FACULTY OF ELECTRICAL
ENGINEERING
UNIVERSITY OF WEST BOHEMIA

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

ESF II – project of the University of West Bohemia
reg. no. CZ.02.2.69/0.0/0.0/18_056/0013239

Theory of Electric Machines 2

KEV/TS2

Karel Hruska / r20201029

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Circle Diagram of a Synchronous Machine

Circle Diagram of a Synchronous Machine

- Similarly to the induction machine it describes the movement of the endpoint of a current phasor in complex plane in dependency on the load of the machine
- In contrast to induction machines, there is no slip of a synchronous machine
  => not only voltage, but also speed is constant
- Results directly from the phasor diagram
- From the circle diagram, the behaviour of the machine while changing excitation current / load may be determined
- In case of large machines the armature resistance may be neglected ($R_a = 0$)
- In case of smaller or traction machines it must be considered ($R_a \neq 0$)
- For further progress let us presume cylindrical rotor machine (i.e. $X_d \approx X_q$)...

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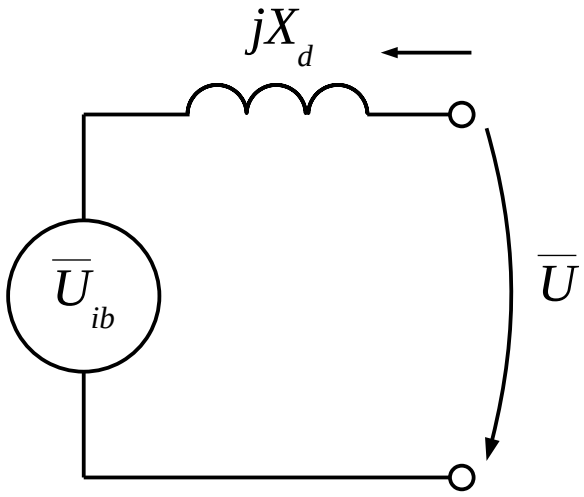
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Circle Diagram of a Synchronous Machine for $R_a = 0$

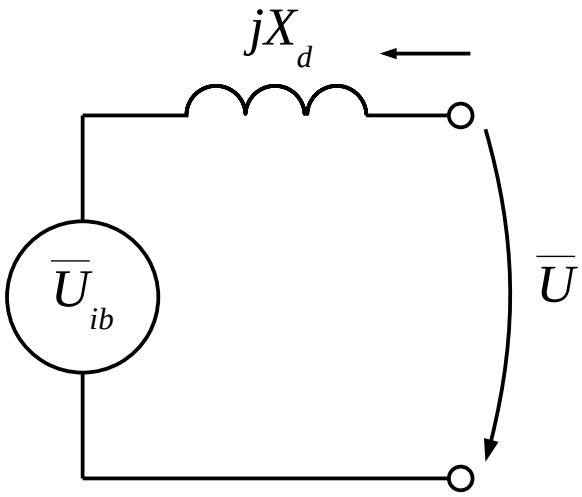
Circle Diagram of a Synchronous Machine for $R_a = 0$

- Results from simplified equivalent circuit:



Circle Diagram of a Synchronous Machine for $R_a = 0$

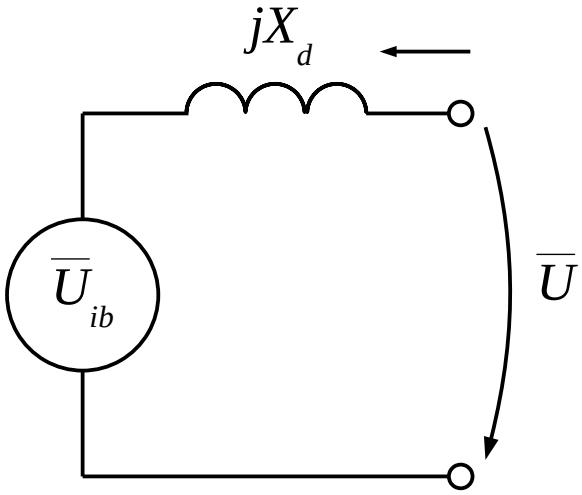
- Results from simplified equivalent circuit:



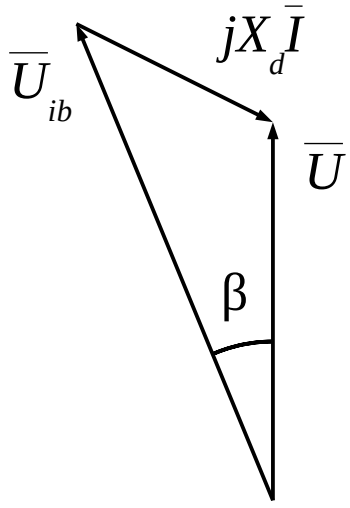
$$\bar{U} = \bar{U}_{ib} + j\,X_d\,\bar{I}$$

Circle Diagram of a Synchronous Machine for $R_a = 0$

- Results from simplified equivalent circuit:

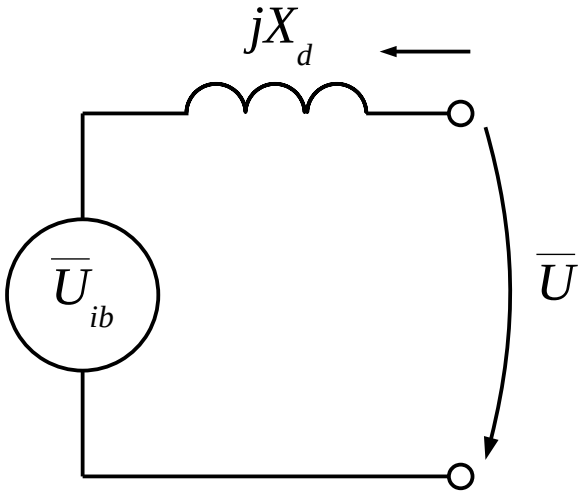


$$\bar{U} = \bar{U}_{ib} + j\,X_d\,\bar{I}$$

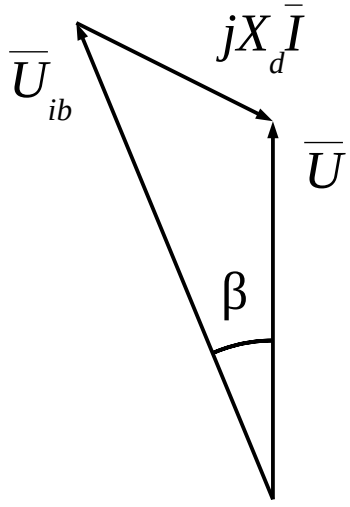


Circle Diagram of a Synchronous Machine for $R_a = 0$

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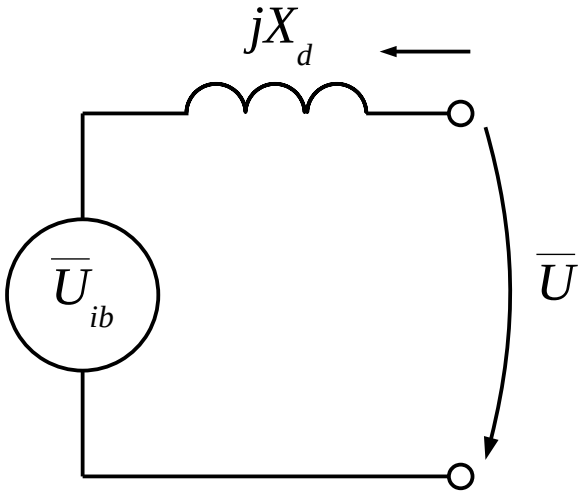


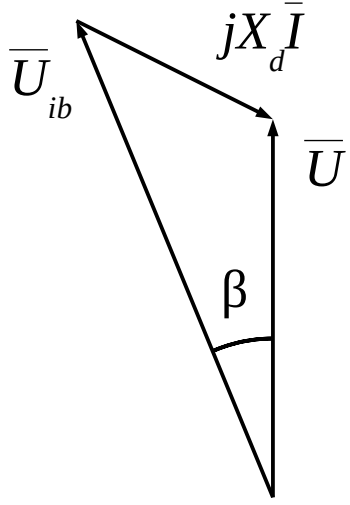
$$\frac{\bar{U}}{jX_d} = \frac{\bar{U}_{ib}}{jX_d} + \bar{I}$$



Circle Diagram of a Synchronous Machine for $R_a = 0$

- Results from simplified equivalent circuit:



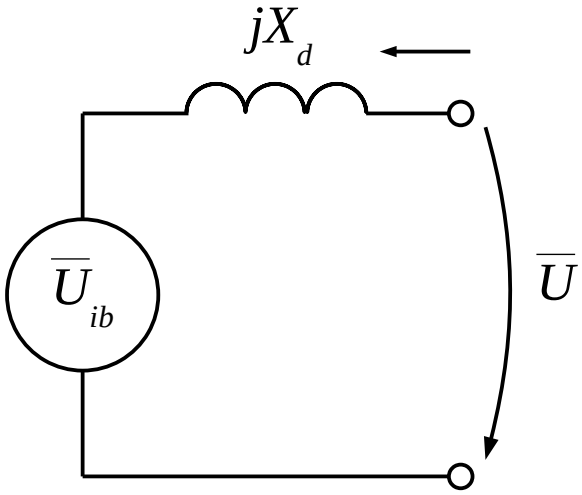


$$\frac{\bar{U}}{jX_d} = \frac{\bar{U}_{ib}}{jX_d} + \bar{I}$$

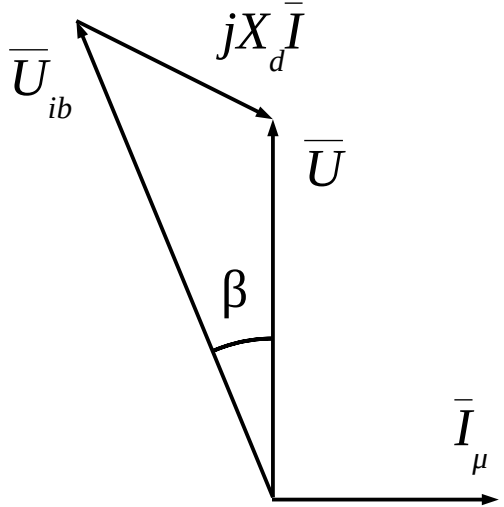
$$\frac{\bar{U}}{jX_d} = \bar{I}_{\mu}$$

Circle Diagram of a Synchronous Machine for $R_a = 0$

- Results from simplified equivalent circuit:



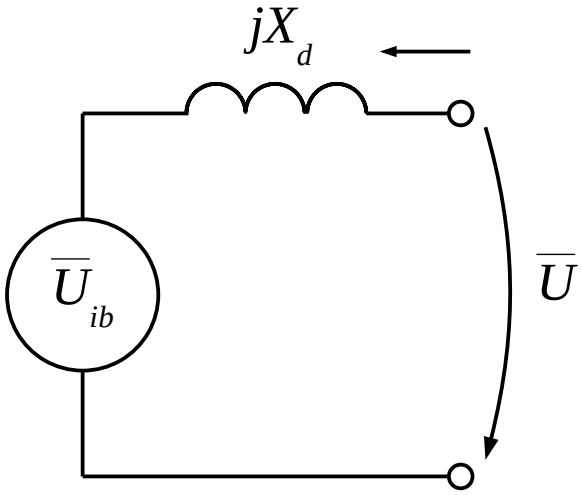
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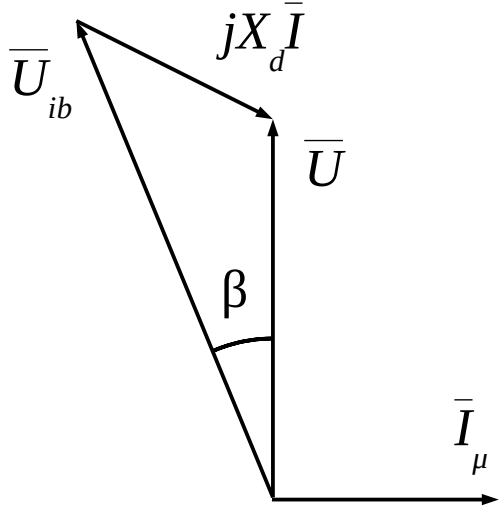


$$\frac{\bar{U}}{jX_d} = \bar{I}_\mu$$

Circle Diagram of a Synchronous Machine for $R_a = 0$

- Results from simplified equivalent circuit:



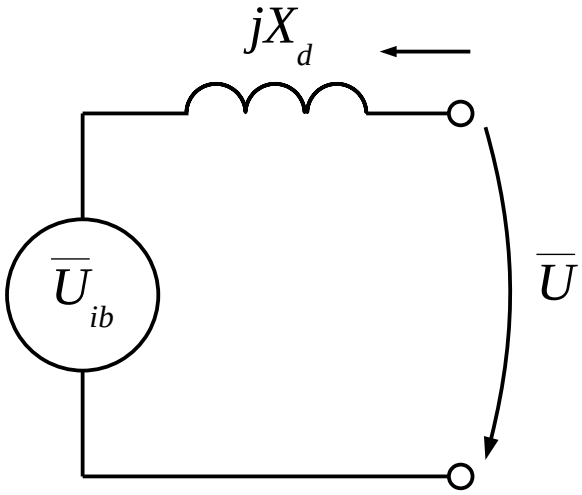


$$\bar{I}_\mu = \frac{\bar{U}_{ib}}{j X_d} + \bar{I}$$

$$\frac{\bar{U}_{ib}}{j X_d} = \bar{I}_k$$

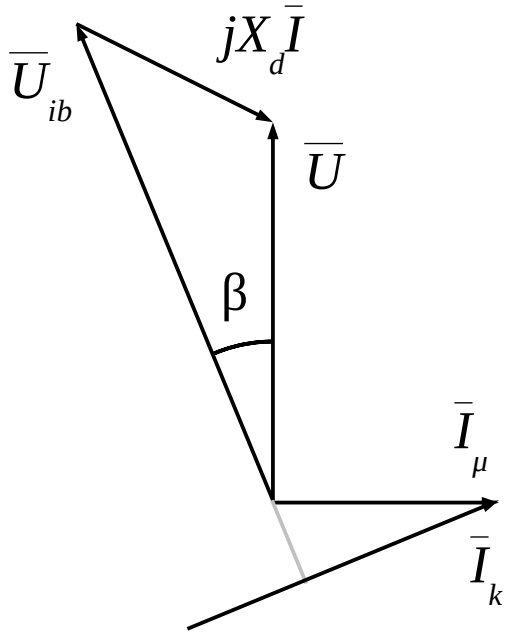
Circle Diagram of a Synchronous Machine for $R_a = 0$

- Results from simplified equivalent circuit:



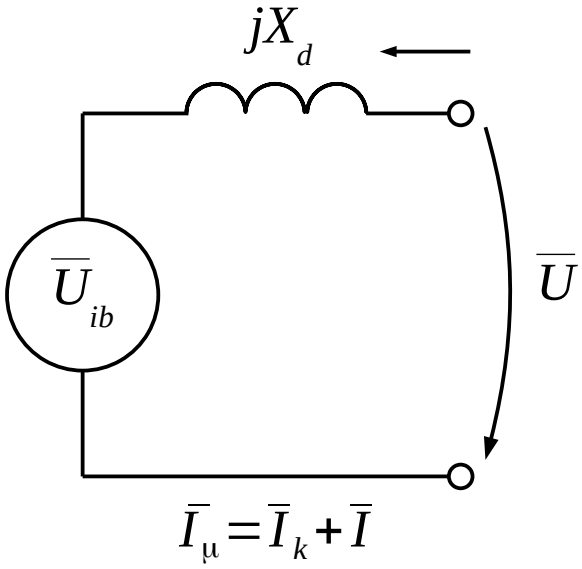
$$\bar{I}_\mu = \frac{\bar{U}_{ib}}{j X_d} + \bar{I}$$

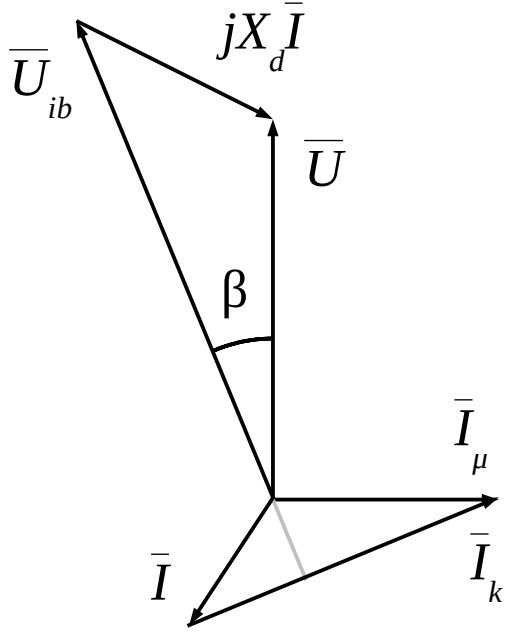
$$\frac{\bar{U}_{ib}}{j X_d} = \bar{I}_k$$



Circle Diagram of a Synchronous Machine for $R_a = 0$

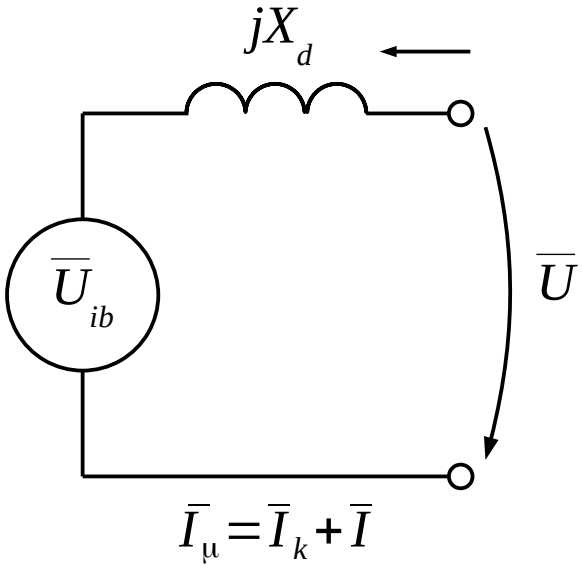
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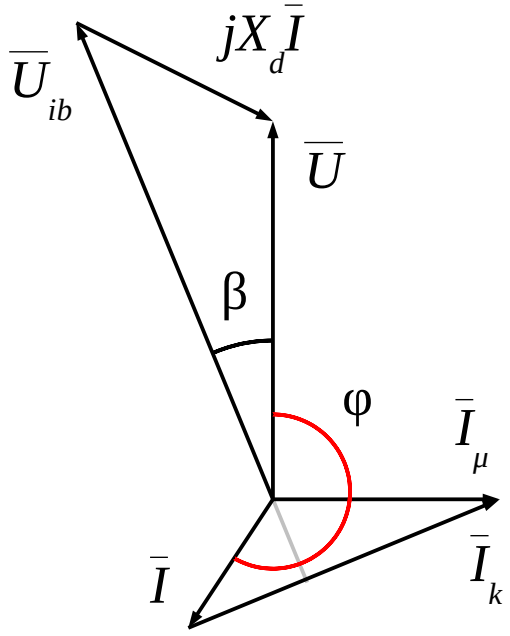




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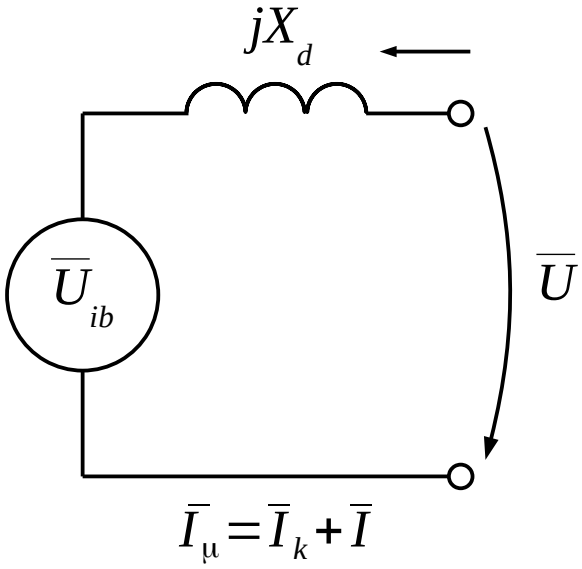
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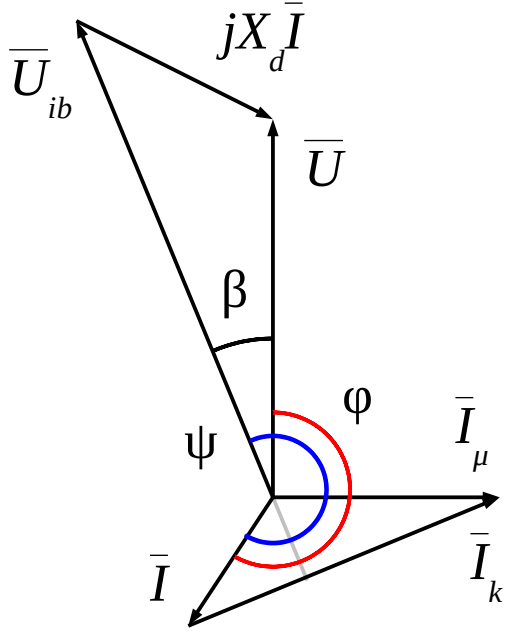




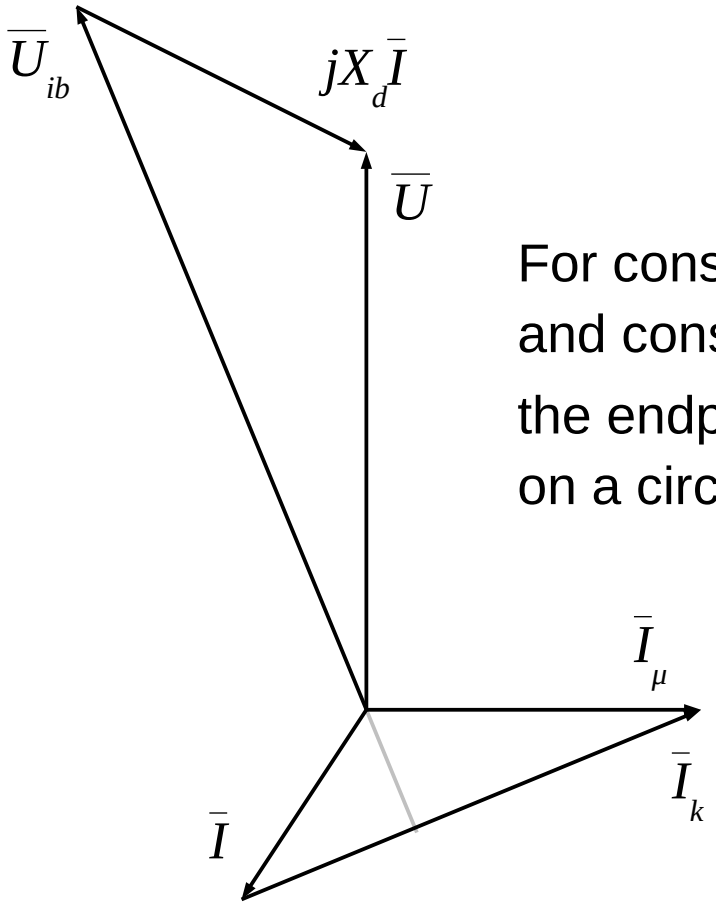
Circle Diagram of a Synchronous Machine for $R_a = 0$

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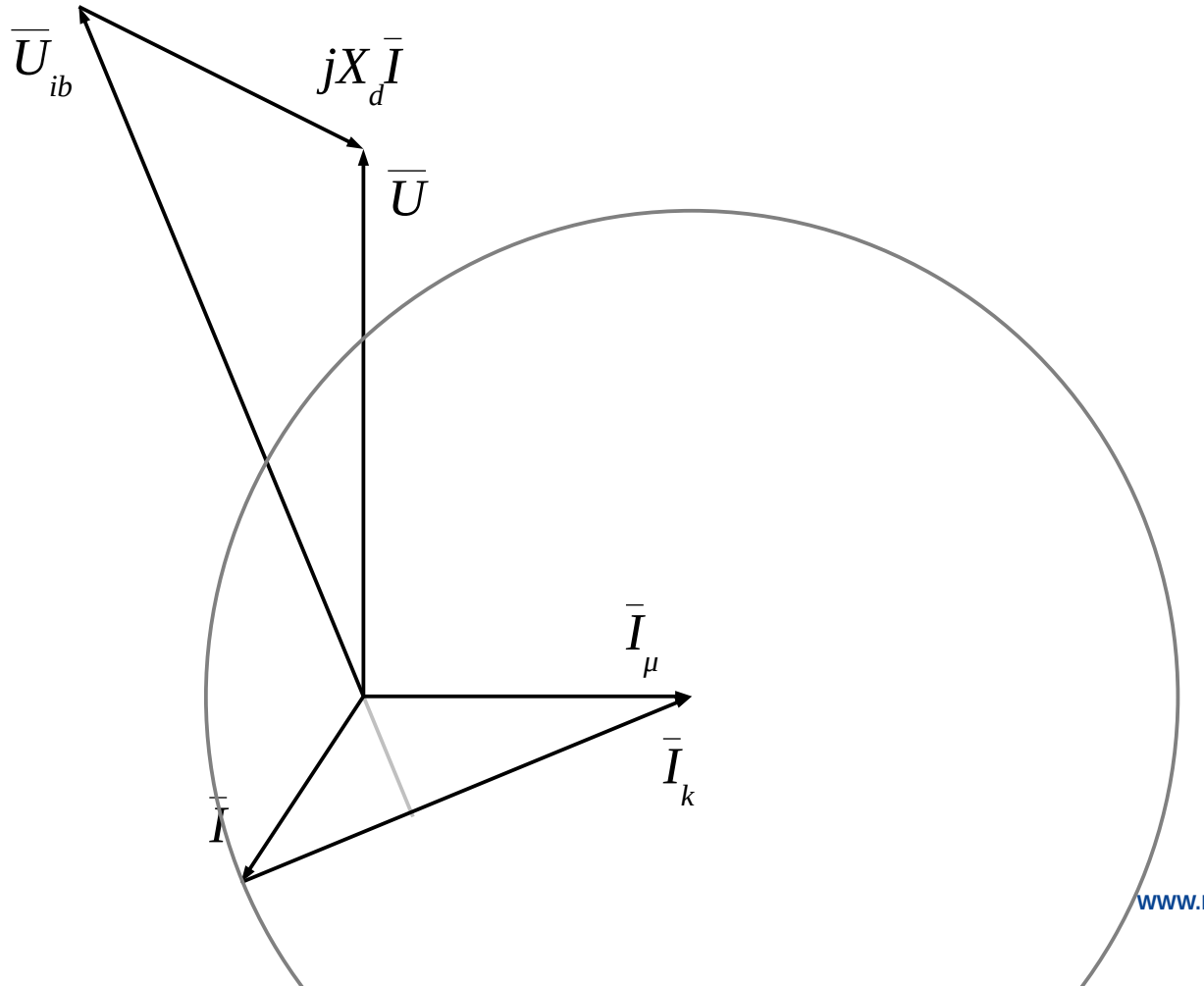


Circle Diagram of a Synchronous Machine for $R_a = 0$

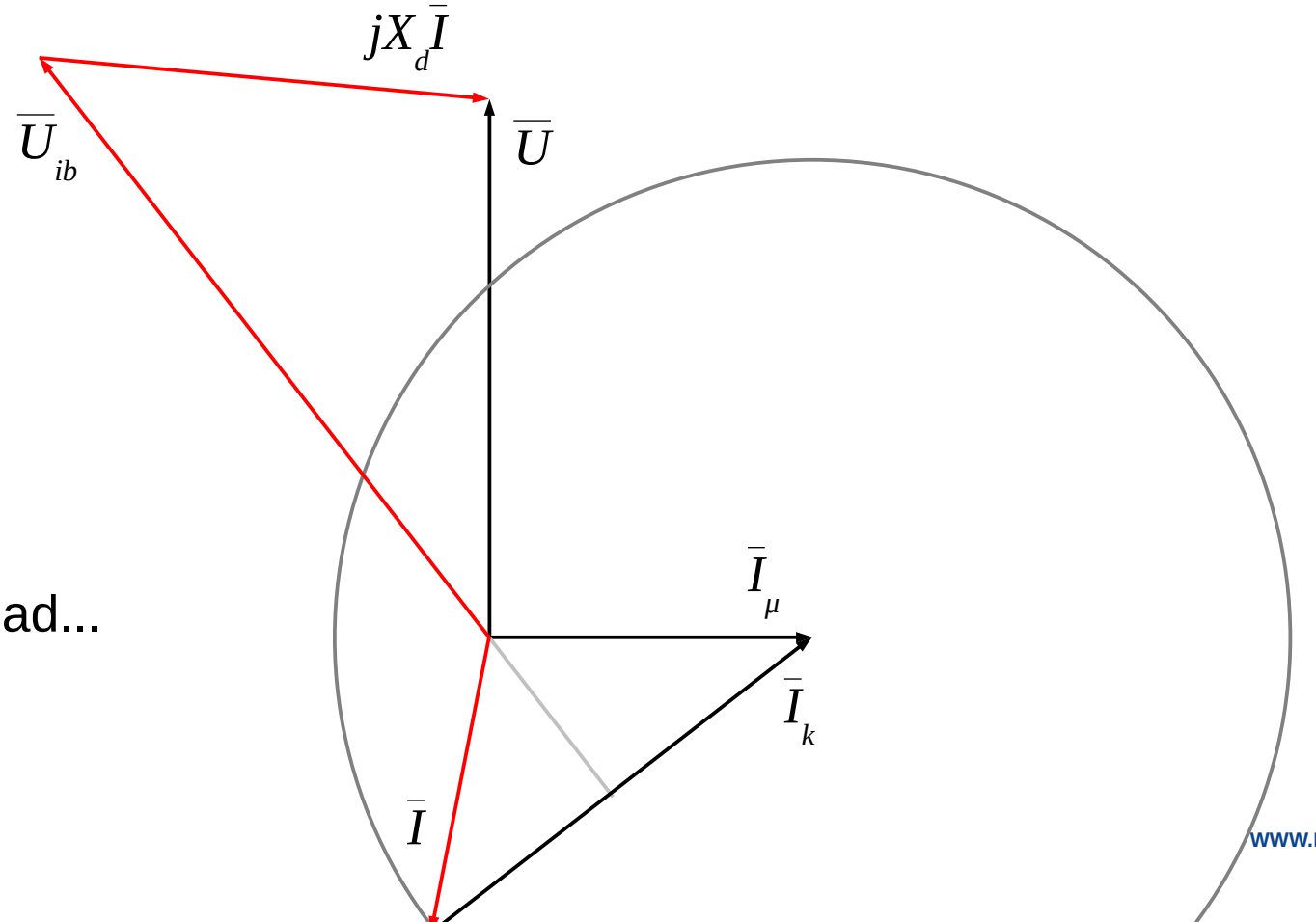


For constant voltage U (network) and constant induced voltage U_{ib} the endpoint of vector $\bar{I}$ moves on a circle with diameter $\bar{I}_k$

Circle Diagram of a Synchronous Machine for $R_a = 0$



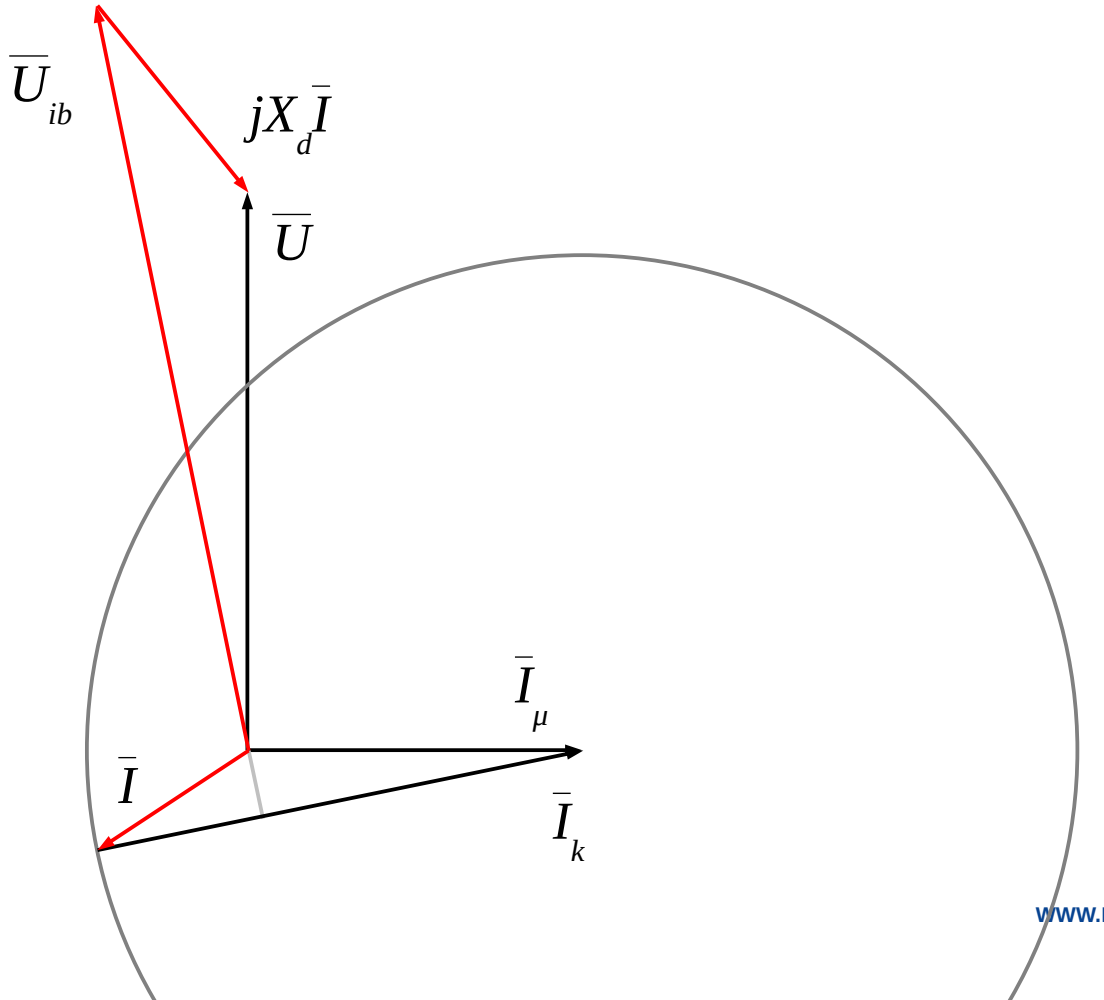
Circle Diagram of a Synchronous Machine for $R_a = 0$



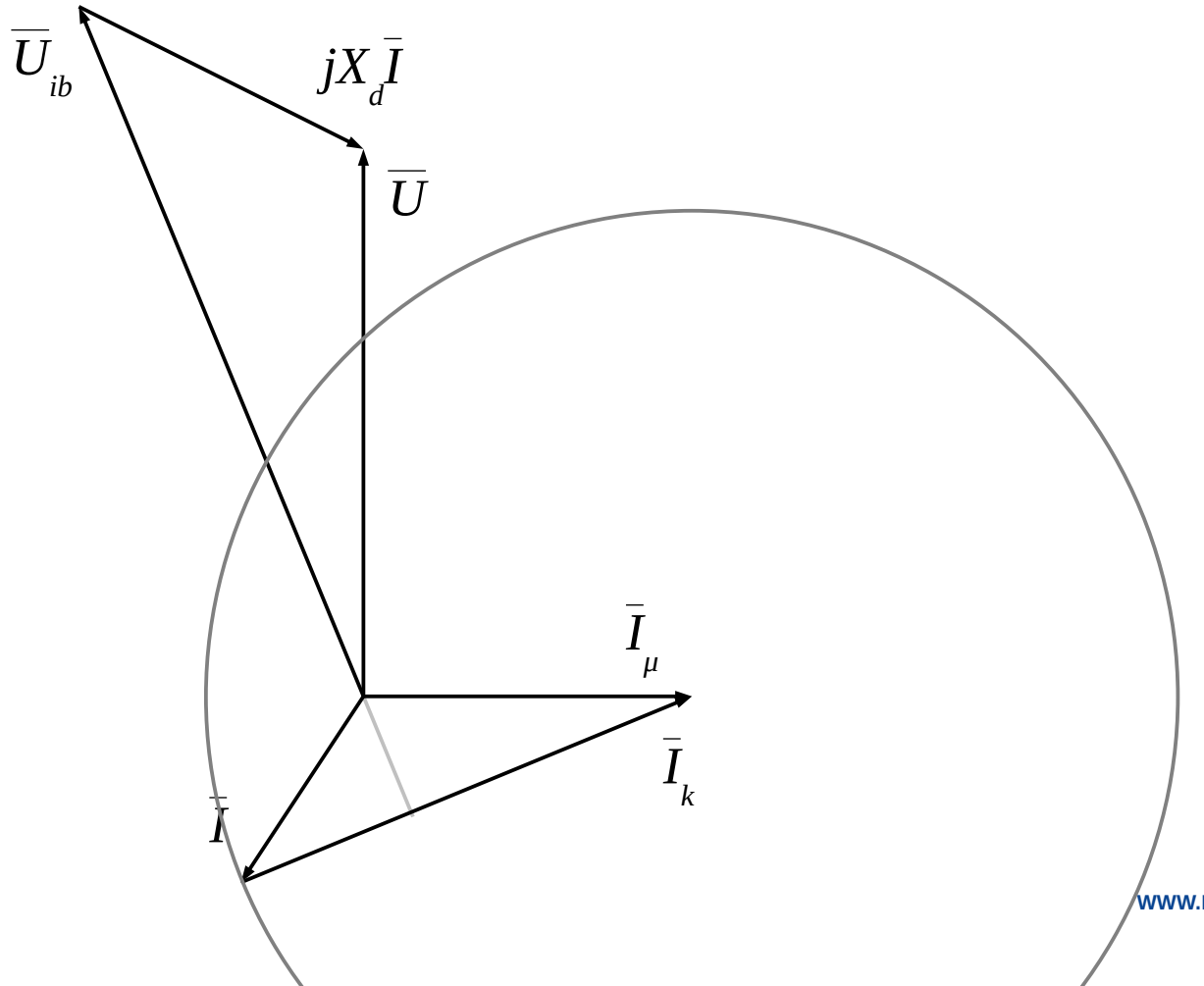
...for higher load...

Circle Diagram of a Synchronous Machine for $R_a = 0$

...for lower load...

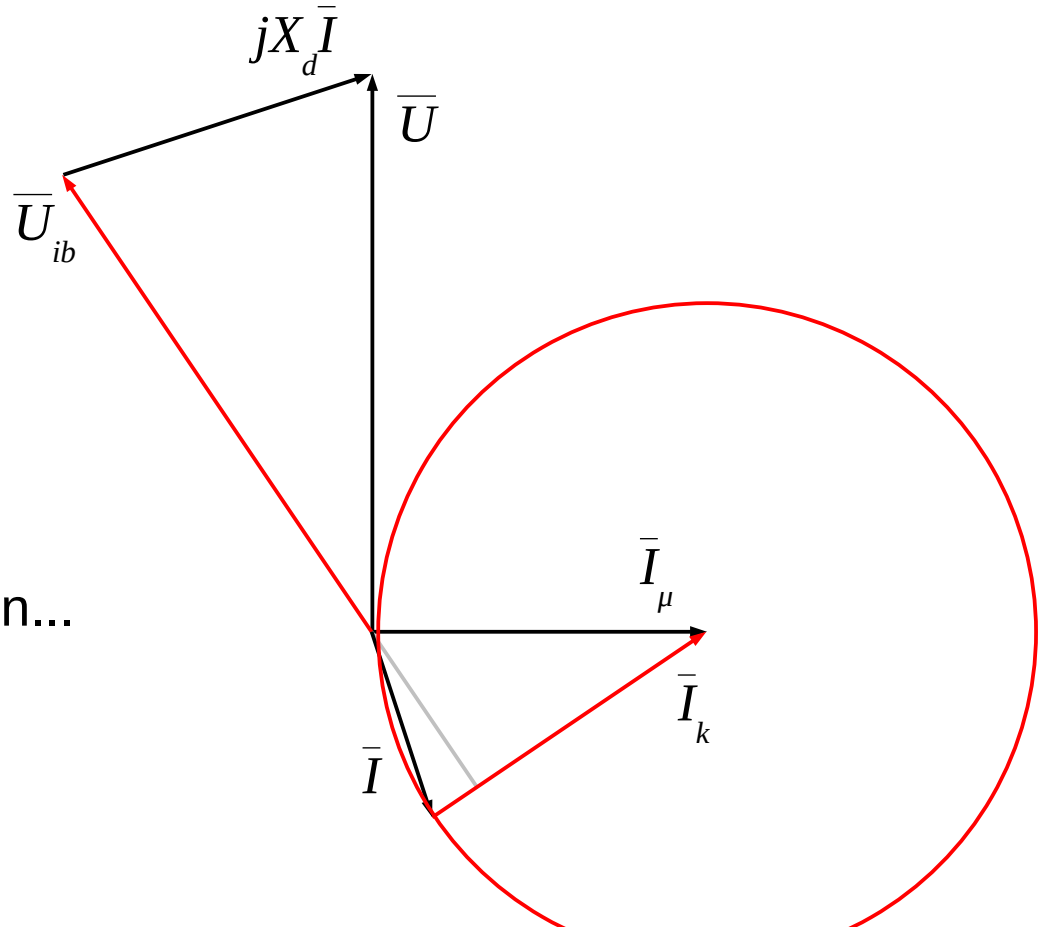


Circle Diagram of a Synchronous Machine for $R_a = 0$

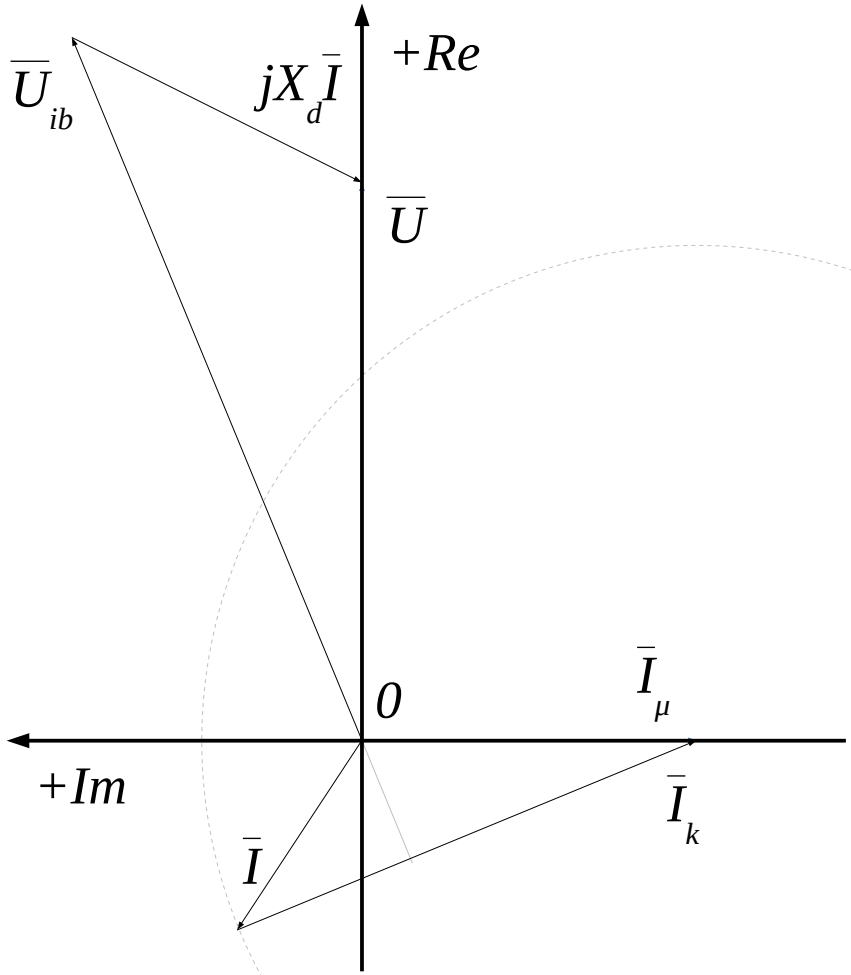


Circle Diagram of a Synchronous Machine for $R_a = 0$

...for lower excitation...



Circle Diagram of a Synchronous Machine for $R_a = 0$



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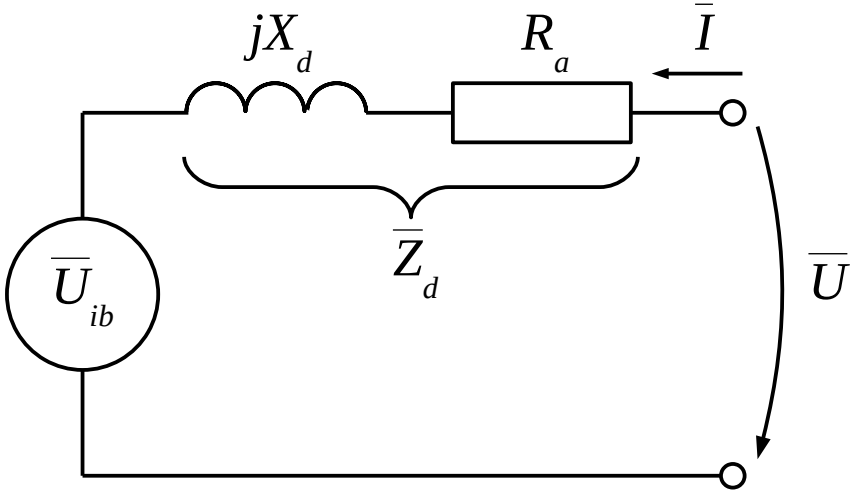
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Circle Diagram of a Synchronous Machine for $R_a \neq 0$

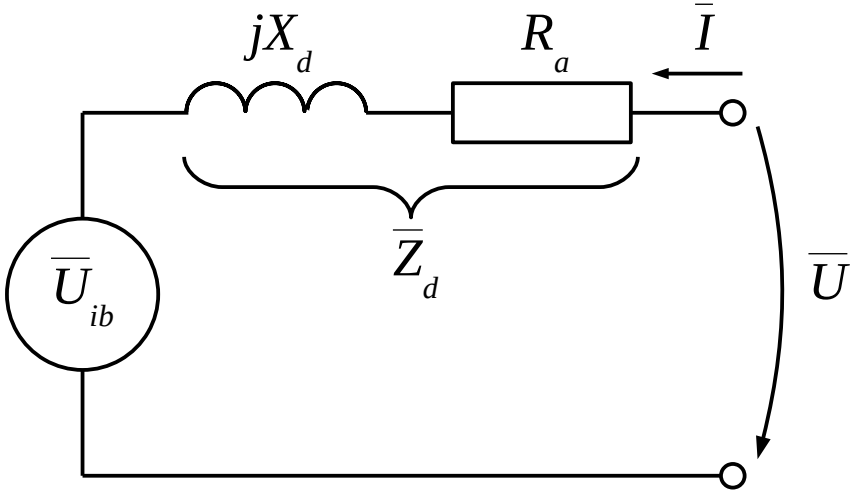
Circle Diagram of a Synchronous Machine for $R_a \neq 0$

- It is necessary to consider full equivalent circuit:



Circle Diagram of a Synchronous Machine for $R_a \neq 0$

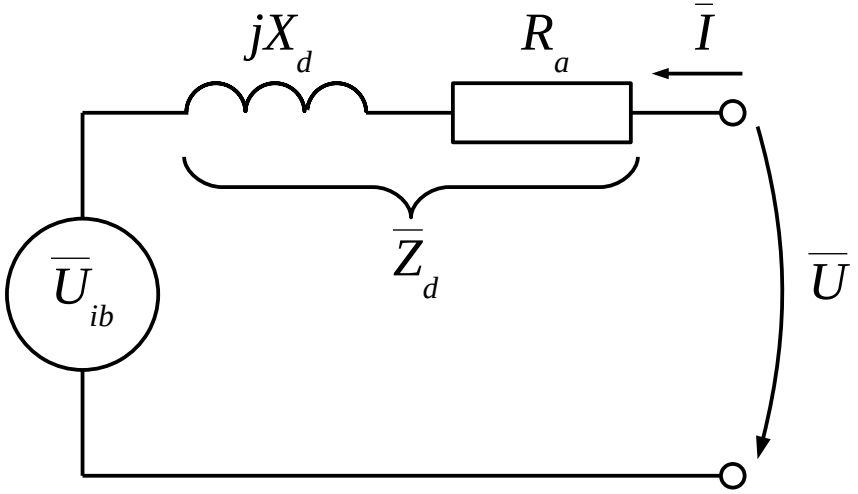
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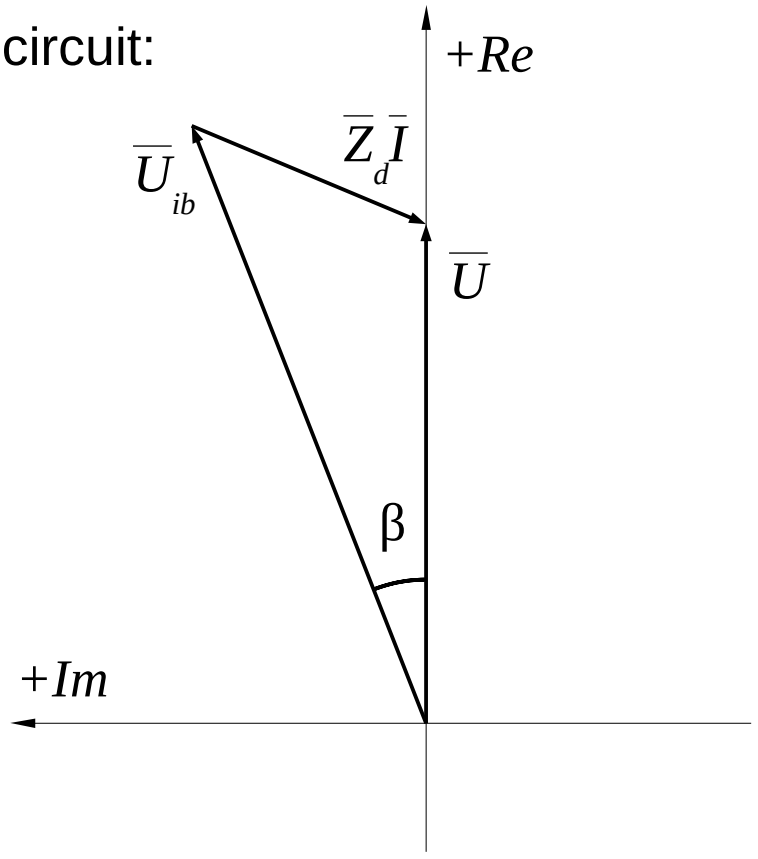
$$\bar{U} = \bar{U}_{ib} + \bar{Z}_d \bar{I}$$

Circle Diagram of a Synchronous Machine for $R_a \neq 0$

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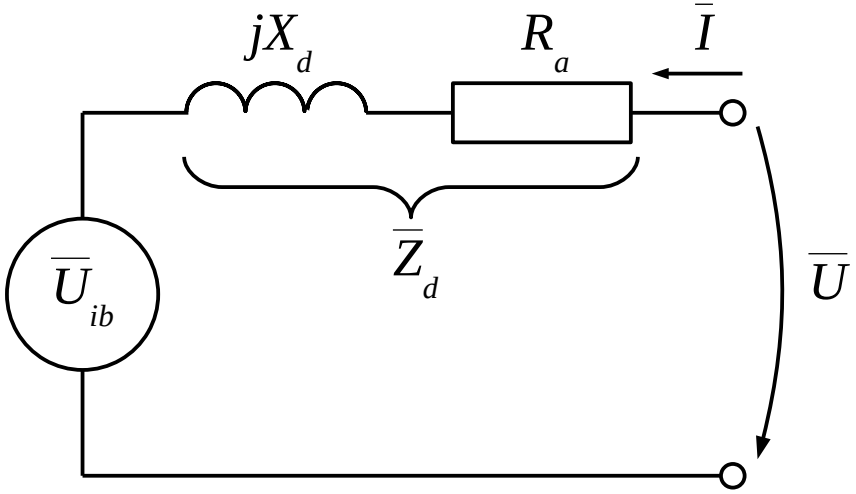


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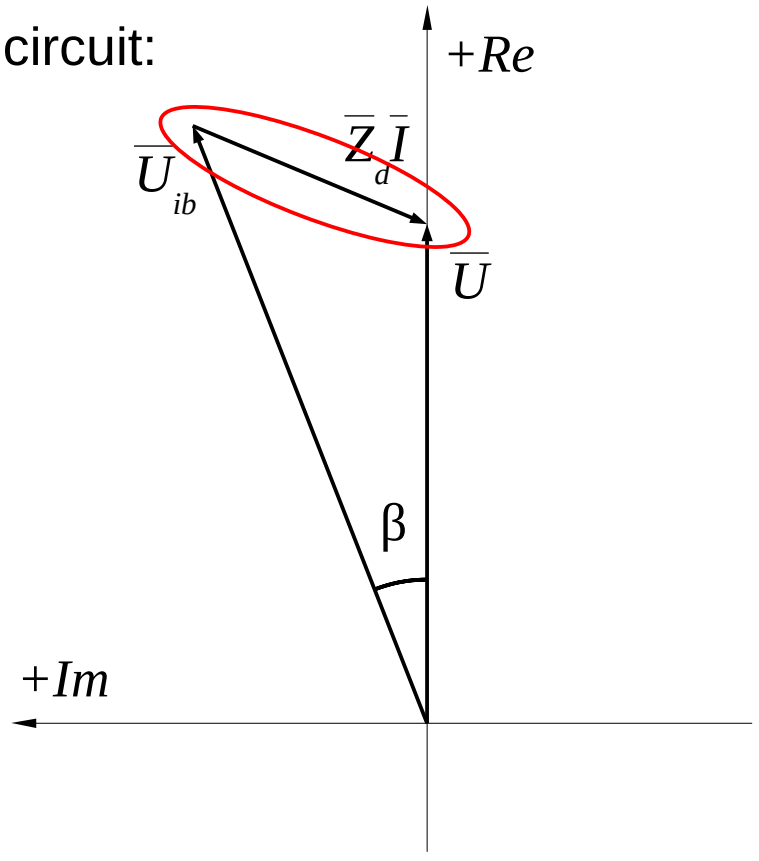


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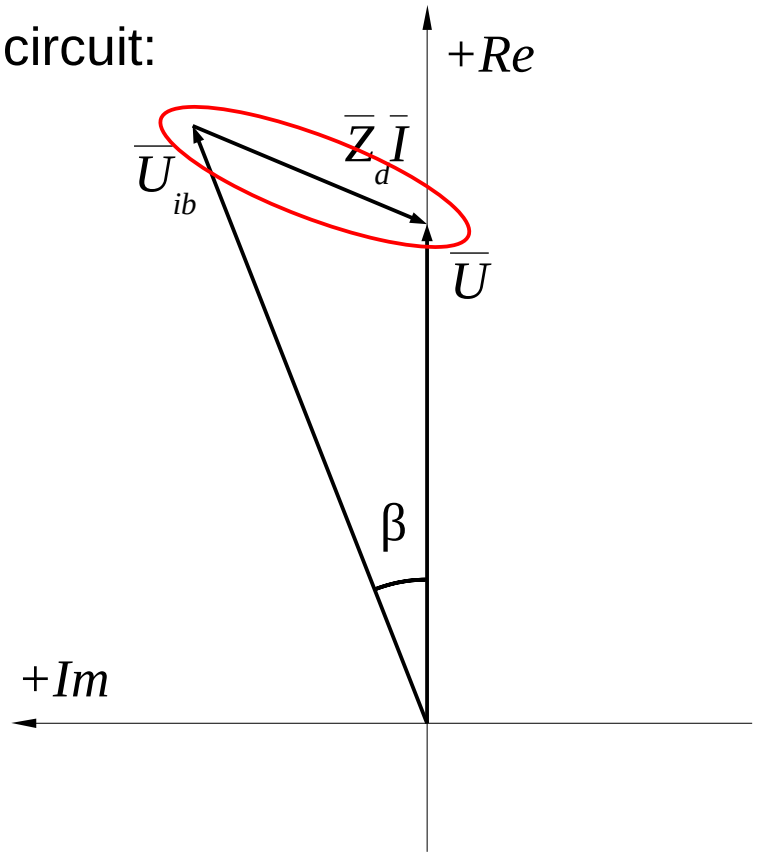


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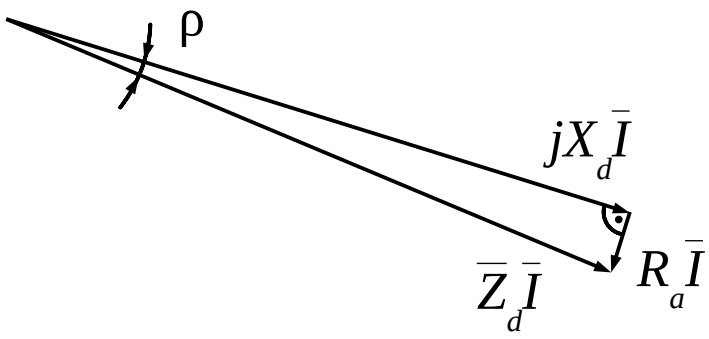
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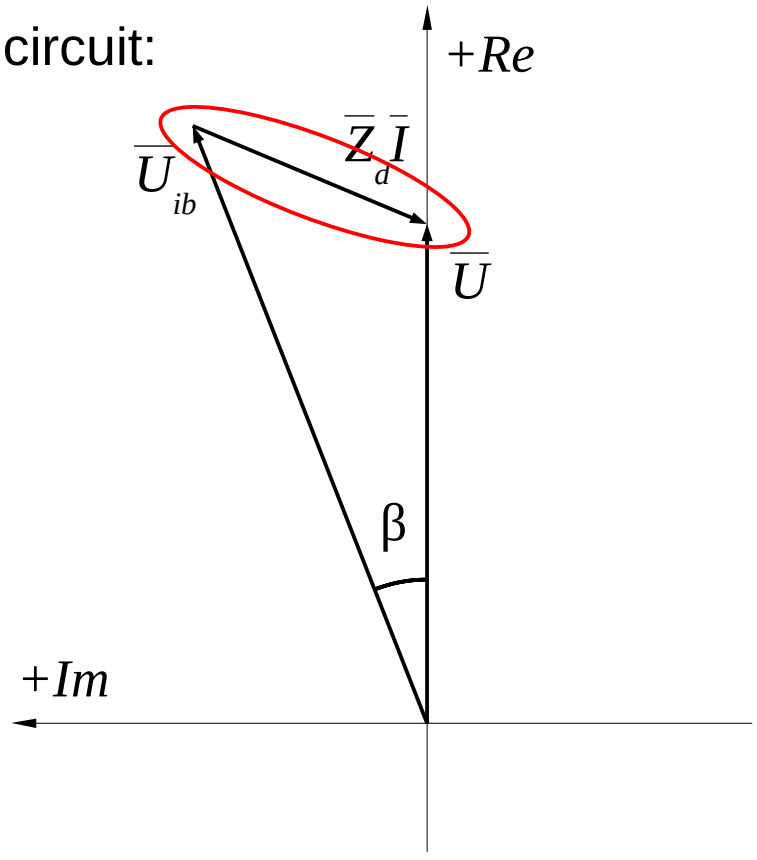
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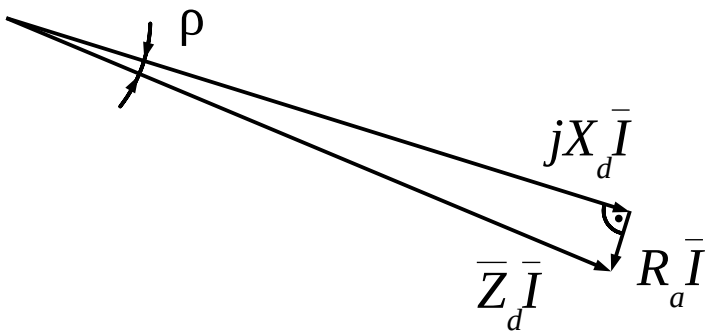
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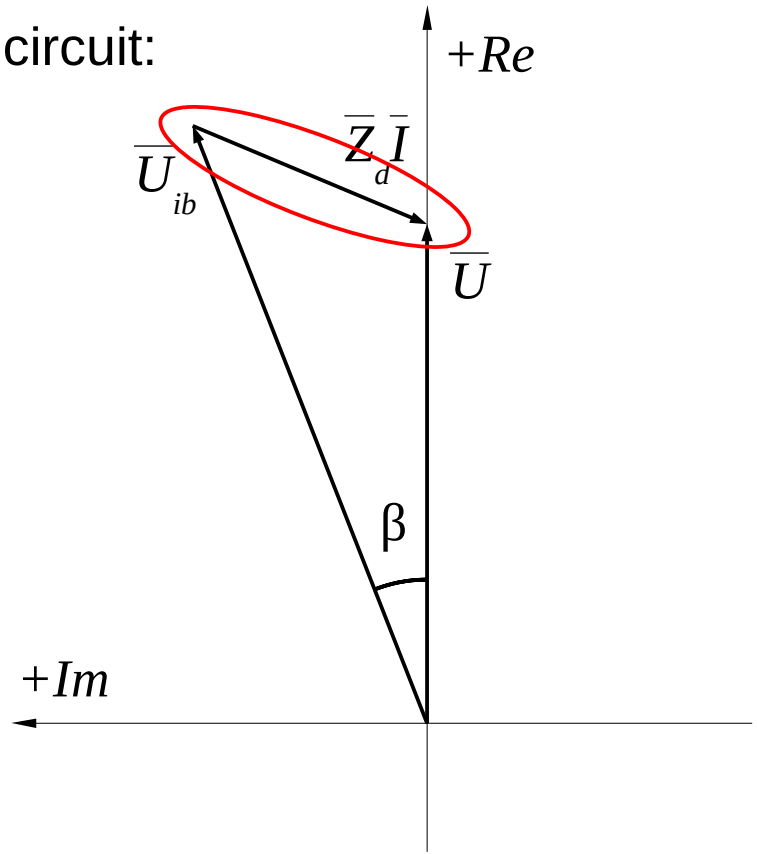


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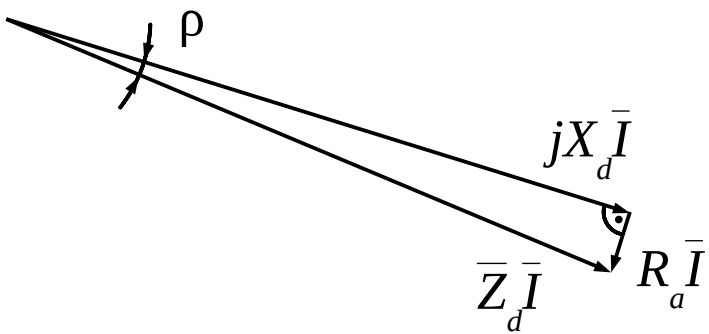


$$\bar{Z}_d\bar{I} = R_a\bar{I} + jX_d\bar{I}$$

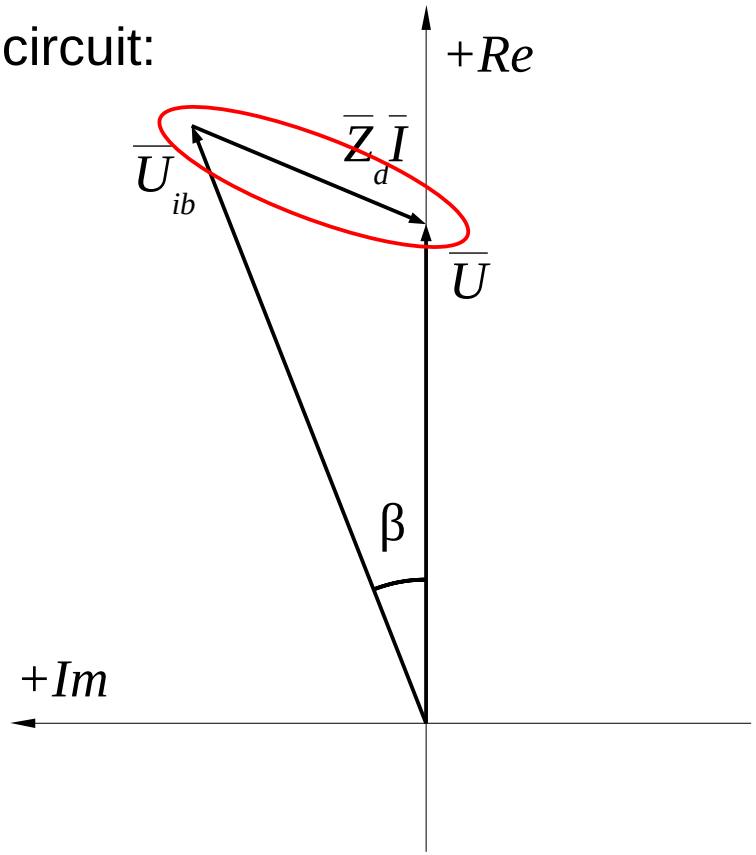


Circle Diagram of a Synchronous Machine for $R_a \neq 0$

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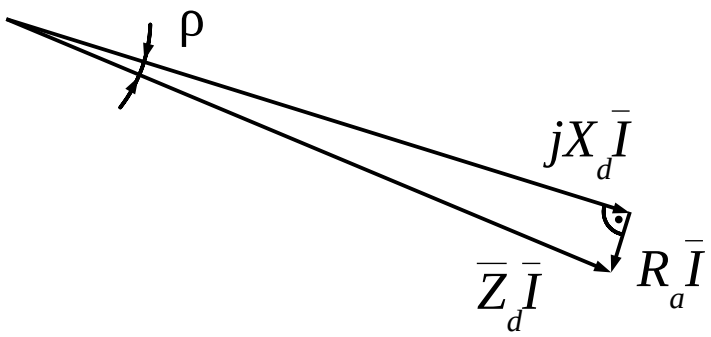


$$\bar{Z}_d = R_a + j\,X_d$$



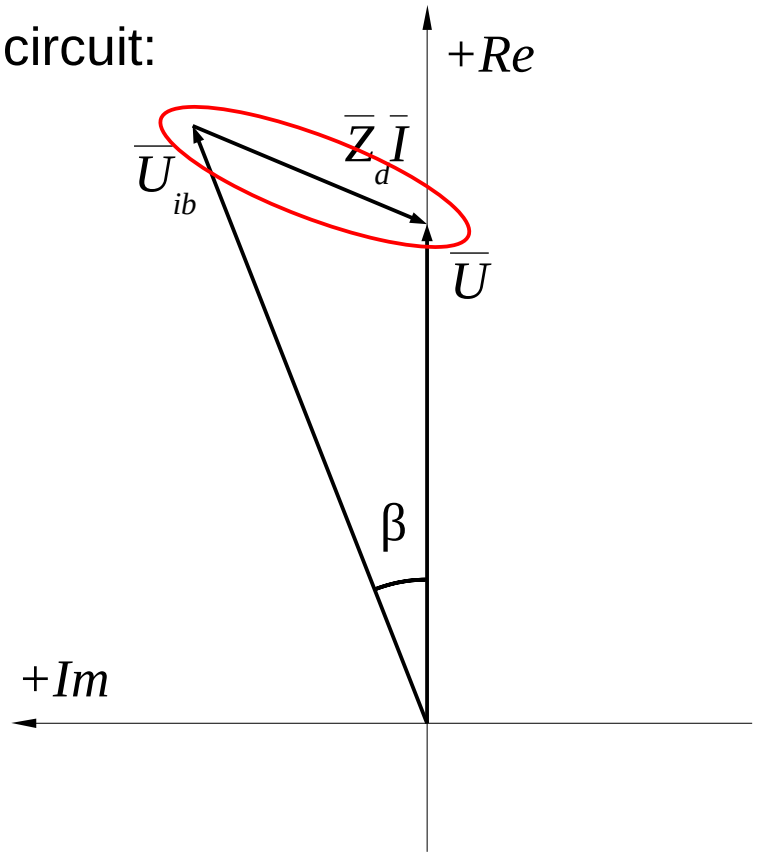
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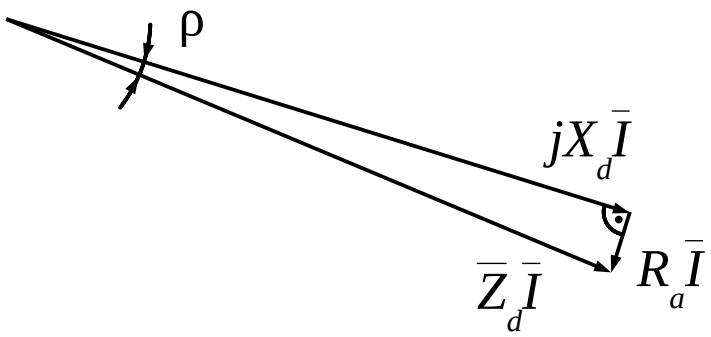
$$\bar{Z}_d = R_a + j\,X_d$$

$$\operatorname{tg}\rho = \frac{R_a}{X_d}$$



Circle Diagram of a Synchronous Machine for $R_a \neq 0$

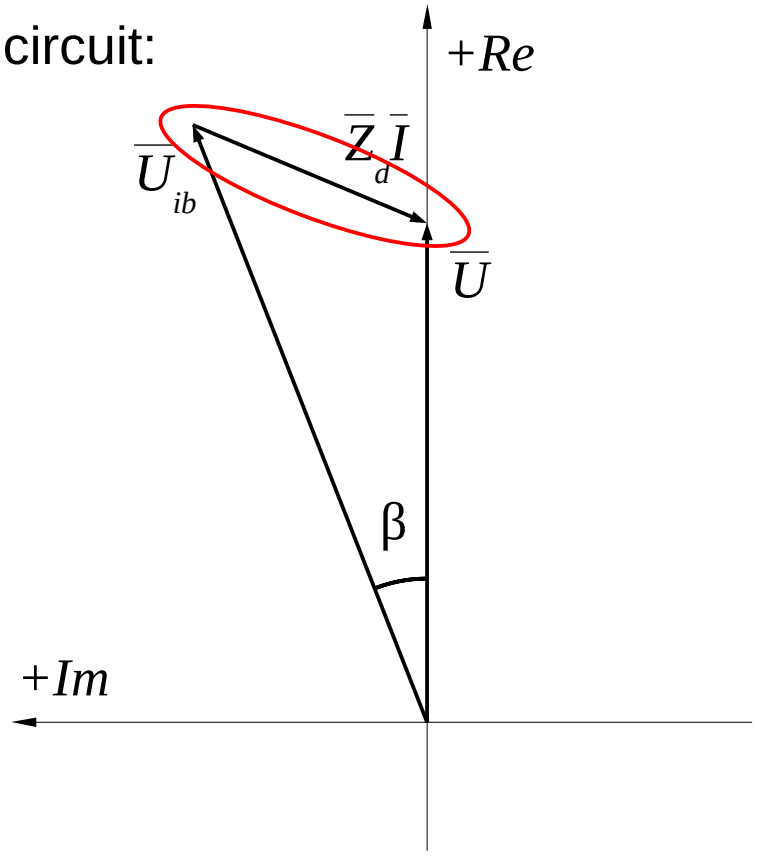
- It is necessary to consider full equivalent circuit:



$$\bar{Z}_d = R_a + j\,X_d$$

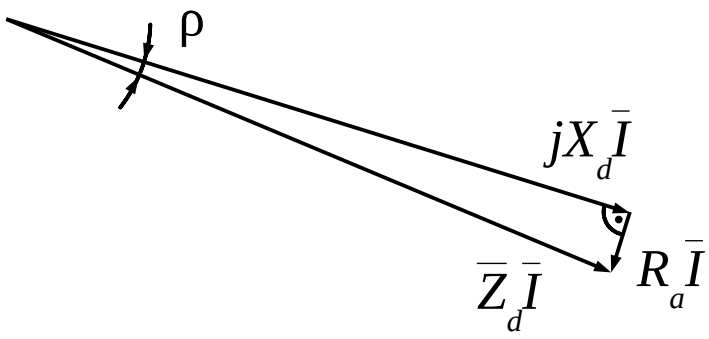
$$\operatorname{tg}\rho = \frac{R_a}{X_d}$$

$$Z_d = \sqrt{R_a^2 + X_d^2}$$



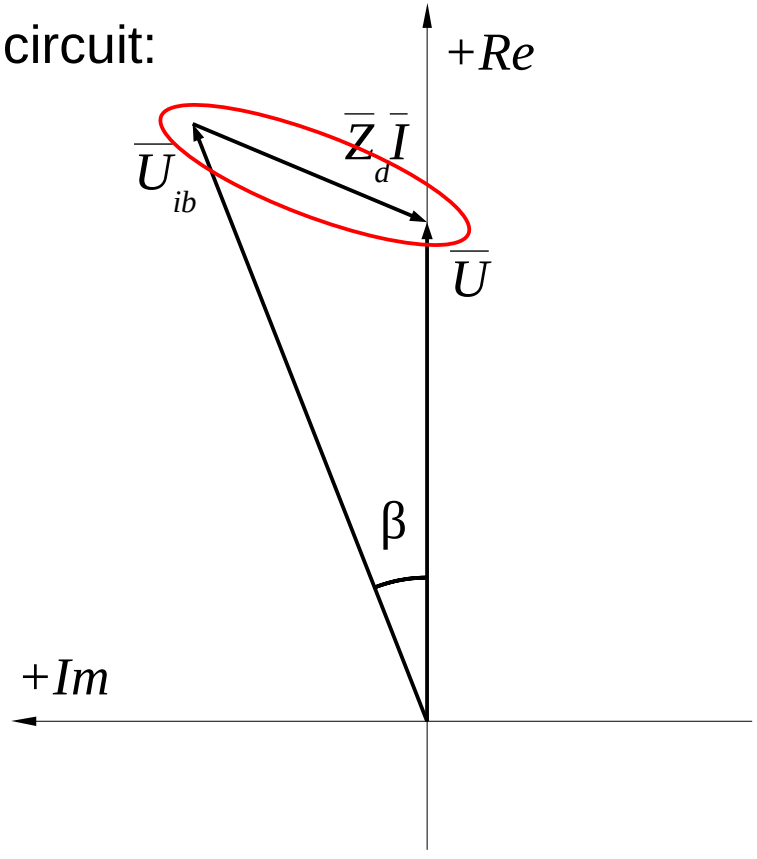
Circle Diagram of a Synchronous Machine for $R_a \neq 0$

- It is necessary to consider full equivalent circuit:



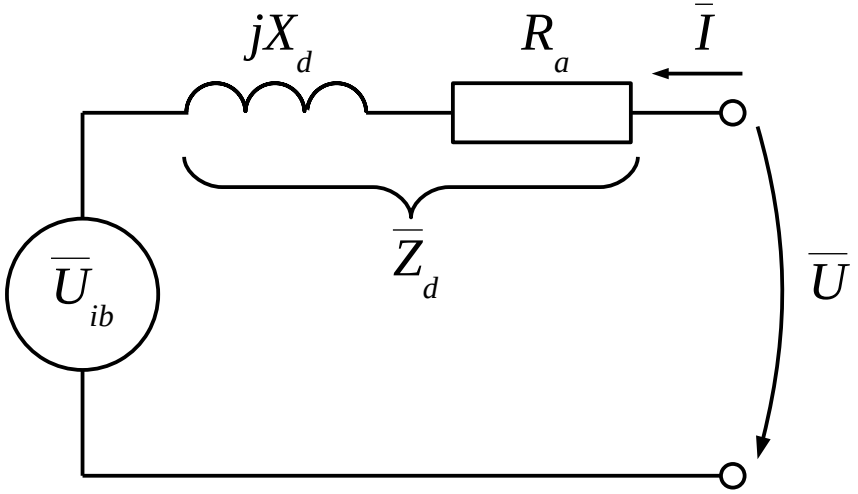
$$\bar{Z}_d = R_a + jX_d = Z_d e^{j\left(\frac{\pi}{2}-\rho\right)}$$

$$\rho = \operatorname{arctg}\frac{R_a}{X_d} \qquad Z_d = \sqrt{R_a^2 + X_d^2}$$

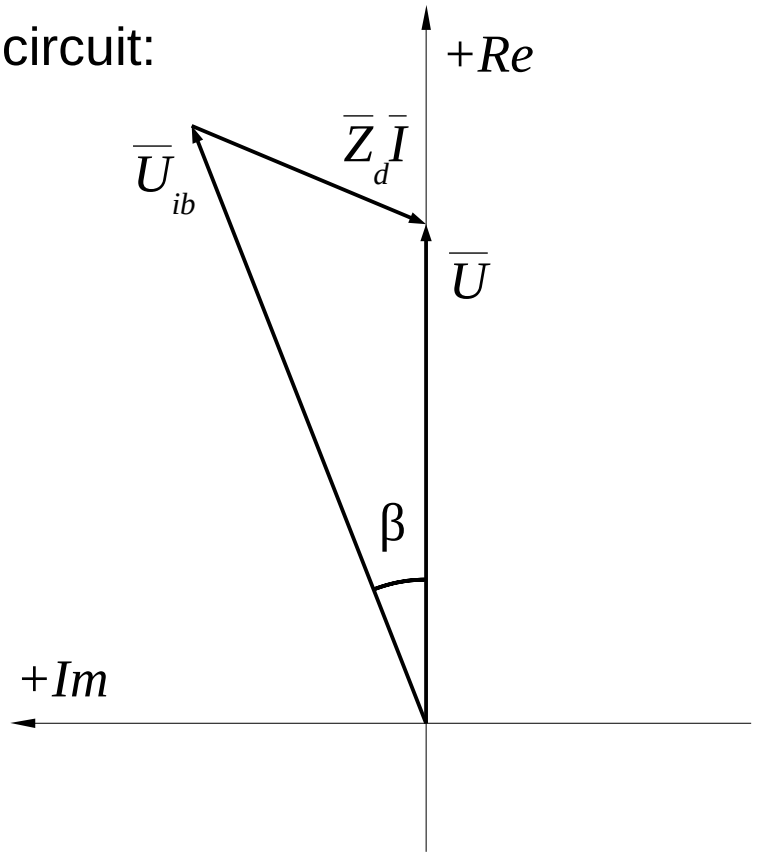


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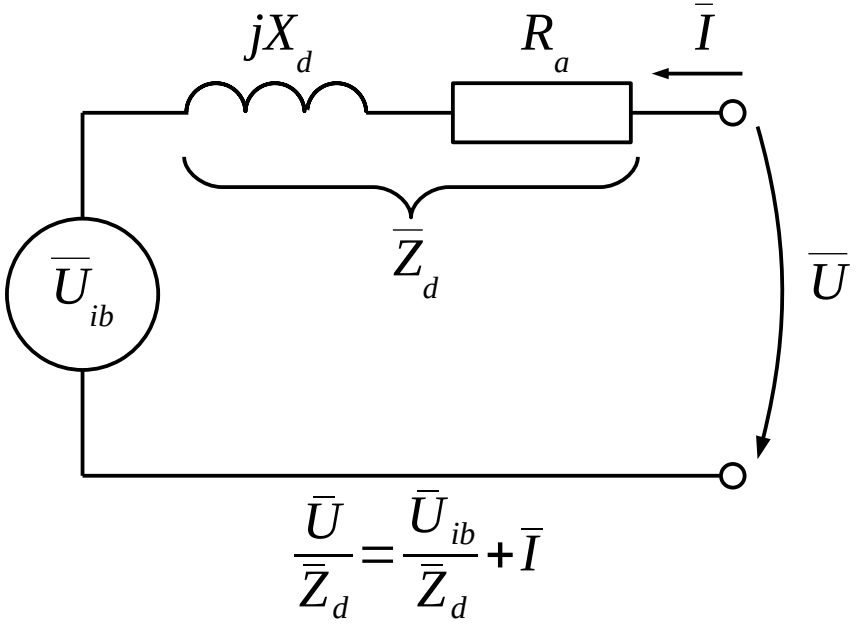


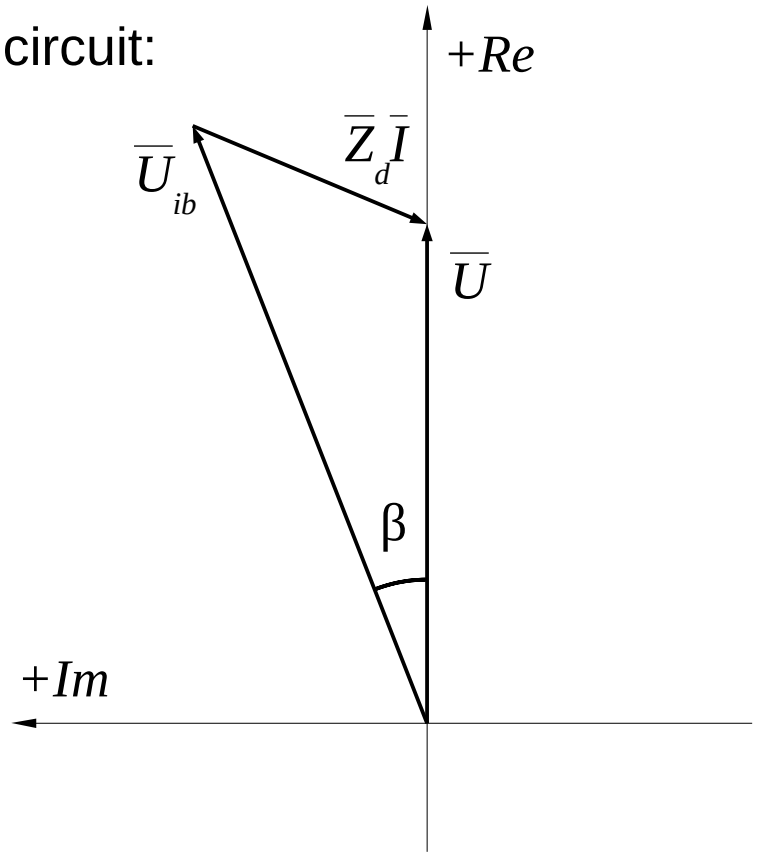
$$\bar{U} = \bar{U}_{ib} + \bar{Z}_d \bar{I}$$



Circle Diagram of a Synchronous Machine for $R_a \neq 0$

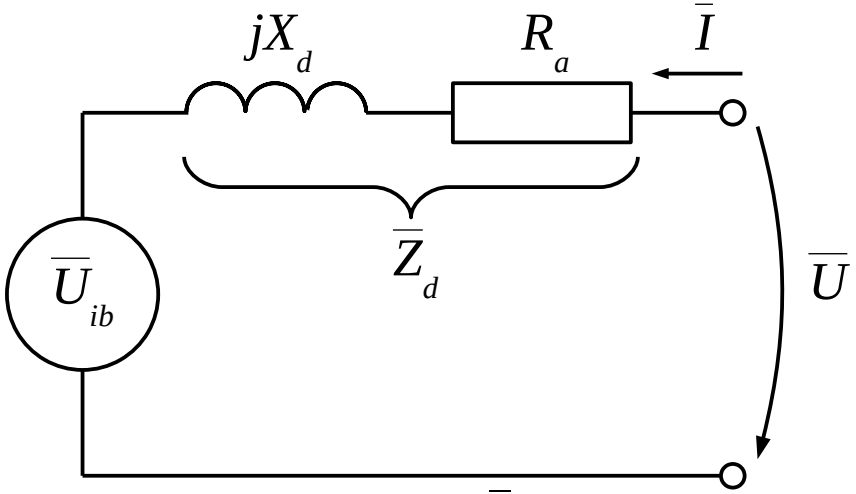
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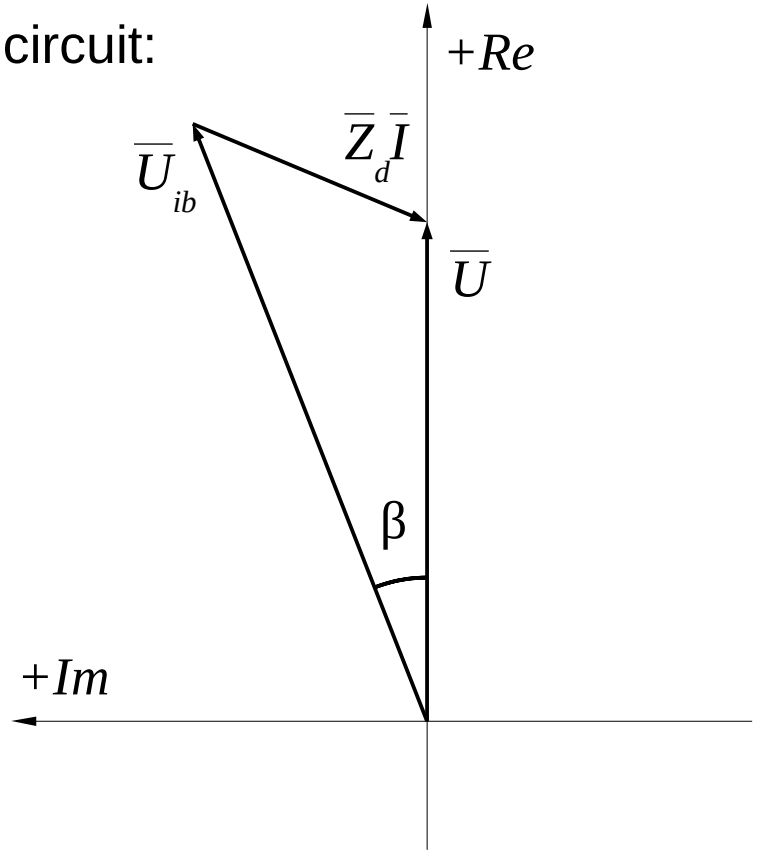


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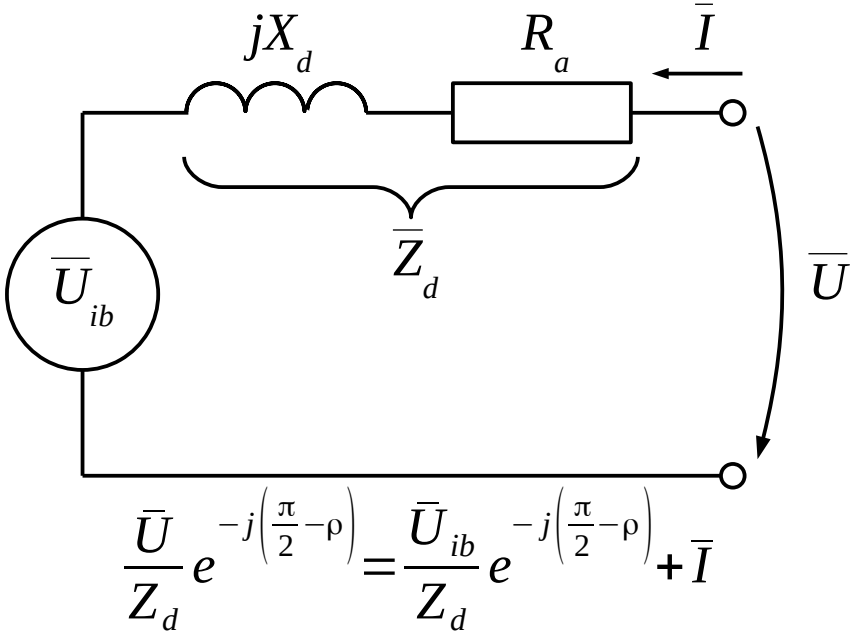


$$\frac{\bar{U}}{Z_d e^{j\left(\frac{\pi}{2}-\rho\right)}} = \frac{\bar{U}_{ib}}{Z_d e^{j\left(\frac{\pi}{2}-\rho\right)}} + \bar{I}$$

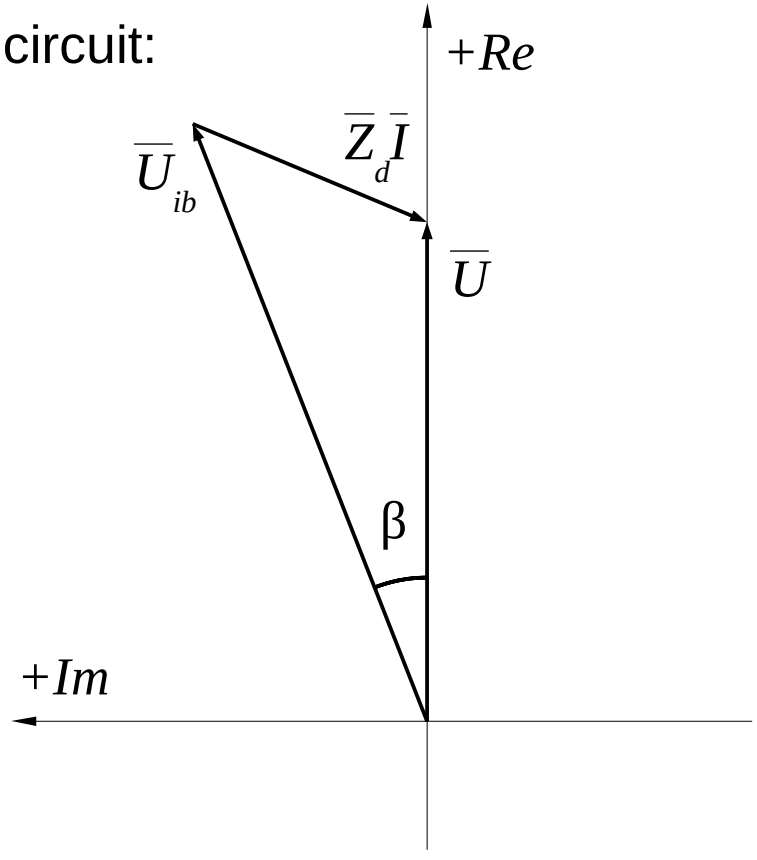


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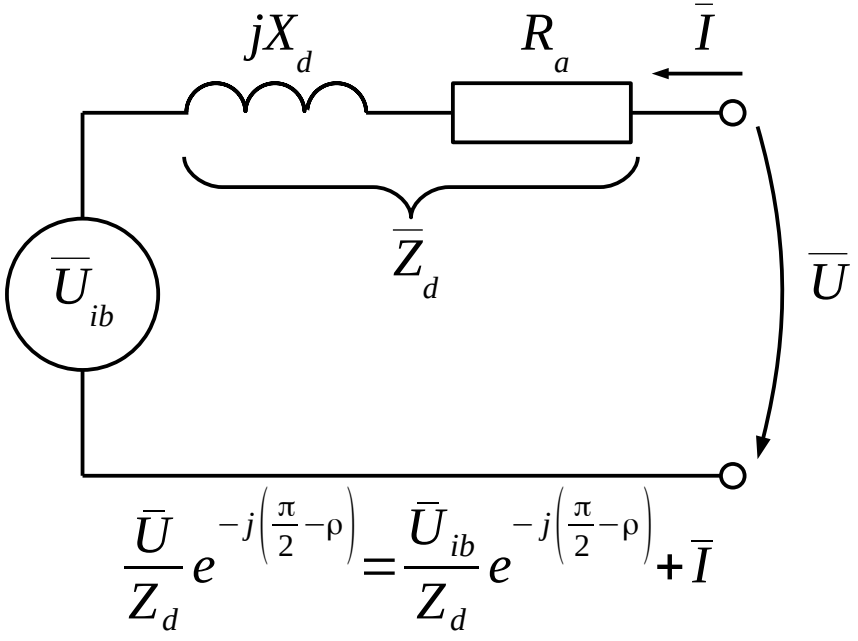


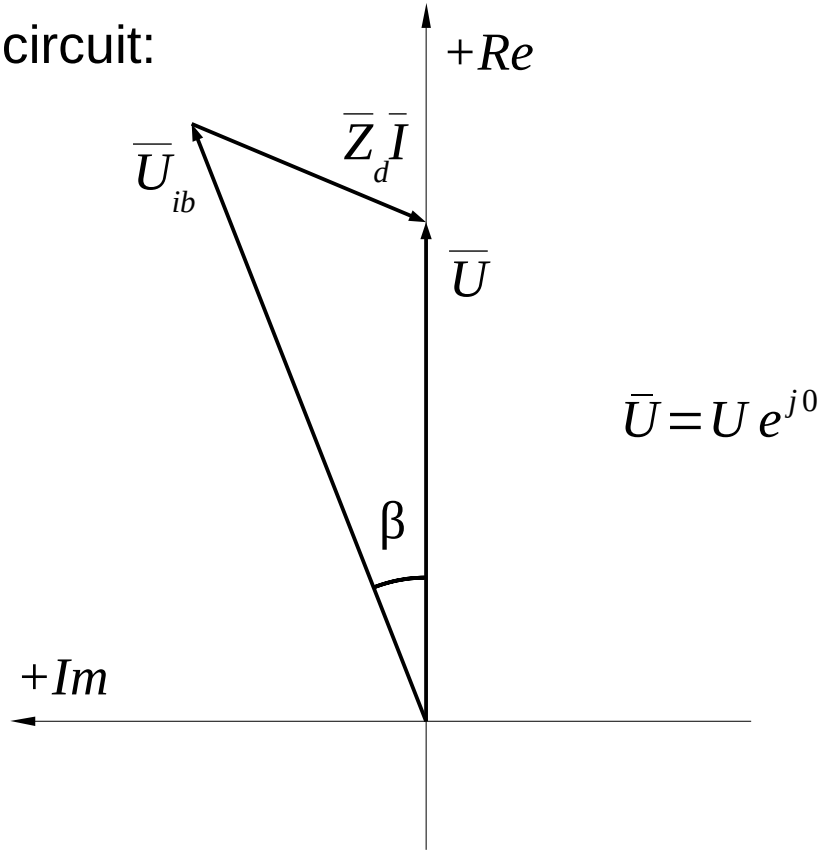
$$\frac{\bar{U}}{Z_d} e^{-j\left(\frac{\pi}{2}-\rho\right)} = \frac{\bar{U}_{ib}}{Z_d} e^{-j\left(\frac{\pi}{2}-\rho\right)} + \bar{I}$$



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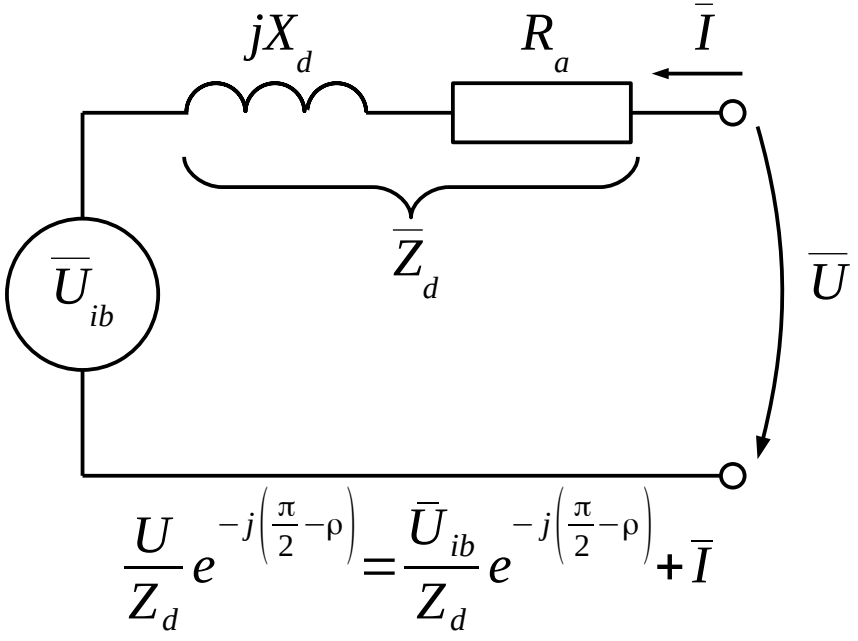
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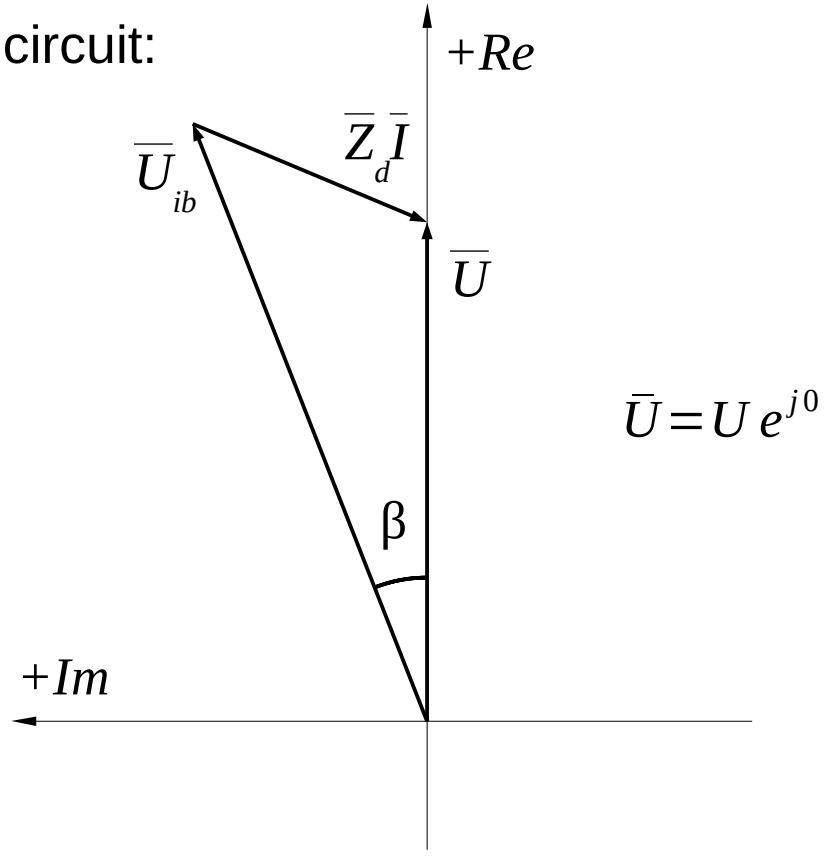




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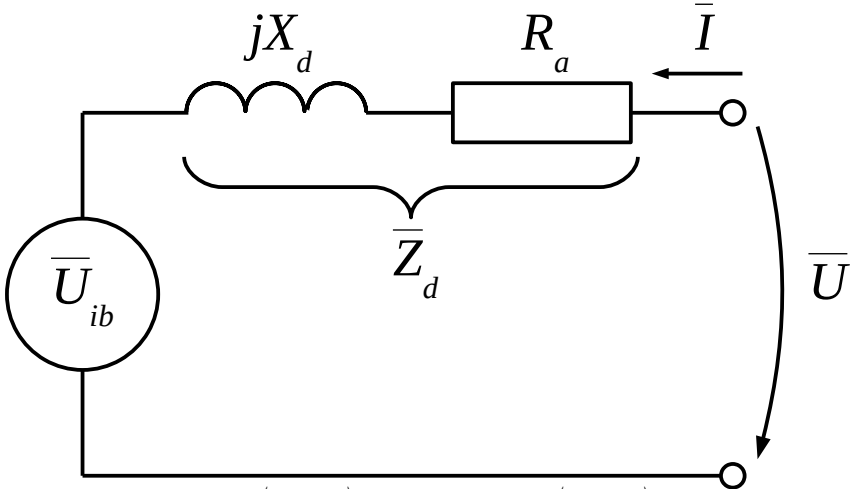
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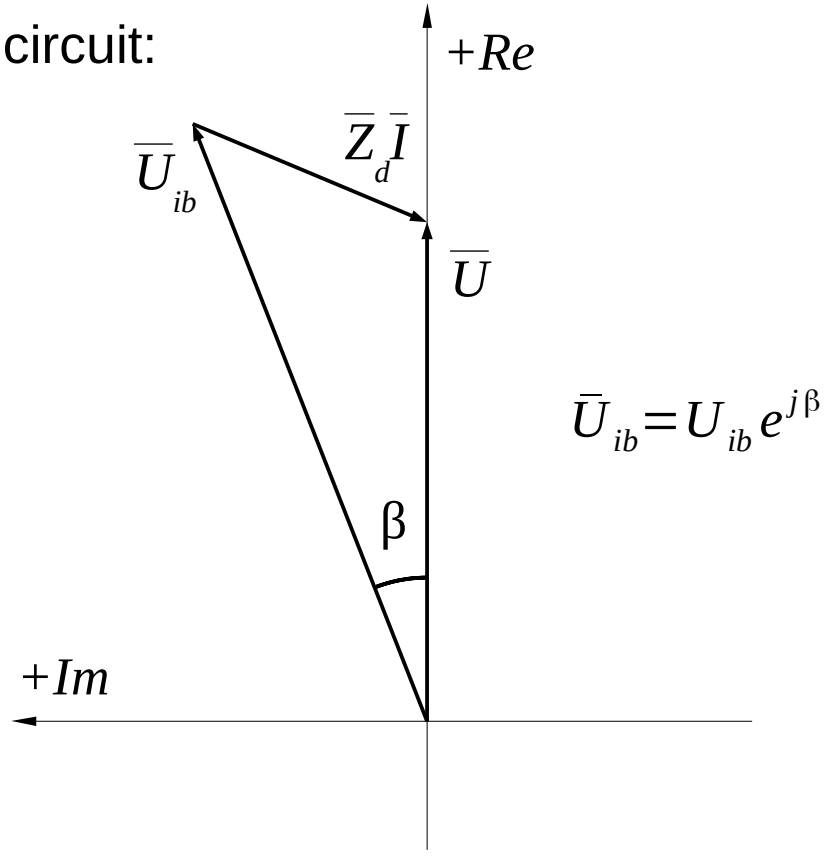


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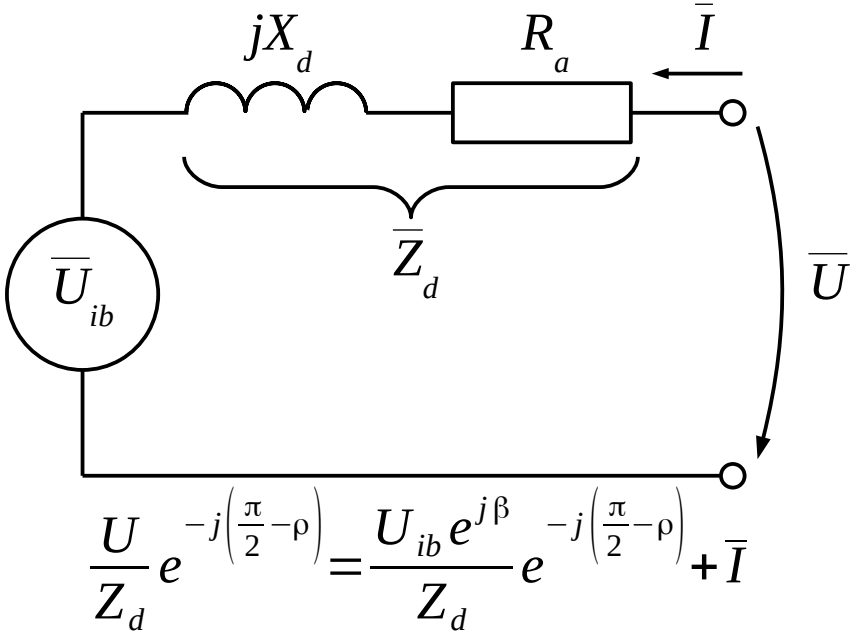


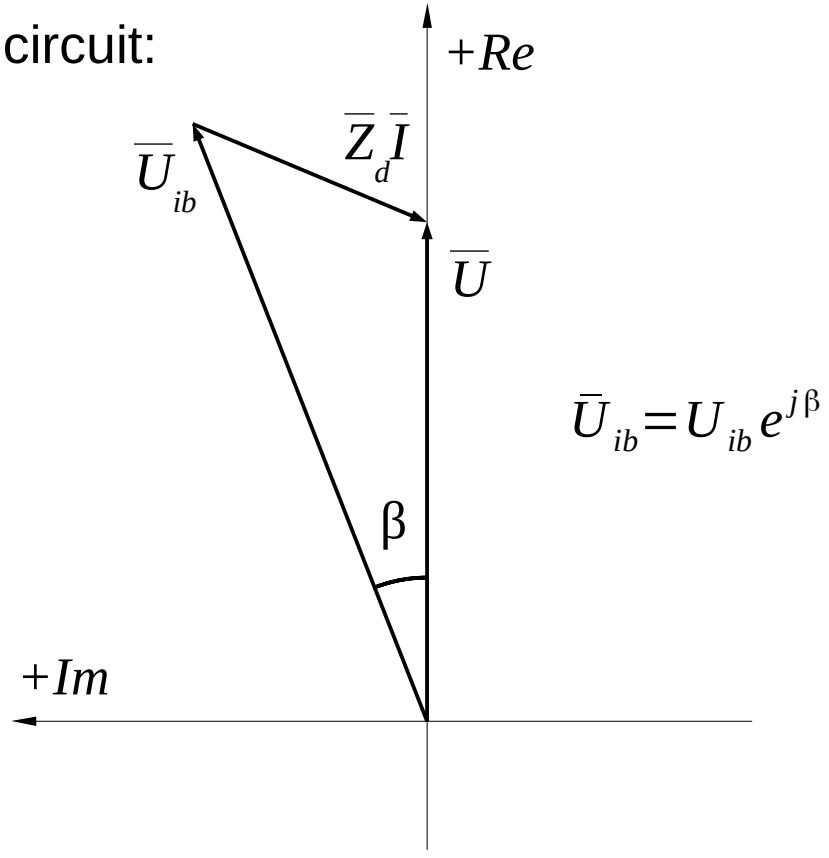
$$\frac{U}{Z_d} e^{-j\left(\frac{\pi}{2}-\rho\right)} = \frac{\bar{U}_{ib}}{Z_d} e^{-j\left(\frac{\pi}{2}-\rho\right)} + \bar{I}$$



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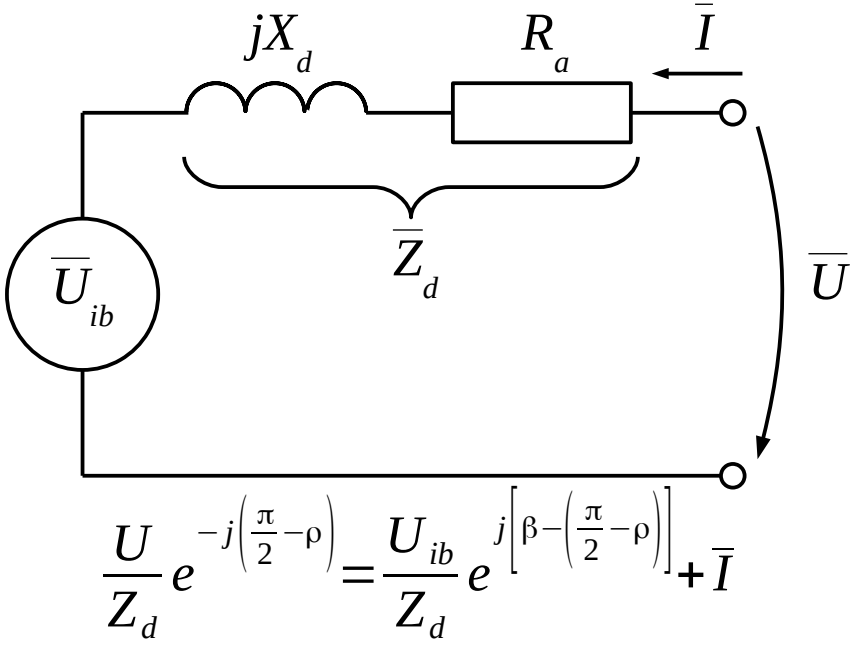
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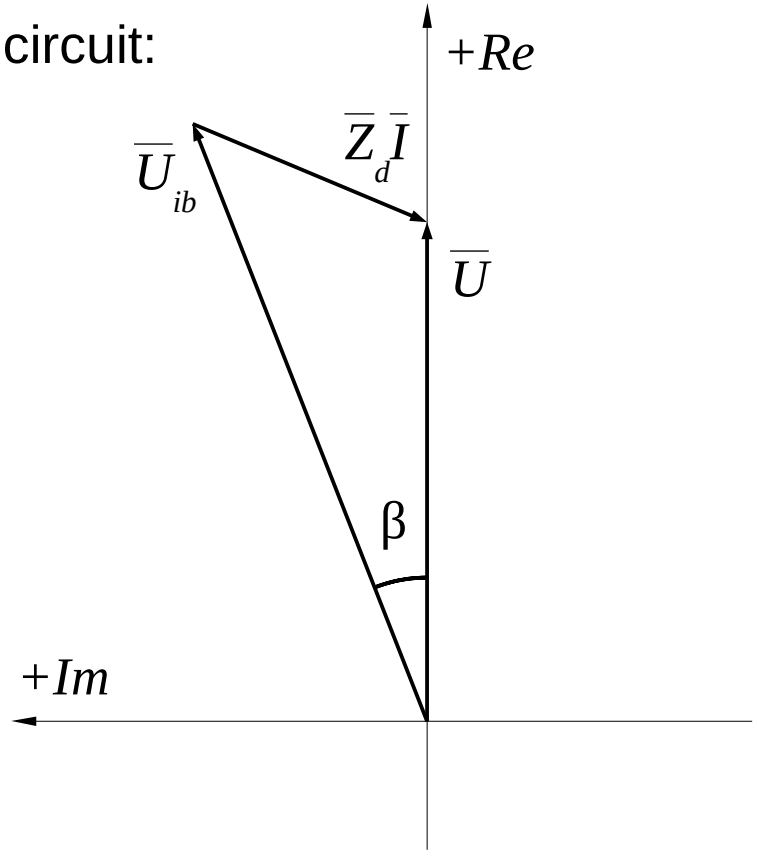




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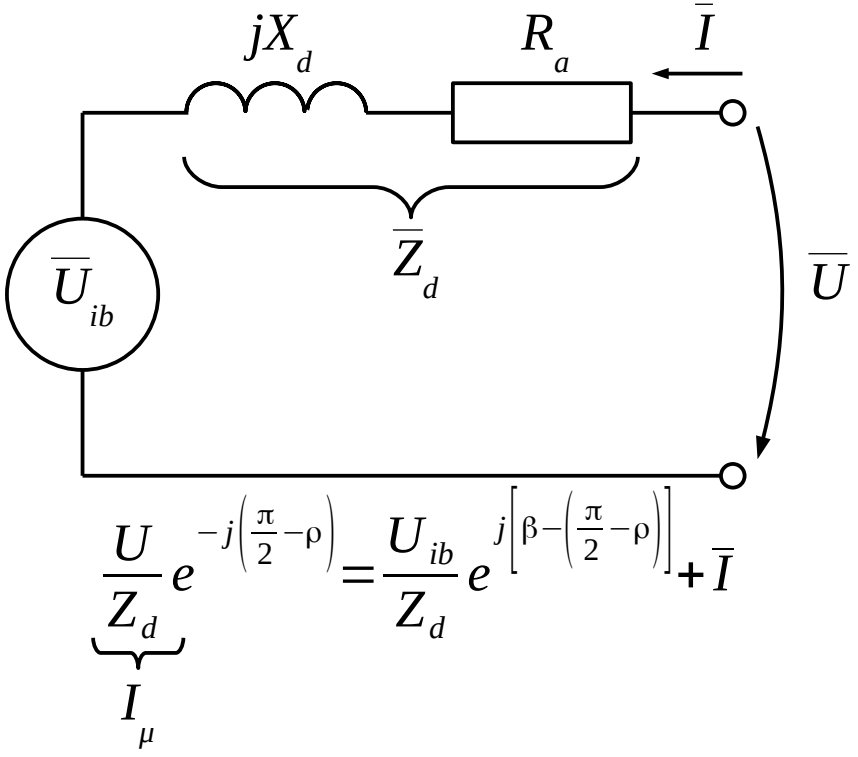
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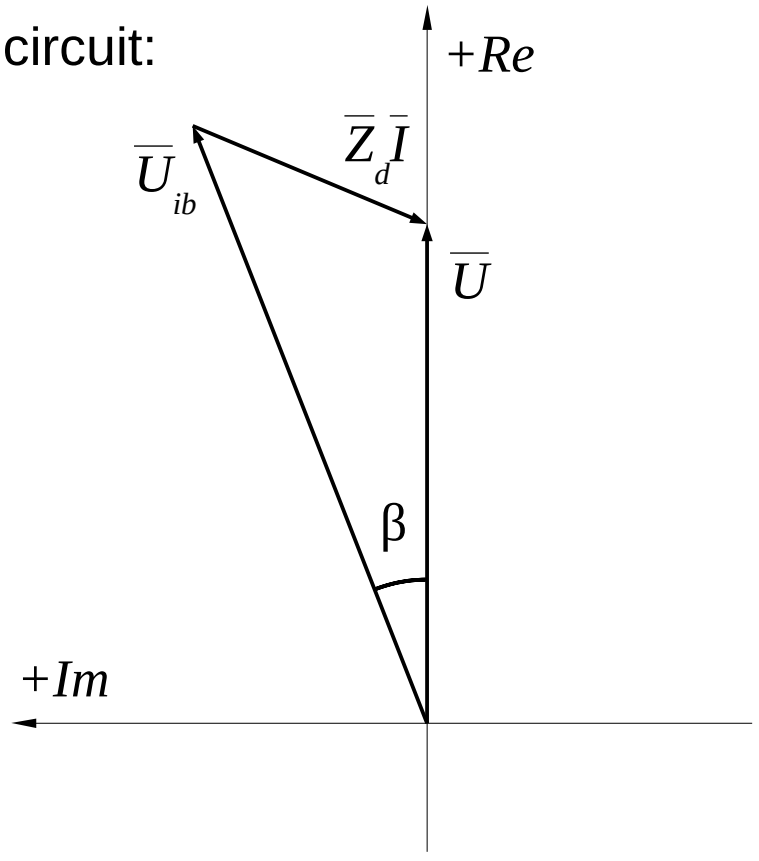




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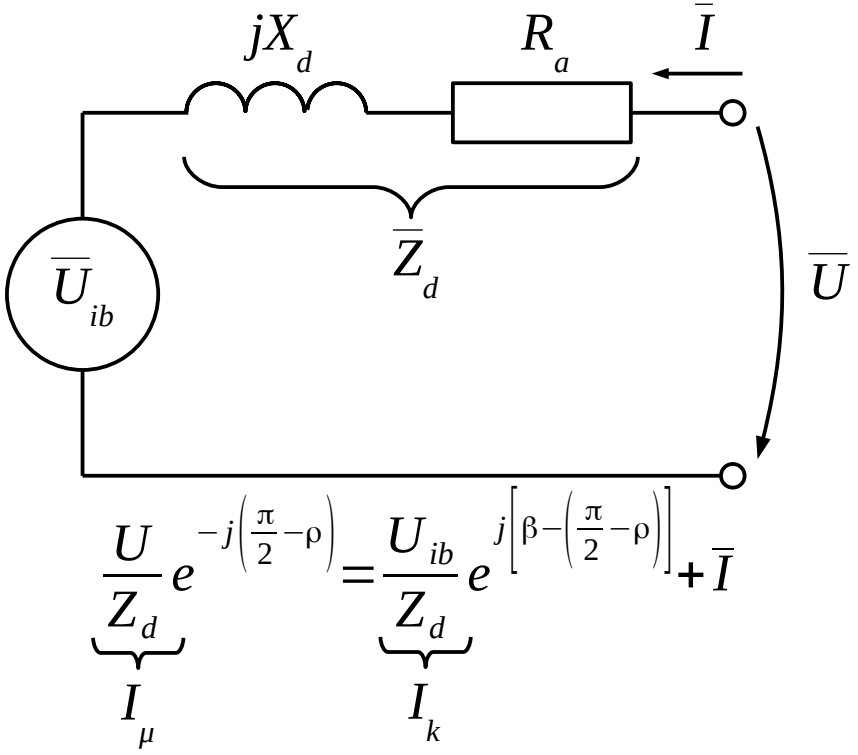
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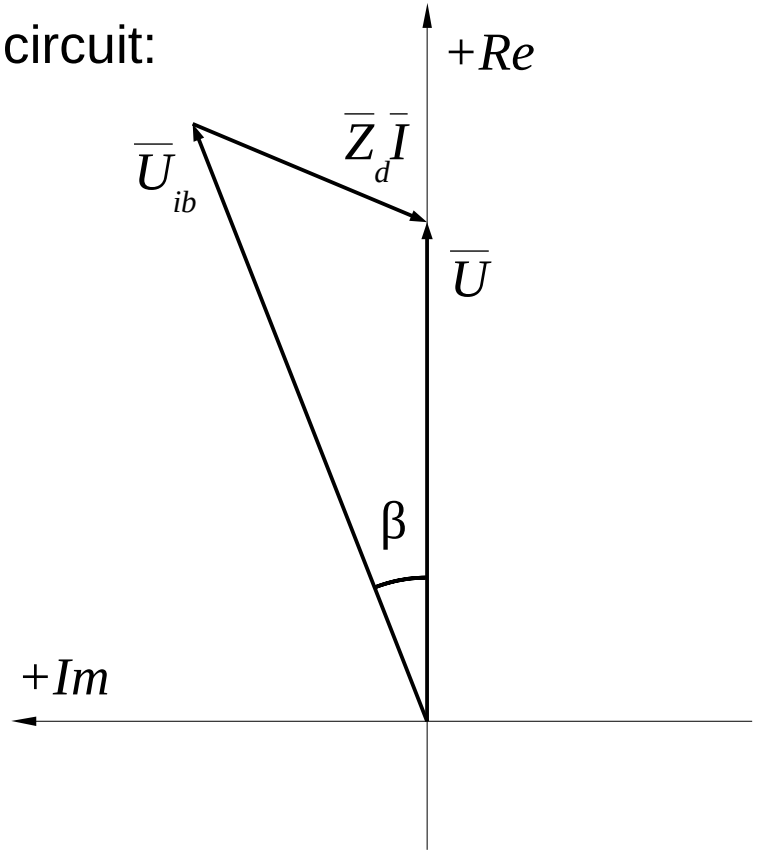




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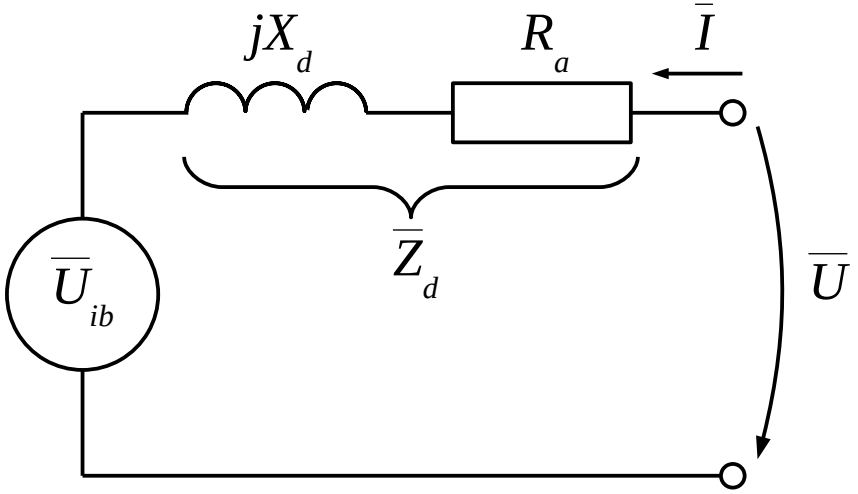
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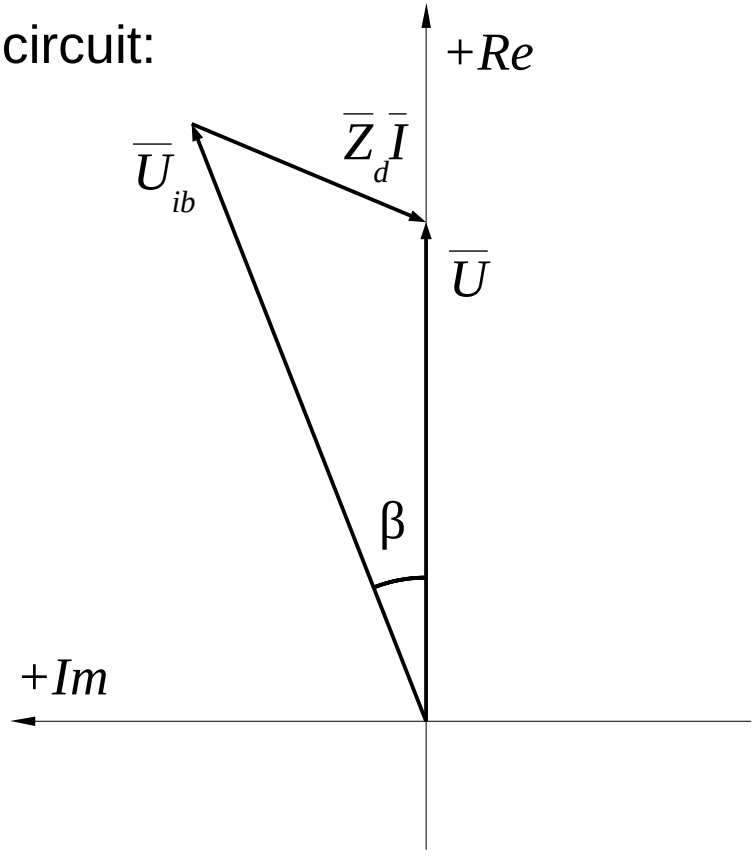


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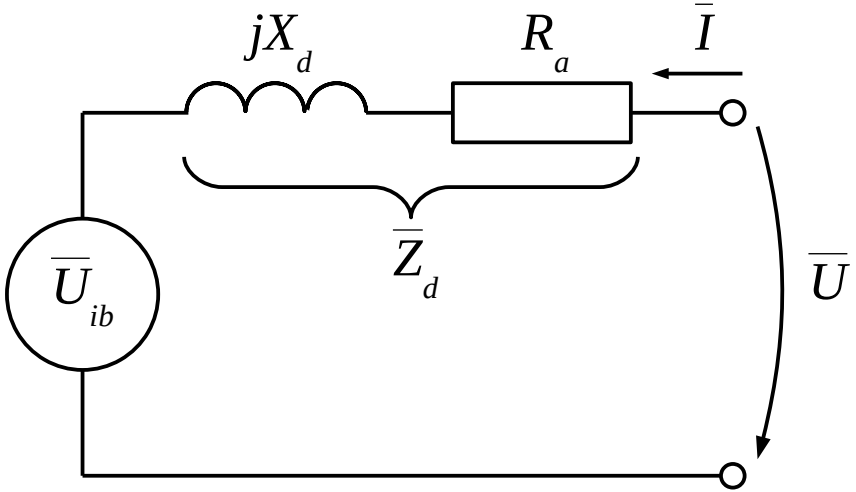


$$I_\mu e^{-j\left(\frac{\pi}{2}-\rho\right)} = I_k e^{j\left[\beta-\left(\frac{\pi}{2}-\rho\right)\right]} + \bar{I}$$

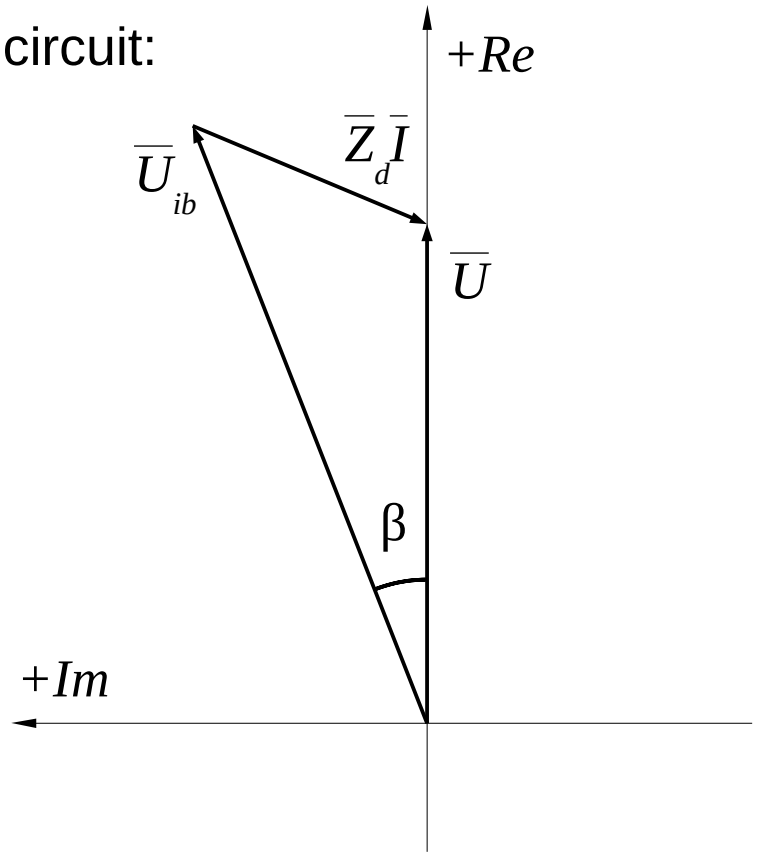


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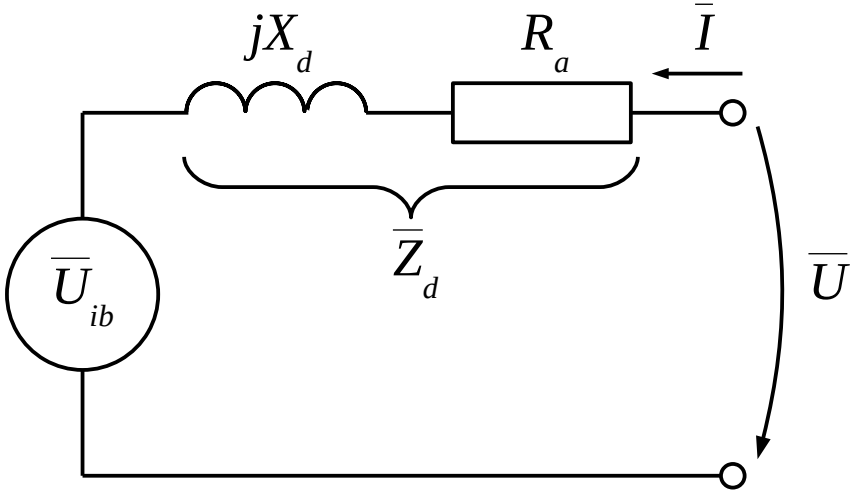


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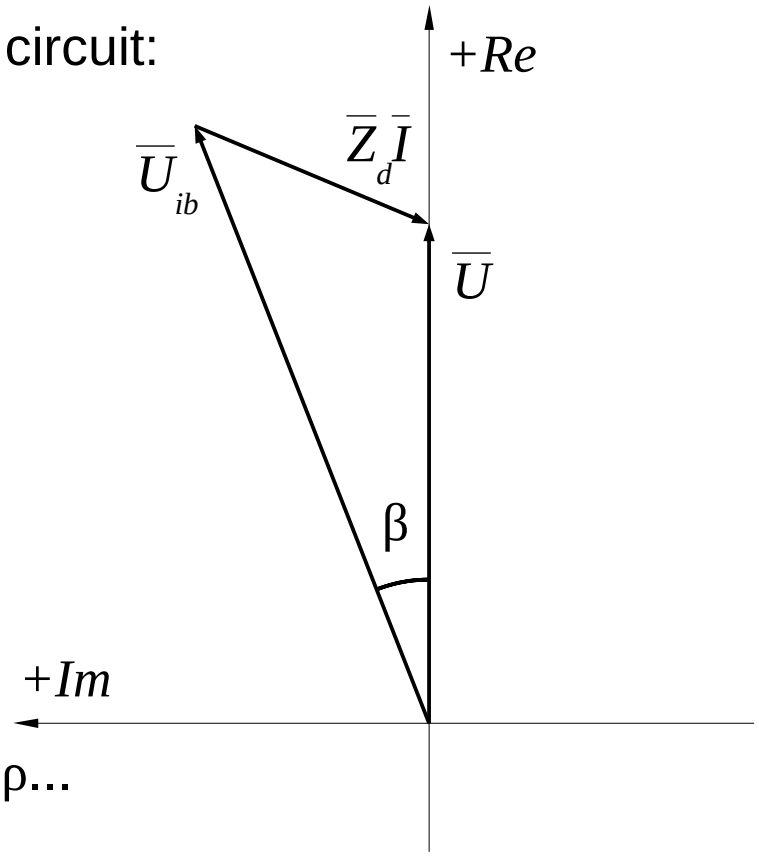


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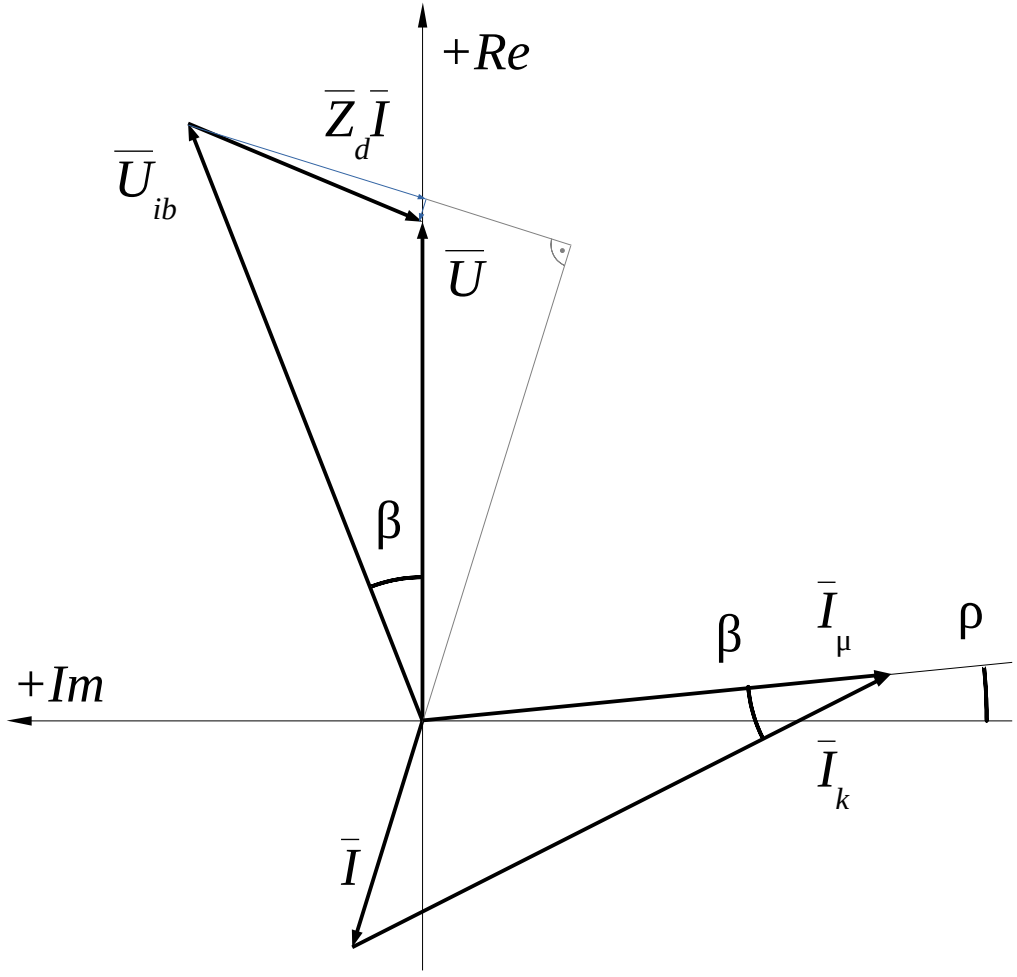


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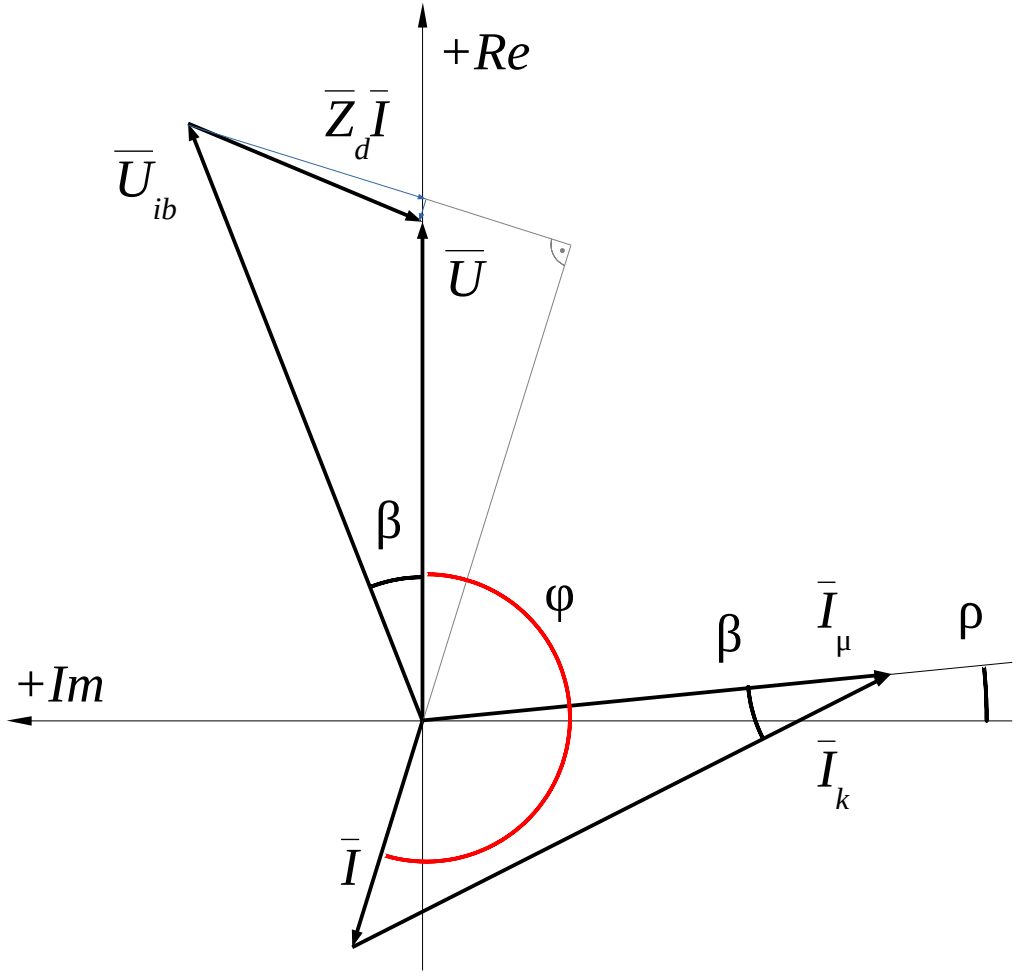


…the triangle of currents is shifted by angle ρ...

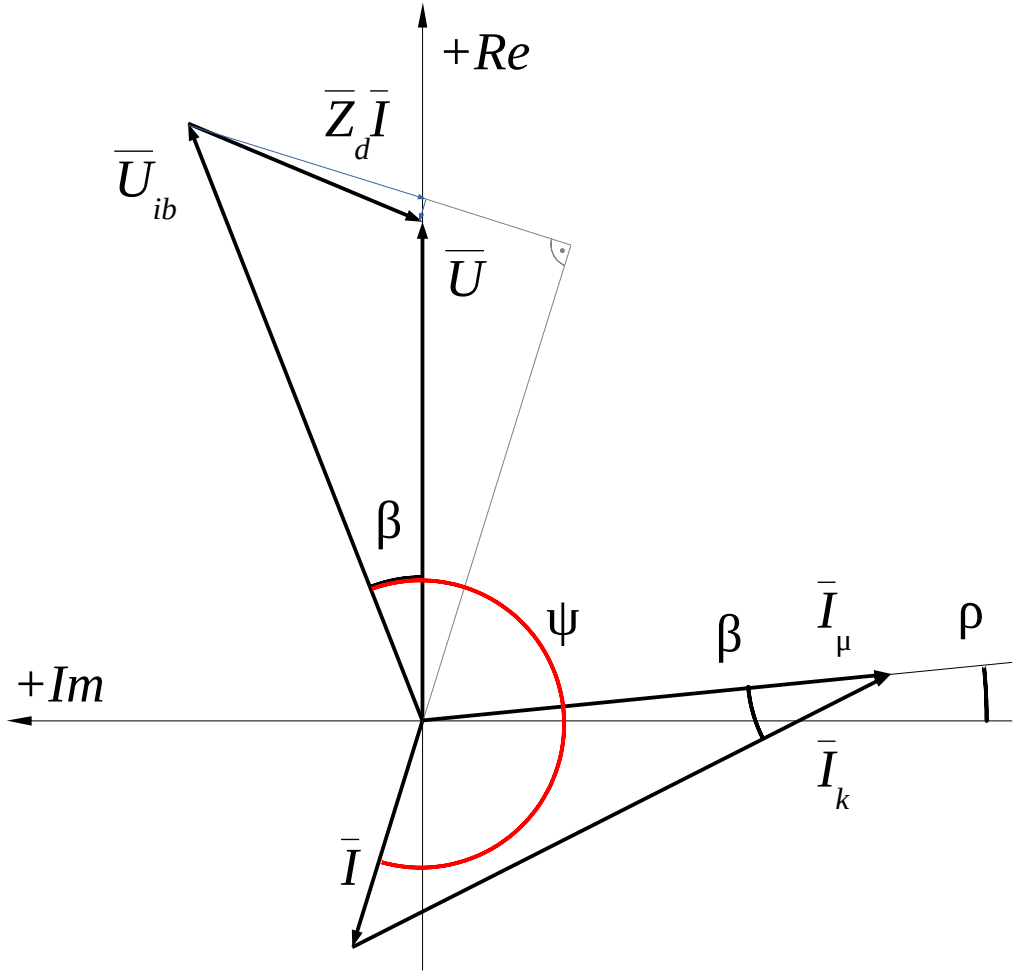
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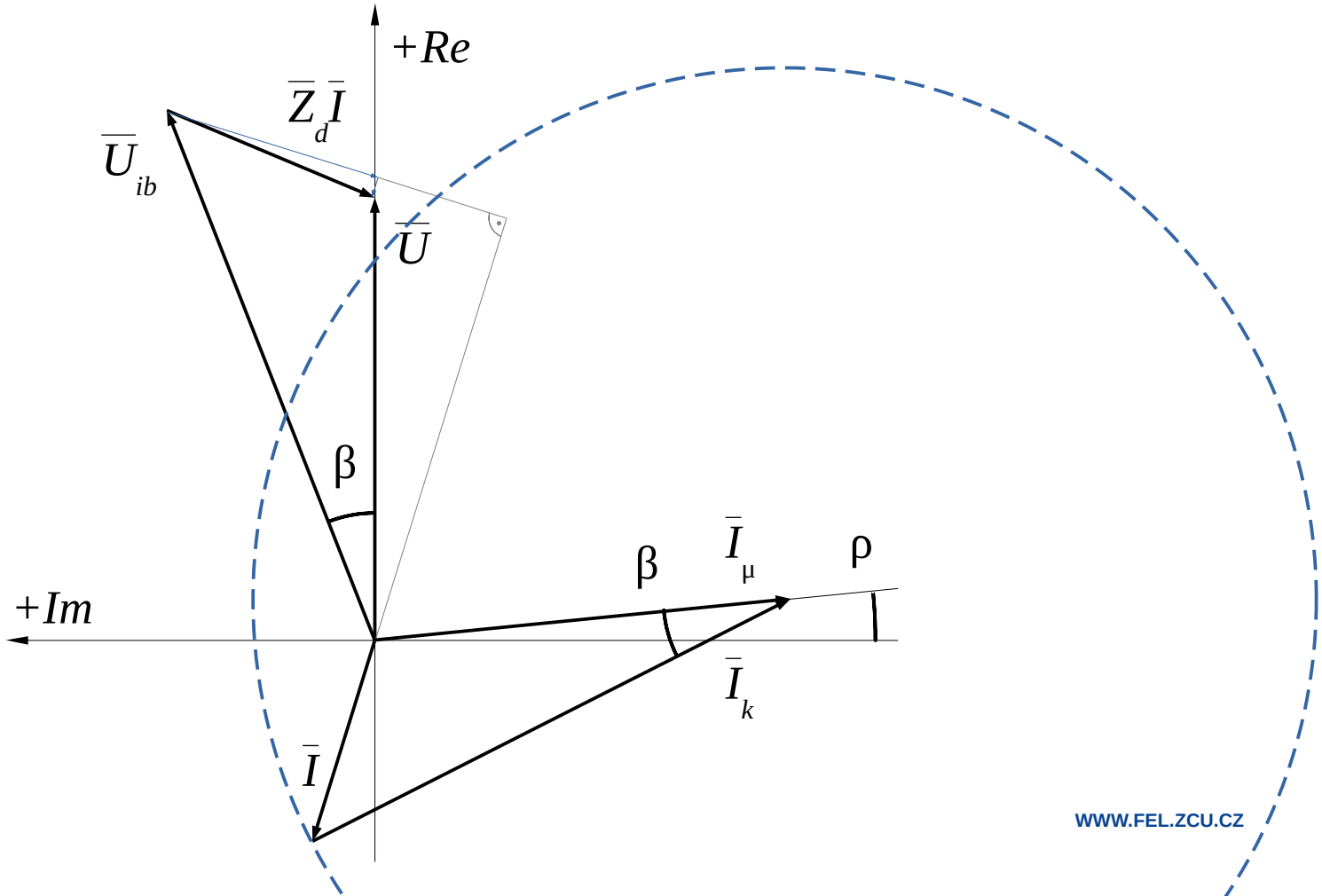
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Circle Diagram of a Synchronous Machine for $R_a \neq 0$



FACULTY OF ELECTRICAL
ENGINEERING
UNIVERSITY OF WEST BOHEMIA

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

ESF II – project of the University of West Bohemia
reg. no. CZ.02.2.69/0.0/0.0/18_056/0013239

Output Power of a Synchronous Machine while Considering Armature Resistance

Output Power of a SM while Considering Armature Resistance

- The power balance in a machine with non-zero R_a may be derived from the external power provided to the machine

$$P_e = m \Re \left\{ \bar{U}\, \bar{I}^* \right\} = m U I_w$$

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$$P_e = m \Re \left\{ \bar{U}\, \bar{I}^* \right\} = m\, U\, I_w$$

known I^2R losses

$$\Delta P_j = m\, R_a\, I^2$$

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$$P_i = m\,\Re\left\{\bar{U}_{ib}\,\bar{I}^*\right\}$$

- Presuming negligible iron core and mechanical losses (i.e. for ΔP_{Fe}, $\Delta P_{Fe} \rightarrow 0$) we can write

$$P_e = P_i + \Delta P_j$$

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and so-called inner power of the machine

$$P_i = m \Re \left\{ \bar{U}_{ib}\, \bar{I}^* \right\}$$

- The powers in a single phase give

$$U\, I_w = P_i + R_a\, I^2$$

Output Power of a SM while Considering Armature Resistance

- Thus current flowing through stator

$$\frac{U\,I_w}{R_a} = \frac{P_i}{R_a} + I^2$$

Output Power of a SM while Considering Armature Resistance

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which may be written also using real (watt) and imaginary component as

$$I^2 = I_w^2 + I_j^2$$

Output Power of a SM while Considering Armature Resistance

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- Then we have

$$\frac{U\,I_w}{R_a} = \frac{P_i}{R_a} + I_w^2 + I_j^2$$

Output Power of a SM while Considering Armature Resistance

- It is suitable to move currents to a single side of the equation

$$-\frac{P_i}{R_a} = I_w^2 + I_j^2 - \frac{U\,I_w}{R_a}$$

Output Power of a SM while Considering Armature Resistance

- It is suitable to move currents to a single side of the equation

$$-\frac{P_i}{R_a} = I_w^2 + I_j^2 - \frac{U\,I_w}{R_a}$$

or

$$-\frac{P_i}{R_a} = I_w^2 - \frac{U\,I_w}{R_a} + I_j^2$$

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$$-\frac{P_i}{R_a} = I_w^2 + I_j^2 - \frac{U\,I_w}{R_a}$$

or

$$-\frac{P_i}{R_a} = \underbrace{I_w^2 - \frac{U\,I_w}{R_a}} + I_j^2$$

solvable by completing the square

$$\left(I_w - X\right)^2$$

Output Power of a SM while Considering Armature Resistance

$$I_w^2 - \frac{U\, I_w}{R_a} + X^2 - X^2 = \left(I_w - X\right)^2 - X^2$$

Output Power of a SM while Considering Armature Resistance

$$I_w^2 - \frac{U\, I_w}{R_a} + X^2 - X^2 = \left(I_w - X\right)^2 - X^2$$

$$a^2 - 2ab + b^2 - b^2 = \left(a - b\right)^2 + b^2$$

Output Power of a SM while Considering Armature Resistance

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$$2\, I_w\, b = \frac{U\, I_w}{R_a}$$

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$$I_w^2 - \frac{U\,I_w}{R_a} + \left(\frac{U}{2\,R_a}\right)^2 - \left(\frac{U}{2\,R_a}\right)^2 = \left(I_w - \frac{U}{2\,R_a}\right)^2 - \left(\frac{U}{2\,R_a}\right)^2$$

Output Power of a SM while Considering Armature Resistance

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$$I_w^2 - \frac{U\,I_w}{R_a} + \left(\frac{U}{2\,R_a}\right)^2 - \left(\frac{U}{2\,R_a}\right)^2 = \left(I_w - \frac{U}{2\,R_a}\right)^2 - \left(\frac{U}{2\,R_a}\right)^2$$

Output Power of a SM while Considering Armature Resistance

- The equation of currents

$$-\frac{P_i}{R_a} = I_w^2 - \frac{U\,I_w}{R_a} + I_j^2$$

Output Power of a SM while Considering Armature Resistance

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then verges into

$$-\frac{P_i}{R_a} = \left(I_w - \frac{U}{2\,R_a}\right)^2 - \left(\frac{U}{2\,R_a}\right)^2 + I_j^2$$

Output Power of a SM while Considering Armature Resistance

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$$-\frac{P_i}{R_a} = \left(I_w - \frac{U}{2\,R_a}\right)^2 - \left(\frac{U}{2\,R_a}\right)^2 + I_j^2$$

which is after modification

$$\left(I_w - \frac{U}{2\,R_a}\right)^2 + I_j^2 = \left(\frac{U}{2\,R_a}\right)^2 - \frac{P_i}{R_a}$$

Output Power of a SM while Considering Armature Resistance

- The resulting equation $$\left(I_w - \frac{U}{2R_a}\right)^2 + I_j^2 = \left(\frac{U}{2R_a}\right)^2 - \frac{P_i}{R_a}$$

  is then for constant P_i an equation of a circle

$$\left(x - x_0\right)^2 + \left(y - y_0\right)^2 = r^2$$

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  whose centre has location at $[x_0; y_0]$, thus

  $$k_{P_i}(S): S\left[\frac{U}{2R_a};0\right]$$

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  $$k_{P_i}(S): S\left[\frac{U}{2R_a};0\right]$$

  and radius

  $$r_{P_i}=\sqrt{\left(\frac{U}{2R_a}\right)^2-\frac{P_i}{R_a}}$$

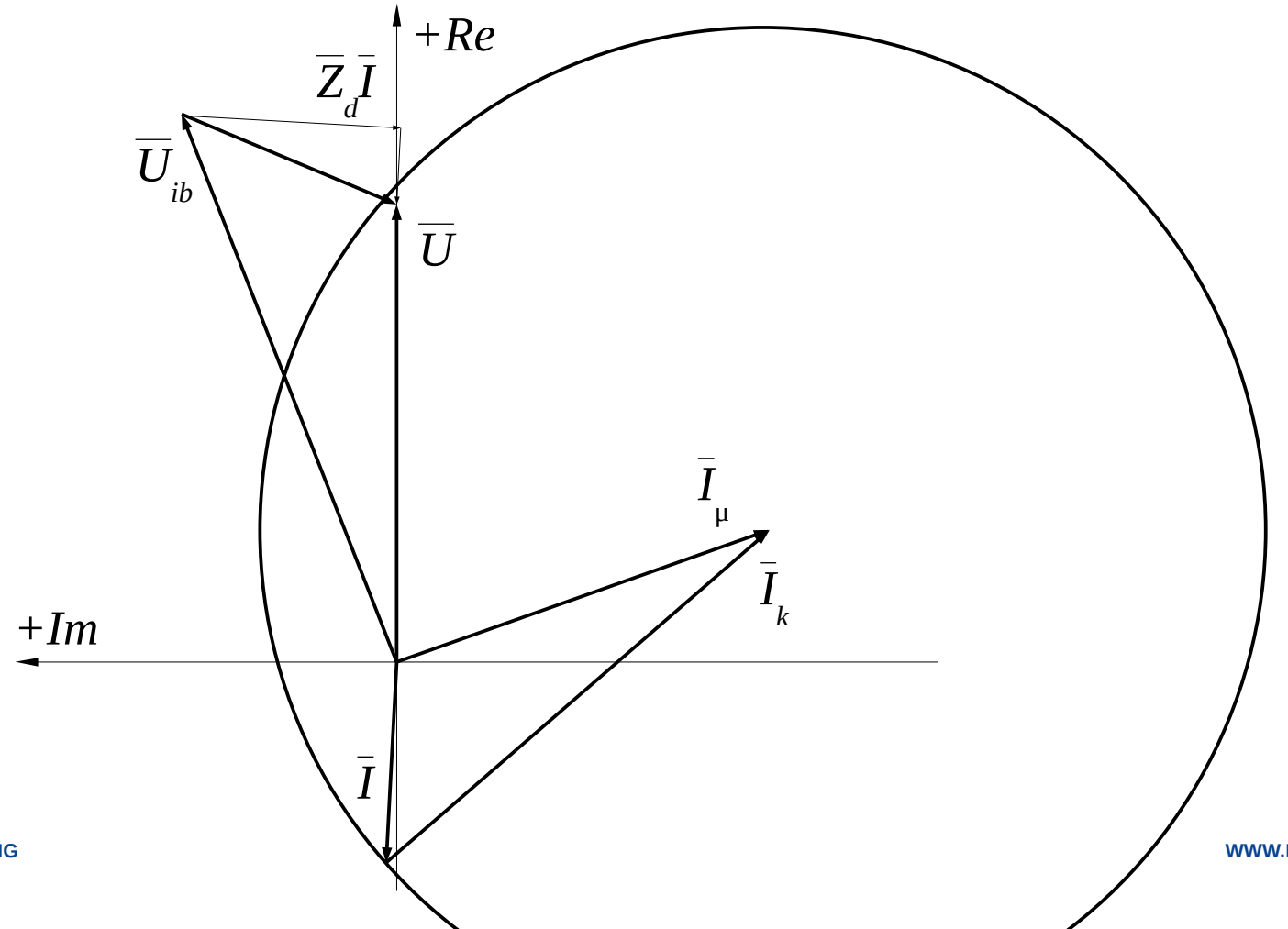
Output Power of a SM while Considering Armature Resistance

- For no input power ($P_i = 0$) then this circle crosses the zero-point of the coordinate system:

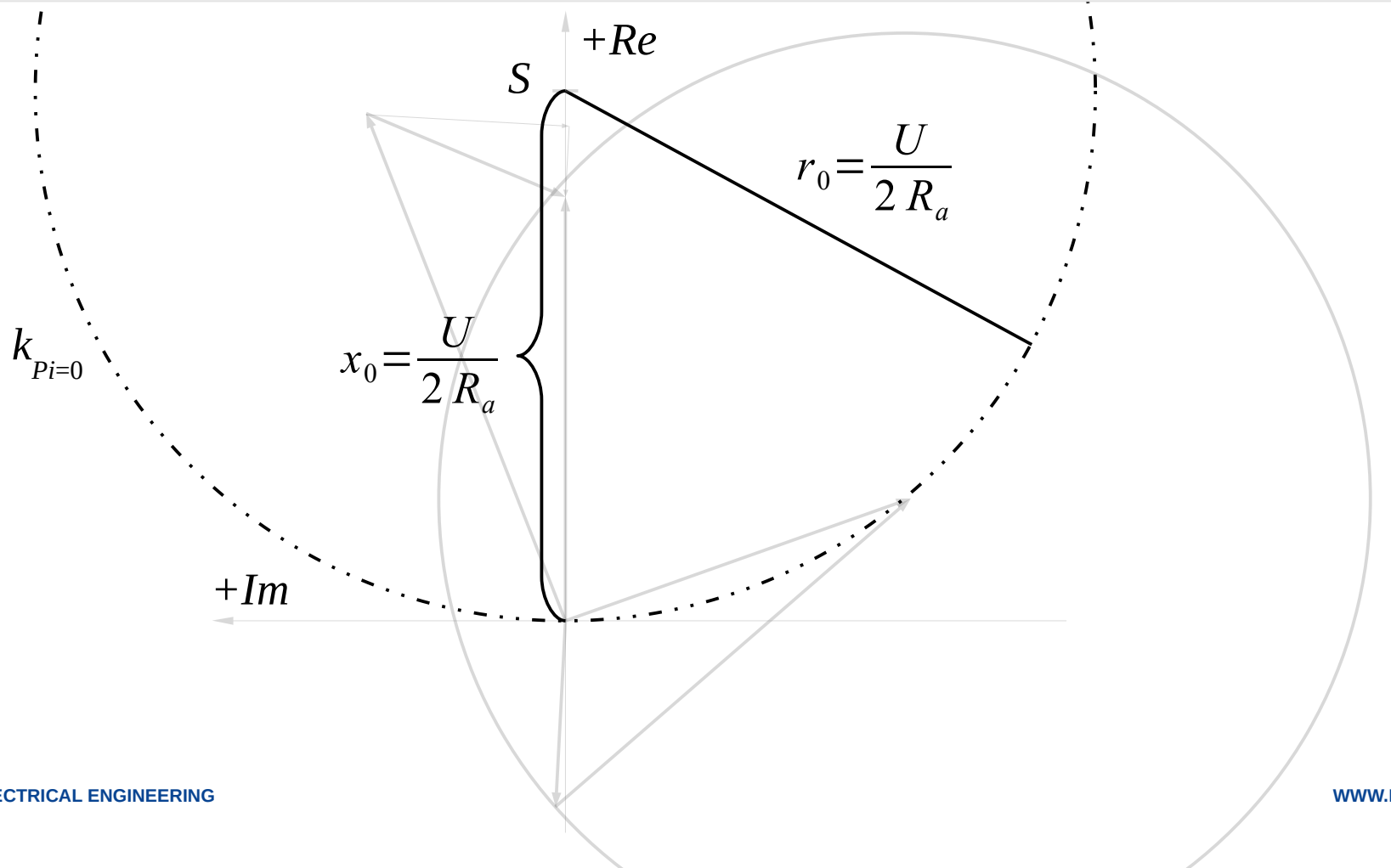
$$r_0 = \sqrt{\left(\frac{U}{2R_a}\right)^2 - \frac{0}{R_a}} = \frac{U}{2R_a} = x_0$$

- Furthermore this circle must go through endpoint of I_μ, because in case of zero excitation we have zero power.

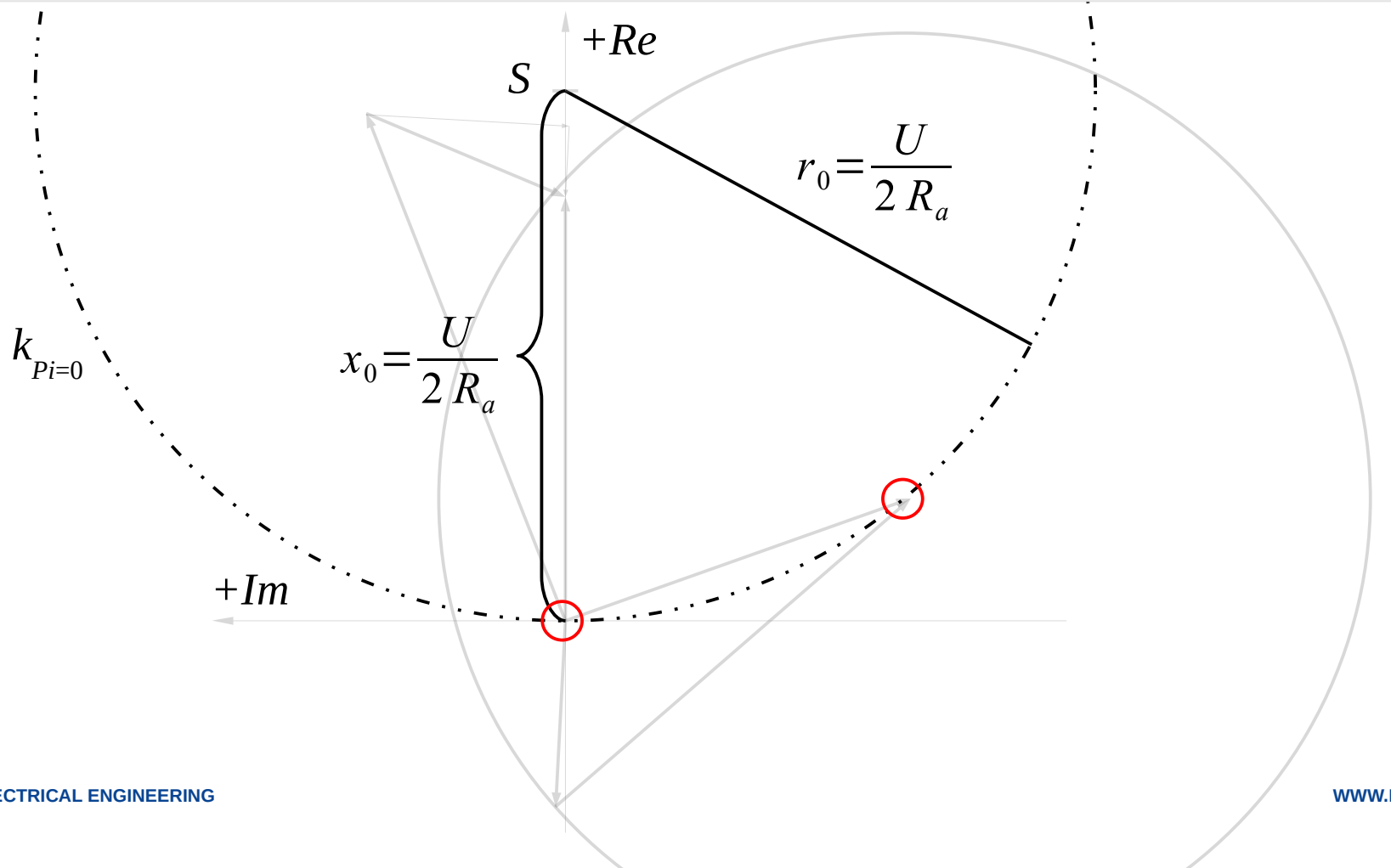
Output Power of a SM while Considering Armature Resistance



Output Power of a SM while Considering Armature Resistance



Output Power of a SM while Considering Armature Resistance

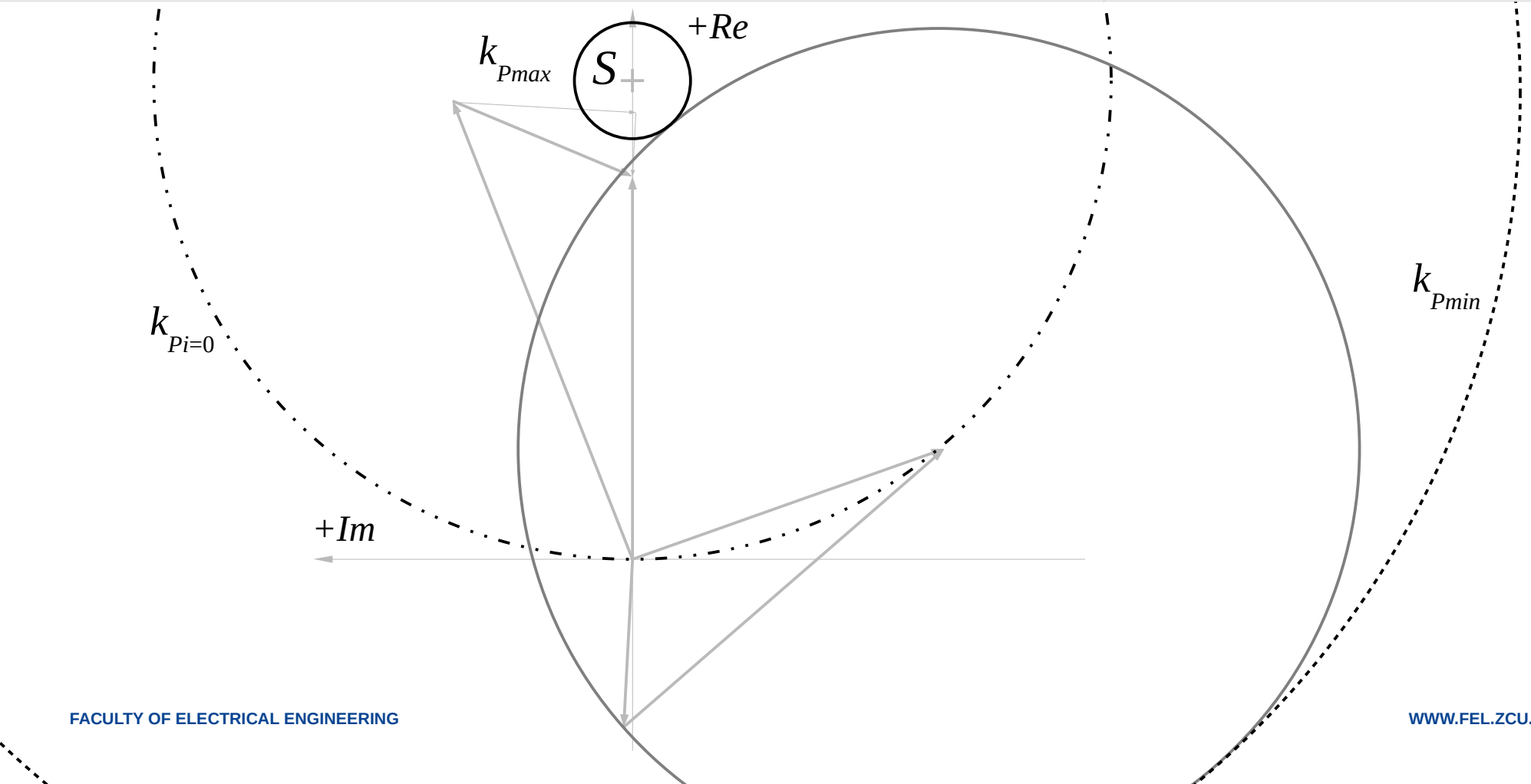


Output Power of a SM while Considering Armature Resistance

- The minimal and the maximal inner power of a machine for given excitation then correspond to maximal and minimal possible radii of circle k_{Pi} so it can cross (or touch) the circle of current phasor endpoints

  => they form tangent circles to the circle of current vectors, whose centres are in point S

Output Power of a SM while Considering Armature Resistance



Output Power of a SM while Considering Armature Resistance

- The radii of these circles are given by subtraction of diameter of the radius of the $P_i = 0$ circle:

$$r_0 = \frac{U}{2R_a}$$

and the current circle radius:

$$r_i = I_k = \frac{U_{ib}}{Z_d}$$

Output Power of a SM while Considering Armature Resistance

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- The radius of the circle for maximal inner power of the machine then results

$$r_{max} = r_0 - r_i = \frac{U}{2R_a} - \frac{U_{ib}}{Z_d}$$

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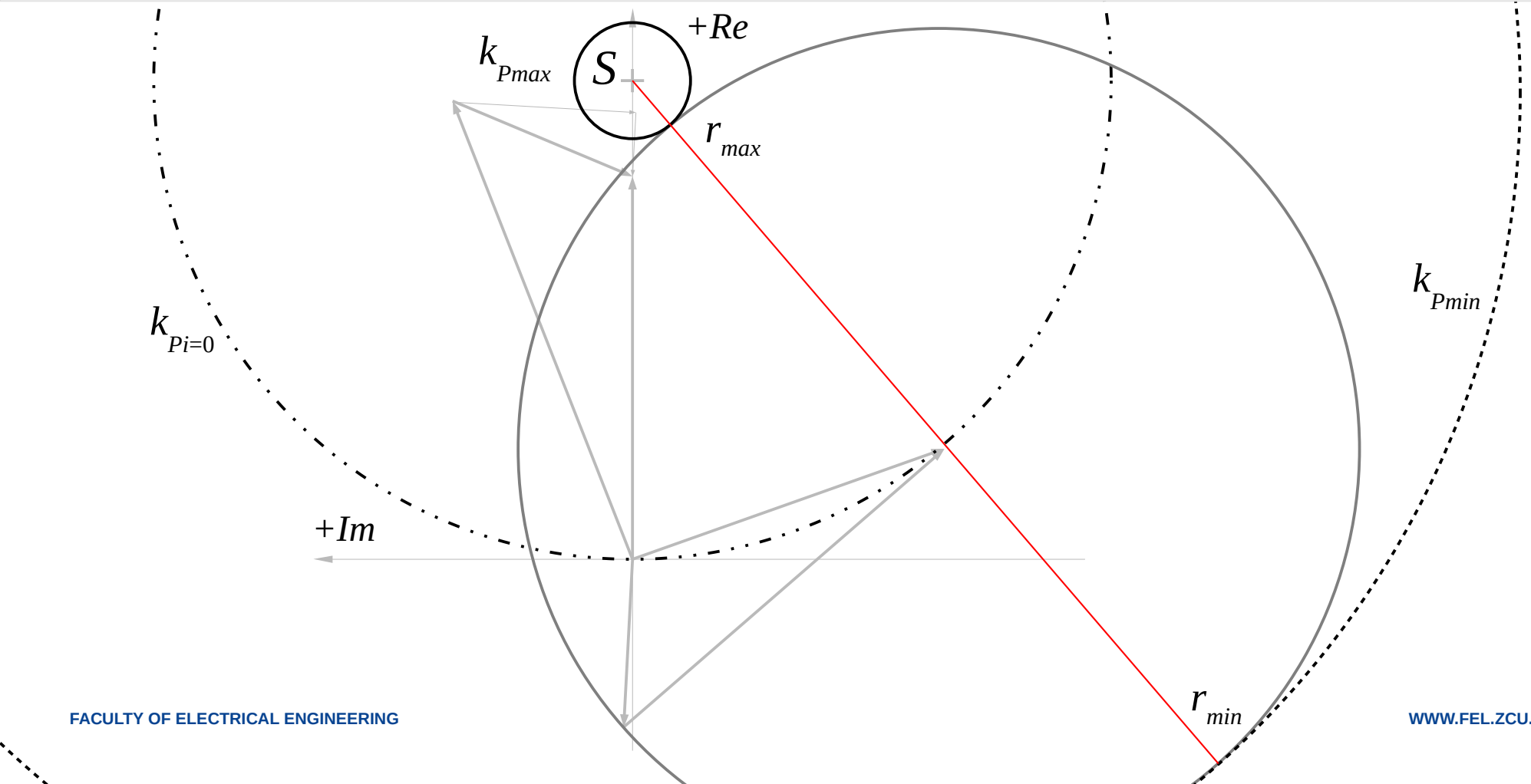
- The radius of the circle for maximal inner power of the machine then results

$$r_{max} = r_0 - r_i = \frac{U}{2R_a} - \frac{U_{ib}}{Z_d}$$

and for minimal output power

$$r_{min} = r_0 + r_i = \frac{U}{2R_a} + \frac{U_{ib}}{Z_d}$$

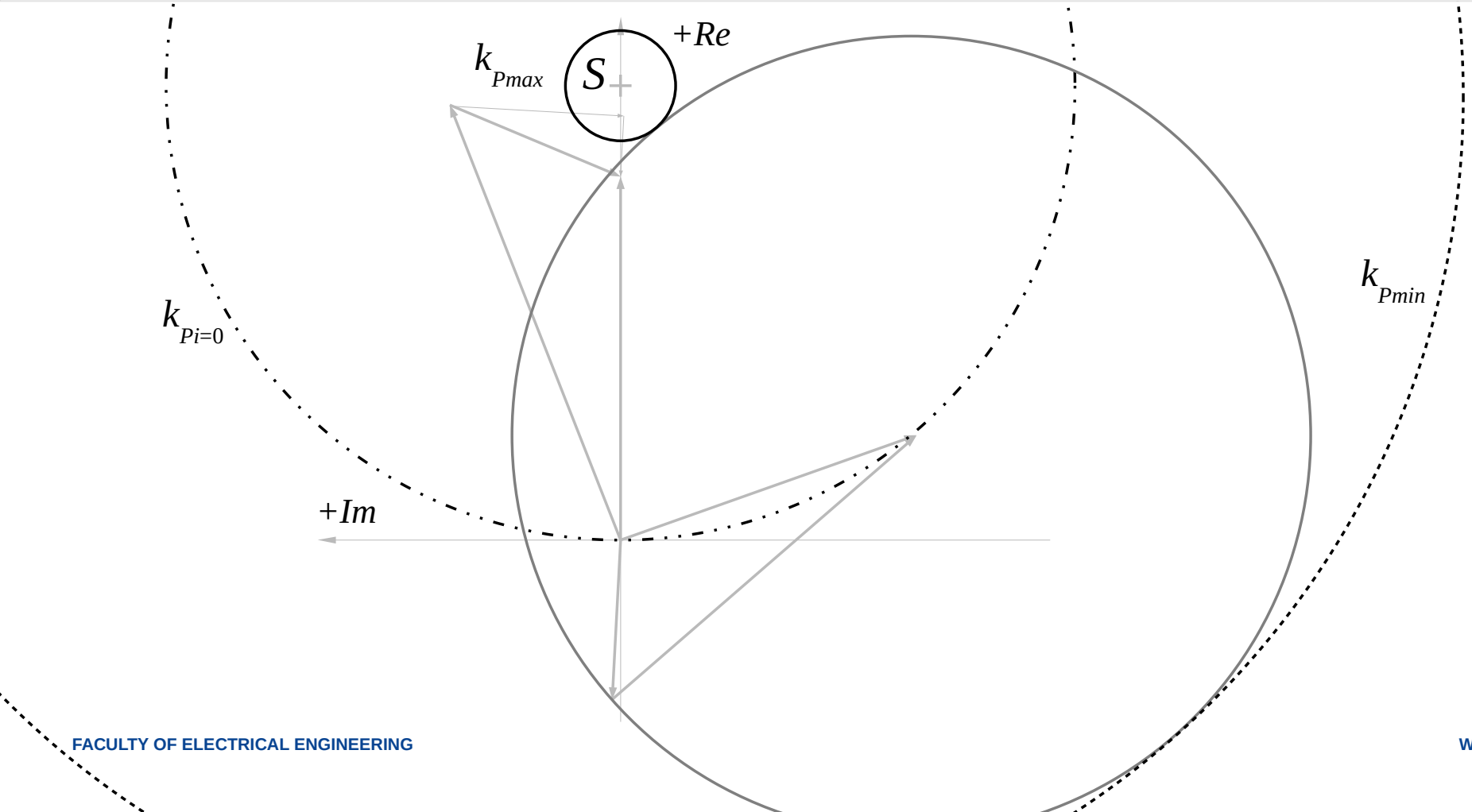
Output Power of a SM while Considering Armature Resistance



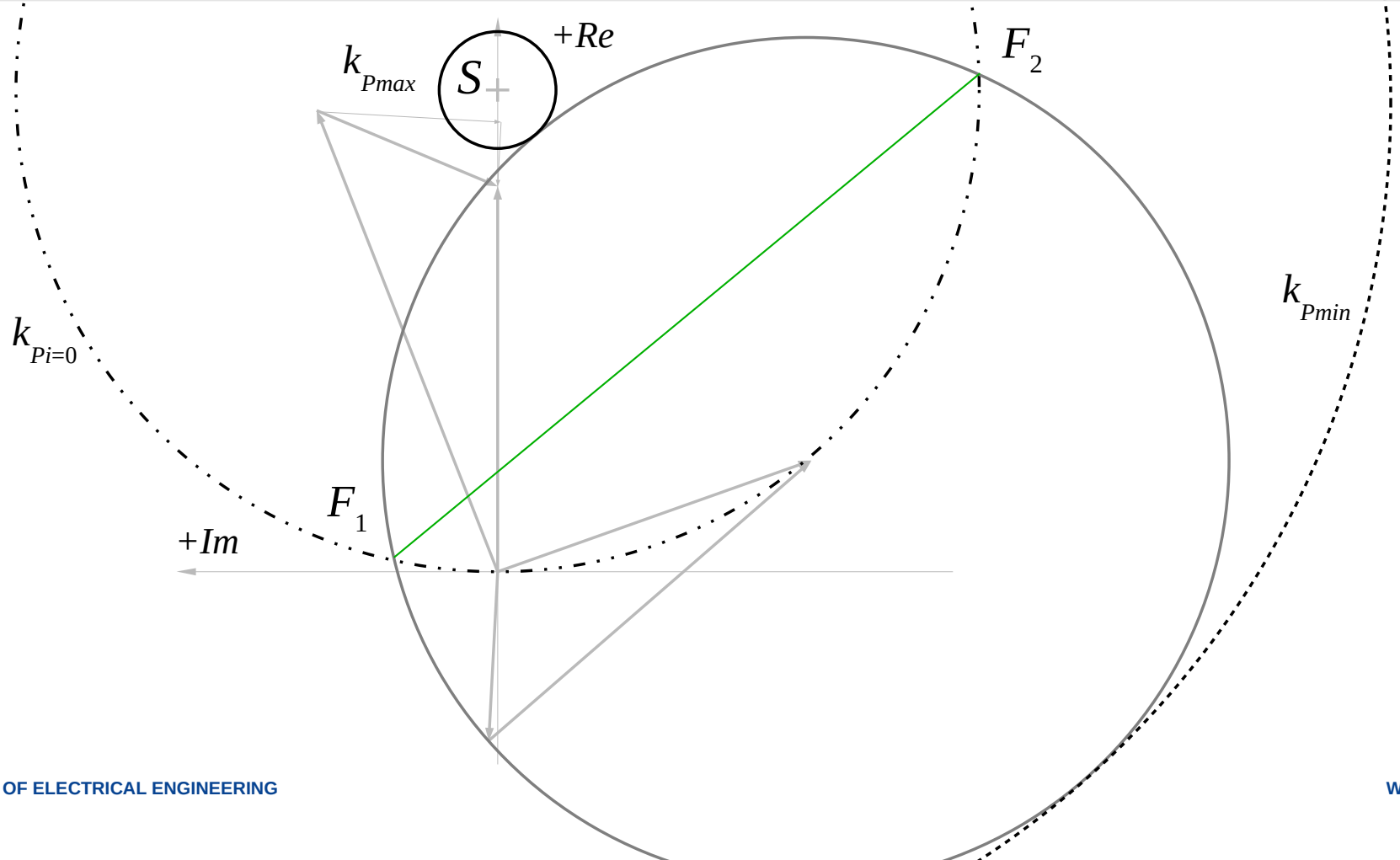
Output Power of a SM while Considering Armature Resistance

- Similarly to the circle diagram of an induction machine, even in this case we can read the powers and torque of the machine
- They are gven by intersection of the curent circle and the circle of zero inner power, point denoted as F_1 a F_2
- The powers are read between the line segment given by these points and the imaginary axis or current circle respectively

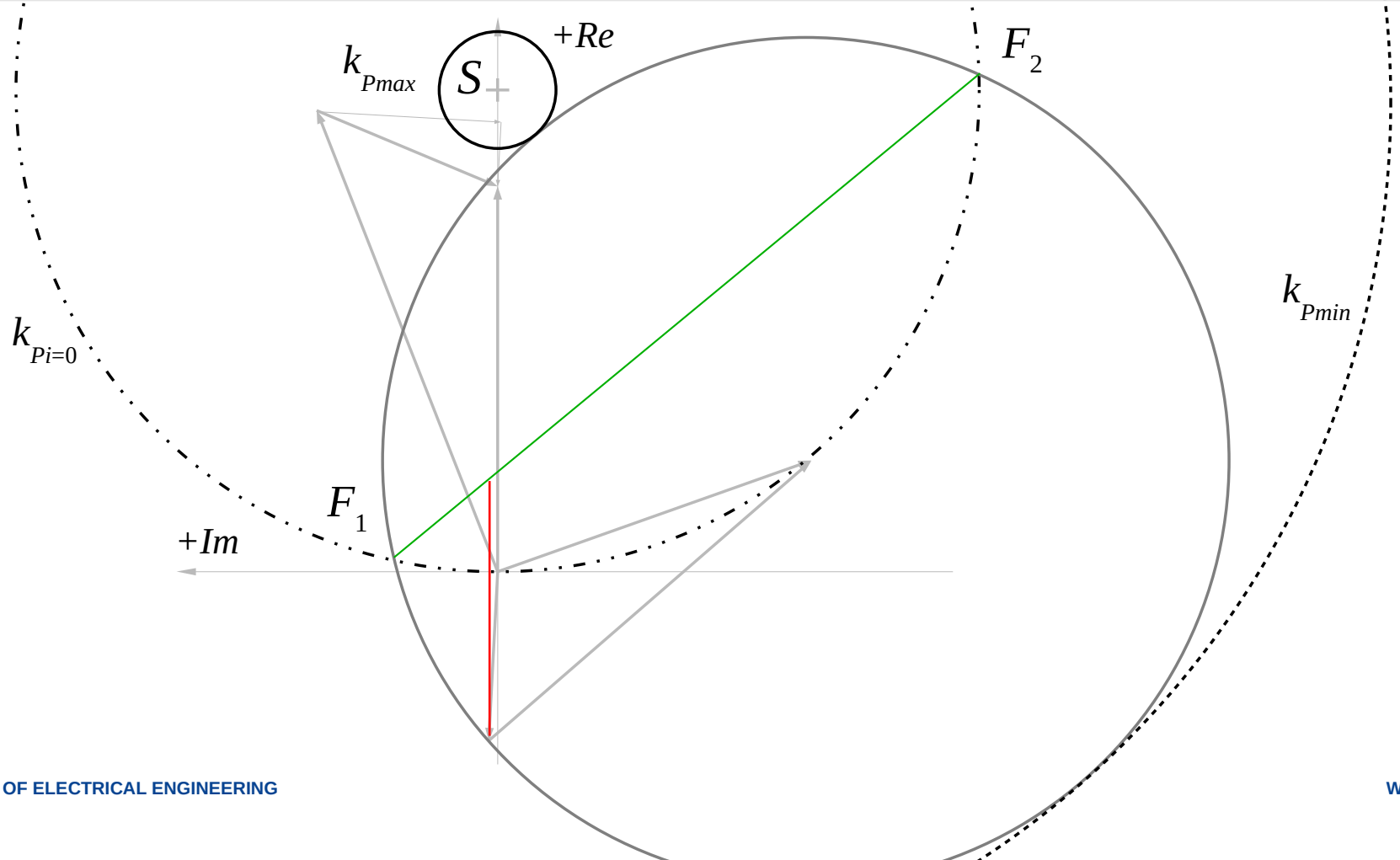
Output Power of a SM while Considering Armature Resistance



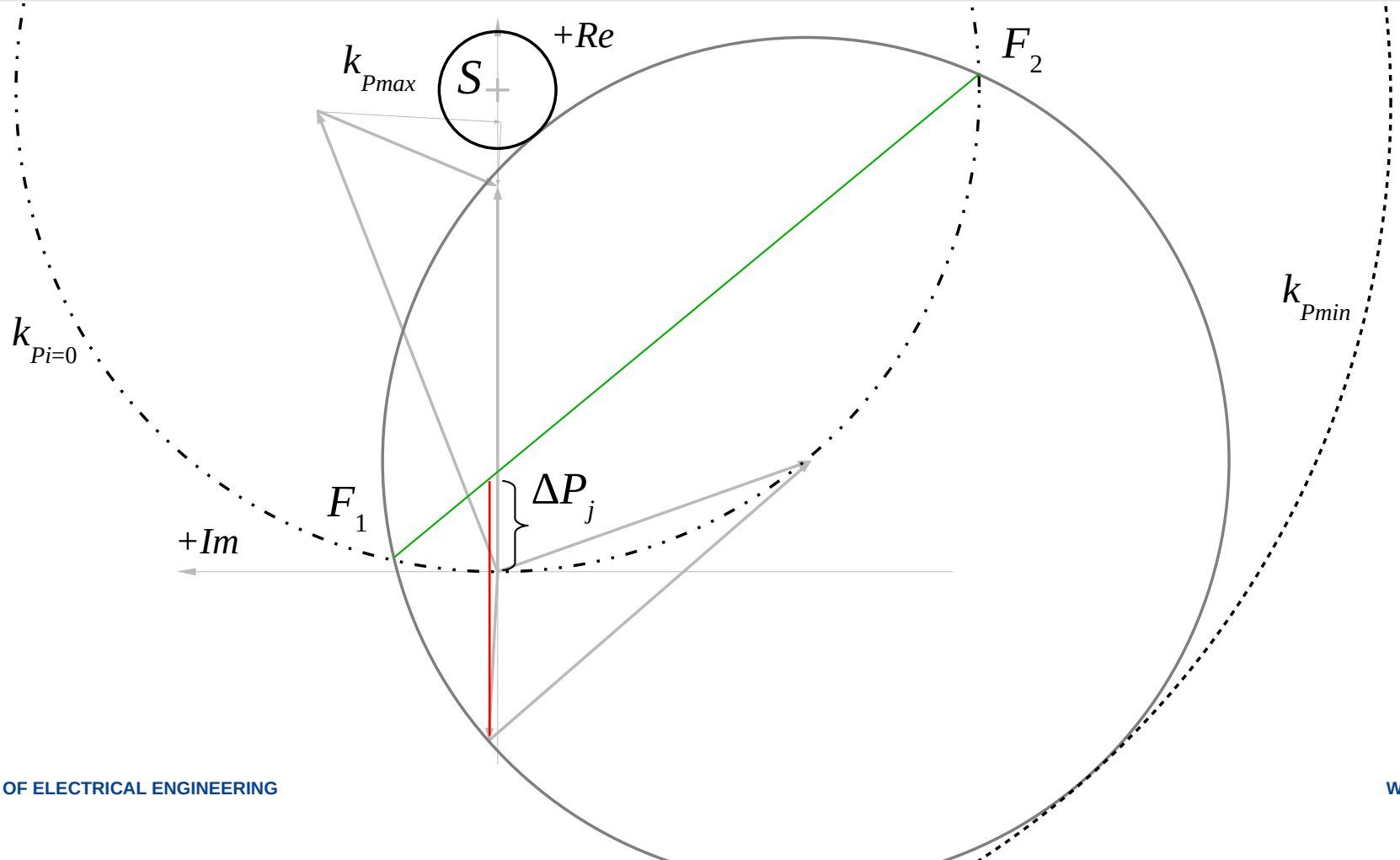
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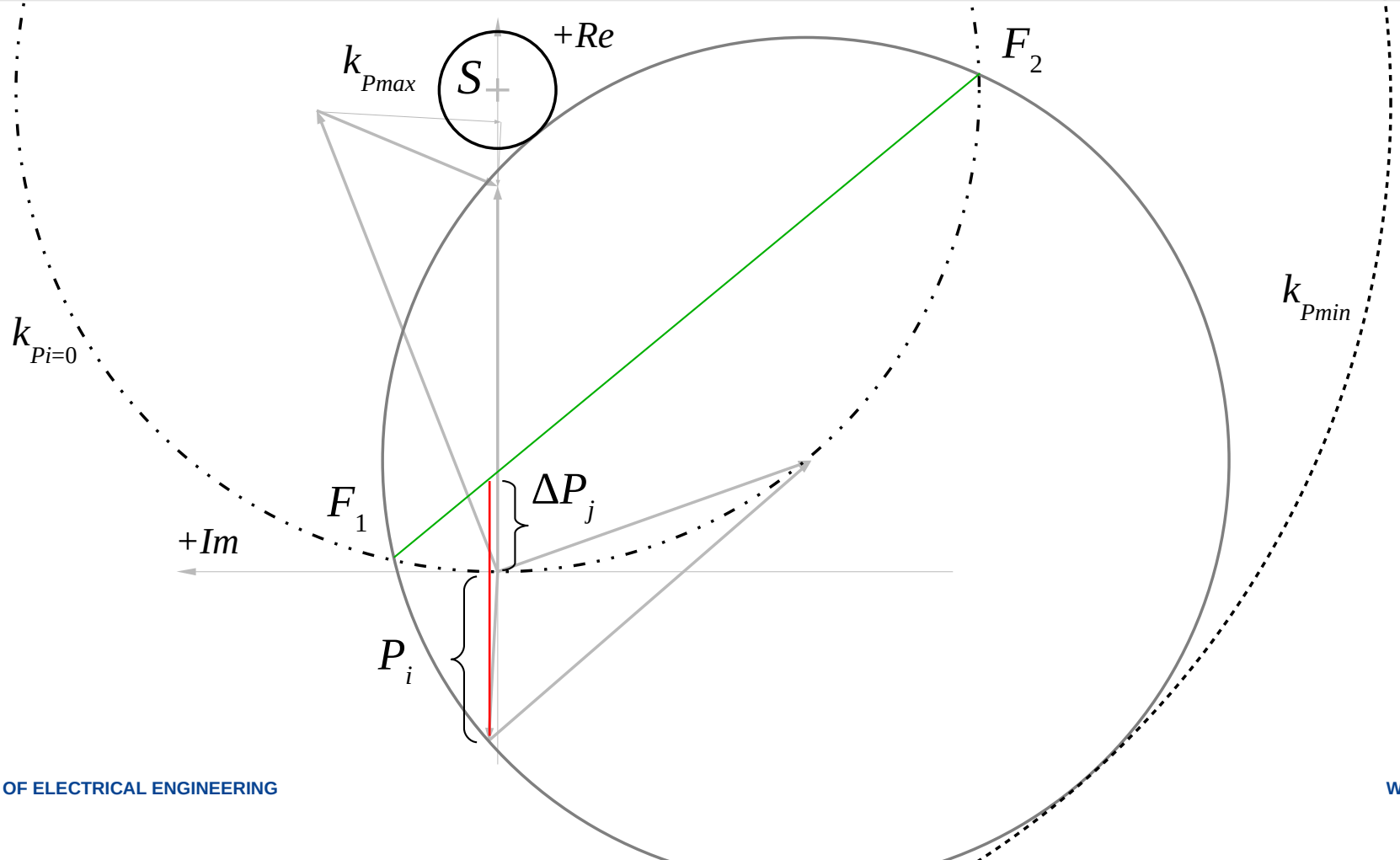
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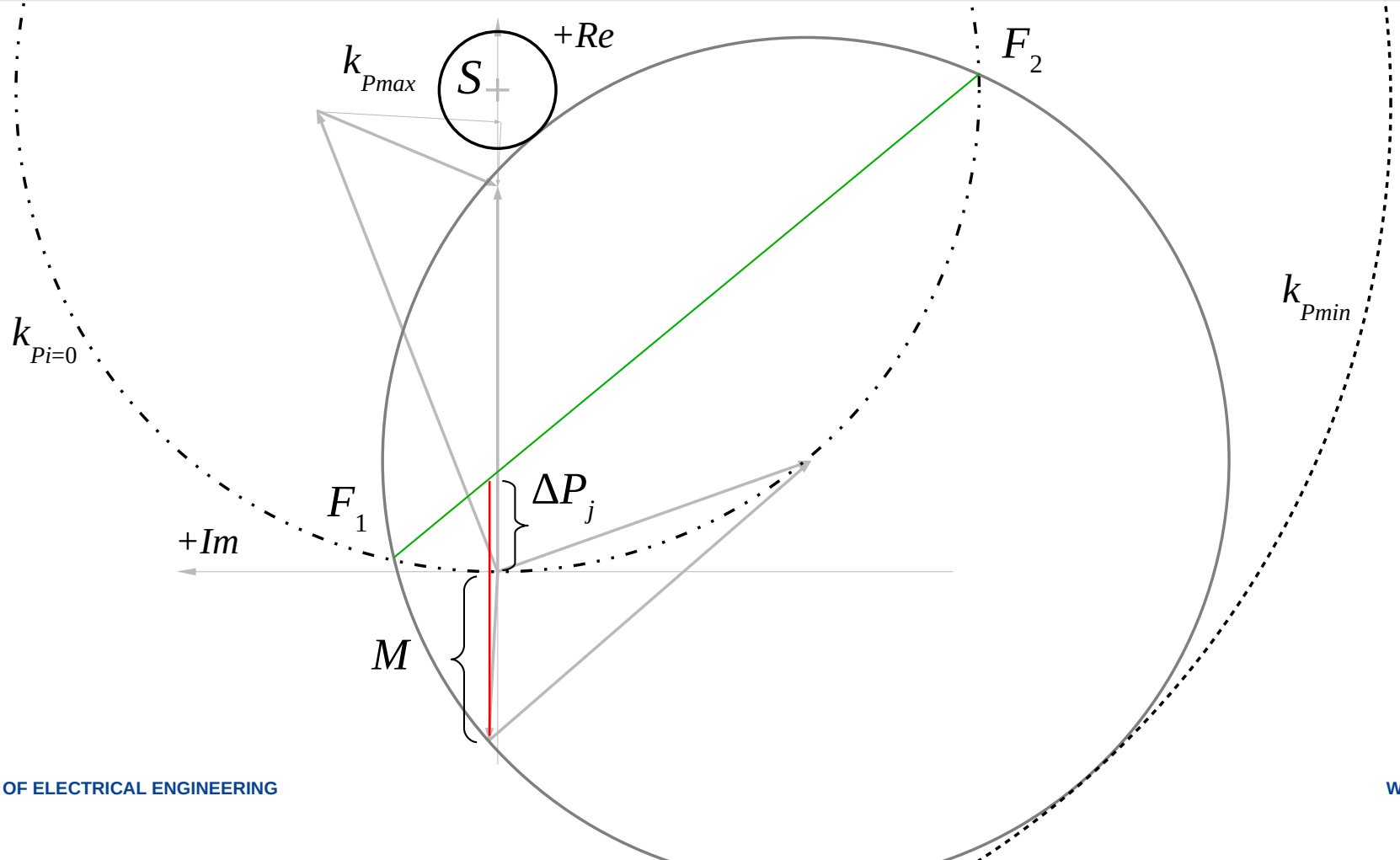
Output Power of a SM while Considering Armature Resistance



Output Power of a SM while Considering Armature Resistance



Output Power of a SM while Considering Armature Resistance



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Torque of a Synchronous Machine while Considering Armature Resistance

Torque of a Synch. Machine while Considering Armature Resistance

- The torque of a synchronous machine results from its inner power

$$P_i = m \Re \left\{ \bar{U}_{ib} \bar{I}^* \right\}$$

where the current of the machine was earlier derived as

$$I_\mu e^{j\left(\rho - \frac{\pi}{2}\right)} = I_k e^{j\left(\beta + \rho - \frac{\pi}{2}\right)} + \bar{I}$$

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and its complex conjugate is

$$\bar{I}^* = I_\mu e^{-j\left(\rho-\frac{\pi}{2}\right)} - I_k e^{-j\left(\beta+\rho-\frac{\pi}{2}\right)}$$

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$$\bar{I}^* = I_\mu e^{-j\left(\rho-\frac{\pi}{2}\right)} - I_k e^{-j\left(\beta+\rho-\frac{\pi}{2}\right)} = I_\mu e^{j\left(\frac{\pi}{2}-\rho\right)} - I_k e^{j\left(\frac{\pi}{2}-\beta-\rho\right)}$$

Torque of a Synch. Machine while Considering Armature Resistance

- After substitution into the relation for inner power we obtain

$$P_i = m\Re\left\{U_{ib}\, e^{j\beta}\left[I_\mu\, e^{j\left(\frac{\pi}{2}-\rho\right)} - I_k\, e^{j\left(\frac{\pi}{2}-\beta-\rho\right)}\right]\right\}$$

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which is

$$P_i = m\Re\left\{U_{ib}\left[I_\mu e^{j\left(\frac{\pi}{2}+\beta-\rho\right)} - I_k e^{j\left(\frac{\pi}{2}-\rho\right)}\right]\right\}$$

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$$P_i = m\Re\left\{U_{ib}\left[I_\mu e^{j\left(\frac{\pi}{2}+\beta-\rho\right)} - I_k e^{j\left(\frac{\pi}{2}-\rho\right)}\right]\right\}$$

- The real part of the projection of a phasor into real axis is equal to cosinus of its argument, thus we can futher substitute:

$$\Re\left\{e^{j\left(\frac{\pi}{2}+\beta-\rho\right)}\right\} = \cos\left(\frac{\pi}{2}+\beta-\rho\right) \qquad \text{and} \qquad \Re\left\{e^{j\left(\frac{\pi}{2}-\rho\right)}\right\} = \cos\left(\frac{\pi}{2}-\rho\right)$$

Torque of a Synch. Machine while Considering Armature Resistance

- The result is then

$$P_i = m U_{ib} \left[I_\mu \cos\left(\frac{\pi}{2} + \beta - \rho \right) - I_k \cos\left(\frac{\pi}{2} - \rho \right) \right]$$

Torque of a Synch. Machine while Considering Armature Resistance

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or after transformation sines to cosines

$$P_i = m U_{ib} \left[-I_\mu \sin(\beta - \rho) + I_k \sin(-\rho) \right]$$

Torque of a Synch. Machine while Considering Armature Resistance

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- Since sinus is an odd function, it is valid, that

$$\sin -\alpha = -\sin \alpha$$

and we can write

$$P_i = m U_{ib} \left[I_\mu \sin(\rho - \beta) - I_k \sin \rho \right]$$

Torque of a Synch. Machine while Considering Armature Resistance

- It is possible to substitute instead of both currents and then the resulting relation for calculation of inner power of a synchronous machine considering R_a is

$$P_i = m U_{ib} \left[\frac{U}{Z_d} \sin(\rho - \beta) - \frac{U_{ib}}{Z_d} \sin \rho \right]$$

or

$$P_i = m \frac{U_{ib}}{Z_d} \left[U \sin(\rho - \beta) - U_{ib} \sin \rho \right]$$

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or

$$P_i = m \frac{U_{ib}}{Z_d} \left[U \sin(\rho - \beta) - U_{ib} \sin \rho \right]$$

- The torque of the machine is bound to its power through its mechanical speed

$$M = \frac{P_i}{\omega_s}$$

Torque of a Synch. Machine while Considering Armature Resistance

- The torque of a machine with $L_d = L_q$ and consideration of the armature resistance is then

$$M = \frac{m}{\omega_s} \frac{U_{ib}}{Z_d} \left[U \sin(\rho - \beta) - U_{ib} \sin \rho \right]$$

where the angular speed of the shaft is

$$\omega_s = \frac{\omega_1}{p}$$

and thus for angular frequency of the power supply ω_1 we obtain

$$M = \frac{pm}{\omega_1} \frac{U_{ib}}{Z_d} \left[U \sin(\rho - \beta) - U_{ib} \sin \rho \right]$$

Torque of a Synch. Machine while Considering Armature Resistance

- In comparison to a characteristic neglecting armature resistance we have

$$M = \frac{pm}{\omega_1} \frac{U_{ib}}{Z_d} \left[U \sin(\rho - \beta) - U_{ib} \sin \rho \right]$$

Torque of a Synch. Machine while Considering Armature Resistance

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slightly lower magnitude

Torque of a Synch. Machine while Considering Armature Resistance

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horizontal shift by angle ρ

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Torque of a Synch. Machine while Considering Armature Resistance

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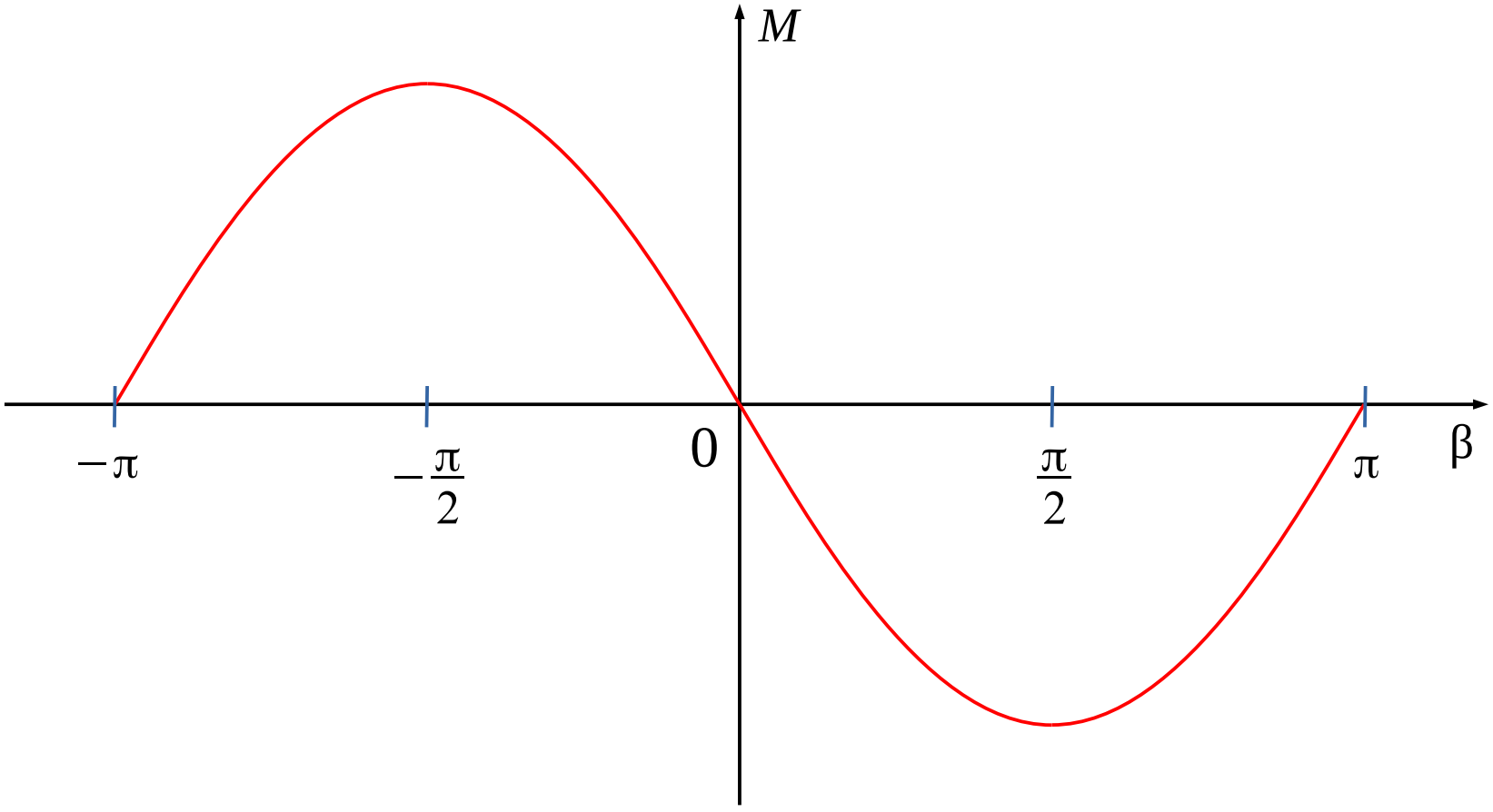
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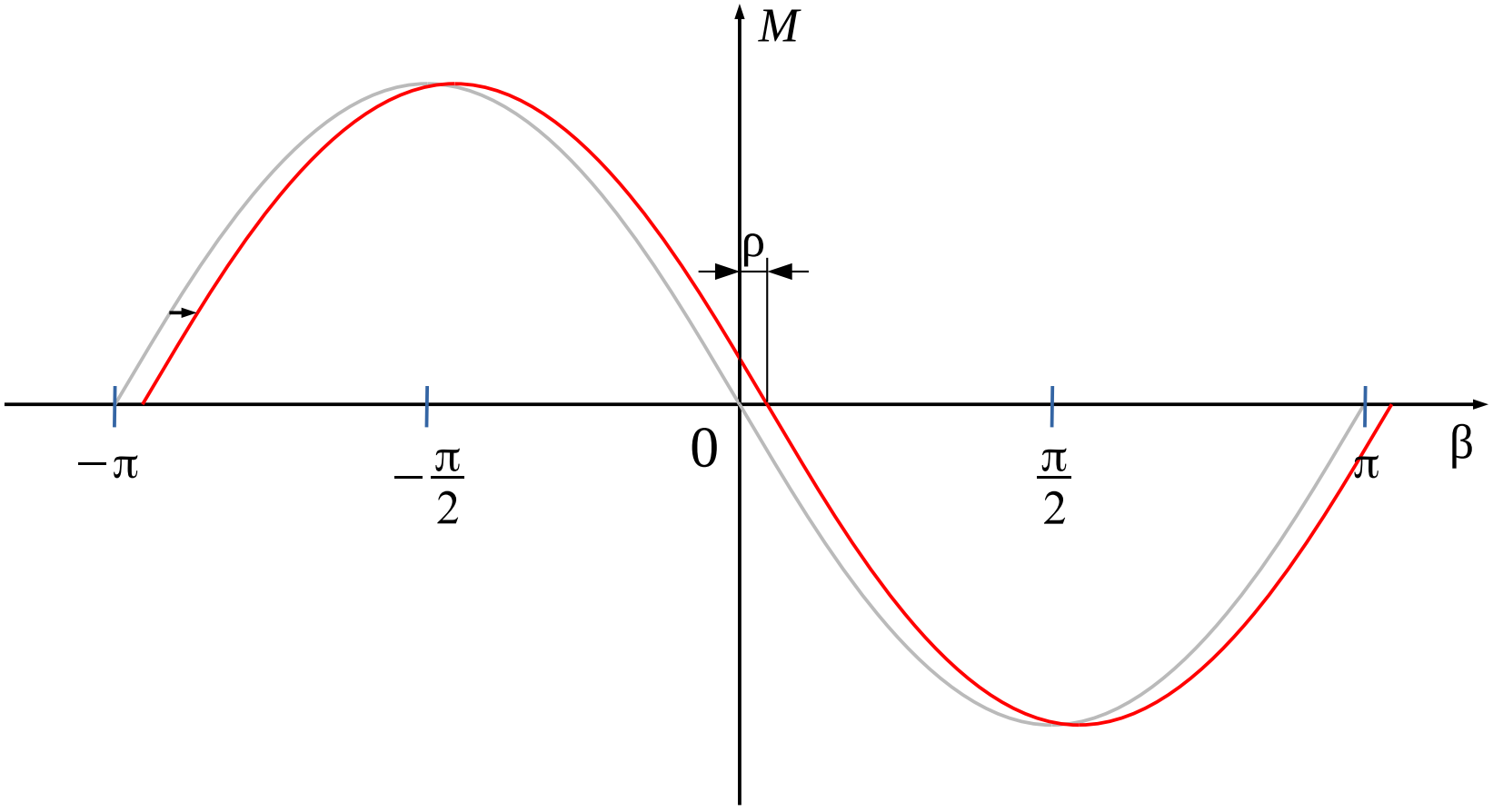
slightly lower magnitude

vertical shift $-\frac{pm}{\omega_1} \frac{U_{ib}^2}{Z_d} \sin \rho$

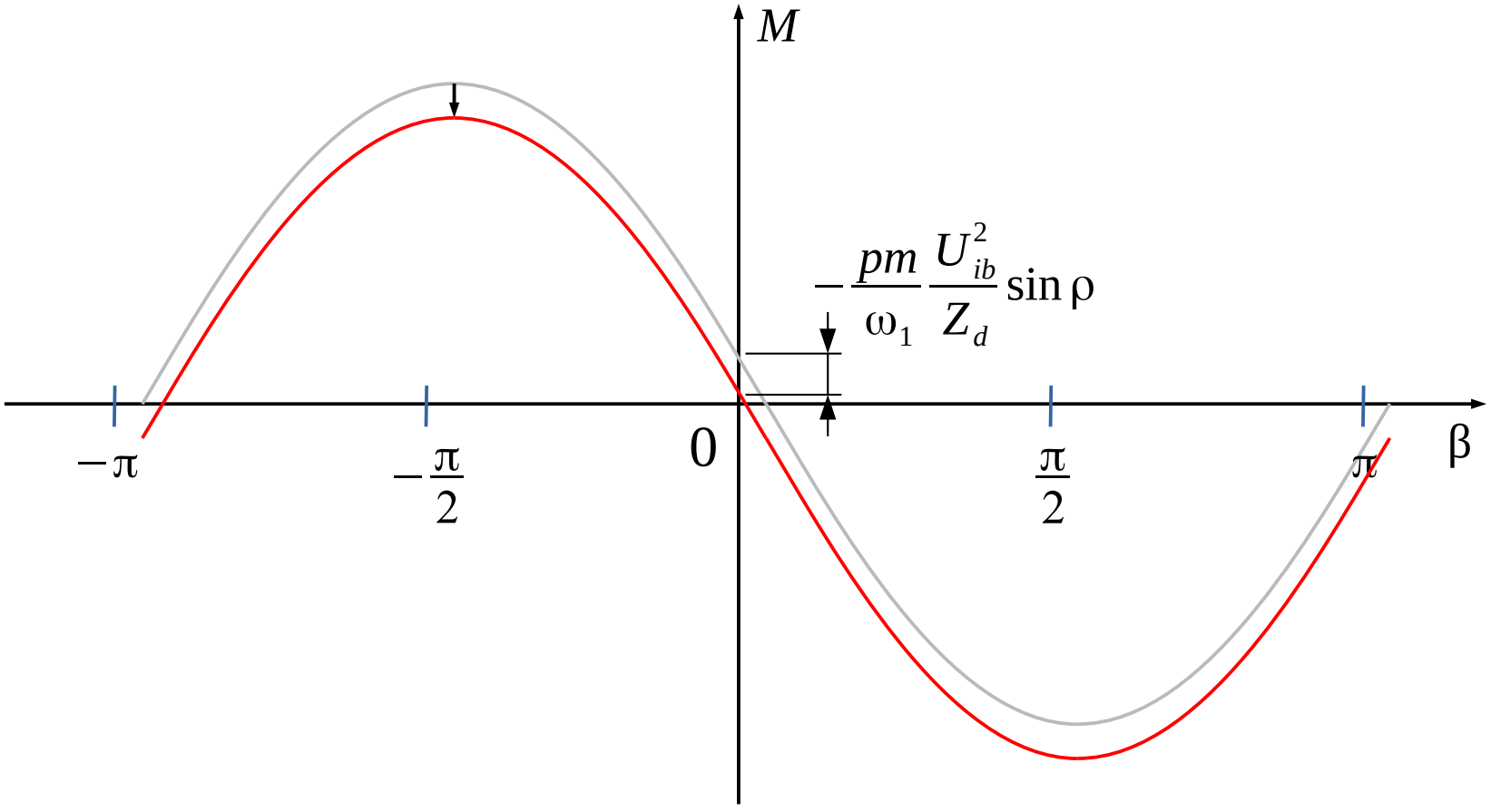
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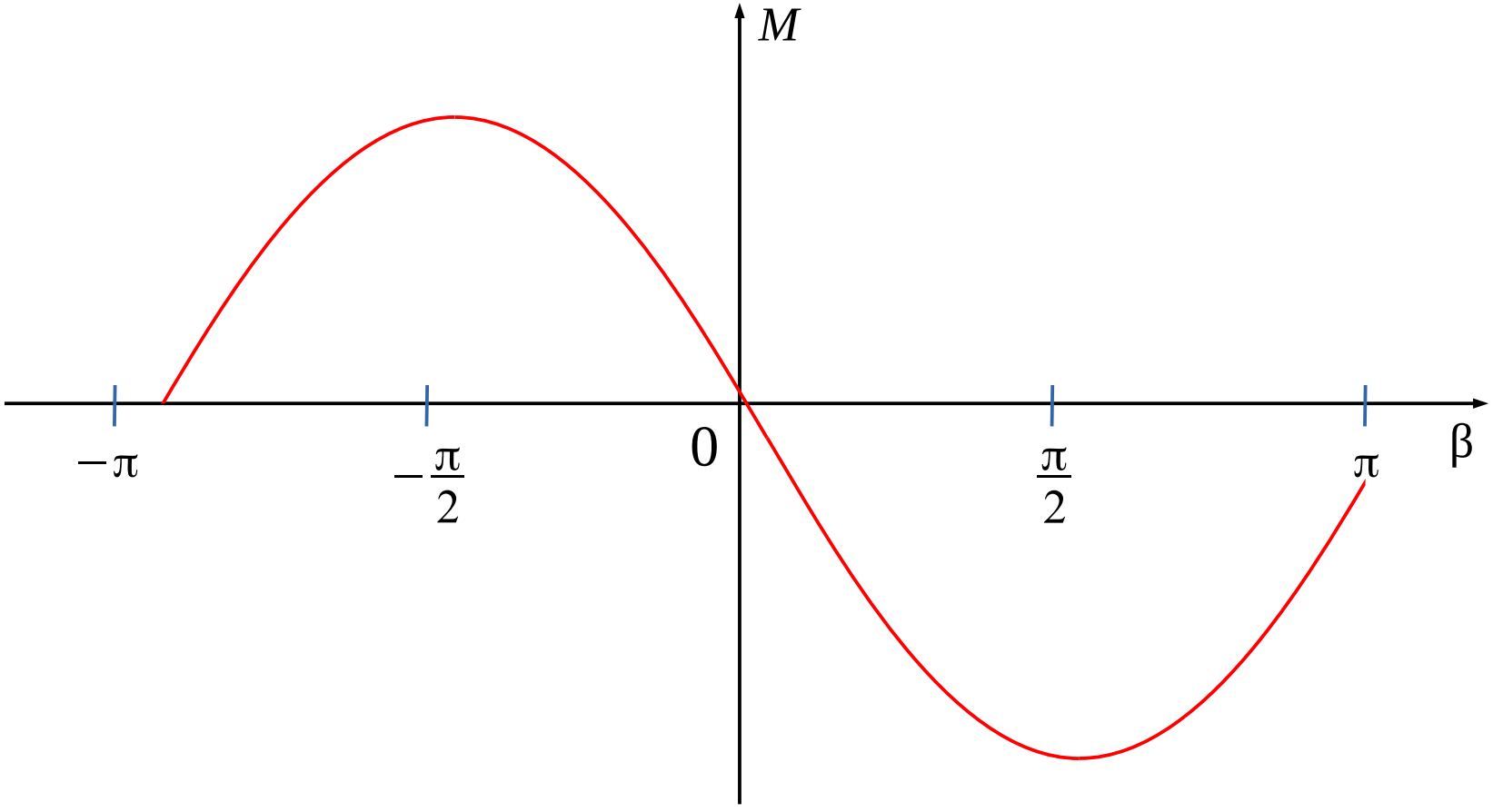
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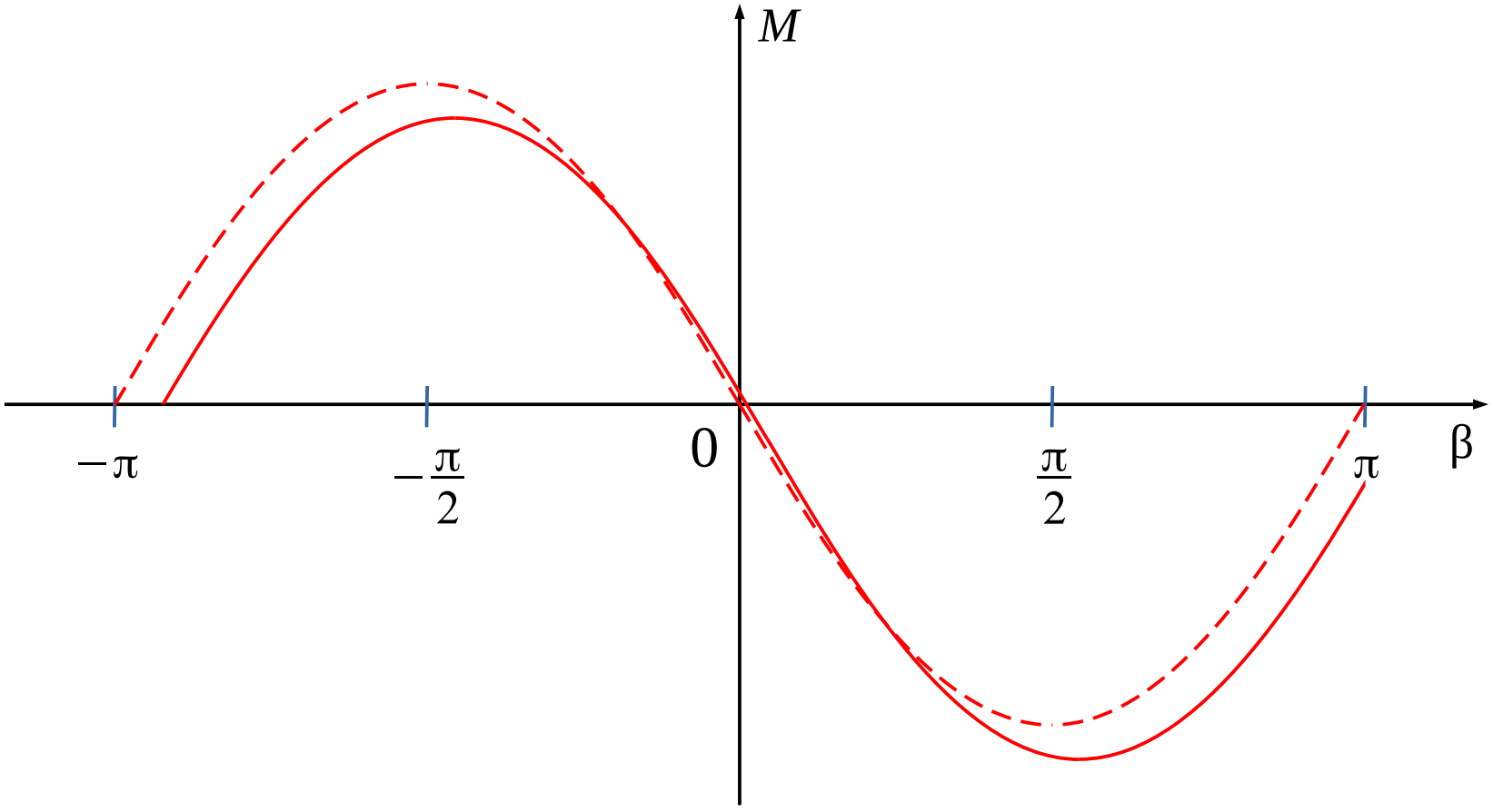
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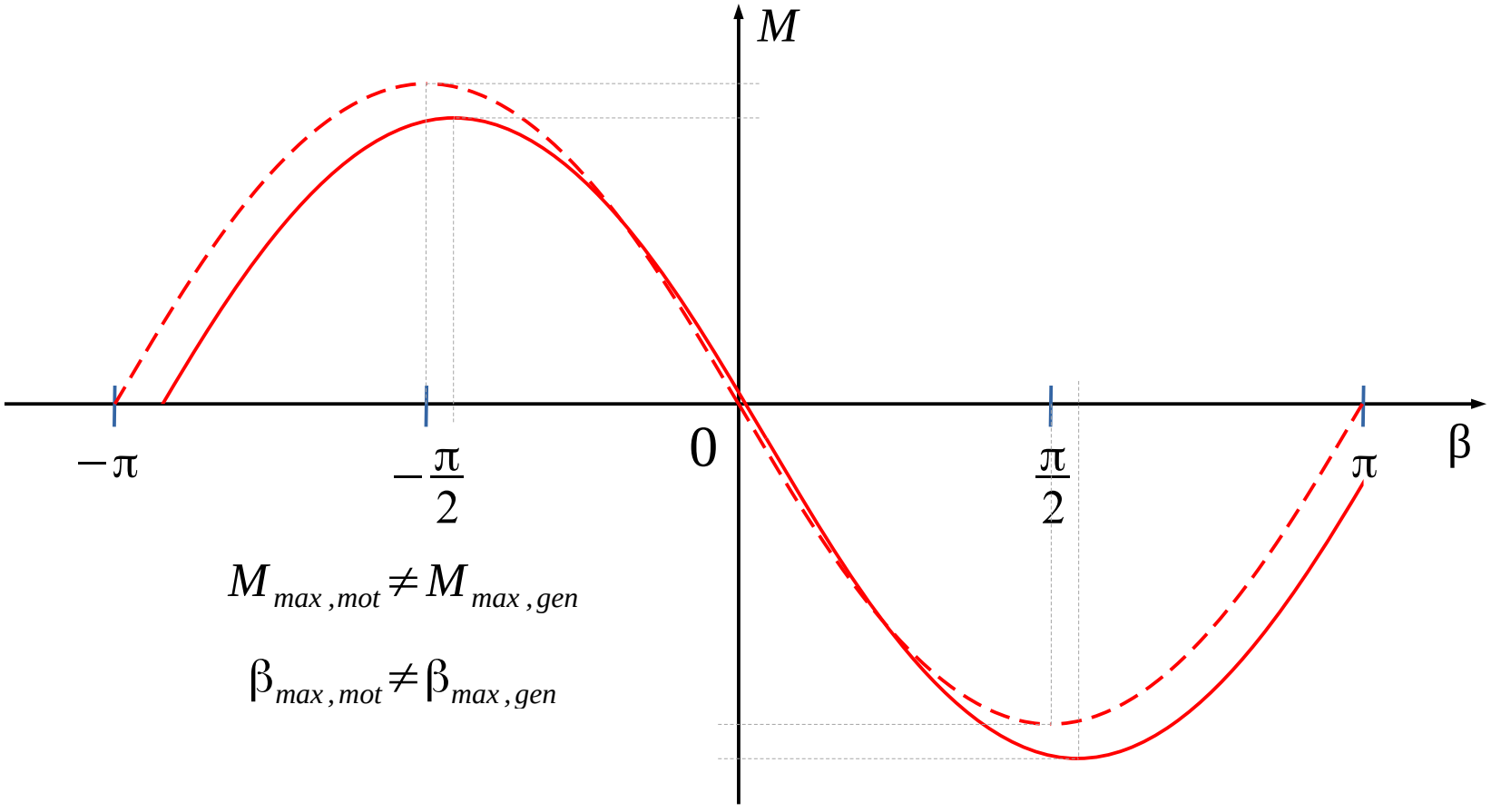
Torque of a Synch. Machine while Considering Armature Resistance



Torque of a Synch. Machine while Considering Armature Resistance



Torque of a Synch. Machine while Considering Armature Resistance



$$M_{max,mot} \neq M_{max,gen}$$

$$\beta_{max,mot} \neq \beta_{max,gen}$$

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Thank You for Your Attention!

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