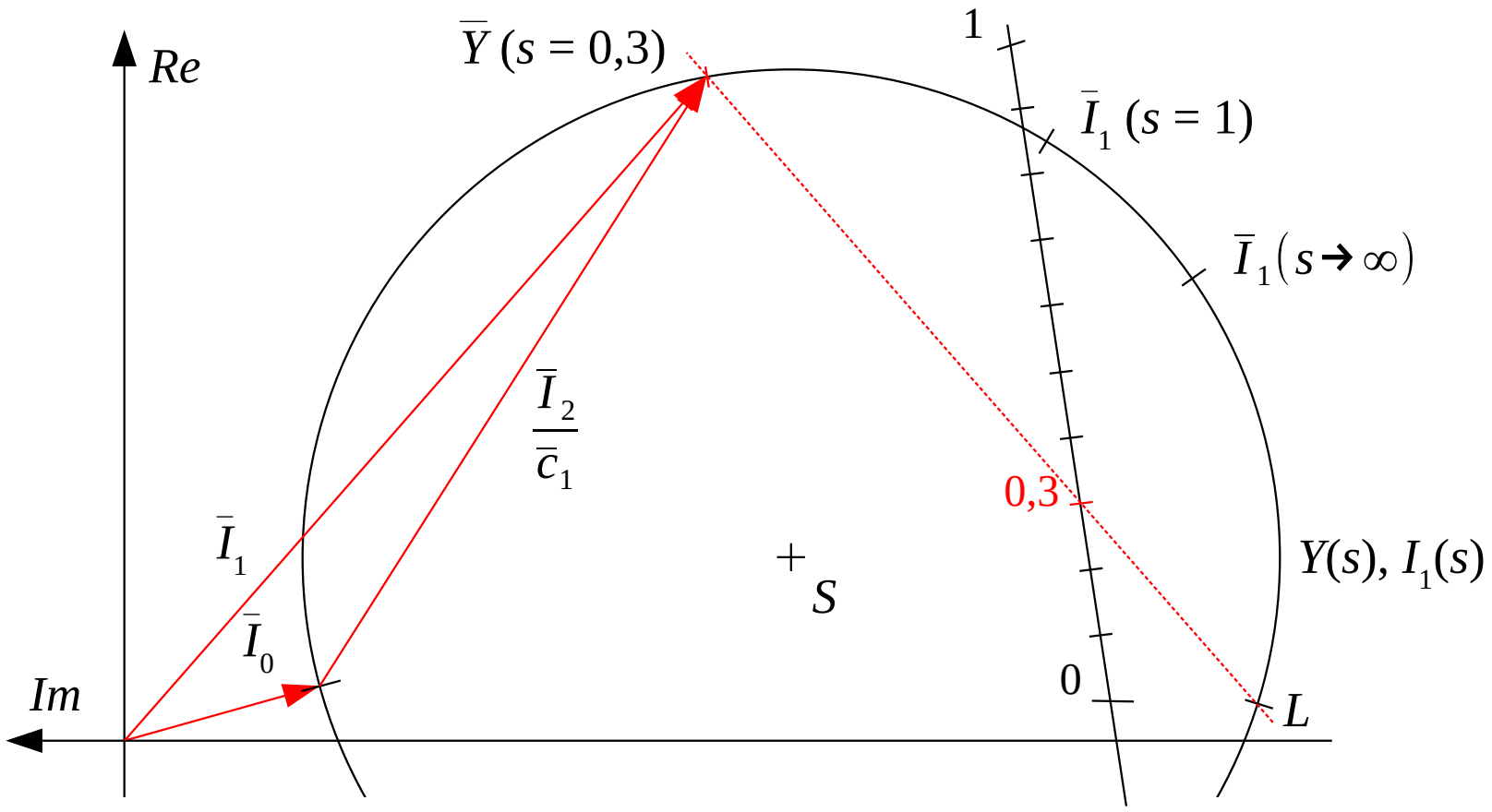
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# Circle Diagram



# Circle Diagram



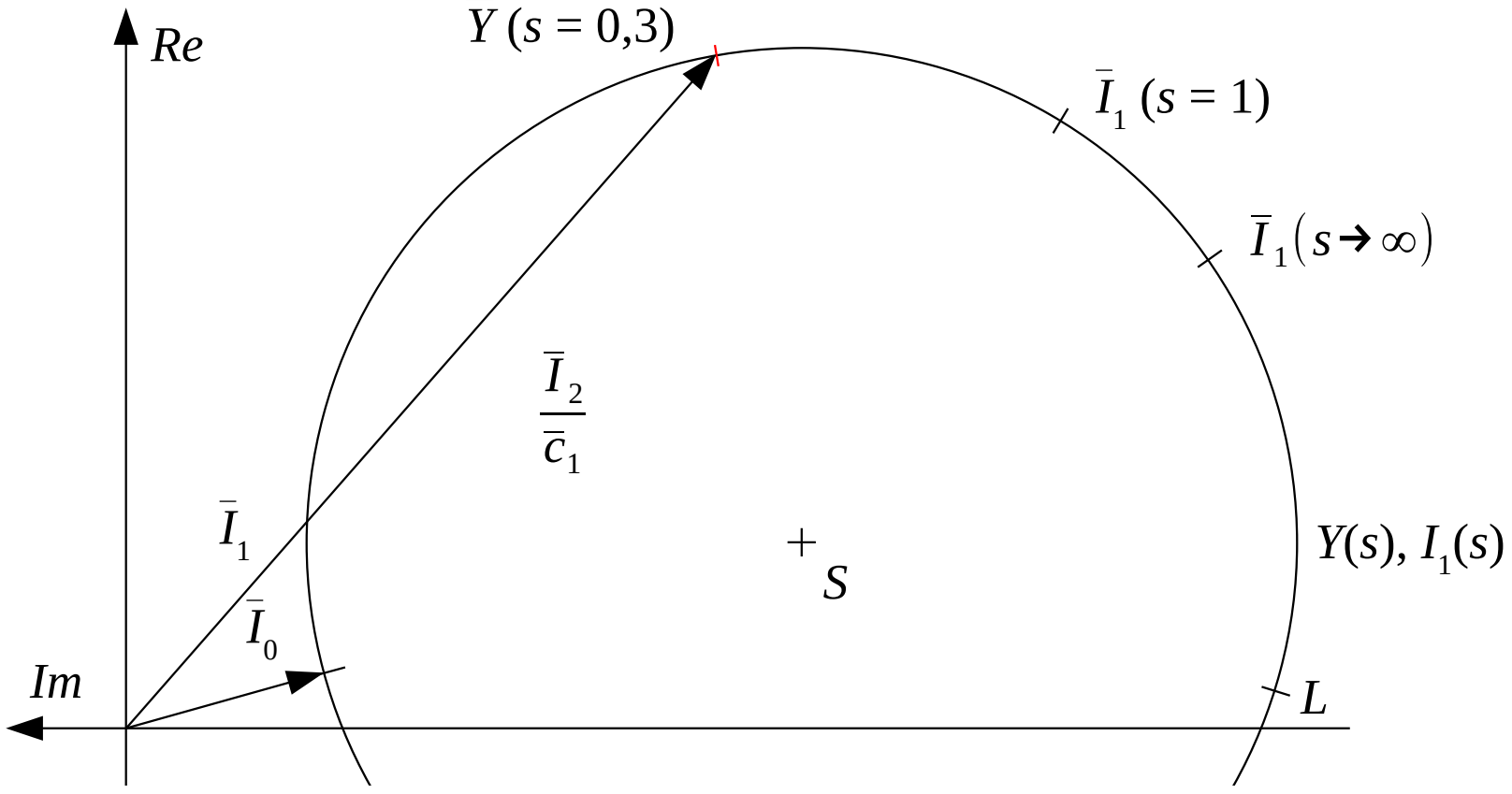
# Circle Diagram

- Reading variables from the circle diagram
  - Power balance:
    - The powers are multiples of real parts of the current and relevant part of the circle diagram
    - The rotor current is shifted by the angle $2\delta$
      - The same is the phase shift of its real part
    - To obtain power balance, it is necessary to use the scale of powers

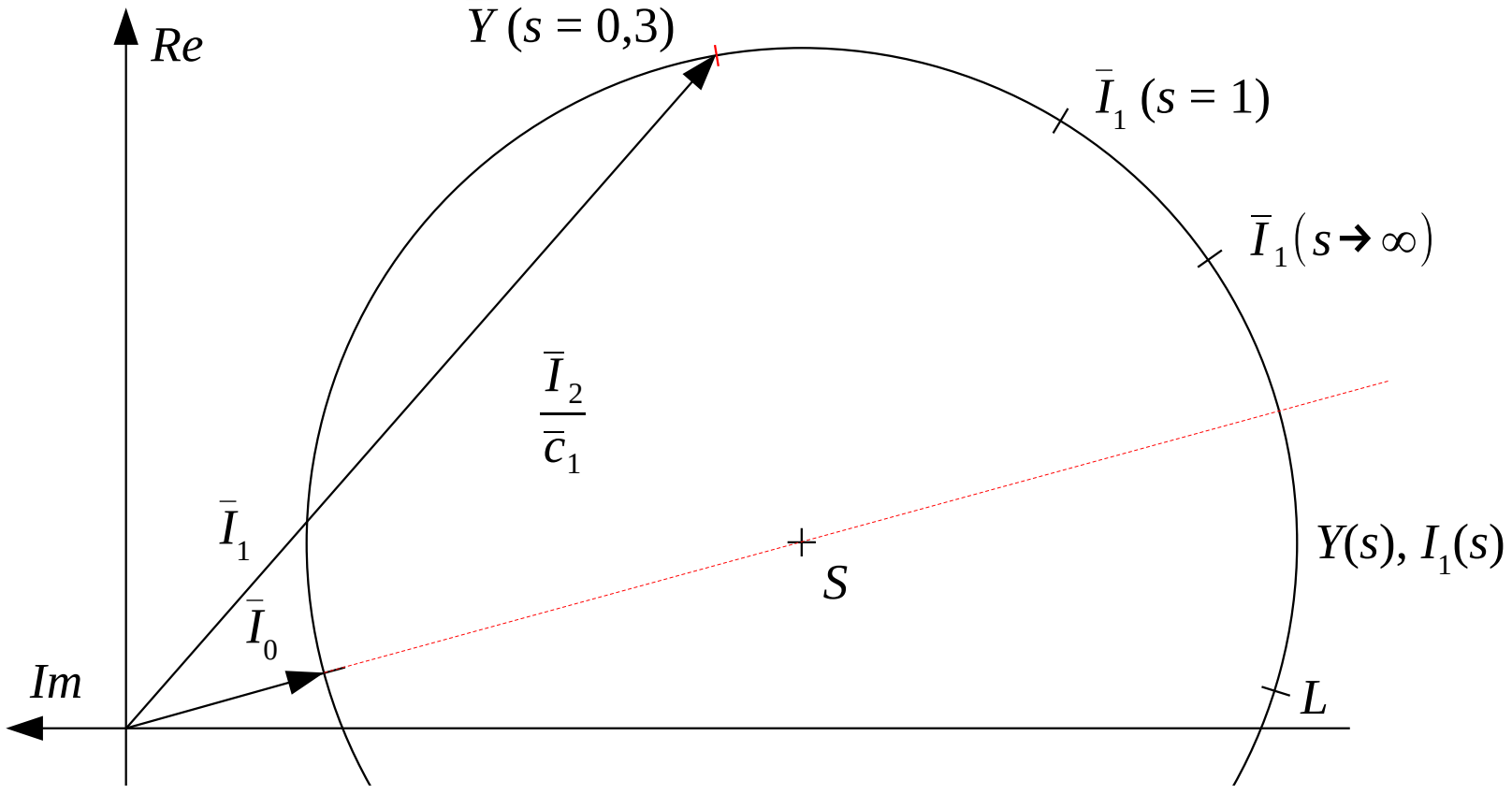
$$m_p = m_1 U_1 m_i$$

… where $m_1$ is the amount of stator winding phases

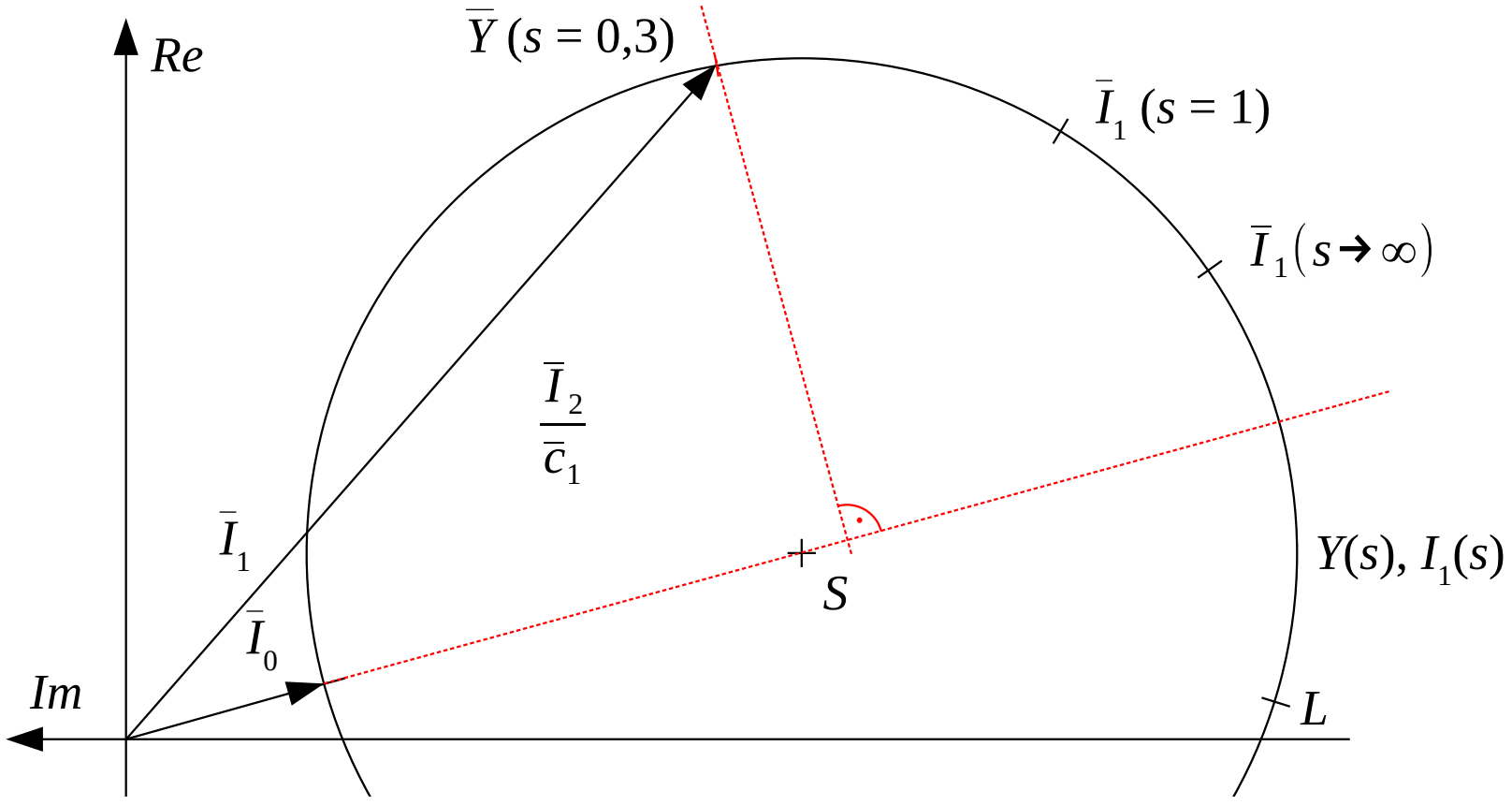
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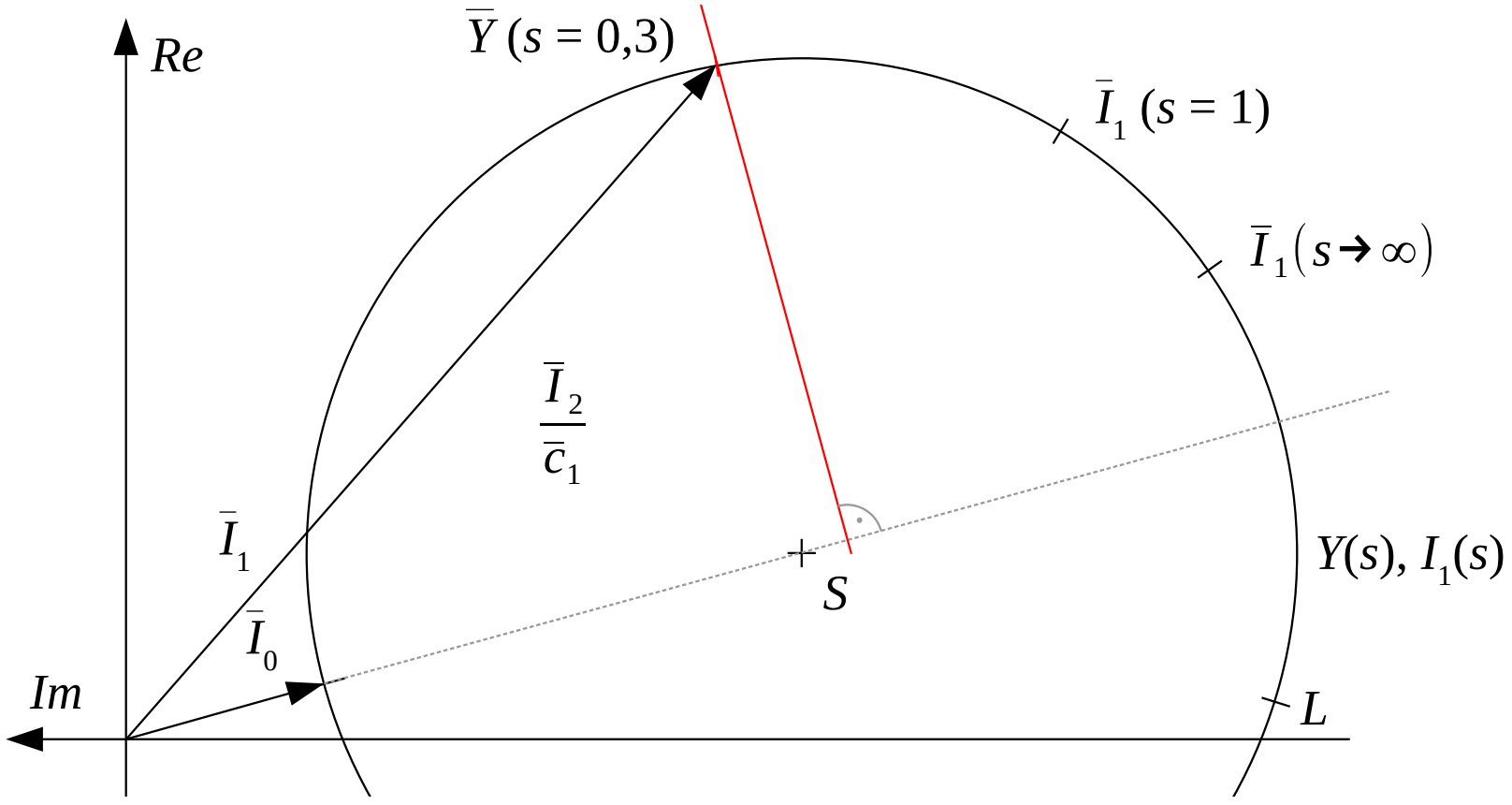
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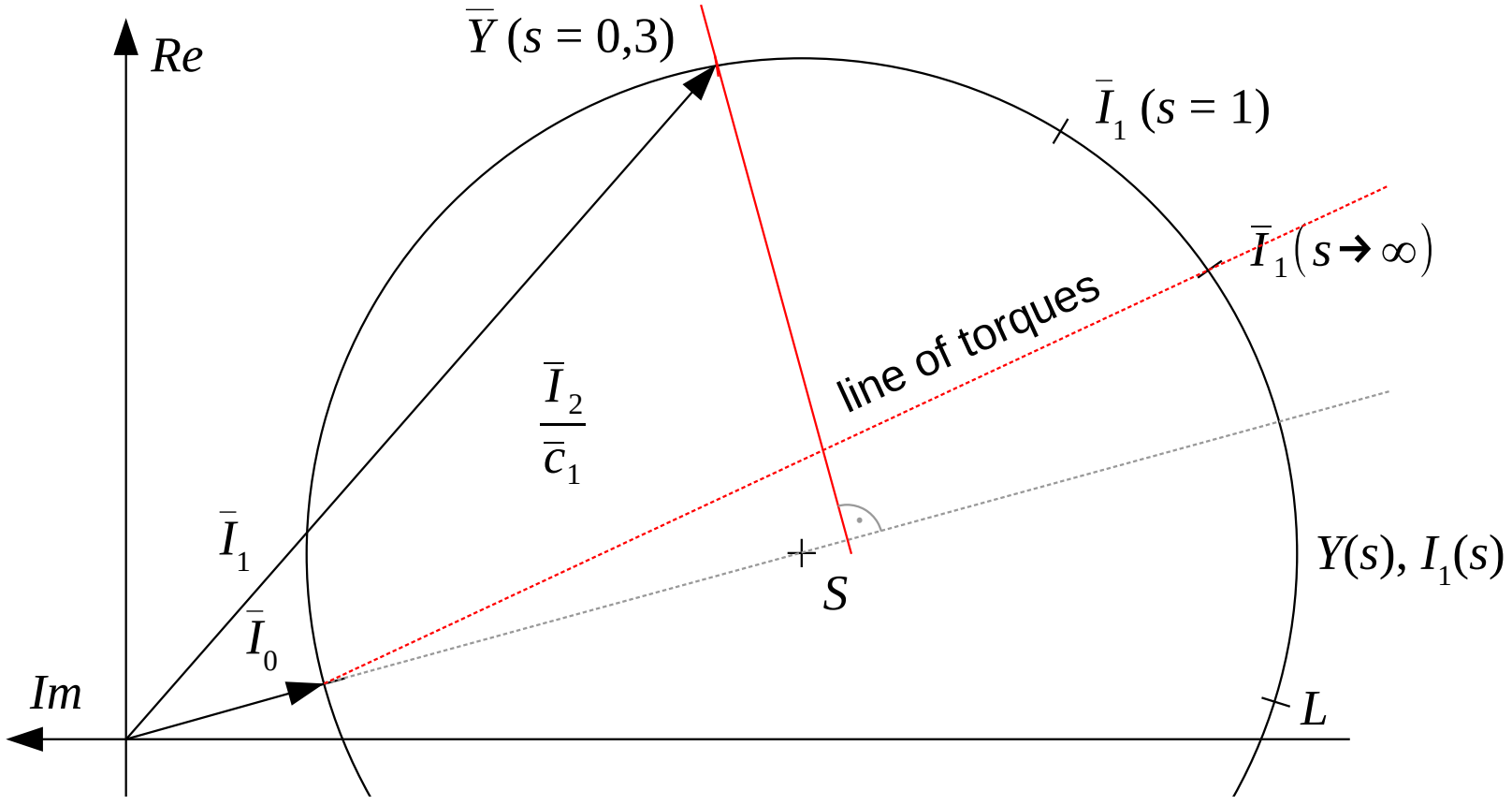
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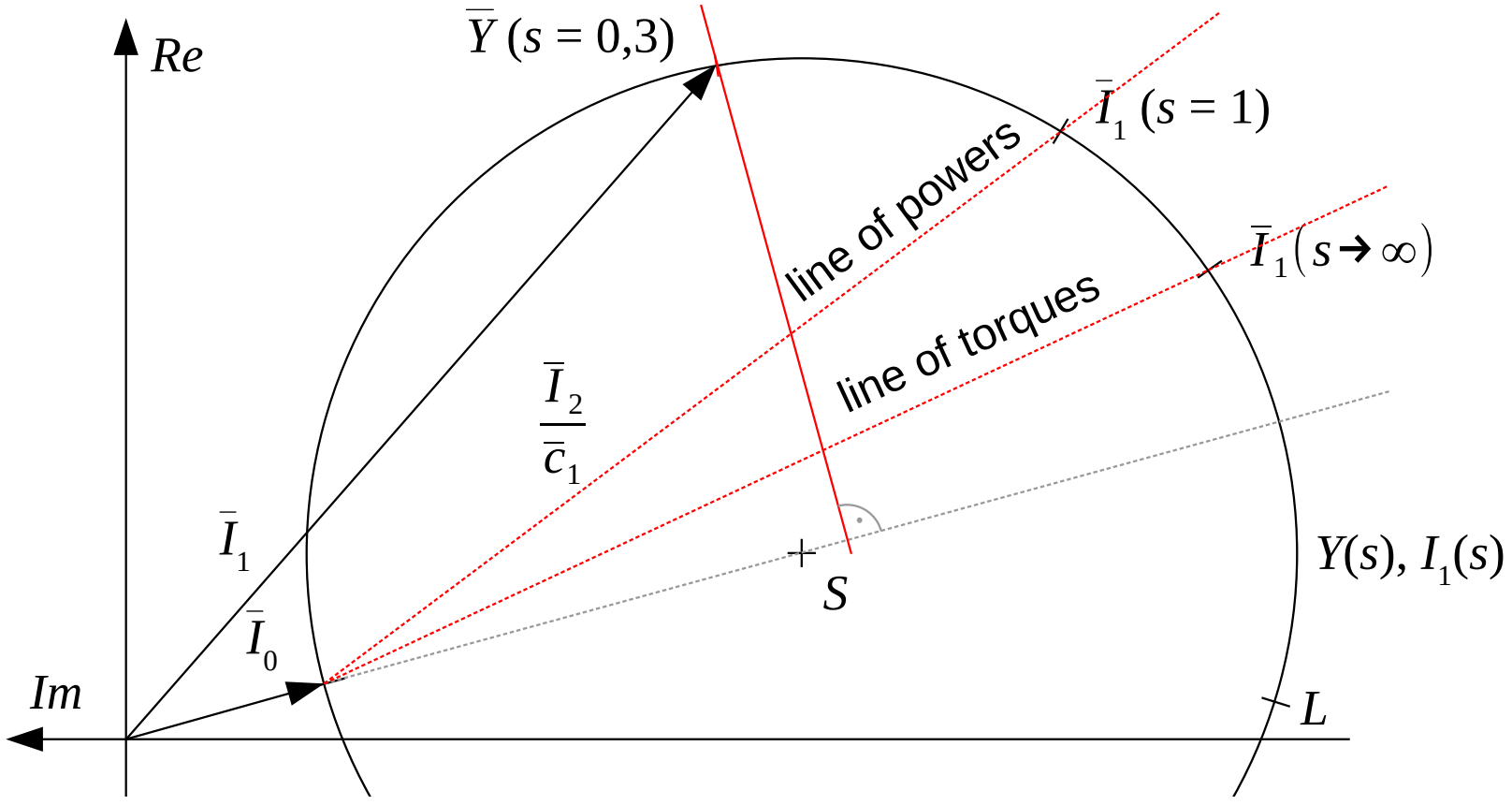
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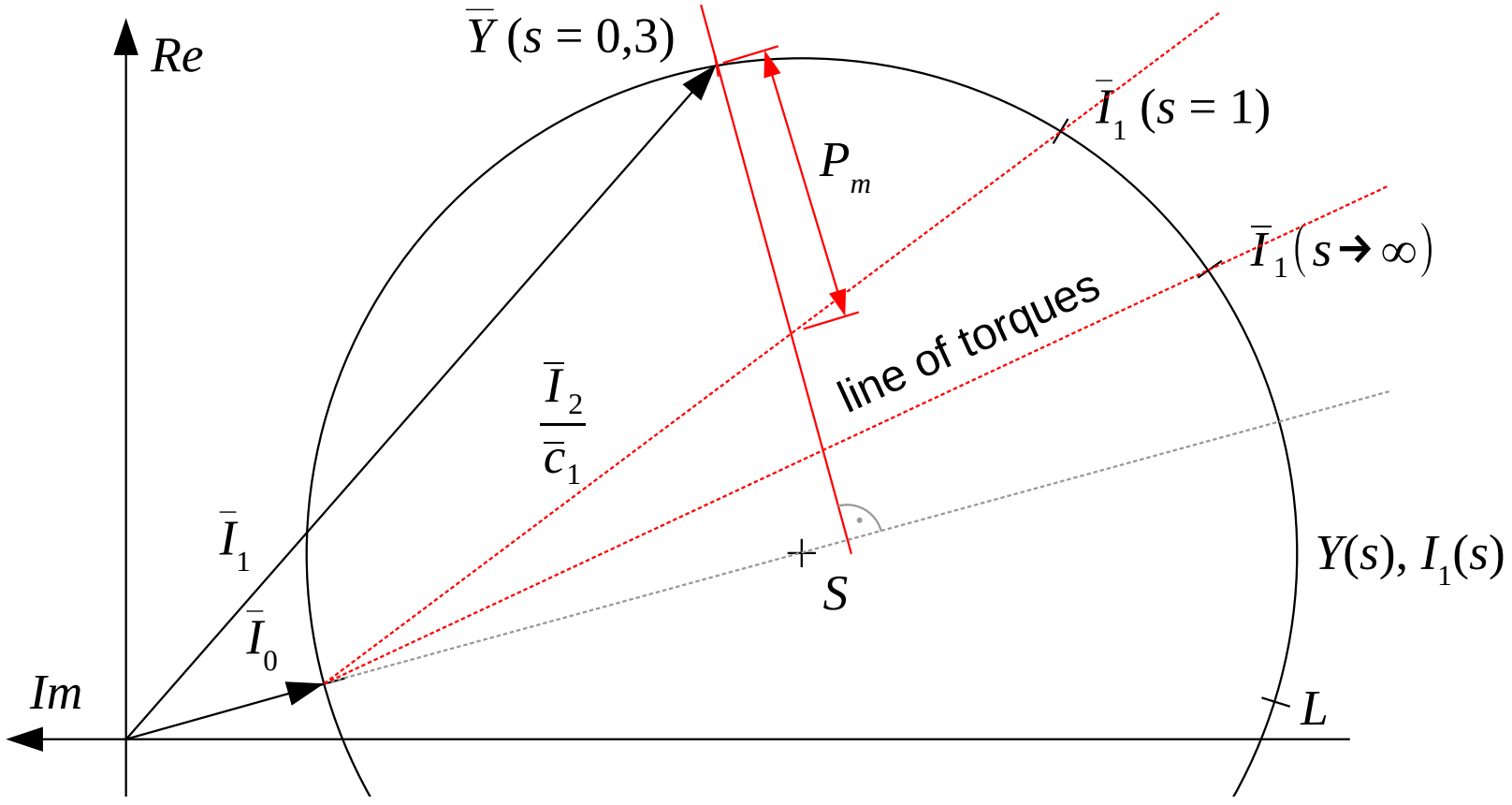
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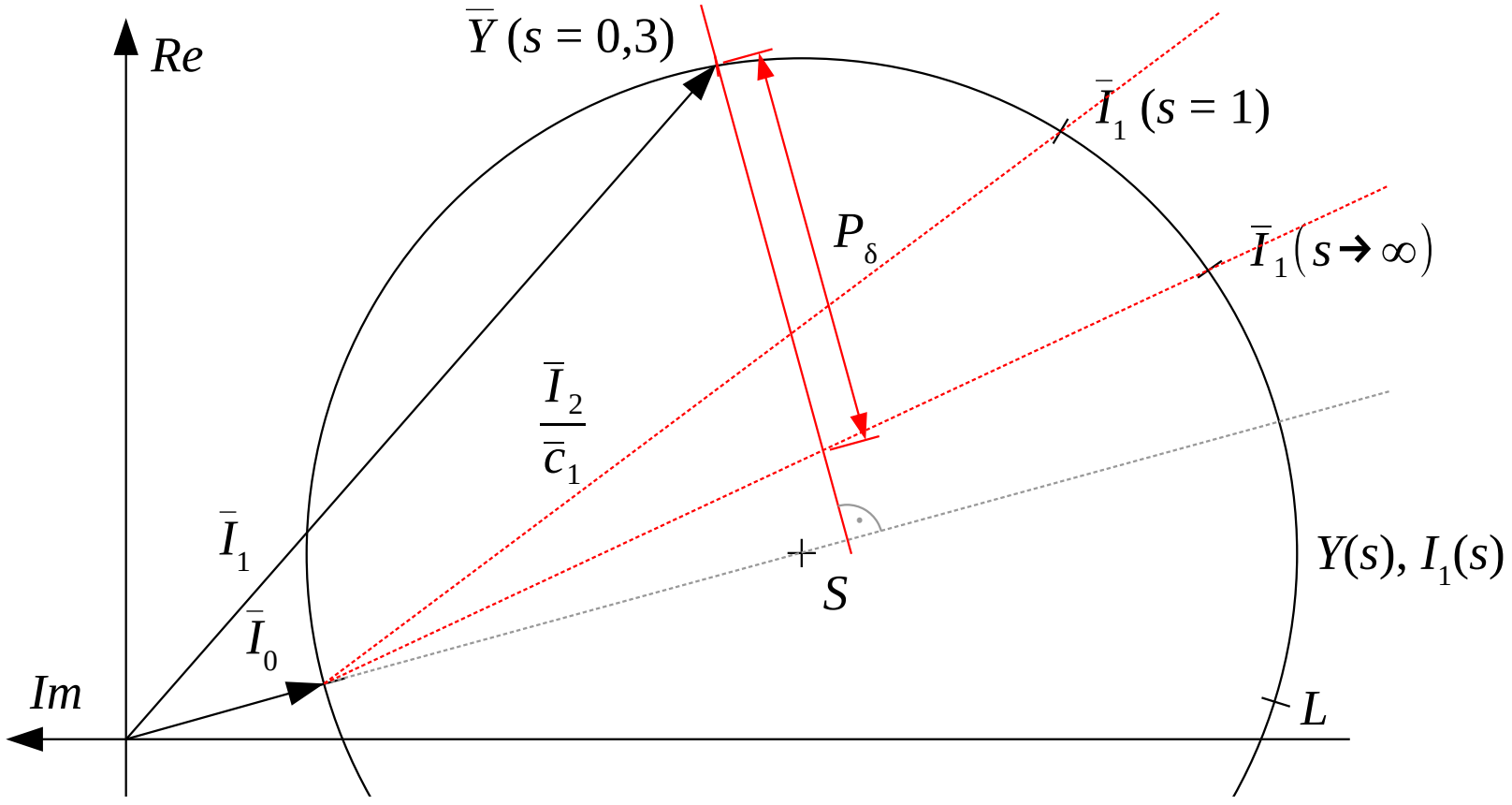
# Circle Diagram



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# Circle Diagram



# Circle Diagram

- Reading variables from the circle diagram
  - Torque:
    - The torque is proportional to the air gap power

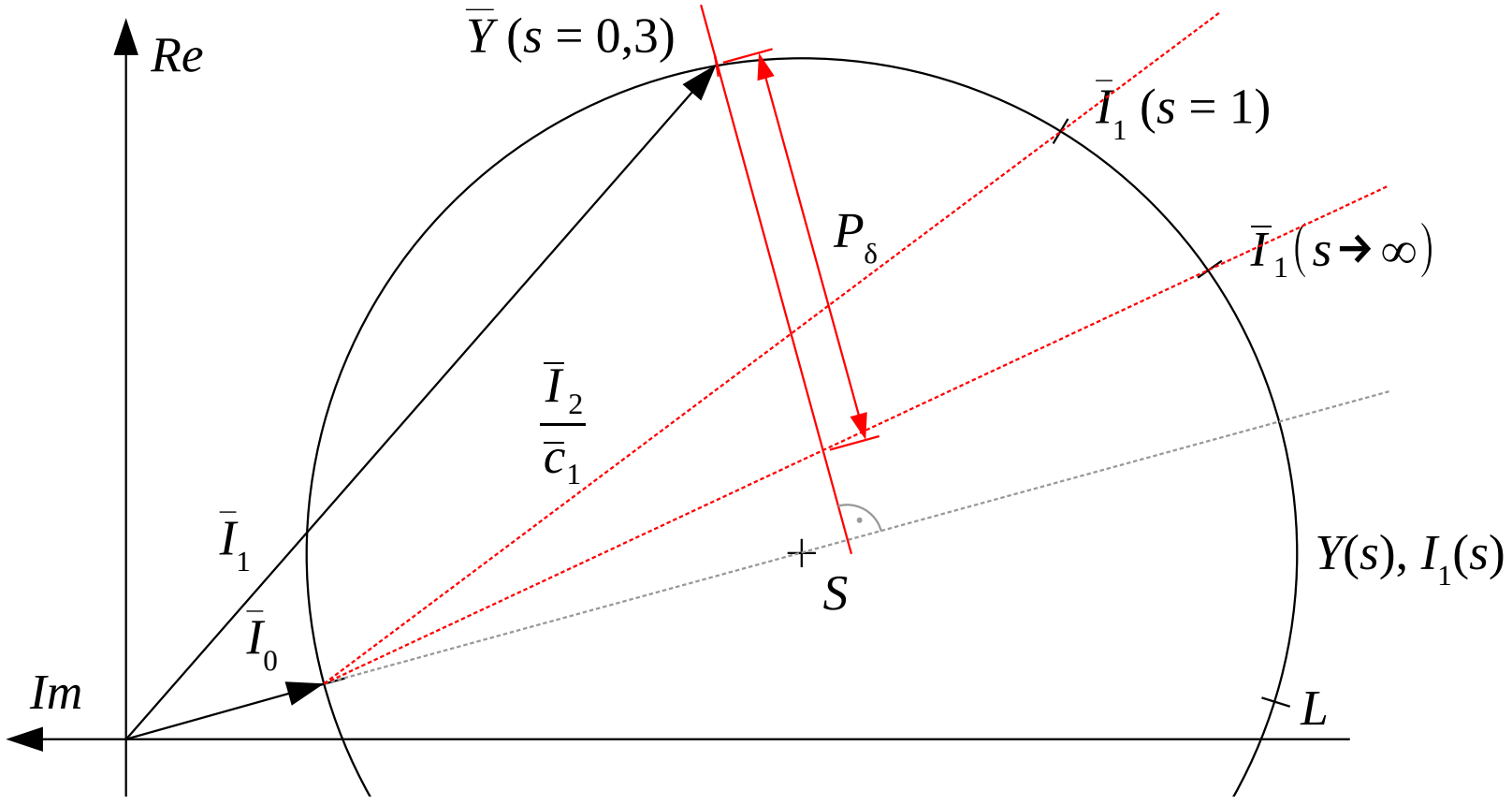
$$M = \frac{P_\delta}{\omega_s}$$

    - Thus the scale of torques:

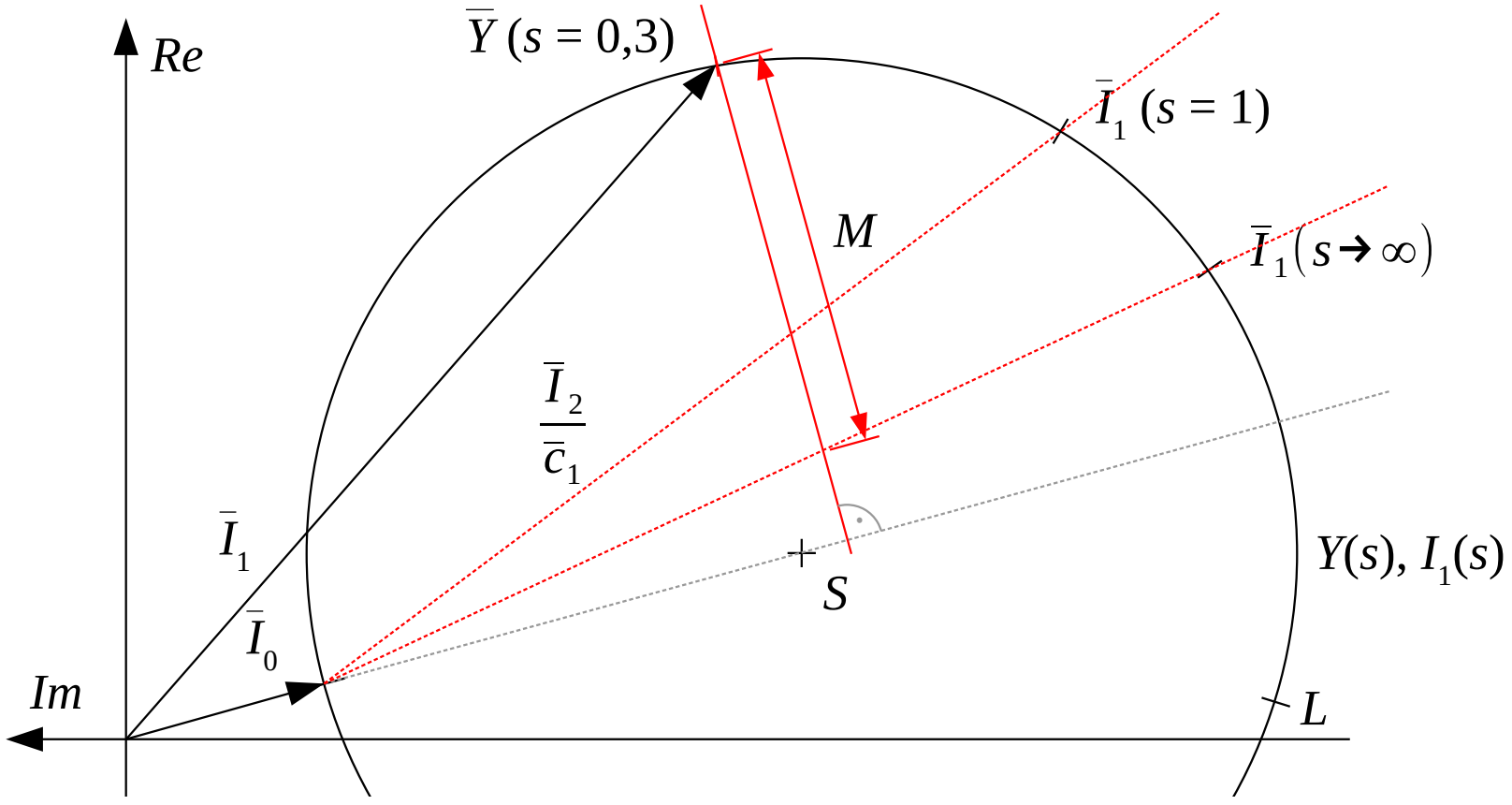
$$m_m = \frac{m_p}{\omega_s}$$

… where $\omega_s$ is the synchronous speed of the machine

# Circle Diagram



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MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY
ESF II – project of the University of West Bohemia
reg. no. CZ.02.2.69/0.0/0.0/18_056/0013239

# Epilogue

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- The circle diagram does not consider saturation effects
  - There are inaccuracies near to no-load and short-circuit points
- In case of machines with significant frequency dependencies it deforms
  - Double and Boucherot squirrel cages
- Doues not consider possible converter power supply
  - For different voltage levels, the circle would differ in diameter
- Yet it is often used for the description of the duty of an induction machine
  - Relatively simple for calculation
  - May be drawn on base of measured data
  - In case of sufficient calculation power it is possible to introduce non-linearity of machine parameters

# Epilogue

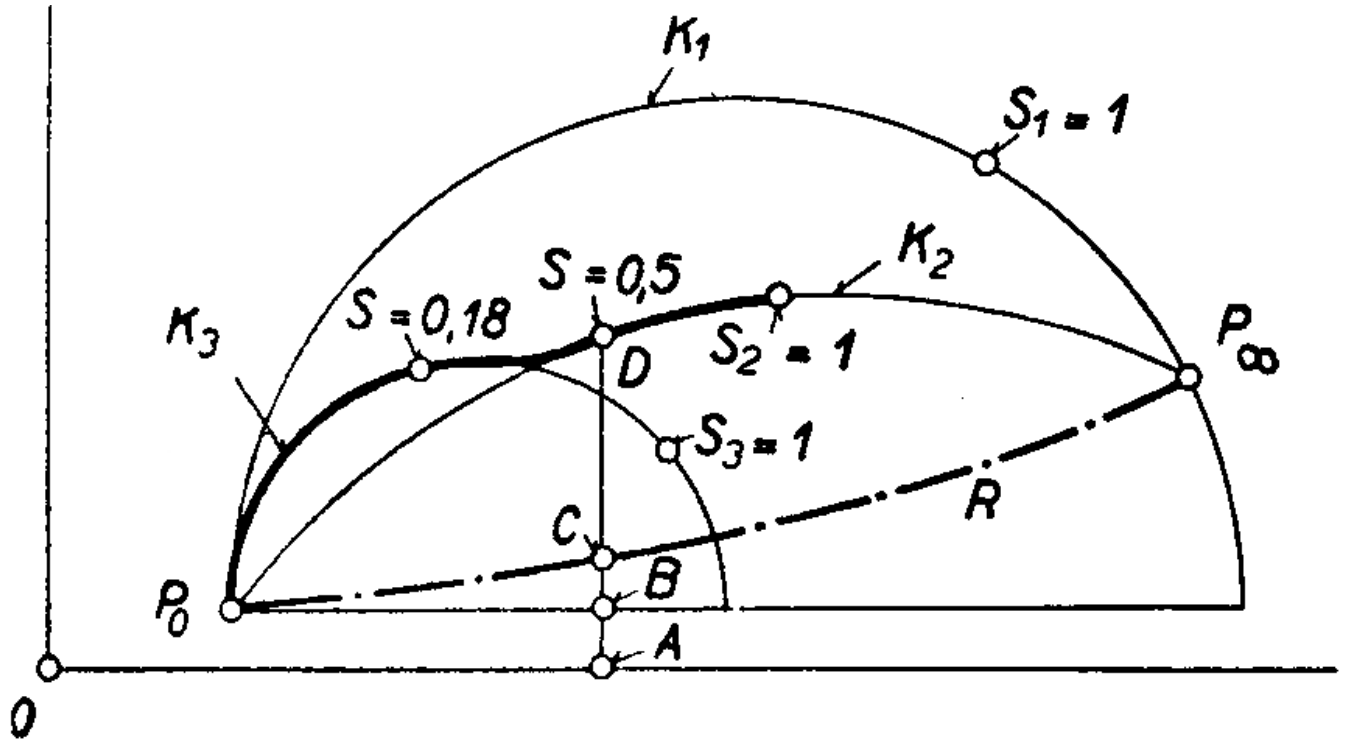



*Fig. D-67. Geometric place of current and torque for a machine with eddy-current squirrel cage.*

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# Thank You for Your Attention!

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