FACULTY OF ELECTRICAL
ENGINEERING
UNIVERSITY OF WEST BOHEMIA

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

ESF II – project of the University of West Bohemia
reg. no. CZ.02.2.69/0.0/0.0/18_056/0013239

Theory of Electric Machines 2

KEV/TS2

Karel Hruska / r20201118

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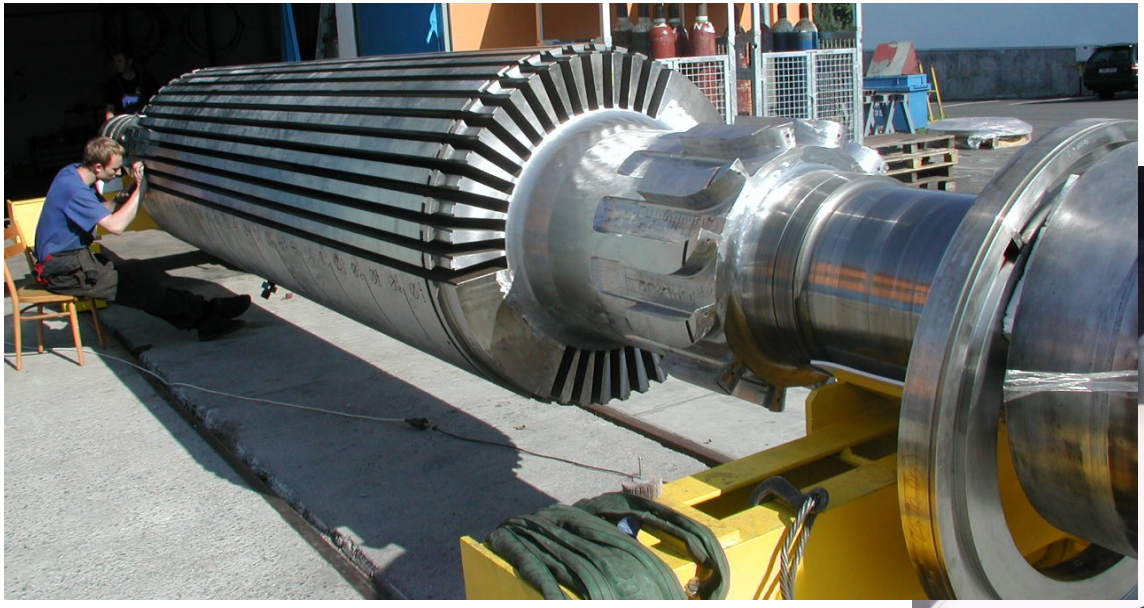
Magnetic Field of Synchronous Machines

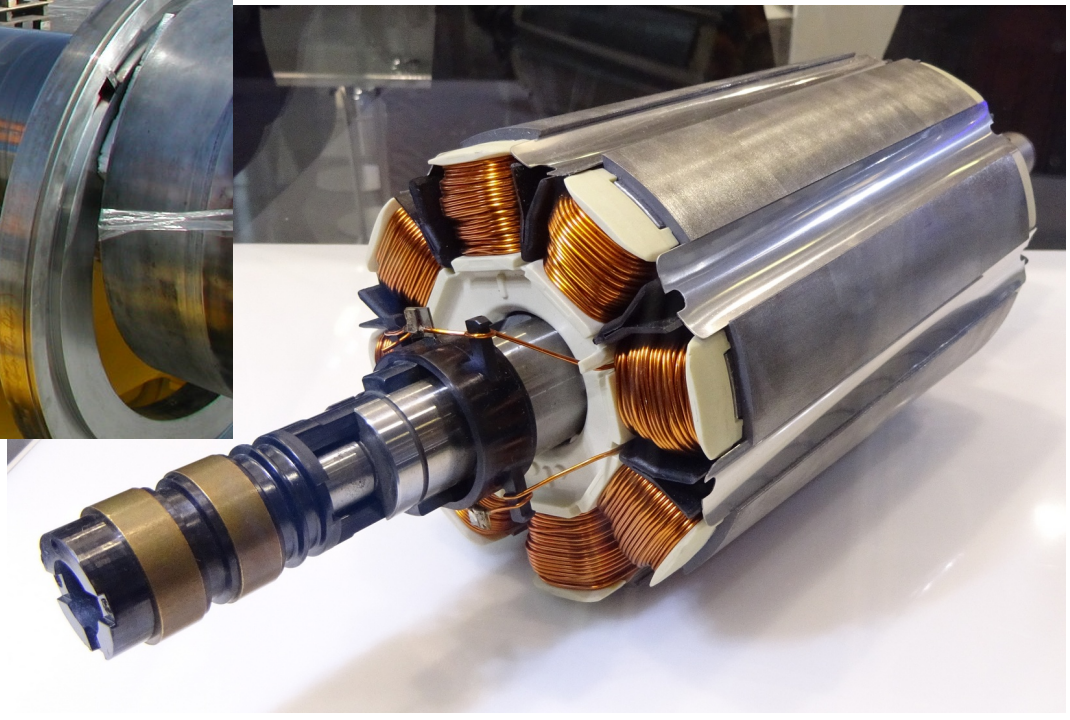
Magnetic Field of Synchronous Machines

- Synchronous machines
  - Electrically excited
    - Cylindrical rotor
    - Salient pole rotor
  - Permanent magnet excited
    - Surface mounted
    - Interior
  - Without excitation
    - Reluctance machines

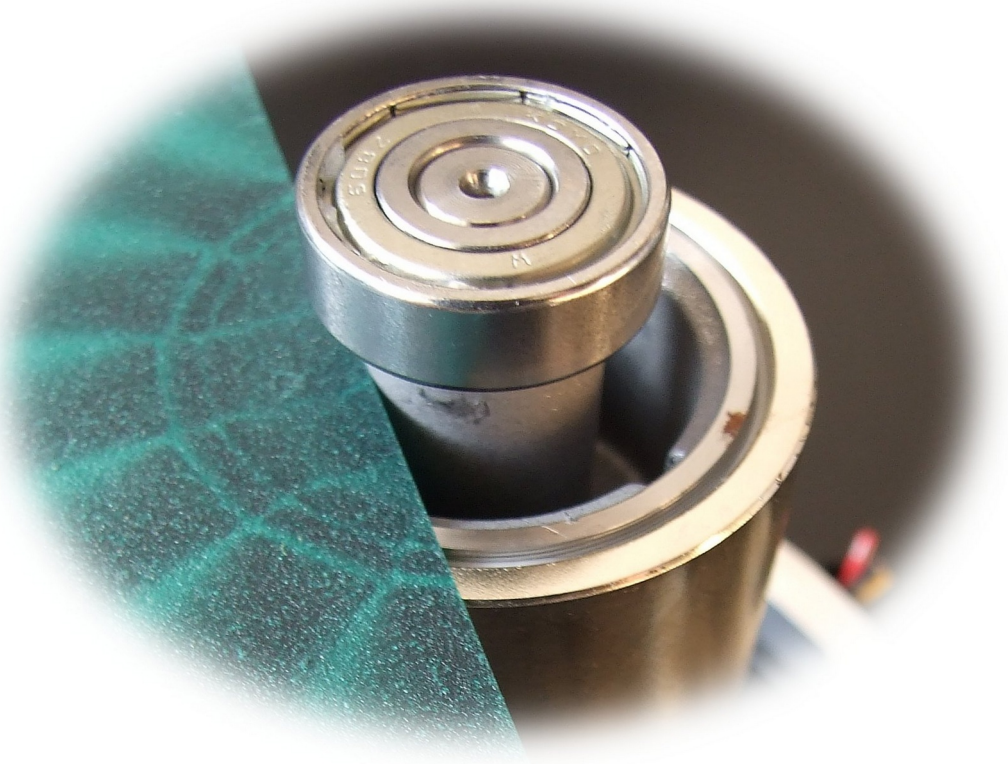


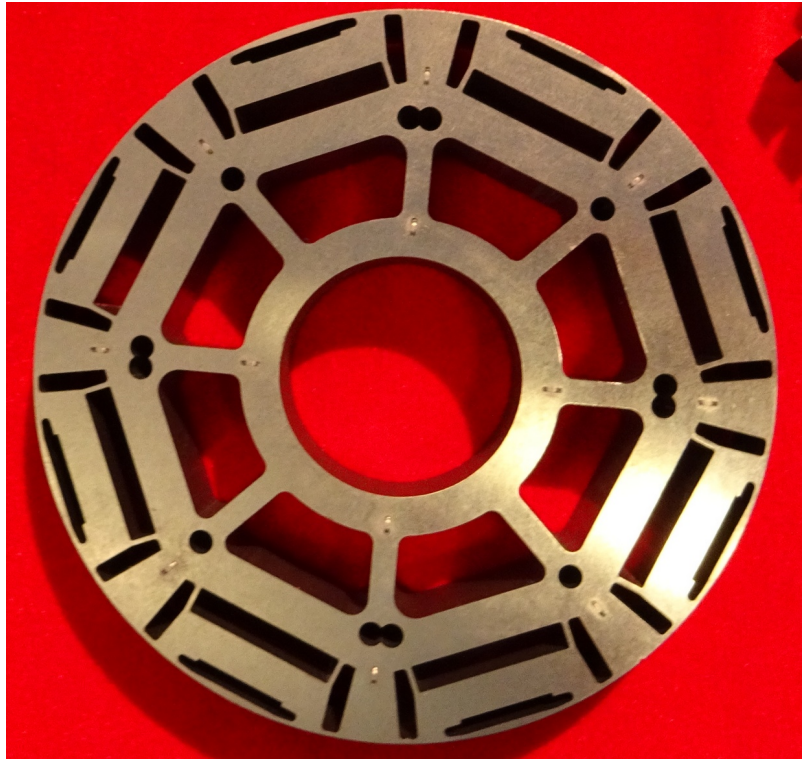
Magnetic Field of Synchronous Machines



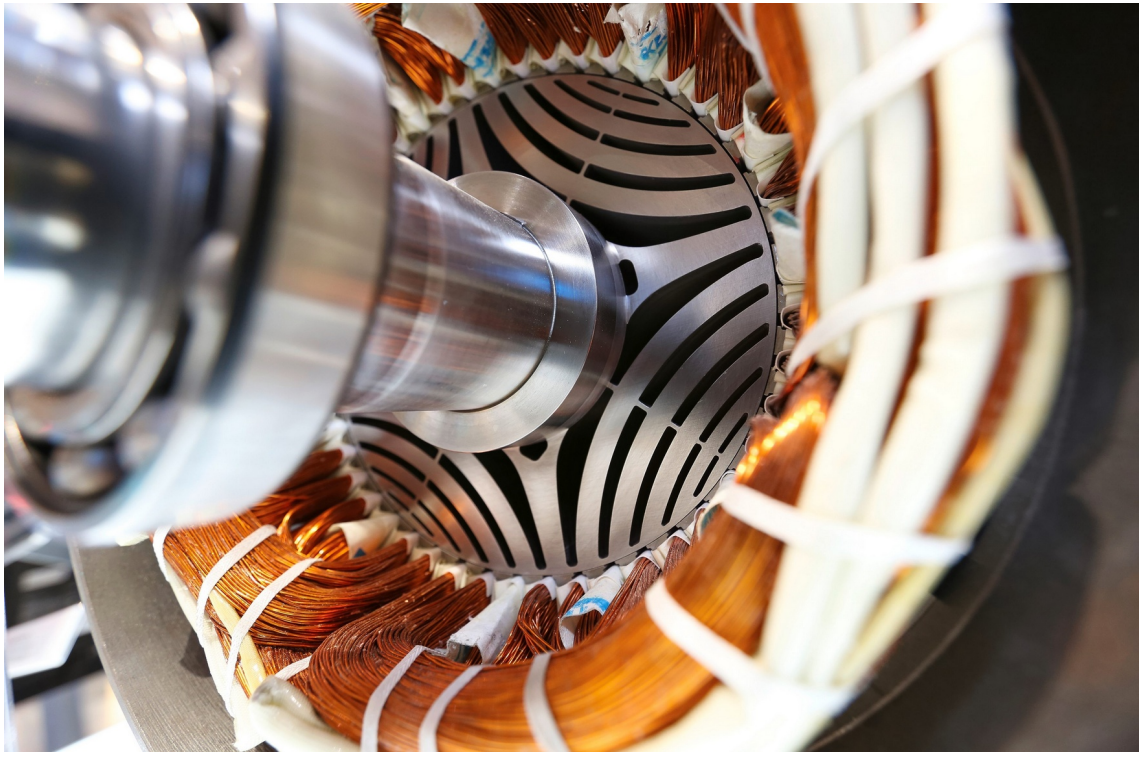


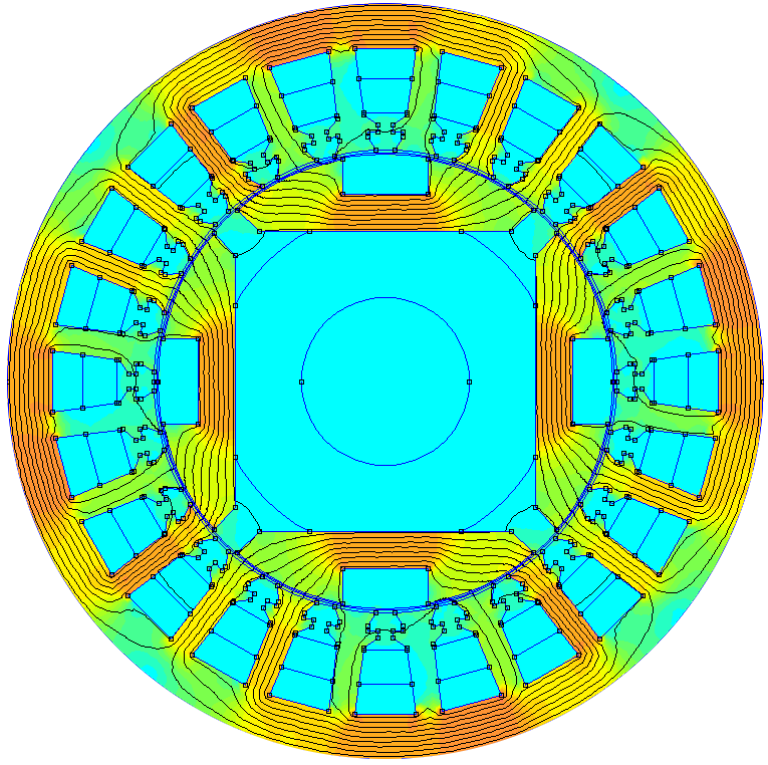
Magnetic Field of Synchronous Machines





Magnetic Field of Synchronous Machines





Magnetic Field of Synchronous Machines

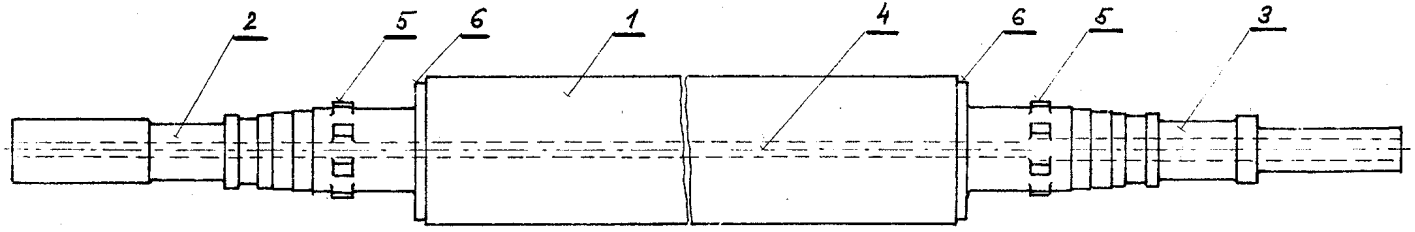
- Synchronous machines
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    - Interior
  - Without excitation
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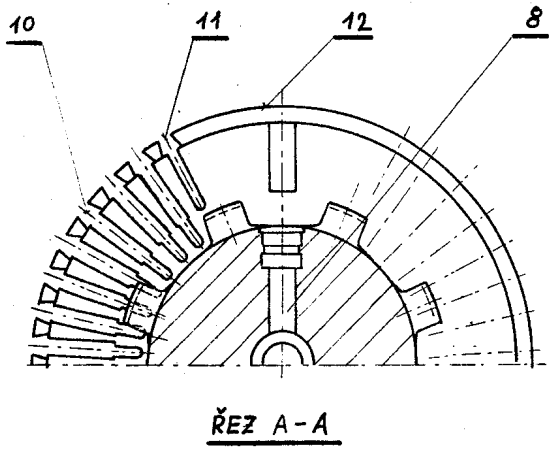
Magnetic Field of Synchronous Machines

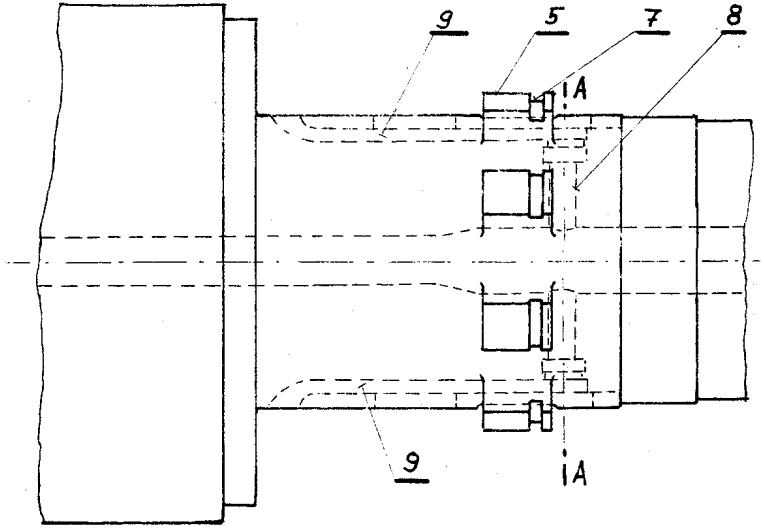
- In case of cylindrical rotor synchronous machines the excitation winding is located in slots
  - Poles are formed by “wide teeth”
  - The magnetic field is further shaped by “blind slots”

- Magnetic field is deformed by slotting, constant air gap in the area of wide tooth and it is shaped by blind teeth

Magnetic Field of Synchronous Machines







1 – rotor body

2, 3 – bearing slip surfaces

4 – axial bore

5 – support for winding overhang bandage

6 – rim for bandage

7 – slots for wells

8 – holes for excitation wire

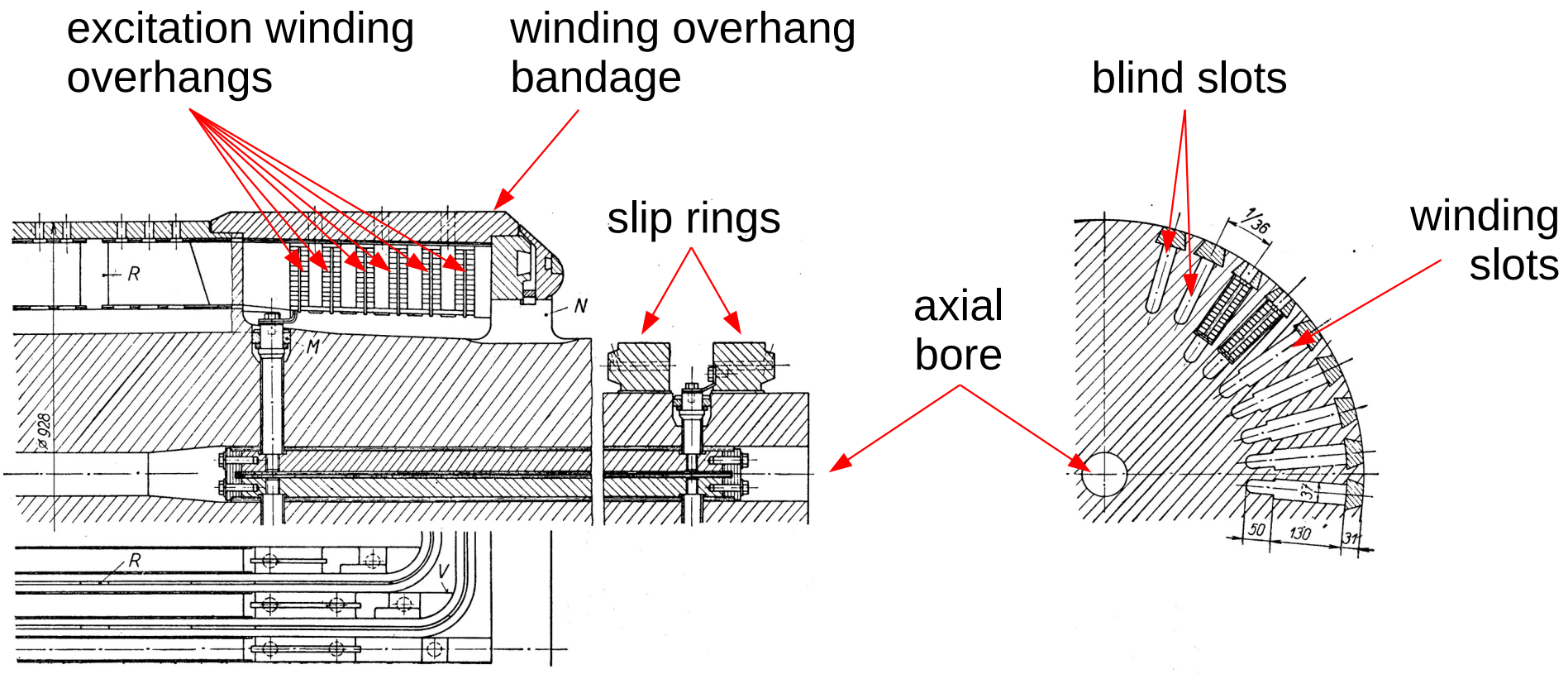
9 – slots for excitation wire

10 – winding slots

11 – blind teeth

12 – wide tooth

Magnetic Field of Synchronous Machines

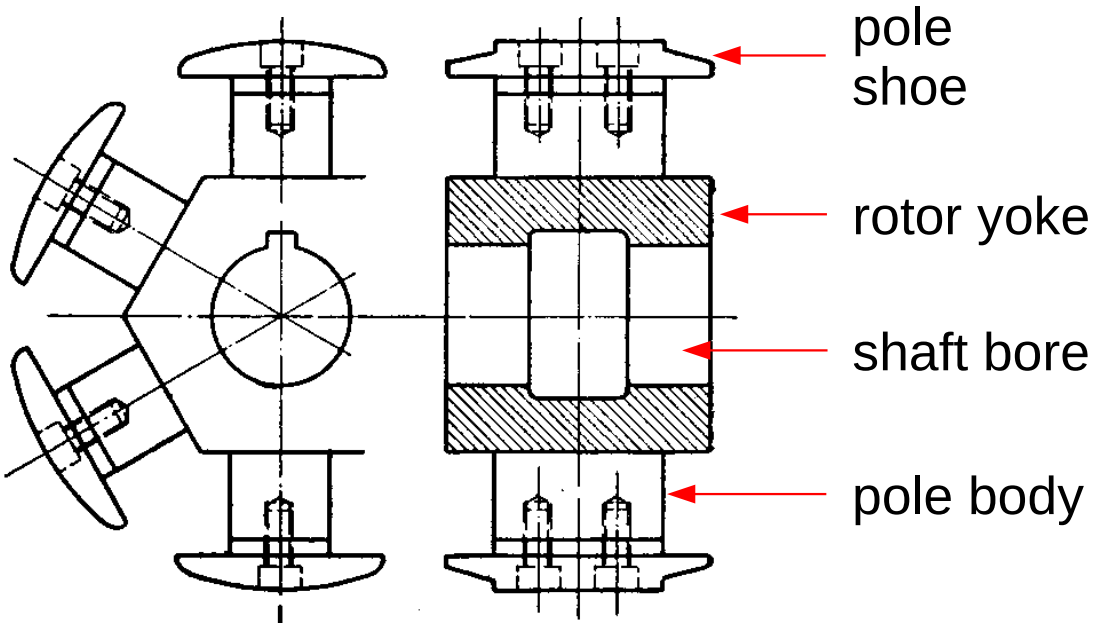


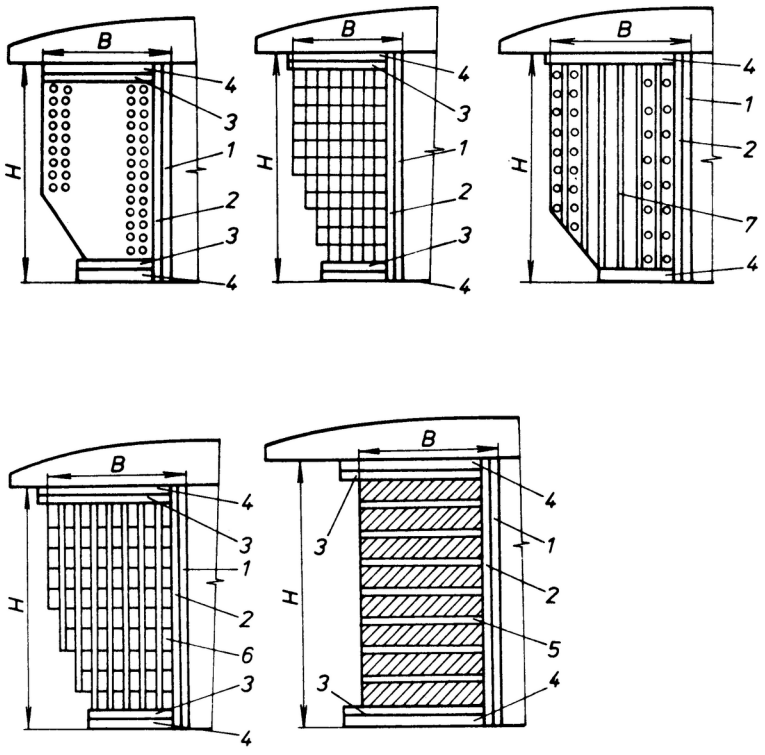
Magnetic Field of Synchronous Machines

- In case of salient pole synchronous machines the excitation winding is wound around pole body and secured by pole shoe
- The pole shoes do not cover the whole pole pitch (α_{mech}) and they are formed for optimal shape of air gap magnetic flux density B_δ

- The magnetic field is located almost fully in the axis of the poles (*d-axis*), in the q-axis it is almost missing.

Magnetic Field of Synchronous Machines





Magnetic Field of Synchronous Machines

- From electromagnetic point of view all synchronous machines have a large amount of similar properties:
  - cylindrical rotor electrically excited synchronous machines
    ≈ surface mounted permanent magnet synchronous machines
  - salient pole electrically excited synchronous machines
    ≈ interior permanent magnet synchronous machines
    ≈ synchronous reluctance machines

- For the purpose of successful design of a synchronous machine it is necessary to handle the analysis of both types of synchronous machines

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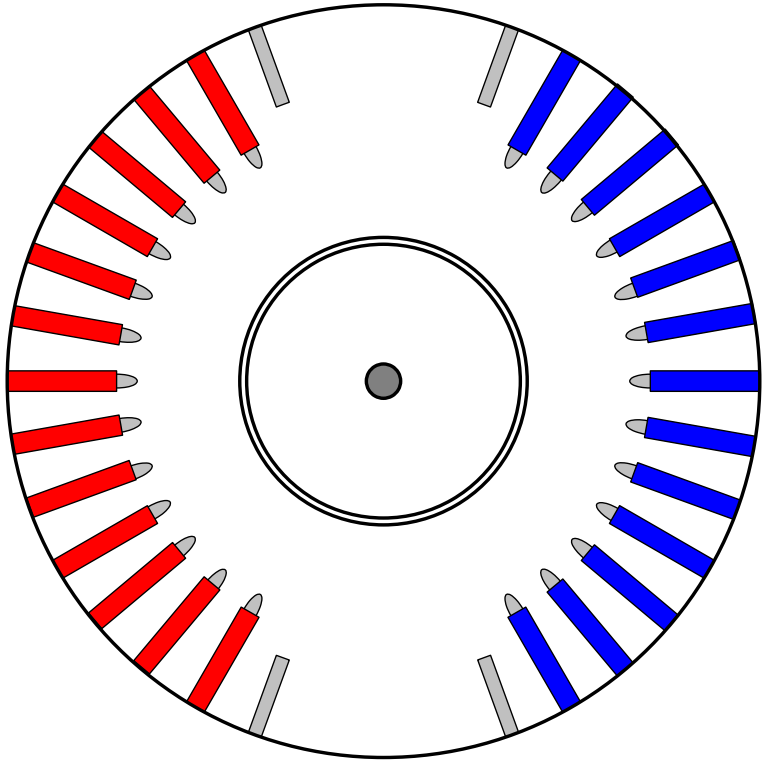
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Magnetic Field of Cylindrical Rotor Synchronous Machines

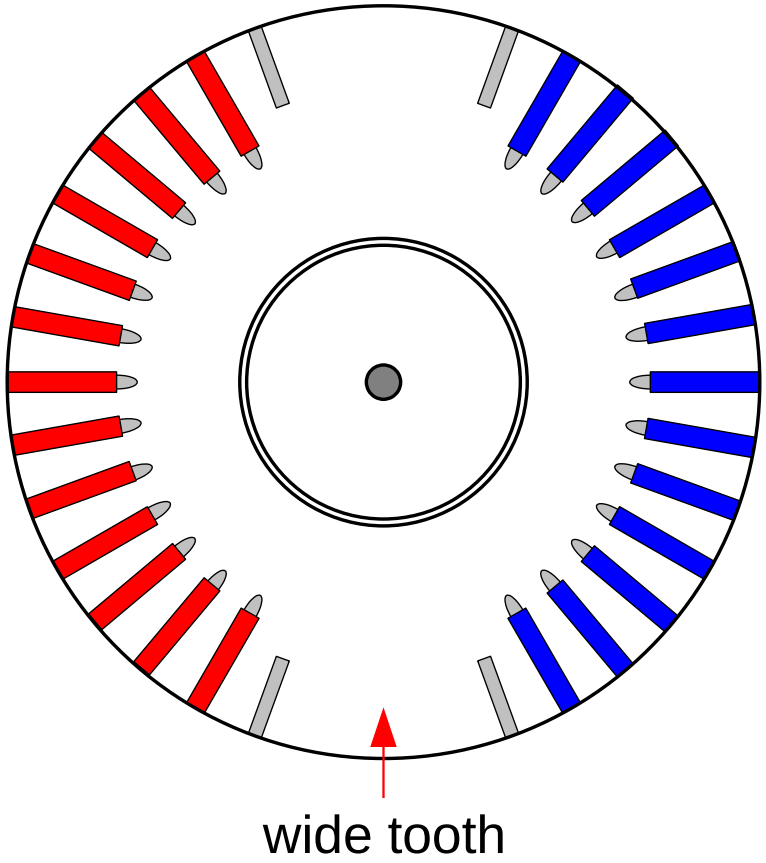
Magnetic Field of Cylindrical Rotor Synchronous Machines

- Let us presume an electrically excited synchronous machine
- A controllable winding is placed on both stator nad the rotor side
  - It is necessary to analyze each field separately
  - The properties of the resulting field are given by superposition of properties of both fields

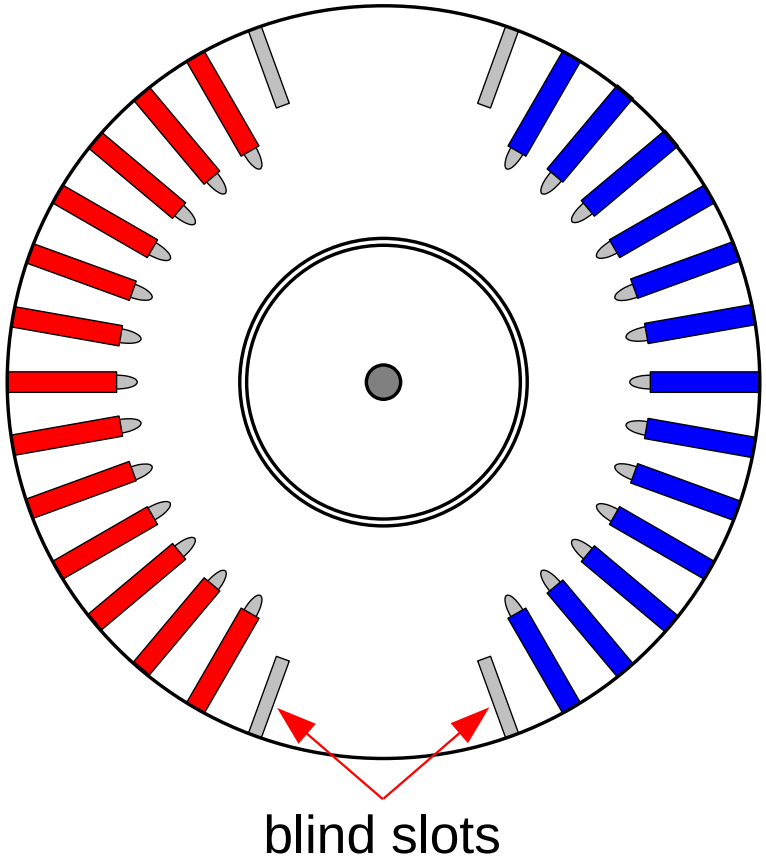
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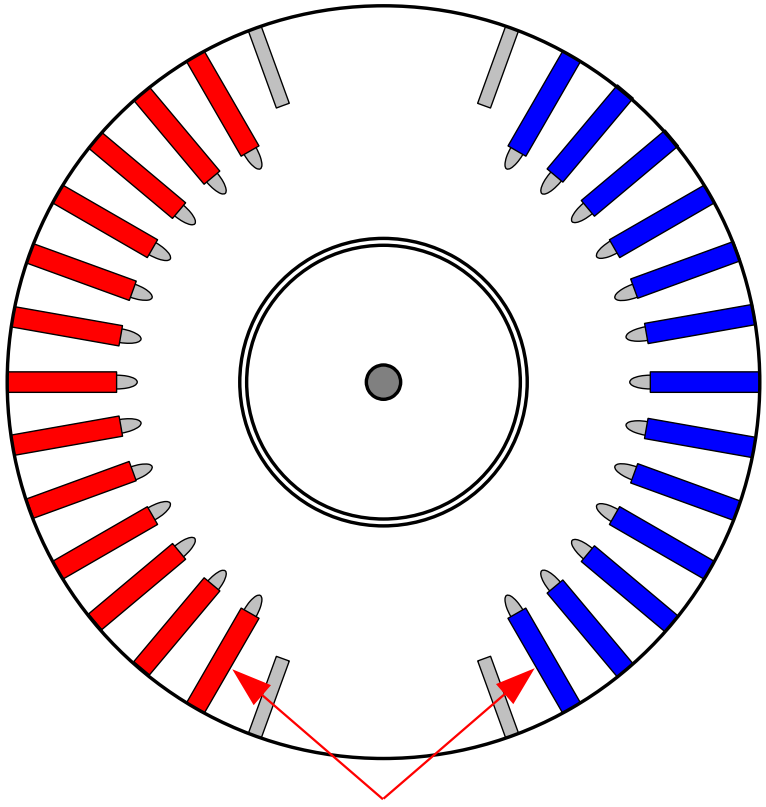
Magnetic Field of Cylindrical Rotor Synchronous Machines



Magnetic Field of Cylindrical Rotor Synchronous Machines

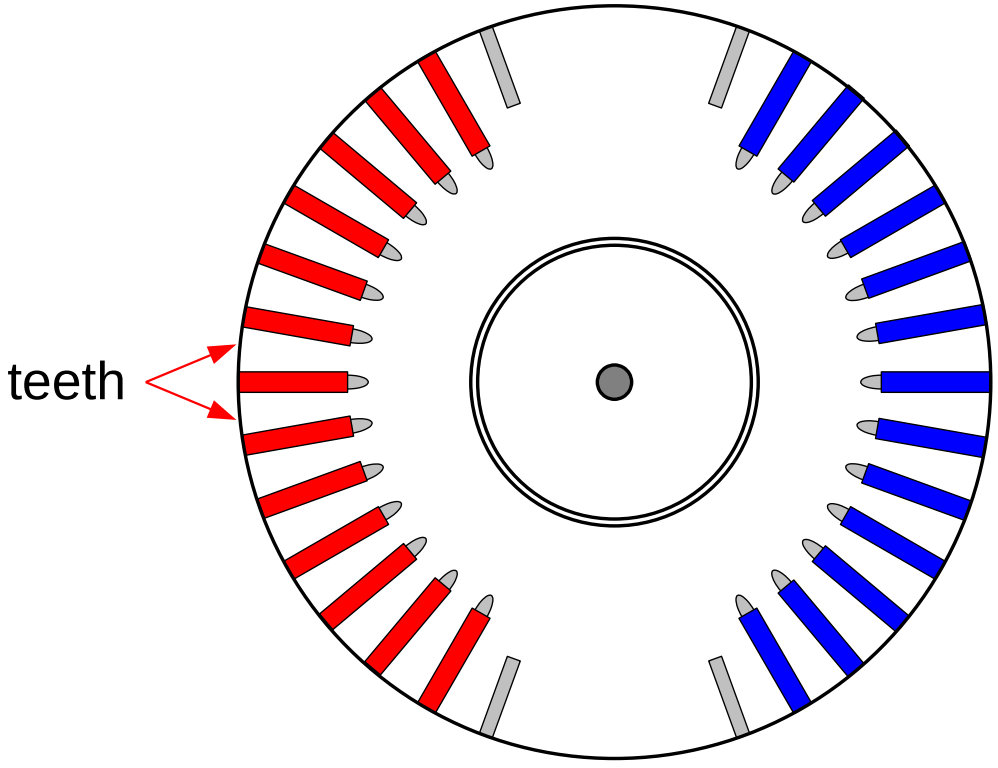


Magnetic Field of Cylindrical Rotor Synchronous Machines

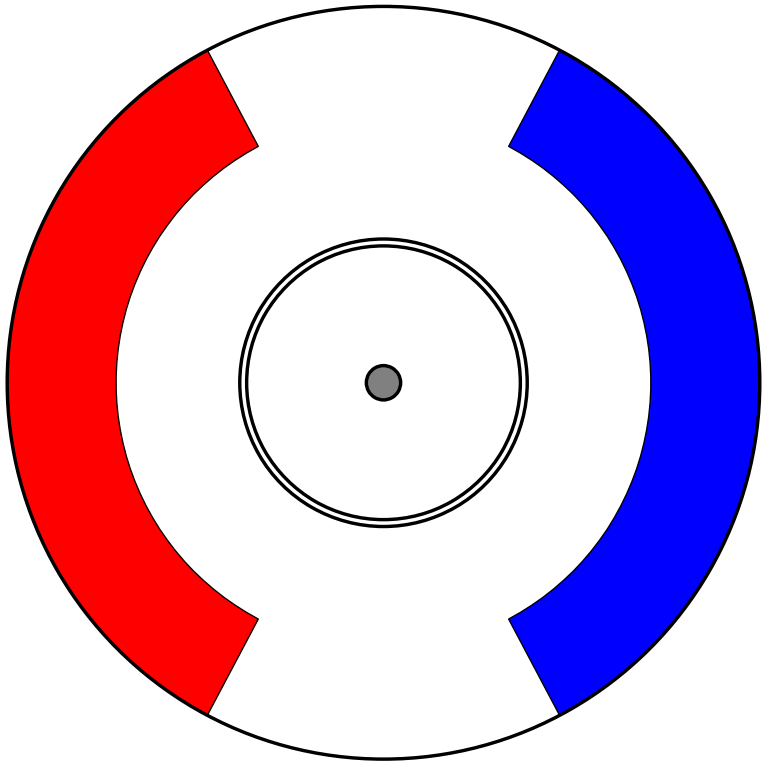


slots with current through winding

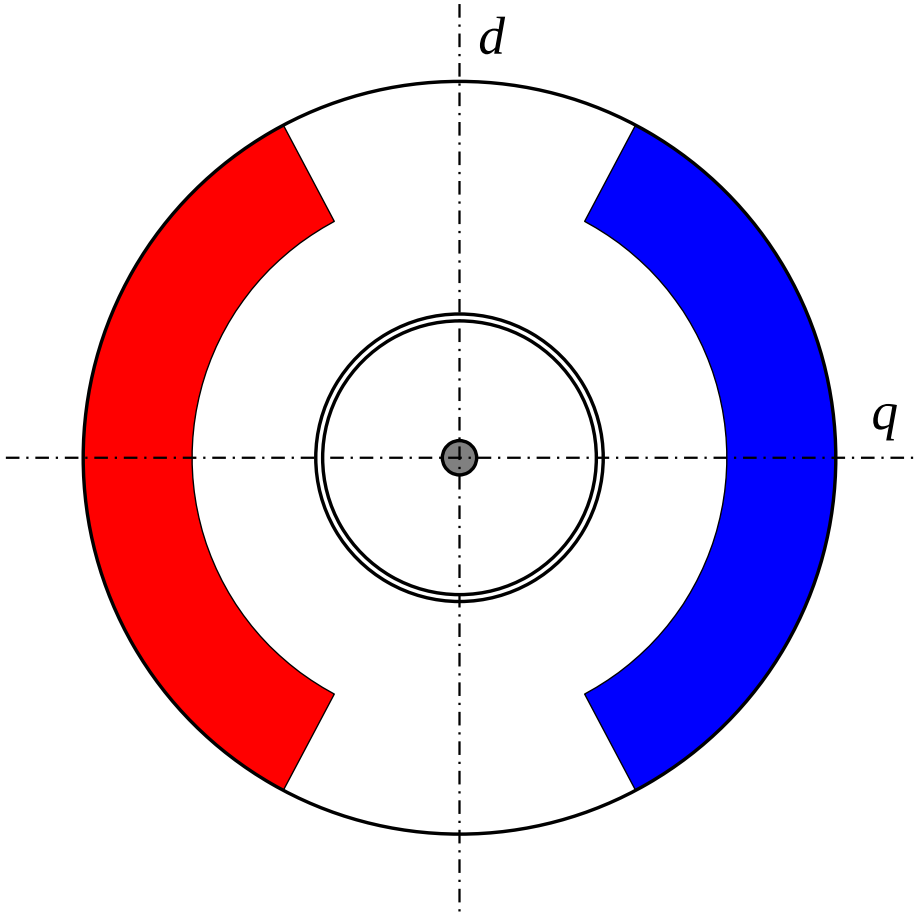
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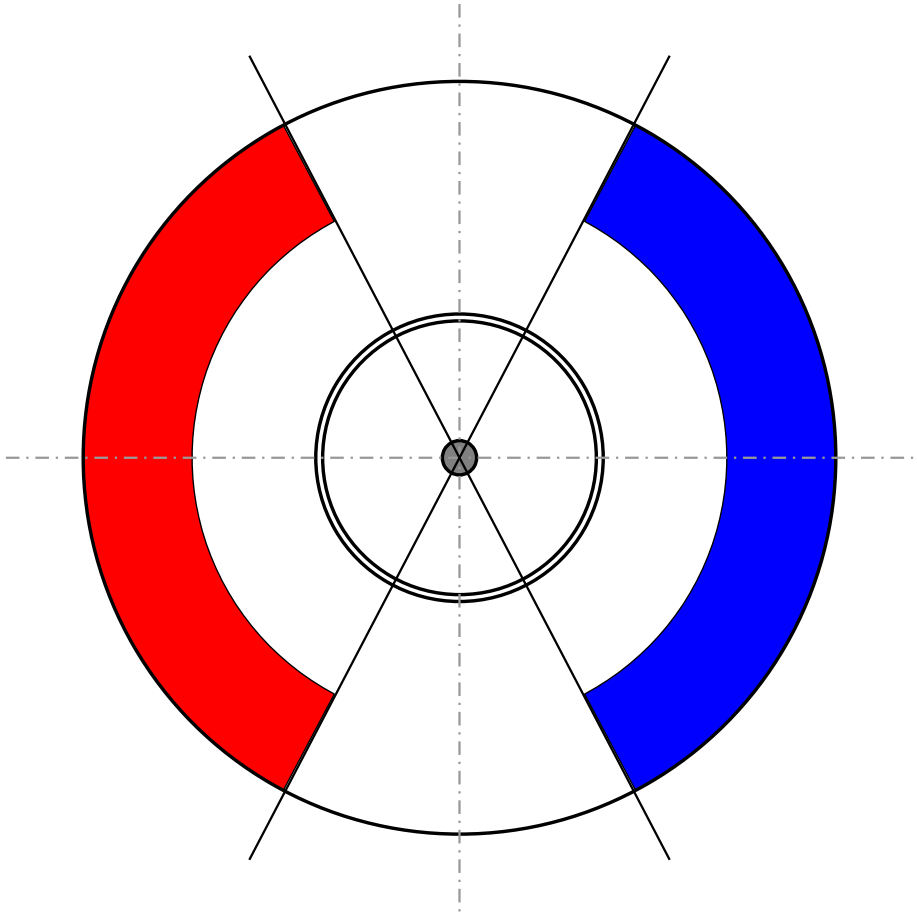
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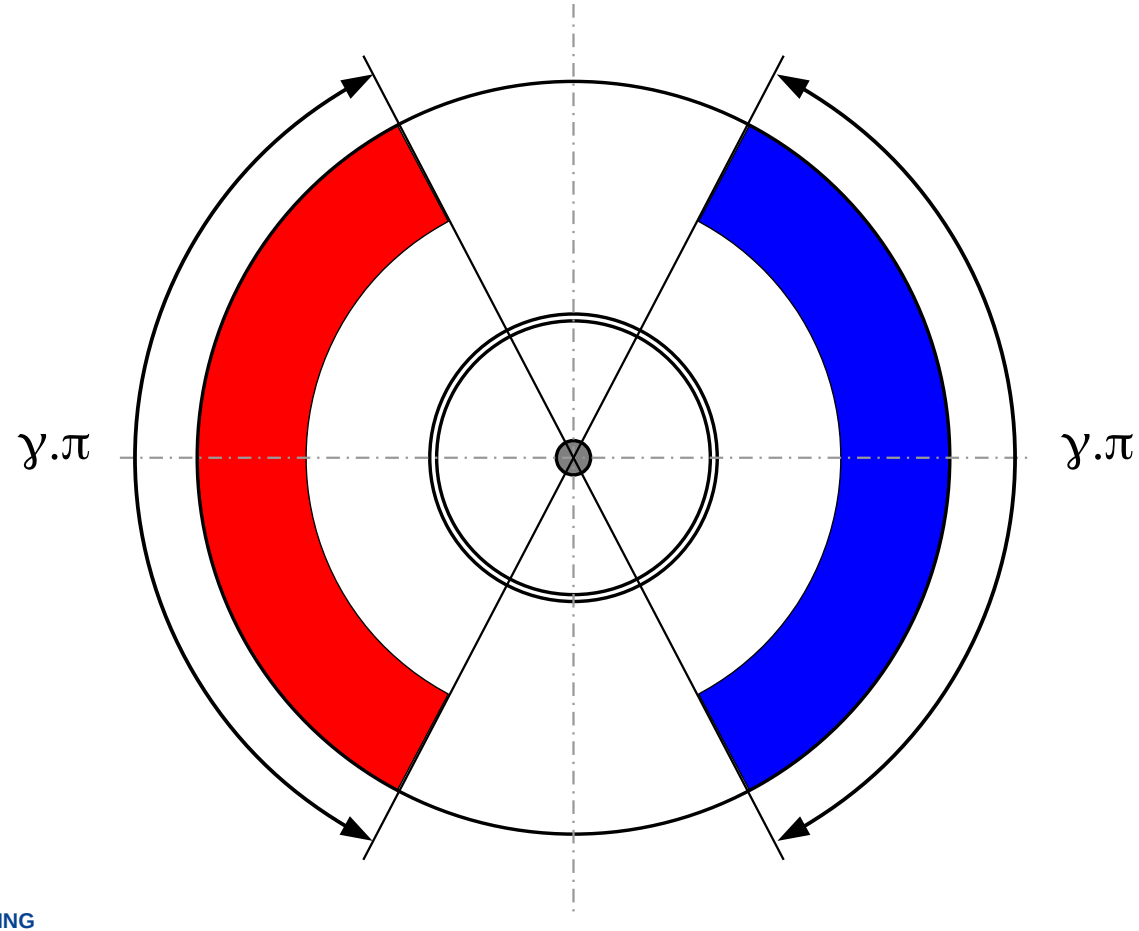
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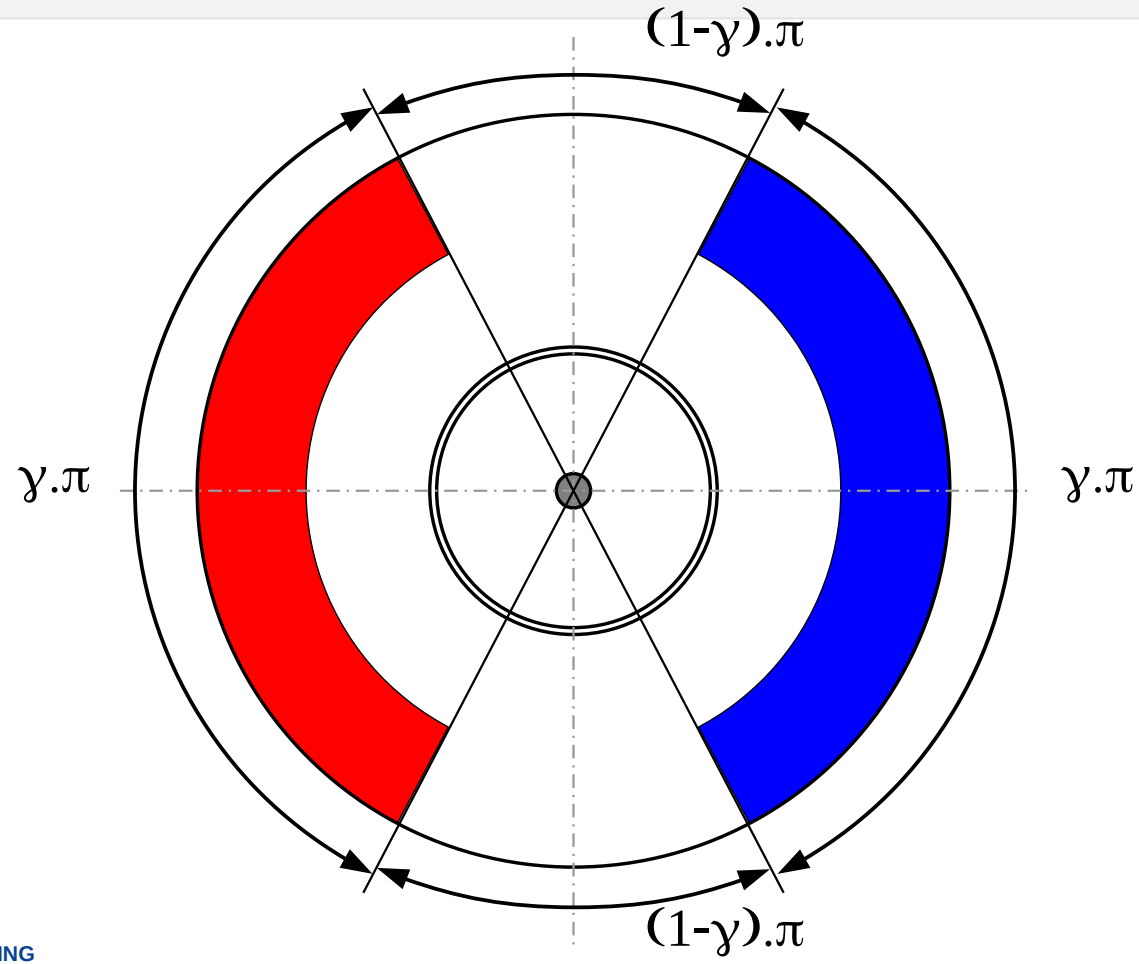
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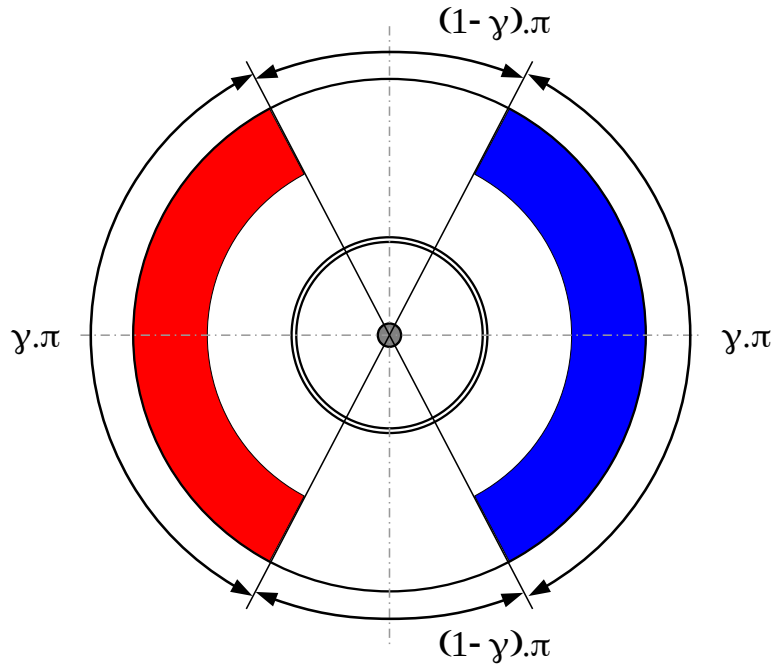


Magnetic Field of Cylindrical Rotor Synchronous Machines



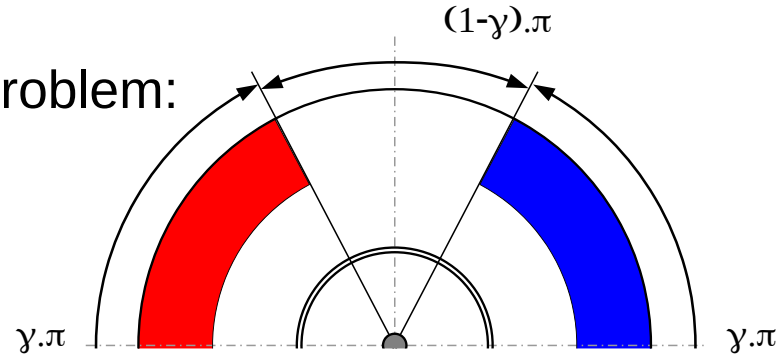
Magnetic Field of Cylindrical Rotor Synchronous Machines

- For the purpose of calculation of shape of the excitation field let us presume:
  - constant mechanical air gap
  - Linear growth of magnetic field strength in the winding area
  - constant magnetic field strength in area of poles (wide teeth)



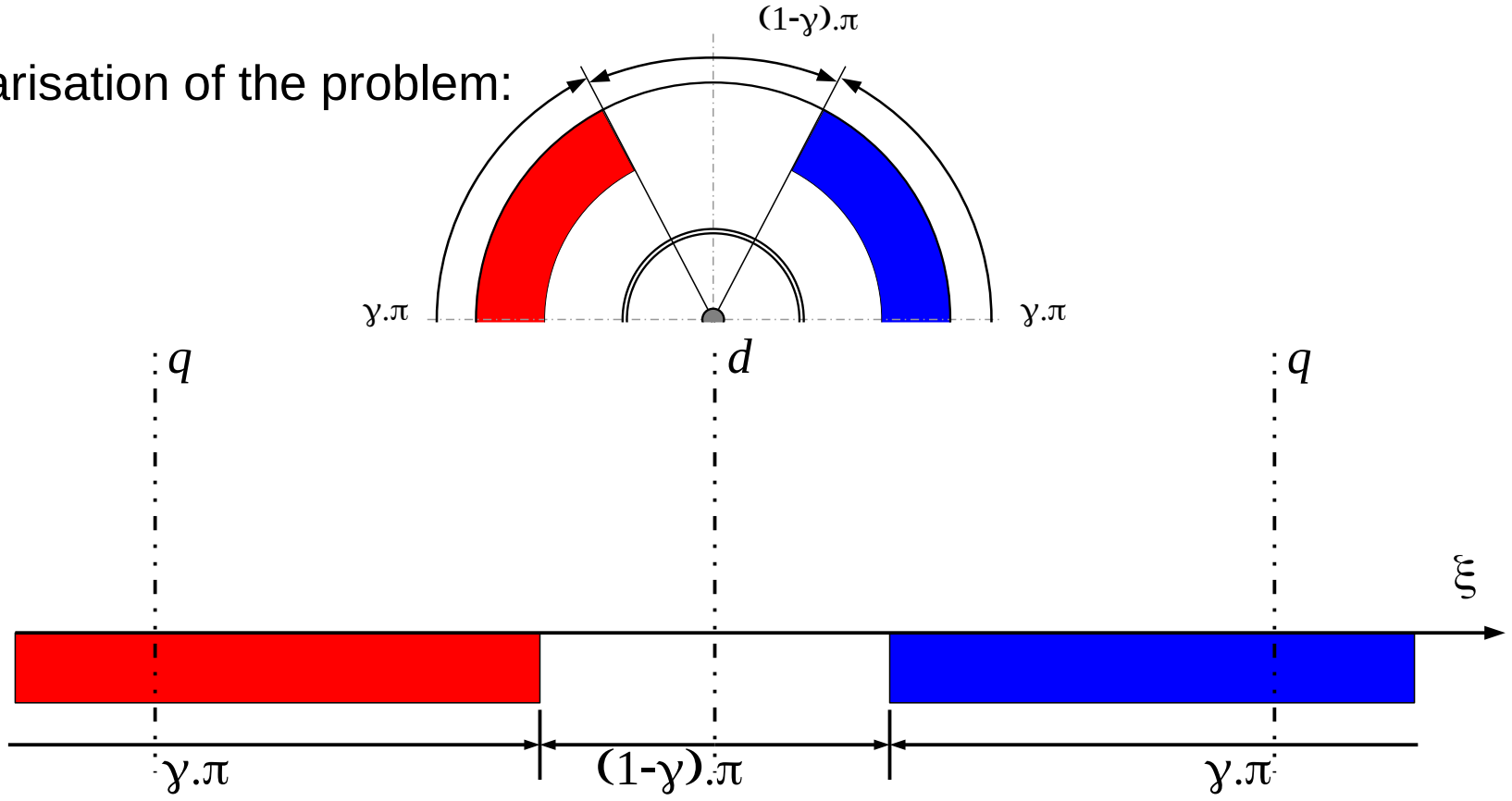
Magnetic Field of Cylindrical Rotor Synchronous Machines

- Linearisation of the problem:



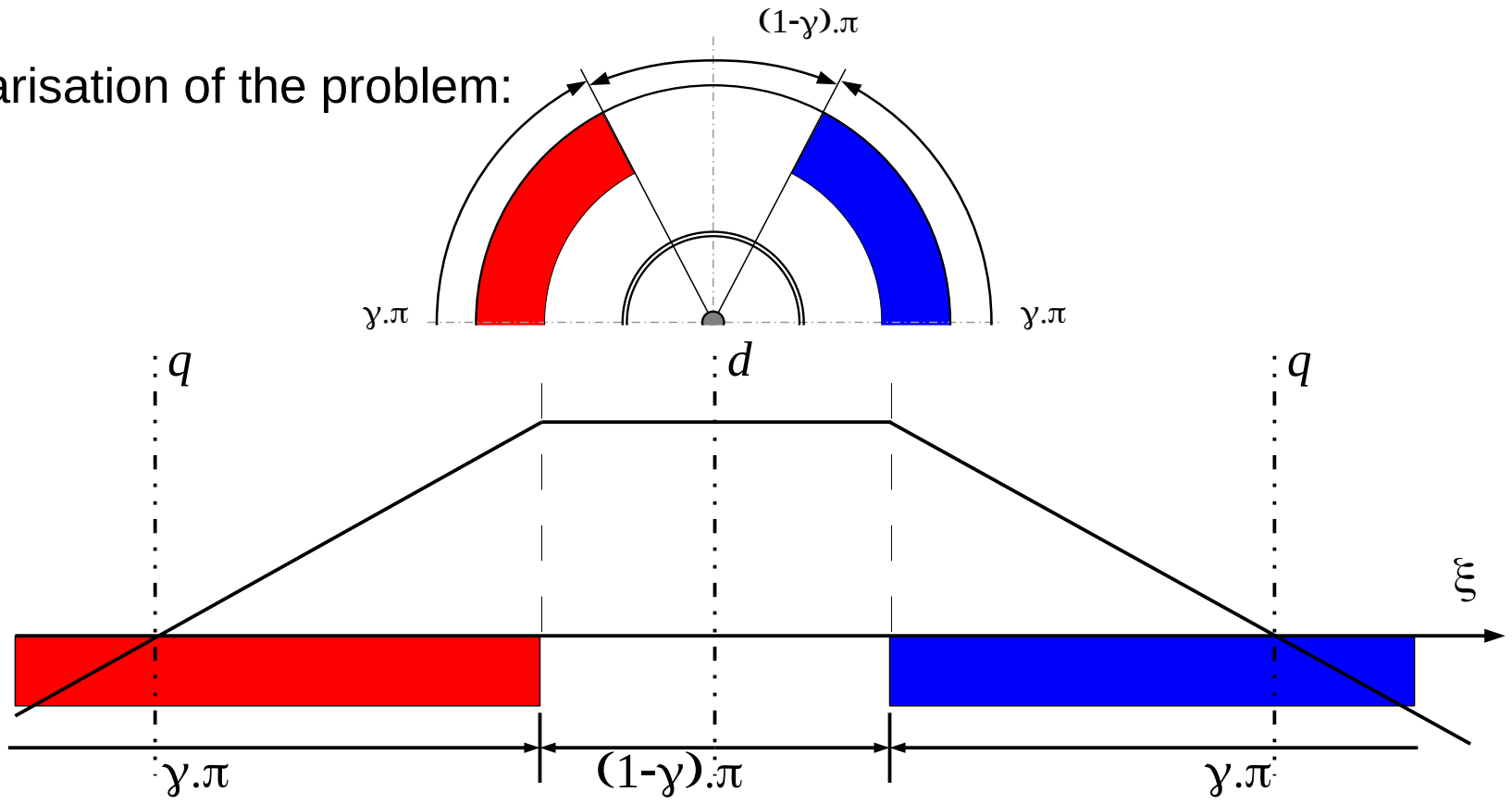
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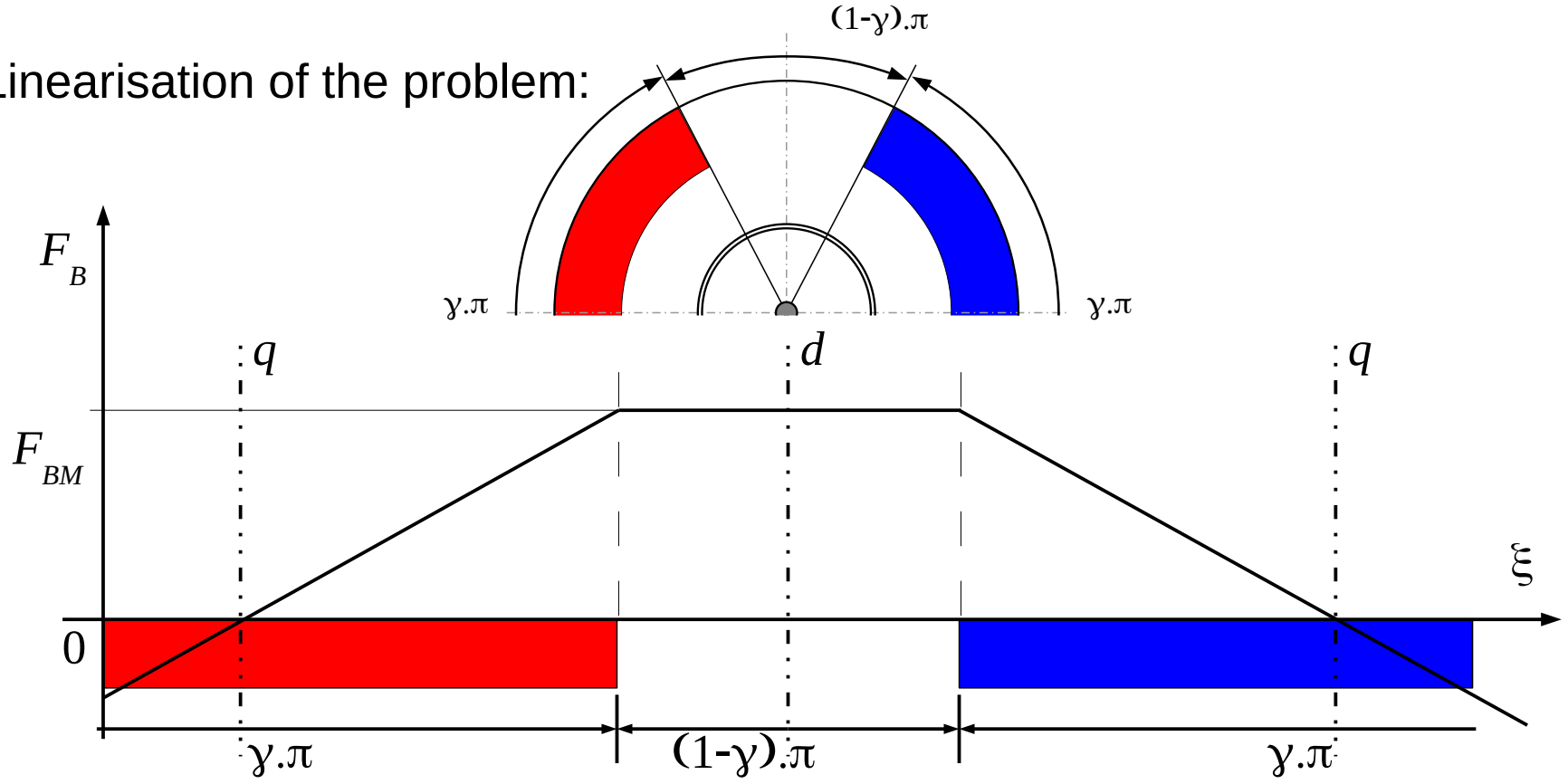
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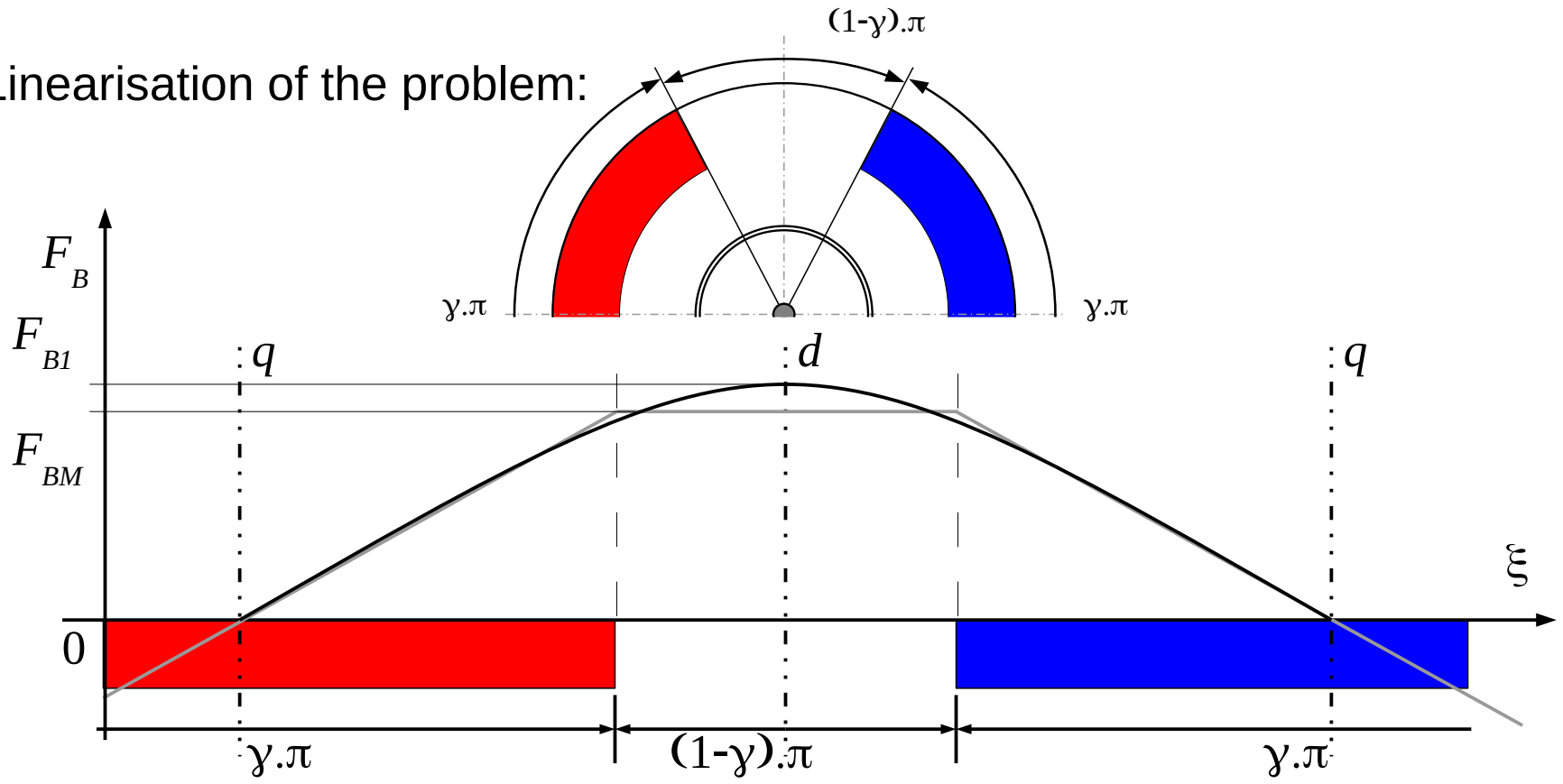
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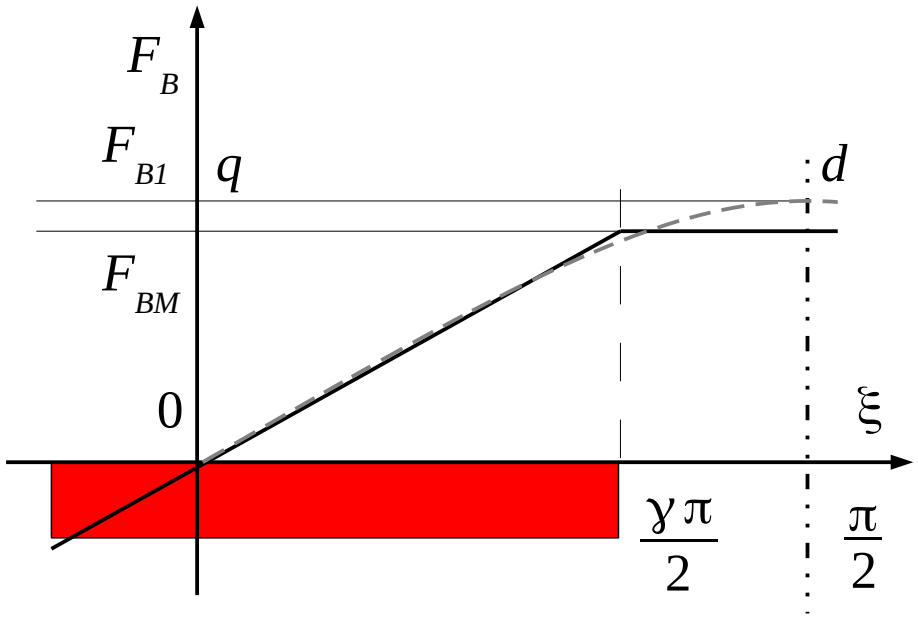
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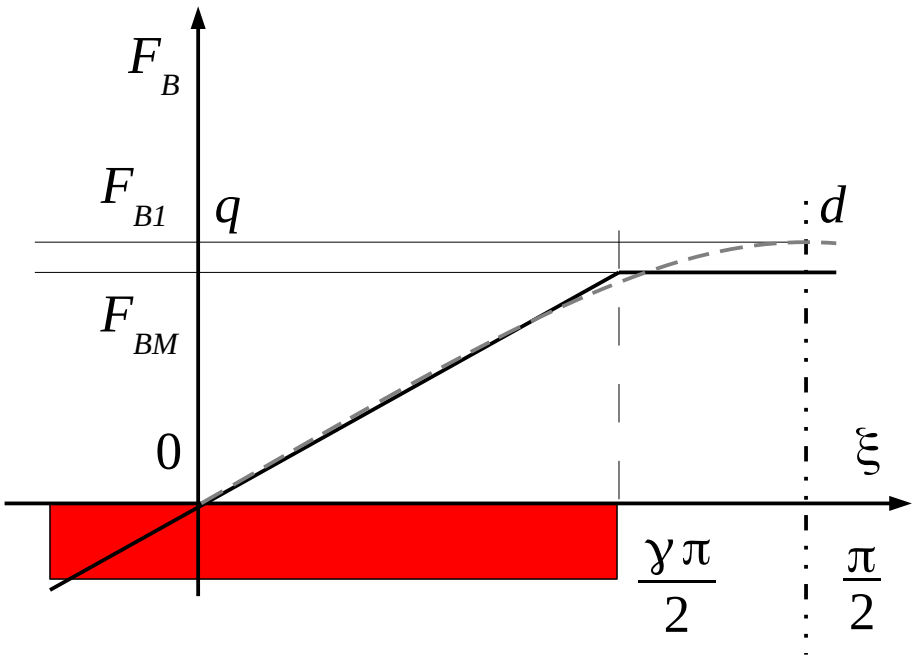
- The shape of the function of the excitation magnetomotive force is periodical and odd, thus it is possible to analyse ¼ only...



Magnetic Field of Cylindrical Rotor Synchronous Machines

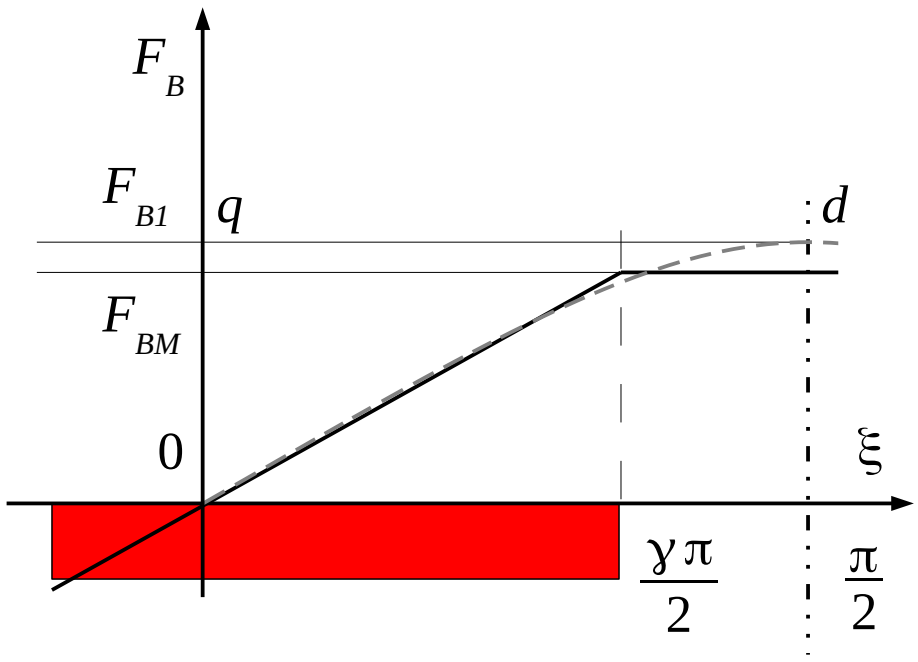
- Fundamental harmonic component magnitude results from Fourier analysis:

$$F_B(\xi) = \frac{F_{B0}}{2} + \sum_{\nu=1}^{\infty} \left(a_\nu \cos \nu \xi + b_\nu \sin \nu \xi \right)$$



Magnetic Field of Cylindrical Rotor Synchronous Machines

- Fundamental harmonic component magnitude results from Fourier analysis:



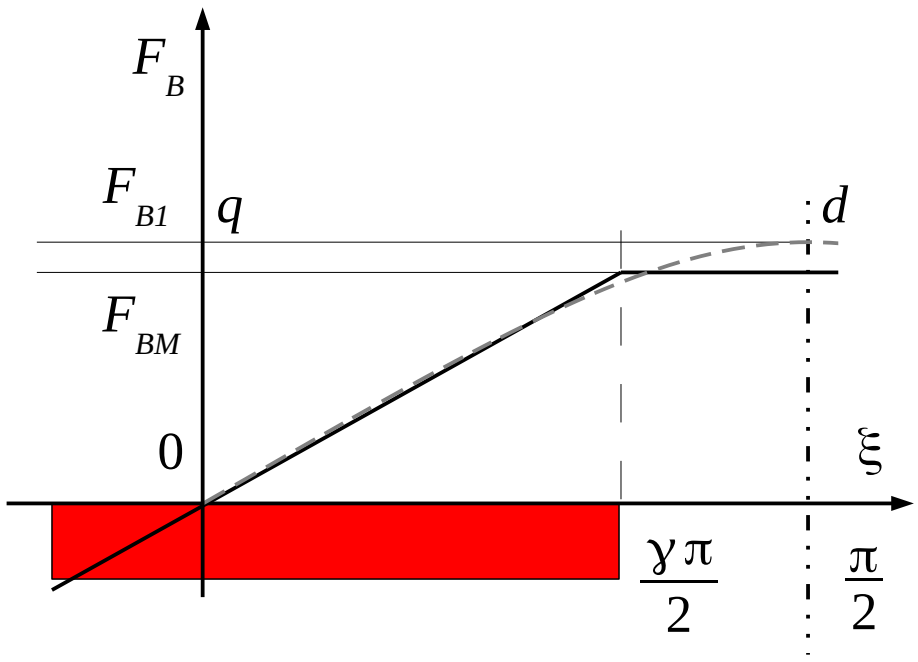
$$F_B(\xi)=\frac{F_{B0}}{2}+\sum_{\nu=1}^{\infty}\left(a_\nu\cos\nu\xi+b_\nu\sin\nu\xi\right)$$

…magnetomotive force $F_B(\xi)$ is symmetric along ξ axis

=> $F_{B0}=0$

Magnetic Field of Cylindrical Rotor Synchronous Machines

- Fundamental harmonic component magnitude results from Fourier analysis:



$$F_B(\xi) = \frac{F_{B0}}{2} + \sum_{\nu=1}^{\infty} \left(a_\nu \cos \nu \xi + b_\nu \sin \nu \xi \right)$$

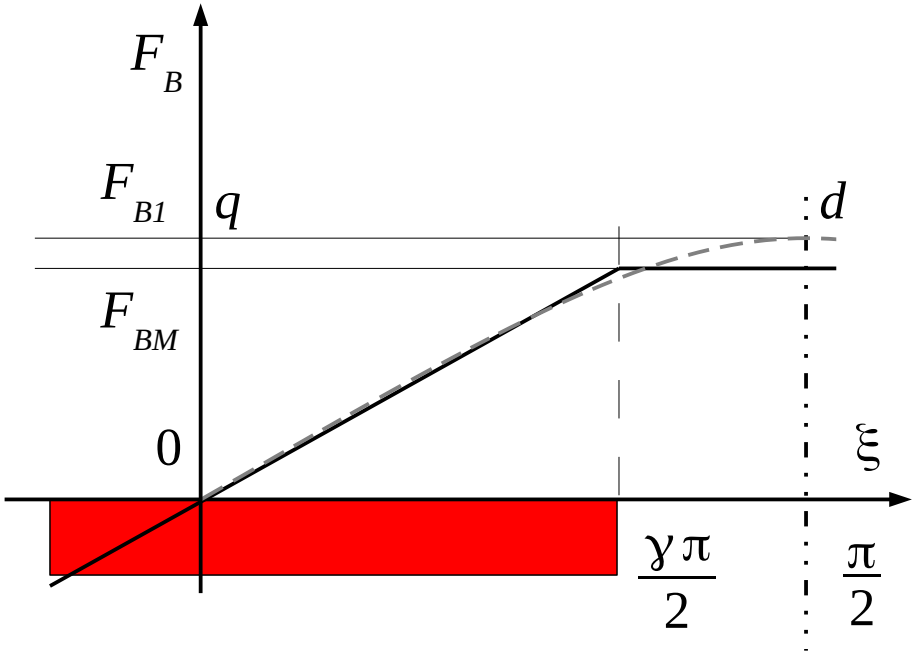
…magnetomotive force $F_B(\xi)$ is an odd function

=> there will be no cosine members

Magnetic Field of Cylindrical Rotor Synchronous Machines

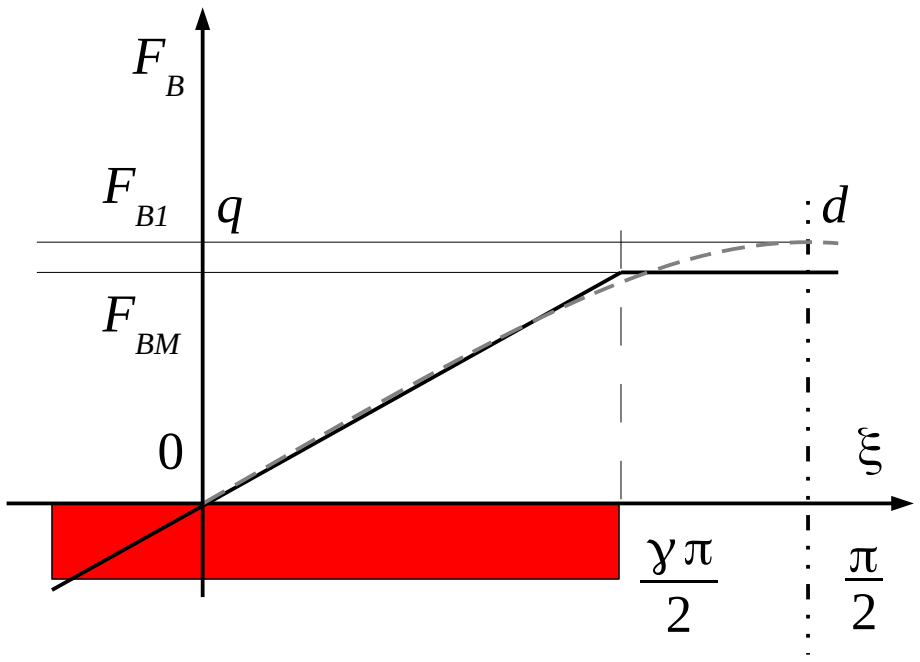
- Fundamental harmonic component magnitude results from Fourier analysis:

$$F_B(\xi) = \sum_{\nu=1}^{\infty} b_\nu \sin \nu \xi$$



Magnetic Field of Cylindrical Rotor Synchronous Machines

- Fundamental harmonic component magnitude results from Fourier analysis:



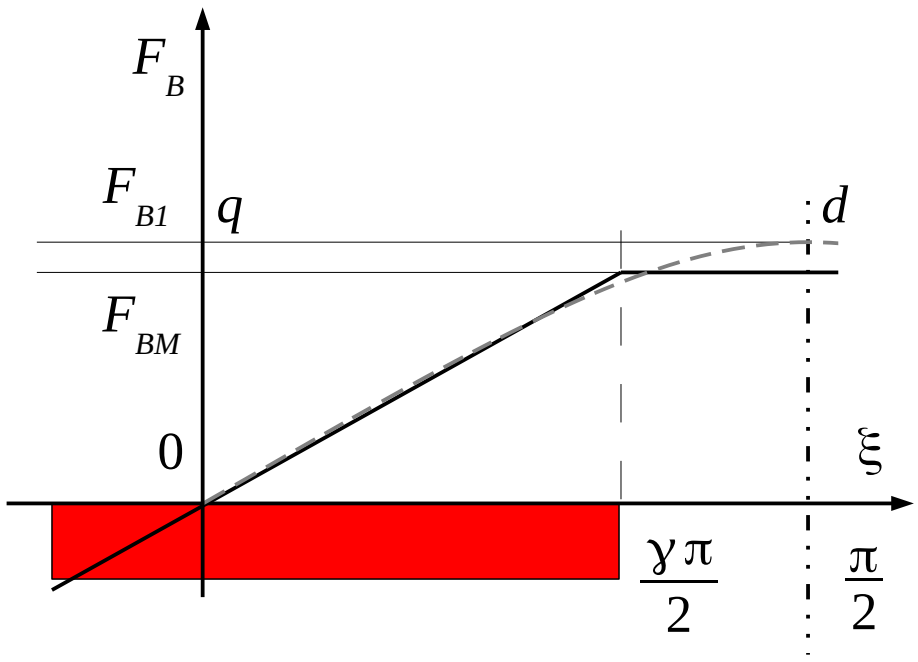
$$F_B(\xi) = \sum_{\nu=1}^{\infty} b_\nu \sin \nu \xi$$

…the magnitudes of each of harmonic components are

$$b_\nu = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} F_B(\xi) \sin \nu \xi \, d\xi$$

Magnetic Field of Cylindrical Rotor Synchronous Machines

- Fundamental harmonic component magnitude results from Fourier analysis:



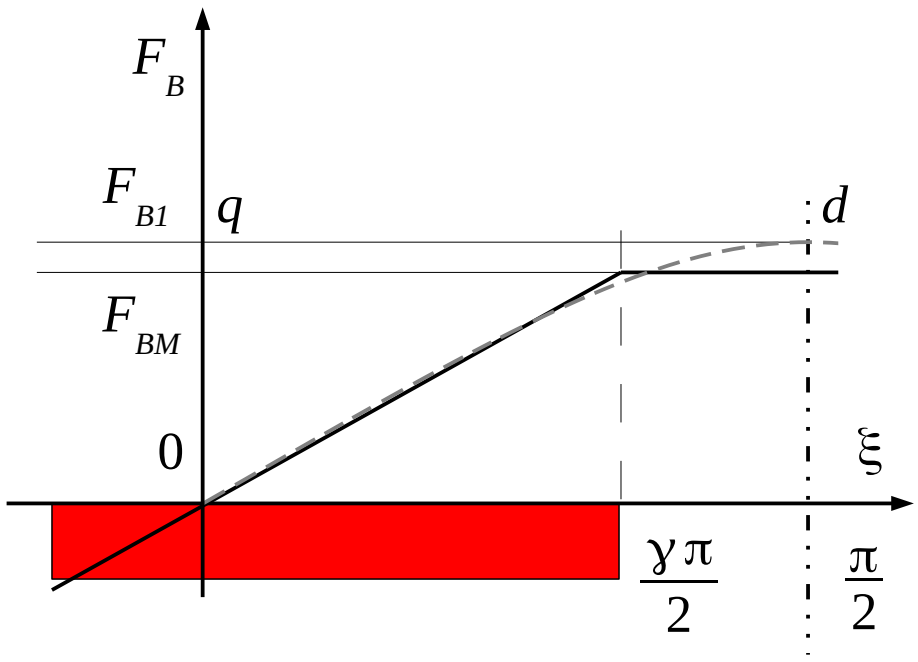
$$F_B(\xi) = \sum_{\nu=1}^{\infty} b_\nu \sin \nu \xi$$

…the task is to calculate the magnitude of the fundamental harmonic component

$$b_1 = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} F_B(\xi) \sin \xi \, d\xi$$

Magnetic Field of Cylindrical Rotor Synchronous Machines

- Fundamental harmonic component magnitude results from Fourier analysis:



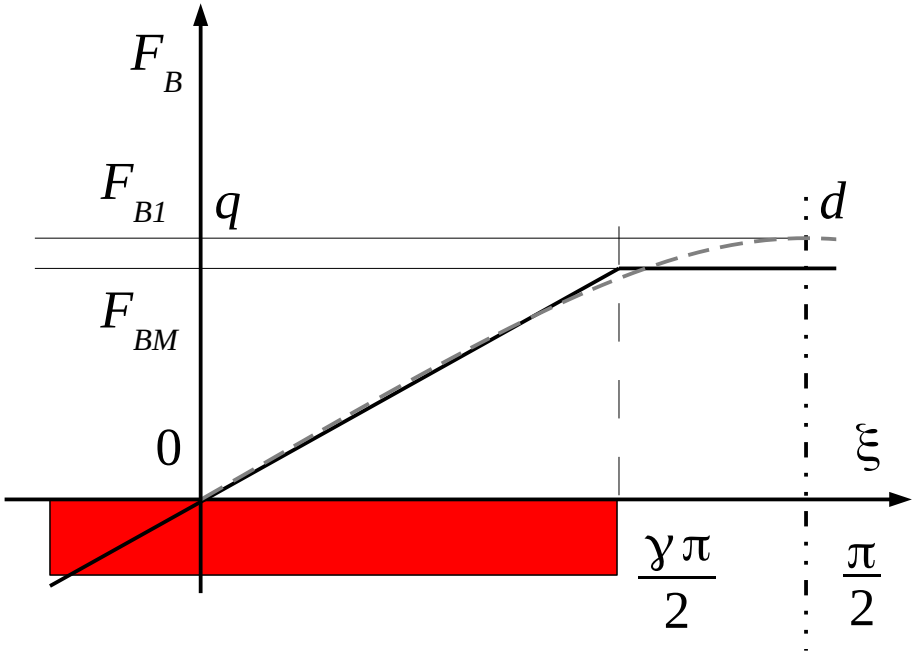
$$F_B(\xi) = \sum_{\nu=1}^{\infty} b_\nu \sin \nu \xi$$

…in an interval from 0 to $\pi/2$

$$b_1 = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} F_B(\xi) \sin \xi \, d\xi$$

Magnetic Field of Cylindrical Rotor Synchronous Machines

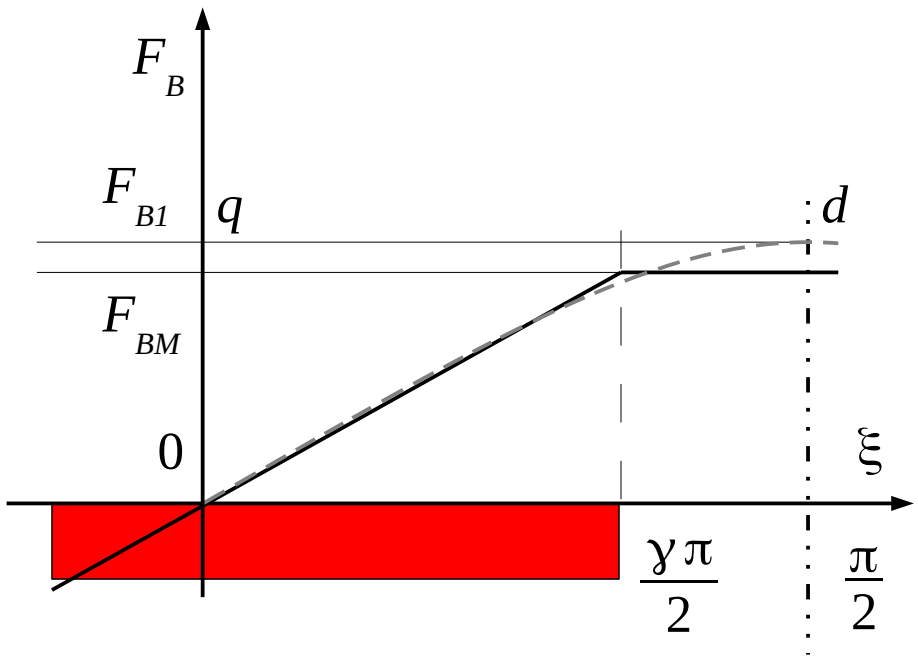
$$b_1 = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} F_B(\xi) \sin \xi \, d\xi$$



- Solution intervals:

Magnetic Field of Cylindrical Rotor Synchronous Machines

$$b_1 = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} F_B(\xi) \sin \xi \, d\xi$$



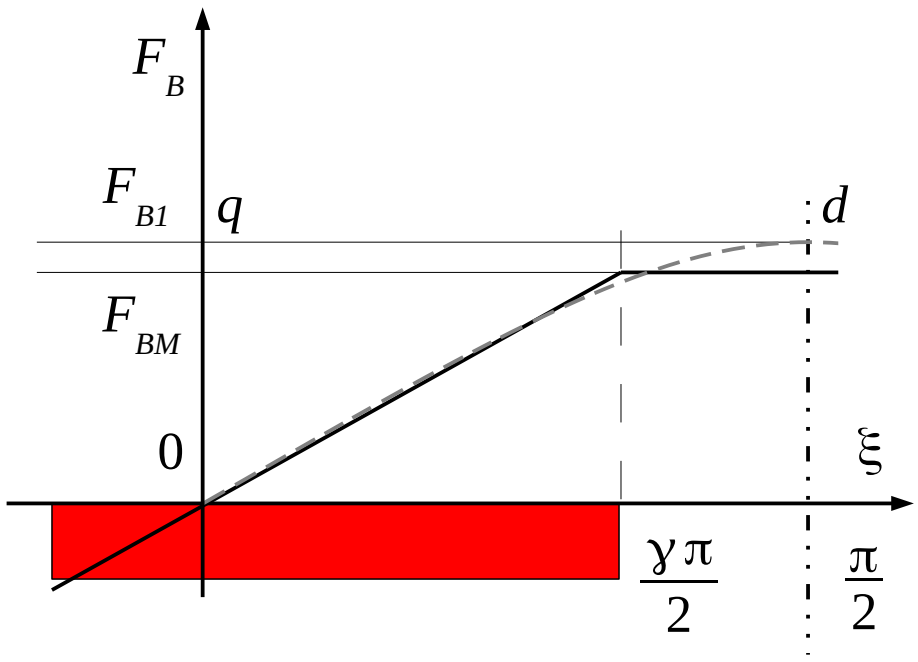
- Solution intervals:
  - I. interval of ascending MMF:

$$\xi \in \left\langle 0 ; \frac{\gamma \pi}{2} \right\rangle$$

$$F_B(\xi) = F_{BM} \frac{2\xi}{\gamma \pi}$$

Magnetic Field of Cylindrical Rotor Synchronous Machines

$$b_1 = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} F_B(\xi) \sin \xi \, d\xi$$



- Solution intervals:
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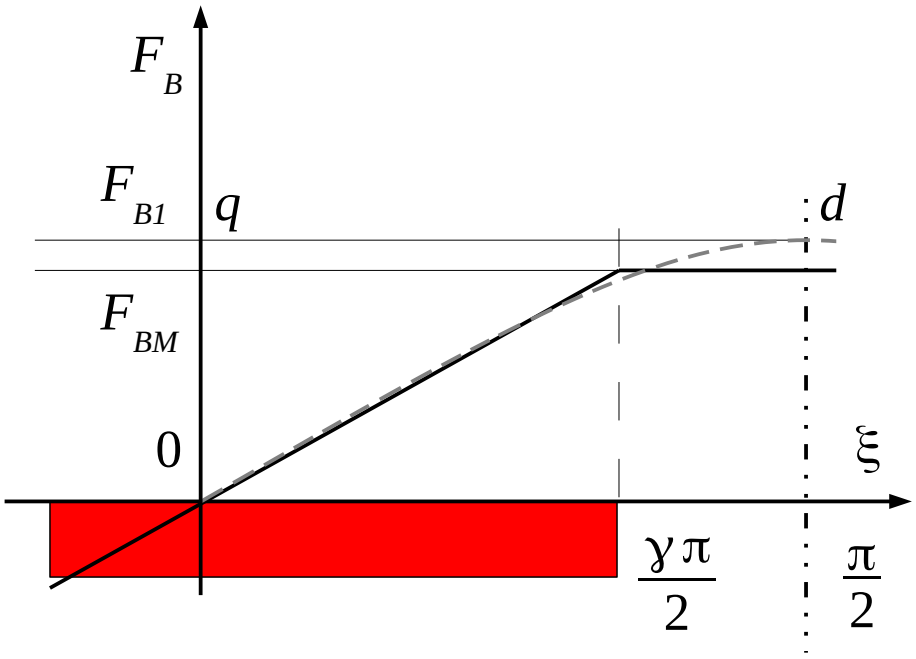
  - II. interval of constant MMF:

$$\xi \in \left\langle \frac{\gamma \pi}{2} ; \frac{\pi}{2} \right\rangle$$

$$F_B(\xi) = F_{BM}$$

Magnetic Field of Cylindrical Rotor Synchronous Machines

$$b_1 = \frac{4}{\pi}\int_0^{\frac{\gamma\pi}{2}} F_{BM}\frac{2\xi}{\gamma\pi}\sin\xi\,d\xi + \frac{4}{\pi}\int_{\frac{\gamma\pi}{2}}^{\frac{\pi}{2}} F_{BM}\sin\xi\,d\xi$$



- Solution intervals:
  - I. interval of ascending MMF:

$$\xi \in \left\langle 0\,;\frac{\gamma\pi}{2}\right\rangle$$

$$F_B(\xi) = F_{BM}\frac{2\xi}{\gamma\pi}$$

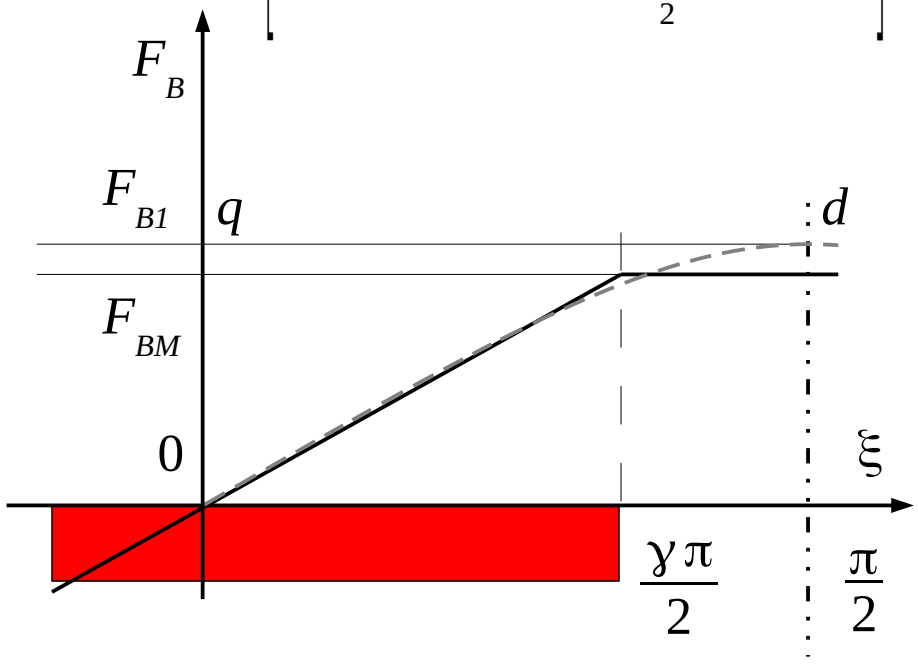
  - II. interval of constant MMF:

$$\xi \in \left\langle \frac{\gamma\pi}{2}\,;\frac{\pi}{2}\right\rangle$$

$$F_B(\xi) = F_{BM}$$

Magnetic Field of Cylindrical Rotor Synchronous Machines

$$b_1 = \frac{4 F_{BM}}{\pi} \left[\int_0^{\frac{\gamma \pi}{2}} \frac{2\xi}{\gamma \pi} \sin \xi \, d\xi + \int_{\frac{\gamma \pi}{2}}^{\frac{\pi}{2}} \sin \xi \, d\xi \right]$$



- Solution intervals:
  - I. interval of ascending MMF:

$$\xi \in \left\langle 0 ; \frac{\gamma \pi}{2} \right\rangle$$

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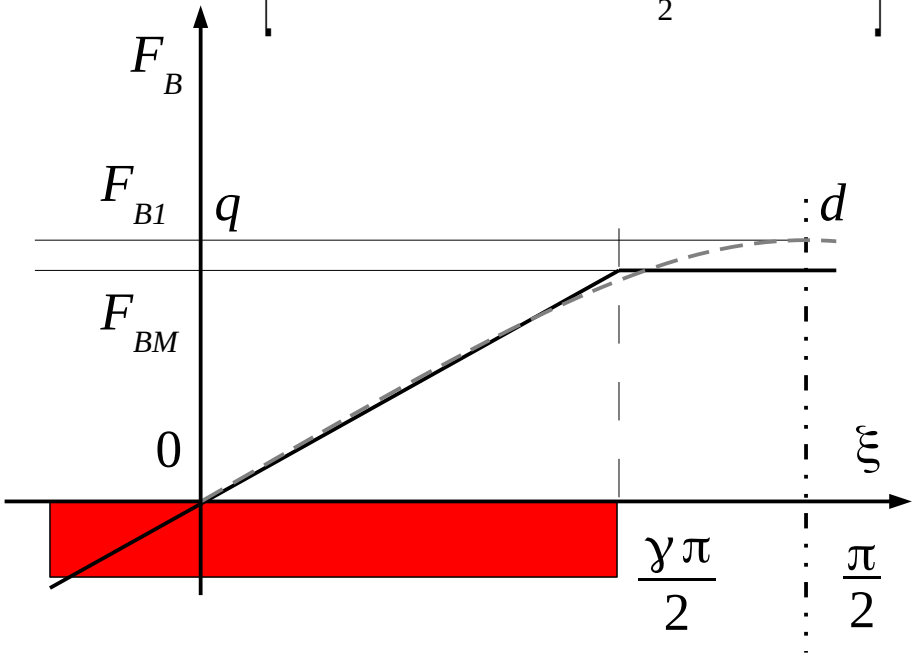
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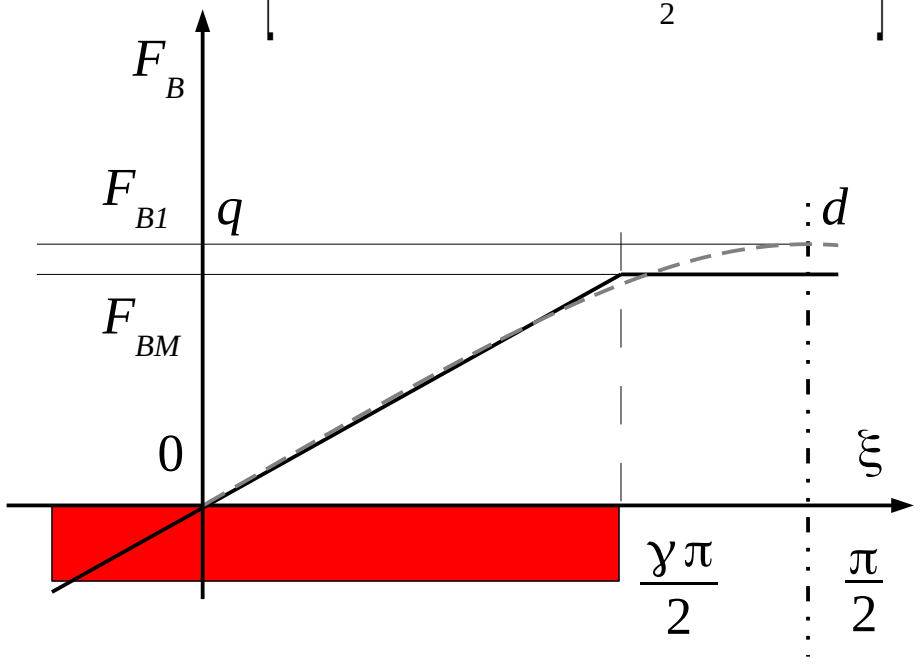


- Now let us integrate:

$$\int_0^{\frac{\gamma \pi}{2}} \frac{2\xi}{\gamma \pi} \sin \xi \, d\xi$$

Magnetic Field of Cylindrical Rotor Synchronous Machines

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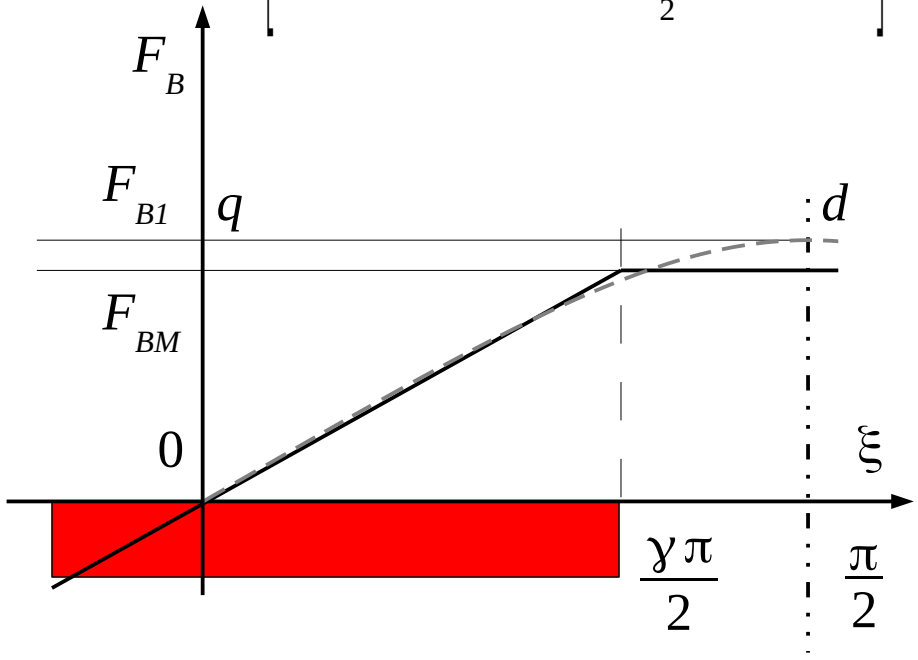
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  - Is a multiplication of two functions => per partes integration

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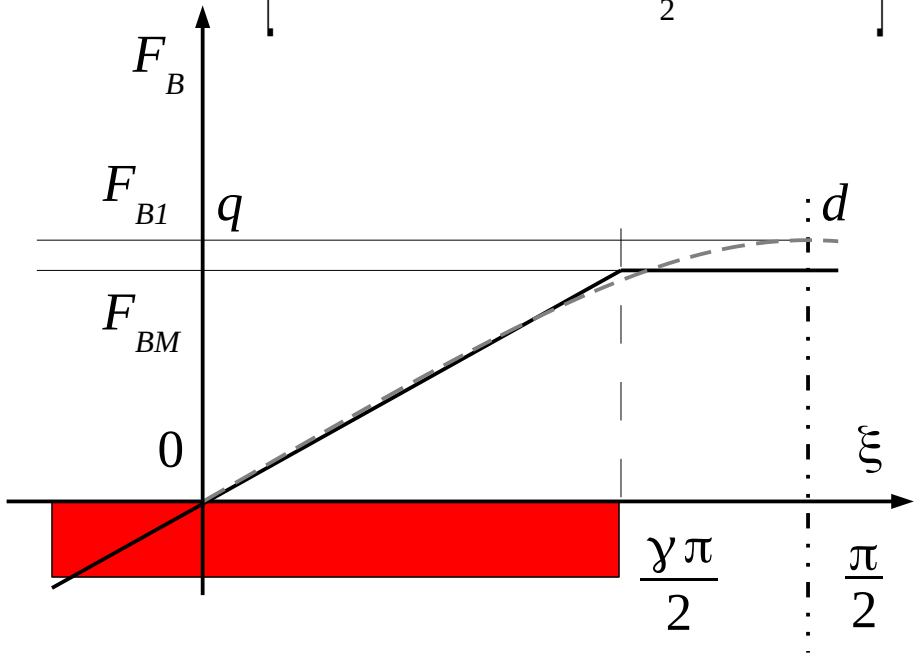
$$(u\,v)' = (u)'\,v + u\,(v)'$$

$$\int (u\,v)'\,dx = \int (u)'\,v\,dx + \int u\,(v)'\,dx$$

$$u\,v = \int (u)'\,v\,dx + \int u\,(v)'\,dx$$

Magnetic Field of Cylindrical Rotor Synchronous Machines

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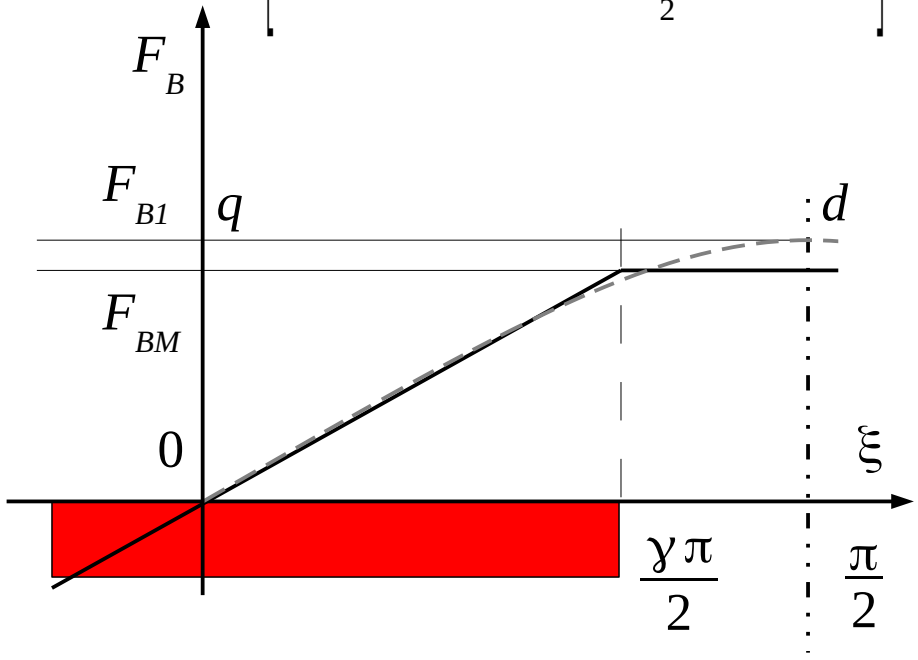
$$(u\,v)' = (u)'\,v + u\,(v)'$$

$$\int (u\,v)'\,dx = \int (u)'\,v\,dx + \int u\,(v)'\,dx$$

$$\int u\,(v)'\,dx = u\,v - \int (u)'\,v\,dx$$

Magnetic Field of Cylindrical Rotor Synchronous Machines

$$b_1 = \frac{4 F_{BM}}{\pi} \left[\int_0^{\frac{\gamma \pi}{2}} \frac{2\xi}{\gamma \pi} \sin \xi \, d\xi + \int_{\frac{\gamma \pi}{2}}^{\frac{\pi}{2}} \sin \xi \, d\xi \right]$$



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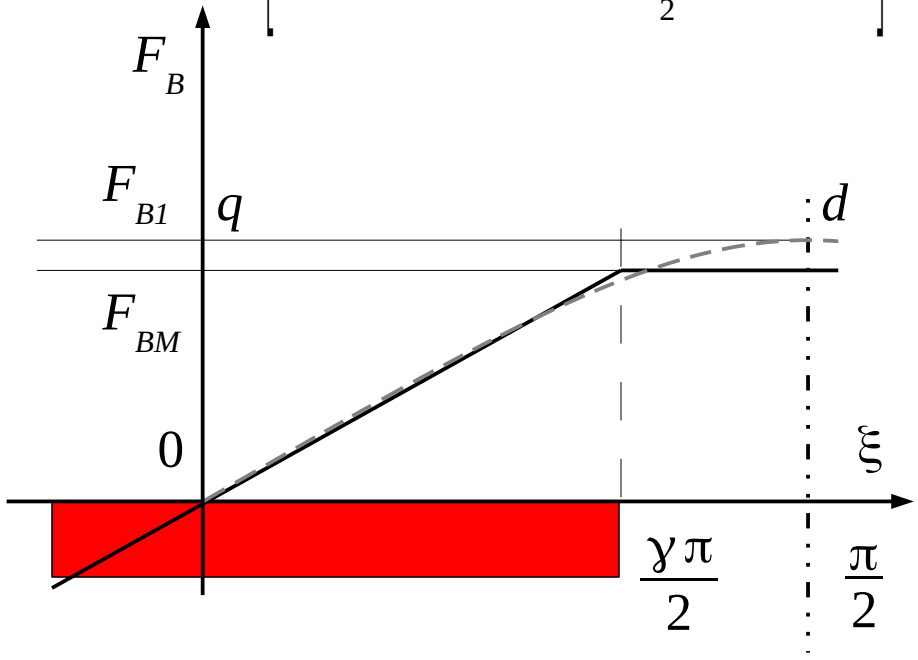
$$\int_0^{\frac{\gamma \pi}{2}} \frac{2\xi}{\gamma \pi} \sin \xi \, d\xi =$$

$$= \frac{2}{\gamma \pi} \left[-\xi \cos \xi \right]_0^{\frac{\gamma \pi}{2}} + \frac{2}{\gamma \pi} \int_0^{\frac{\gamma \pi}{2}} \cos \xi \, d\xi$$

$$\int u (v)' \, dx = u v - \int (u)' v \, dx$$

Magnetic Field of Cylindrical Rotor Synchronous Machines

$$b_1 = \frac{4 F_{BM}}{\pi} \left[\int_0^{\frac{\gamma \pi}{2}} \frac{2\xi}{\gamma \pi} \sin \xi \, d\xi + \int_{\frac{\gamma \pi}{2}}^{\frac{\pi}{2}} \sin \xi \, d\xi \right]$$



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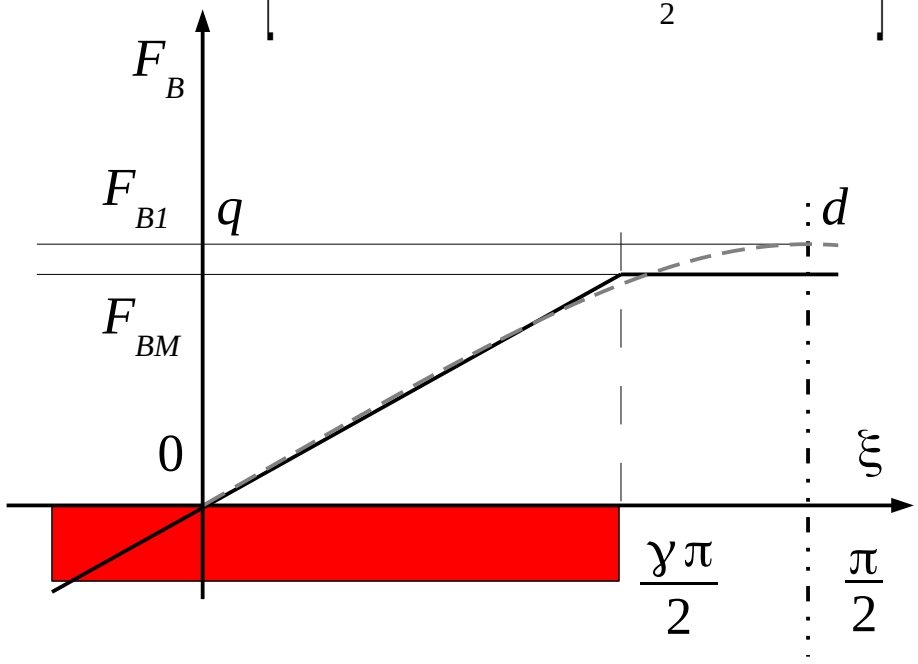
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$$= \frac{2}{\gamma \pi} \left[-\xi \cos \xi + \sin \xi \right]_0^{\frac{\gamma \pi}{2}}$$

Magnetic Field of Cylindrical Rotor Synchronous Machines

$$b_1 = \frac{4 F_{BM}}{\pi} \left[\int_0^{\frac{\gamma \pi}{2}} \frac{2\xi}{\gamma \pi} \sin \xi \, d\xi + \int_{\frac{\gamma \pi}{2}}^{\frac{\pi}{2}} \sin \xi \, d\xi \right]$$



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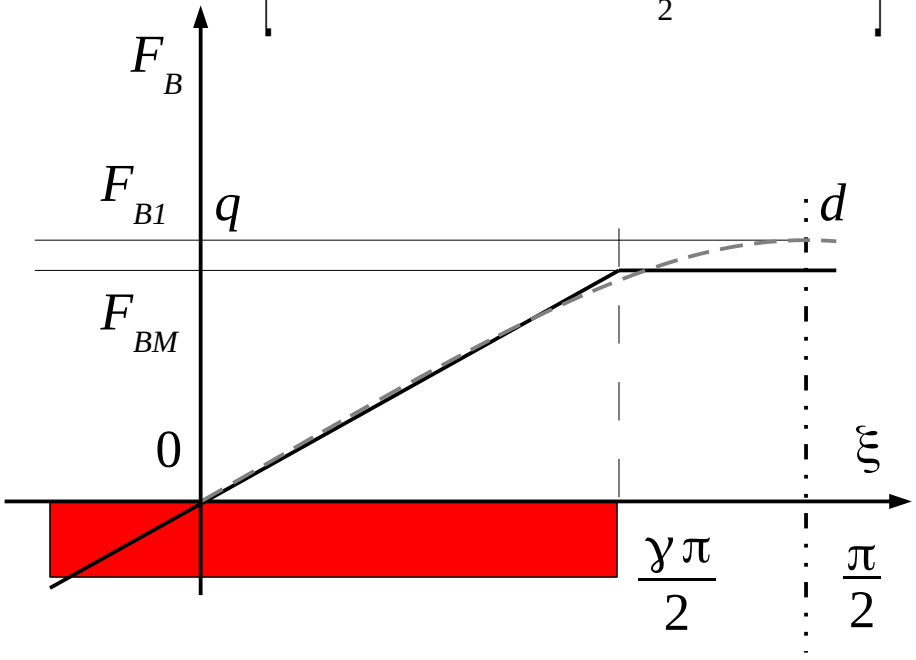
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$$= \frac{2}{\gamma \pi} \left[-\xi \cos \xi + \sin \xi \right]_0^{\frac{\gamma \pi}{2}} =$$

$$= \frac{2}{\gamma \pi} \left(-\frac{\gamma \pi}{2} \cos \frac{\gamma \pi}{2} + \sin \frac{\gamma \pi}{2} \right)$$

Magnetic Field of Cylindrical Rotor Synchronous Machines

$$b_1 = \frac{4 F_{BM}}{\pi} \left[\int_0^{\frac{\gamma \pi}{2}} \frac{2\xi}{\gamma \pi} \sin \xi \, d\xi + \int_{\frac{\gamma \pi}{2}}^{\frac{\pi}{2}} \sin \xi \, d\xi \right]$$

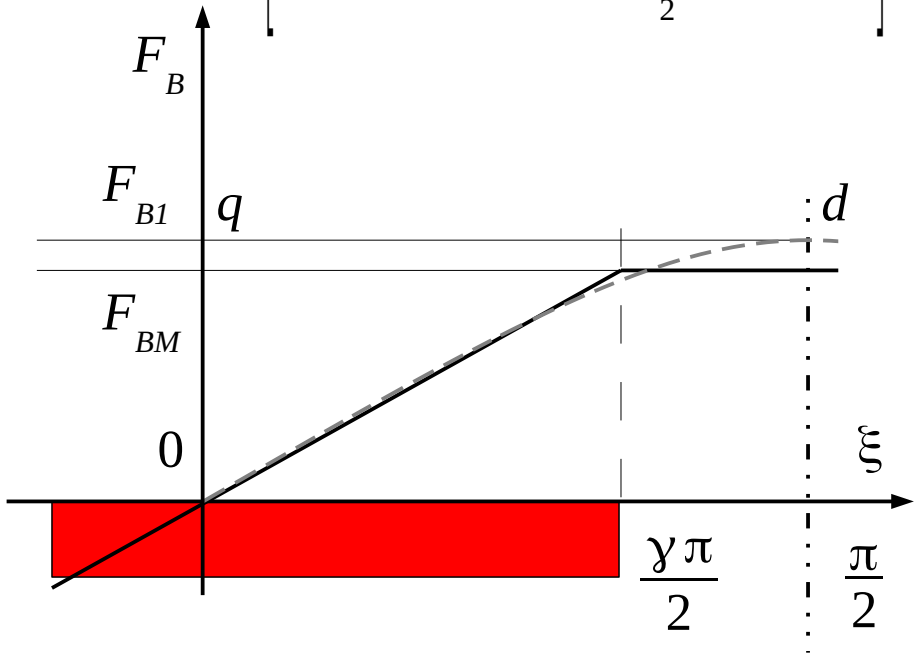


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$$\int_{\frac{\gamma \pi}{2}}^{\frac{\pi}{2}} \sin \xi \, d\xi$$

Magnetic Field of Cylindrical Rotor Synchronous Machines

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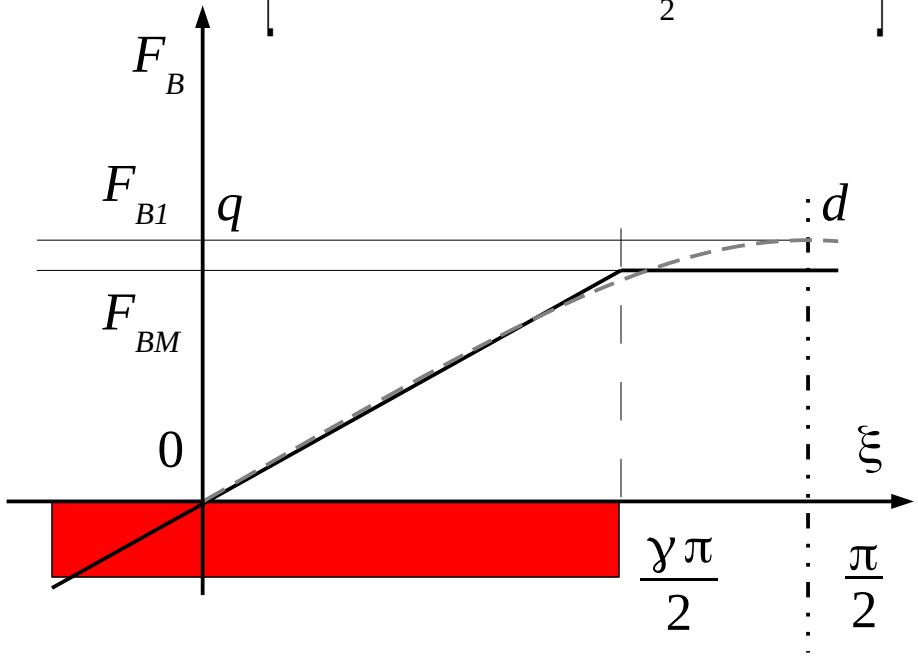


- Now let us integrate:

$$\int_{\frac{\gamma \pi}{2}}^{\frac{\pi}{2}} \sin \xi \, d\xi = \left[-\cos \xi \right]_{\frac{\gamma \pi}{2}}^{\frac{\pi}{2}}$$

Magnetic Field of Cylindrical Rotor Synchronous Machines

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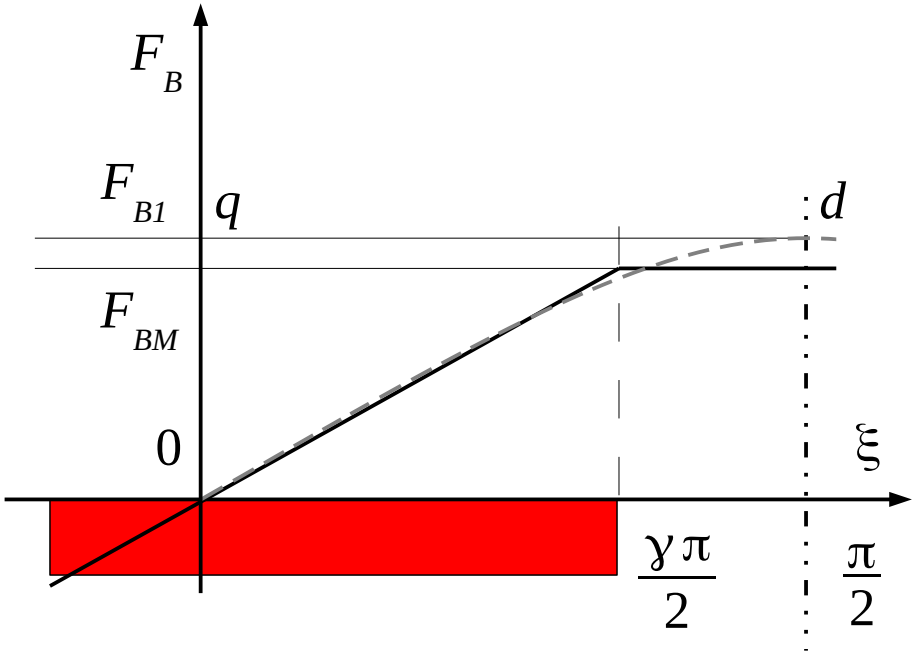
$$\int_{\frac{\gamma \pi}{2}}^{\frac{\pi}{2}} \sin \xi \, d\xi = \left[-\cos \xi \right]_{\frac{\gamma \pi}{2}}^{\frac{\pi}{2}} = 0 + \cos \frac{\gamma \pi}{2}$$

Magnetic Field of Cylindrical Rotor Synchronous Machines

- After substitution of result we have:

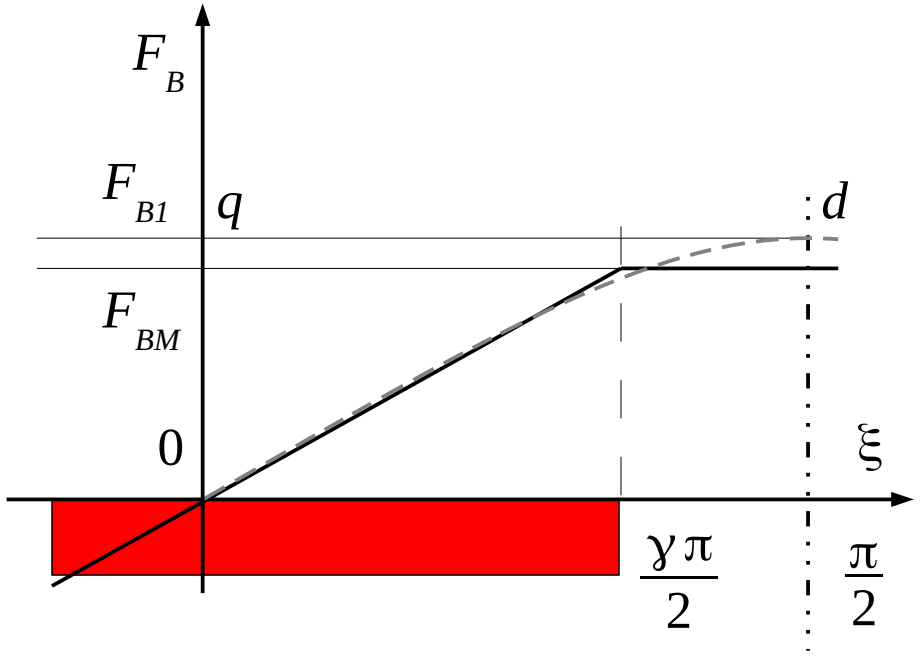
$$b_1 = \frac{4 F_{BM}}{\pi}\left(-\cos\frac{\gamma\pi}{2} + \frac{2}{\gamma\pi}\sin\frac{\gamma\pi}{2} + \cos\frac{\gamma\pi}{2}\right)$$

$$b_1 = \frac{8 F_{BM}}{\gamma\pi^2}\sin\frac{\gamma\pi}{2}$$



Magnetic Field of Cylindrical Rotor Synchronous Machines

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$$b_1 = \frac{8F_{BM}}{\gamma\pi^2}\sin\frac{\gamma\pi}{2}$$

…which is actually a magnitude of the fundamental harmonic component of the rotor magnetomotive force

$$F_{B1} = b_1 = \frac{8F_{BM}}{\gamma\pi^2}\sin\frac{\gamma\pi}{2}$$

Magnetic Field of Cylindrical Rotor Synchronous Machines

- By comparison of the fundamental harmonic component of the MMF with its actual magnitude we obtain excitation winding magnetic field factor C_{b1}

$$C_{b1} = \frac{F_{B1}}{F_{BM}} = \frac{\frac{8 F_{BM}}{\gamma \pi^2} \sin \frac{\gamma \pi}{2}}{F_{BM}}$$

$$C_{b1} = \frac{8}{\gamma \pi^2} \sin \frac{\gamma \pi}{2}$$

Magnetic Field of Cylindrical Rotor Synchronous Machines

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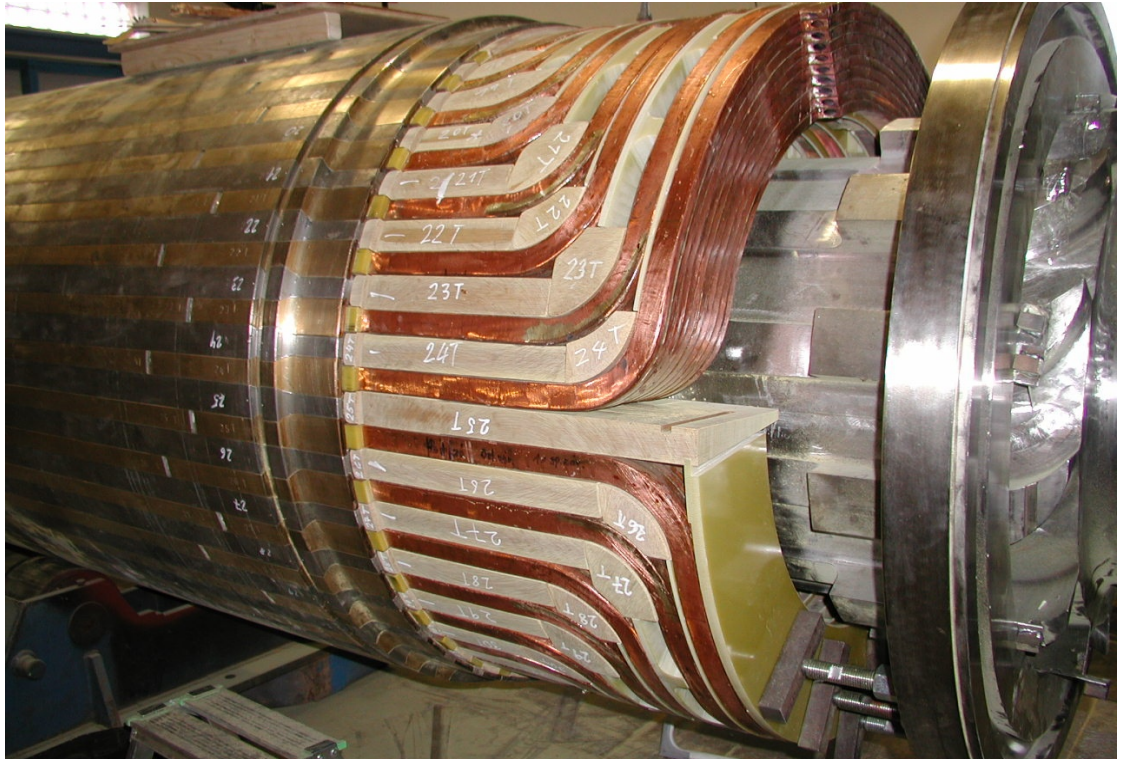
- Some literature further introduces the excitation winding factor k_{vb}, which is a part of the magnetic field factor C_{b1}: $C_{b1} = \frac{4}{\pi} k_{vb}$, where $k_{vb} = \frac{2}{\gamma \pi} \sin \frac{\gamma \pi}{2}$

Magnetic Field of Cylindrical Rotor Synchronous Machines

- The magnetomotive force of the excitation winding per one pole is then

$$F_{BM} = \frac{N_b I_b}{2p}$$

where N_b is the total number of excitation winding turns



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EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

ESF II – project of the University of West Bohemia
reg. no. CZ.02.2.69/0.0/0.0/18_056/0013239

Cylindrical Rotor Machines
Current Transformation Factor

Cylindrical Rotor Machines Current Transformation Factor

- The current transformation factor of a synchronous machine may be obtained in similar way as in case of an induction machine
  => the calculation is based on comparison of stator and rotor magnetomotive forces

Cylindrical Rotor Machines Current Transformation Factor

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- The magnitude of the fundamental harmonic component of the stator magnetomotive force is

$$F_{d1} = C_{d1} \cdot \frac{m}{\pi} \cdot \frac{N_s k_{v1} \sqrt{2} I}{p}$$

- where C_{d1} is the d-axis stator magnetic field shape factor

  → for machines with cylindrical rotor, we may assume $C_{d1} \simeq 1$

Cylindrical Rotor Machines Current Transformation Factor

- The current transformation factor of a synchronous machine may be obtained in similar way as in case of an induction machine
  => the calculation is based on comparison of stator and rotor magnetomotive forces
- The magnitude of the fundamental harmonic component of a machine with cylindrical rotor, the rotor magnetomotive force is

$$F_{B1} = C_{b1} \frac{N_b I_b}{2p}$$

where C_{b1} is the excitation winding magnetic field factor:

$$C_{b1} = \frac{8}{\gamma \pi^2} \sin \frac{\gamma \pi}{2}$$

Cylindrical Rotor Machines Current Transformation Factor

- The current transformation factor of a synchronous machine may be obtained in similar way as in case of an induction machine
  => the calculation is based on comparison of stator and rotor magnetomotive forces
- Presuming equality of magnitudes of both fields, we have

$$F_{d1} = F_{B1}$$

$$C_{d1} \cdot \frac{m}{\pi} \cdot \frac{N_s k_{v1} \sqrt{2} I}{p} = C_{b1} \frac{N_b I_b}{2p}$$

Cylindrical Rotor Machines Current Transformation Factor

- The current transformation factor of a synchronous machine may be obtained in similar way as in case of an induction machine
  => the calculation is based on comparison of stator and rotor magnetomotive forces
- And the transformation factor results:

$$g = \frac{I_b}{I} = \frac{C_{d1} \cdot \frac{m}{\pi} \cdot \frac{N_s k_{v1} \sqrt{2}}{p}}{C_{b1} \frac{N_b}{2p}}$$

$$g = \frac{C_{d1}}{C_{b1}} \cdot \frac{m}{\pi} \cdot \frac{2\sqrt{2} N_s k_{v1}}{N_b}$$

Cylindrical Rotor Machines Current Transformation Factor

- Using excitation winding factor k_{vb}

$$C_{b1} = \frac{4}{\pi} k_{vb}$$

- and by considering d-axis stator magnetic field shape factor $C_{d1} = 1$, the relation simplifies to

$$g = \frac{I_b}{I} = \frac{m}{\sqrt{2}} \cdot \frac{N_s k_{v1}}{N_b k_{vb}}$$

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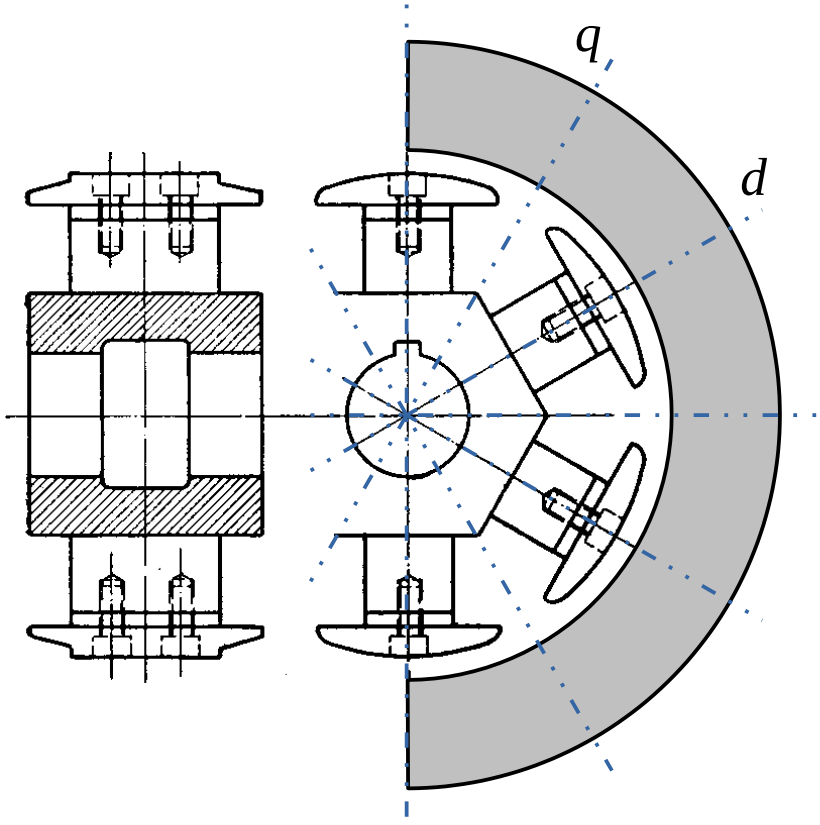
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Excitation Field of Salient Pole Synchronous Machines

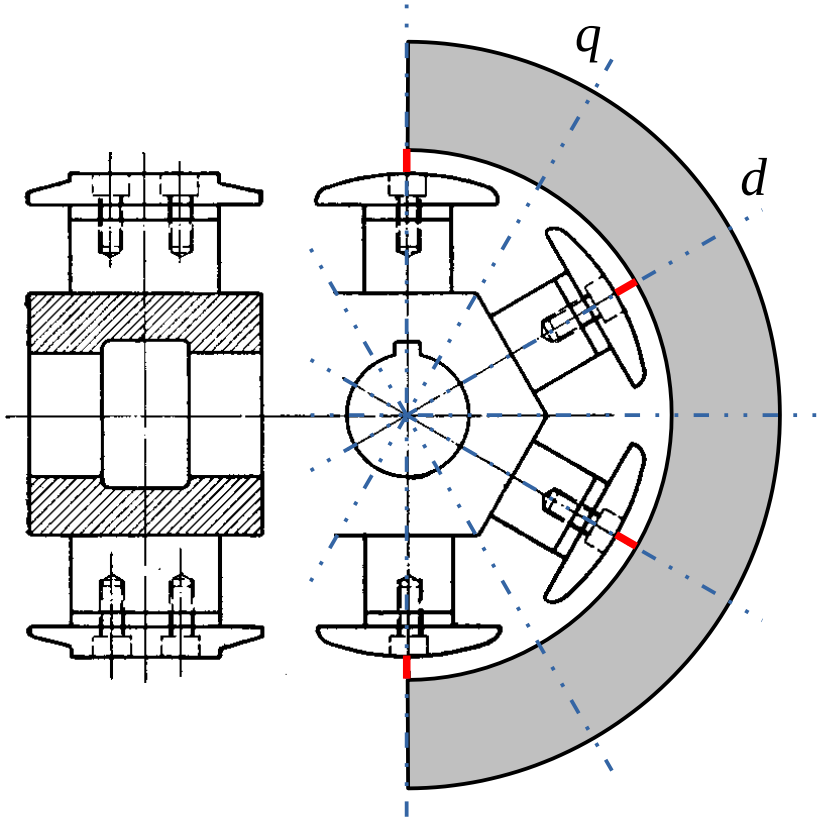
Excitation Field of Salient Pole Synchronous Machines

- In comparison to cylindrical rotor machines it is not possible to neglect variable air gap
- The air gap width changes between d and q axes according to the shape of the pole shoe



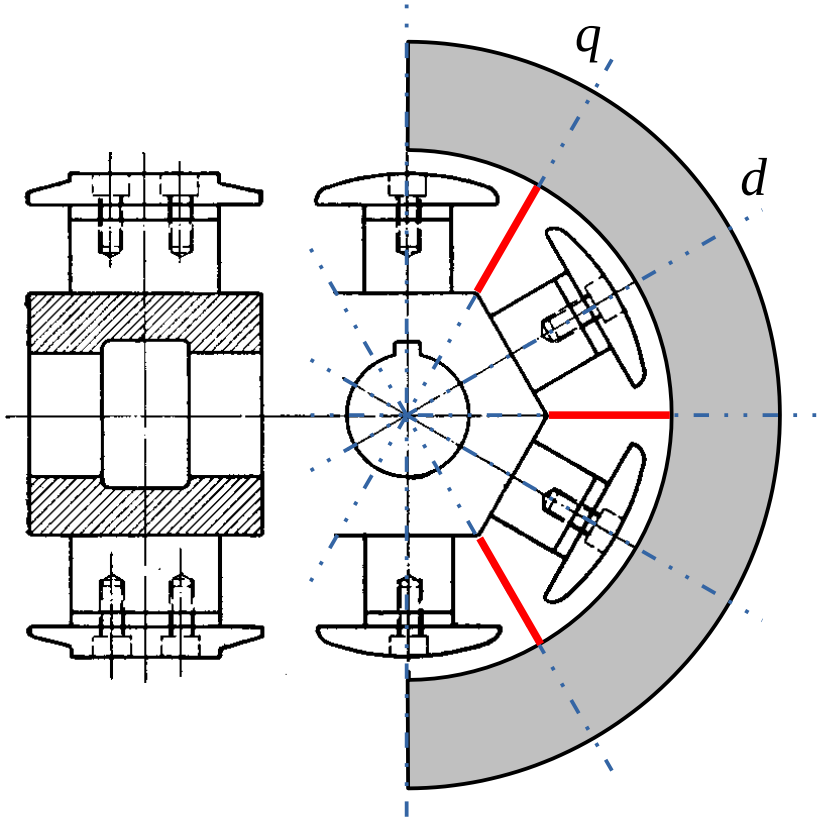
Excitation Field of Salient Pole Synchronous Machines

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- Minimal air gap width is located over the pole shoe (δ_{min})

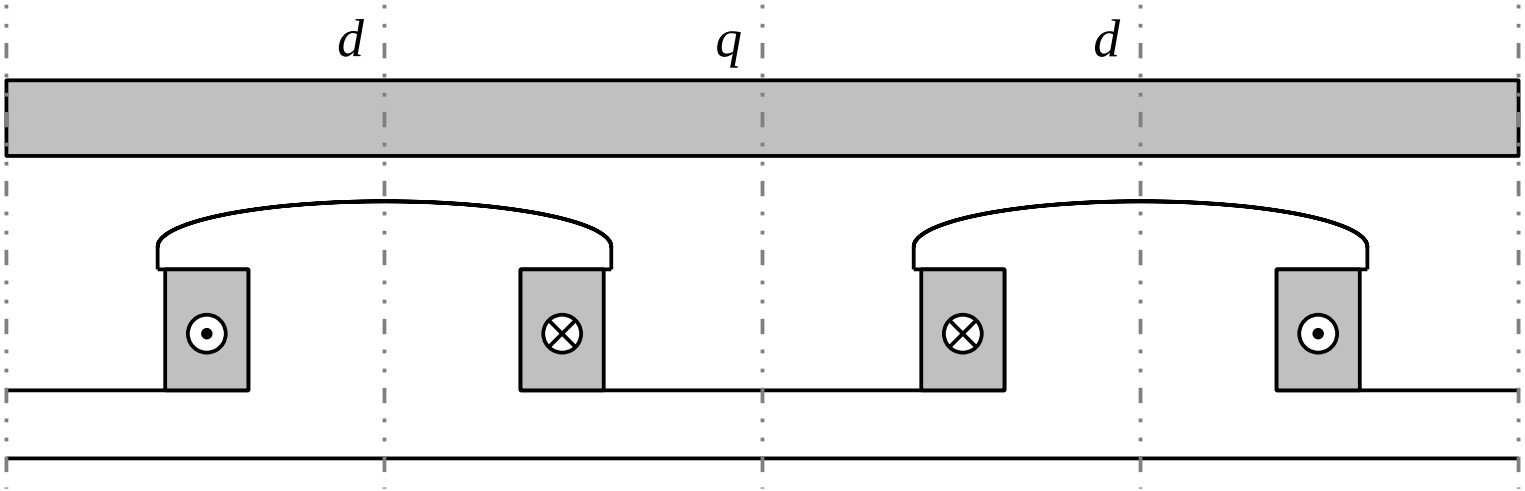


Excitation Field of Salient Pole Synchronous Machines

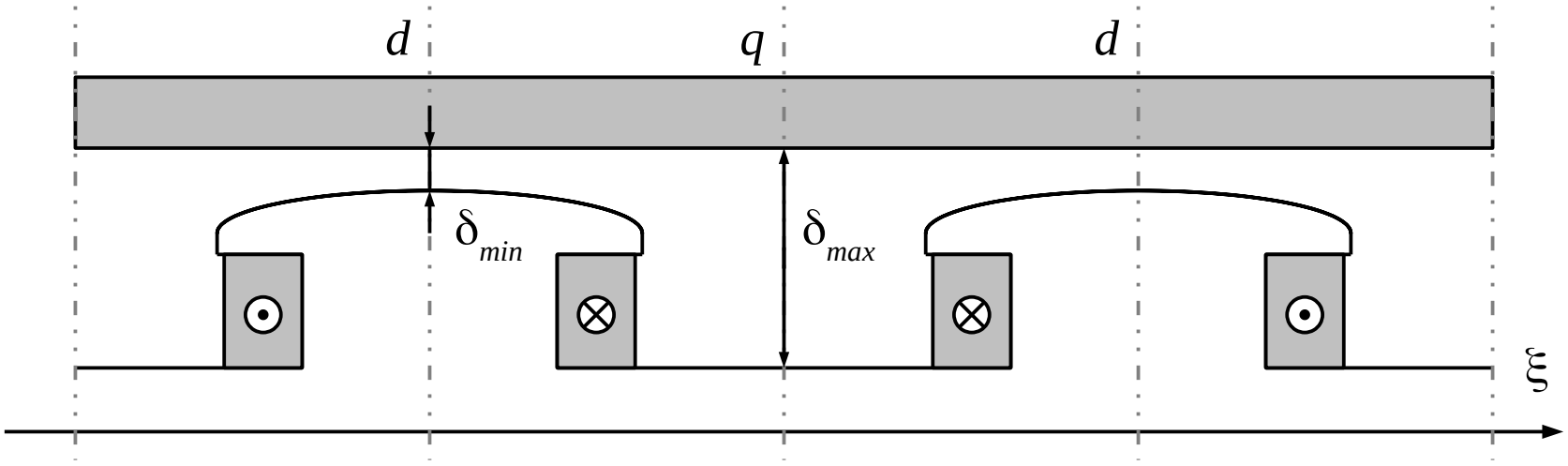
- In comparison to cylindrical rotor machines it is not possible to neglect variable air gap
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- Maximal air gap width is located between rotor poles (δ_{max})



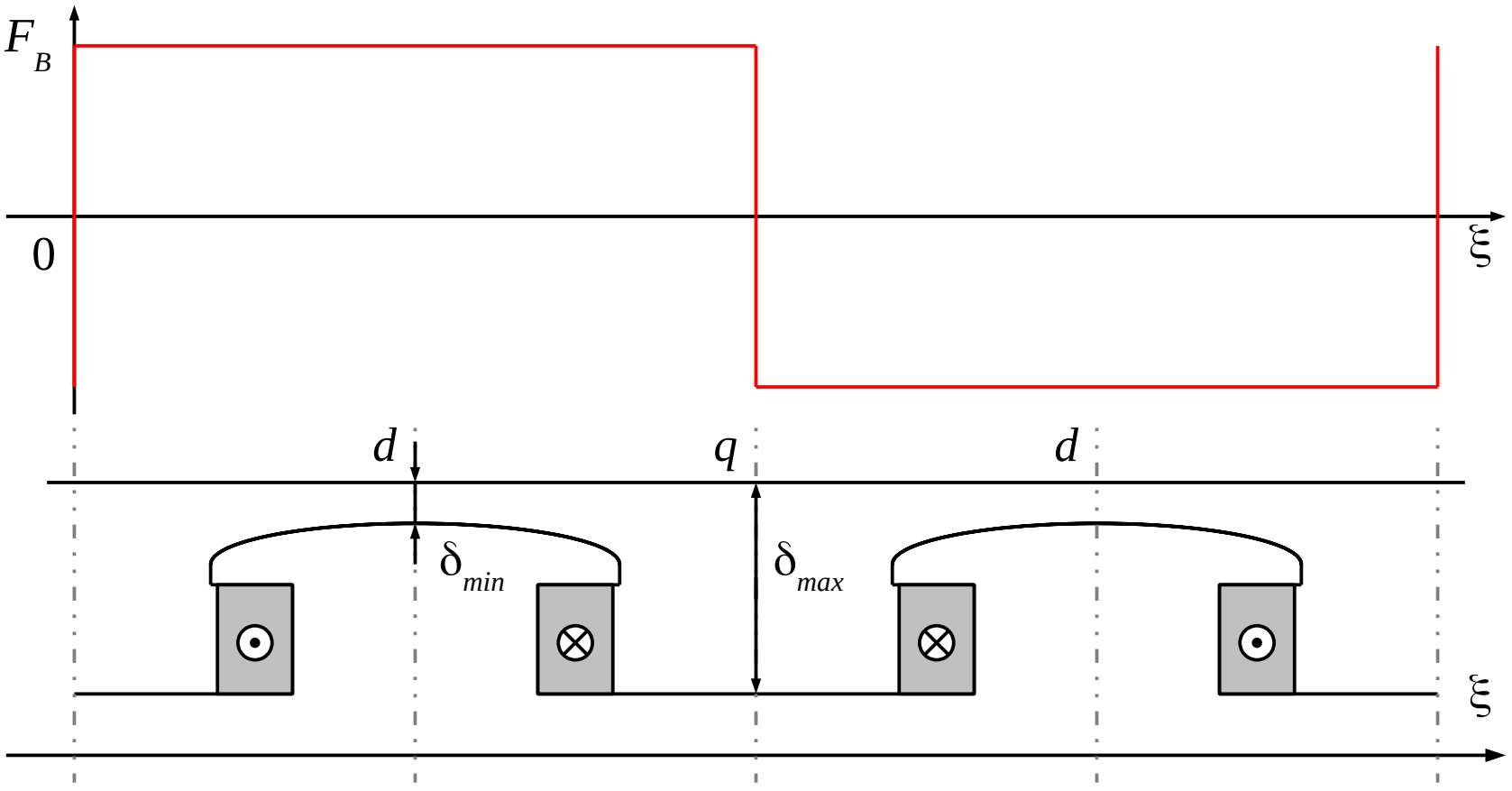
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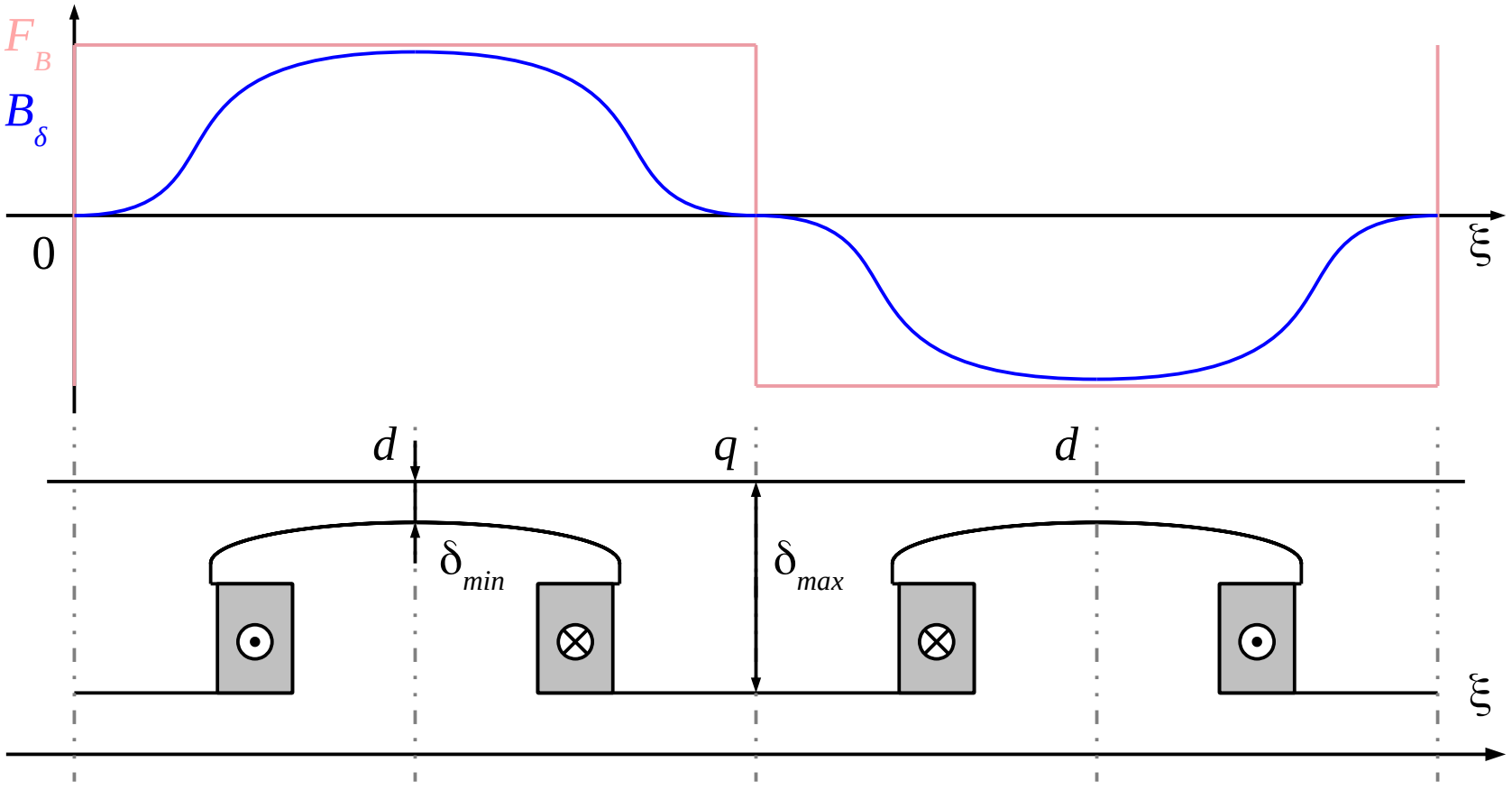
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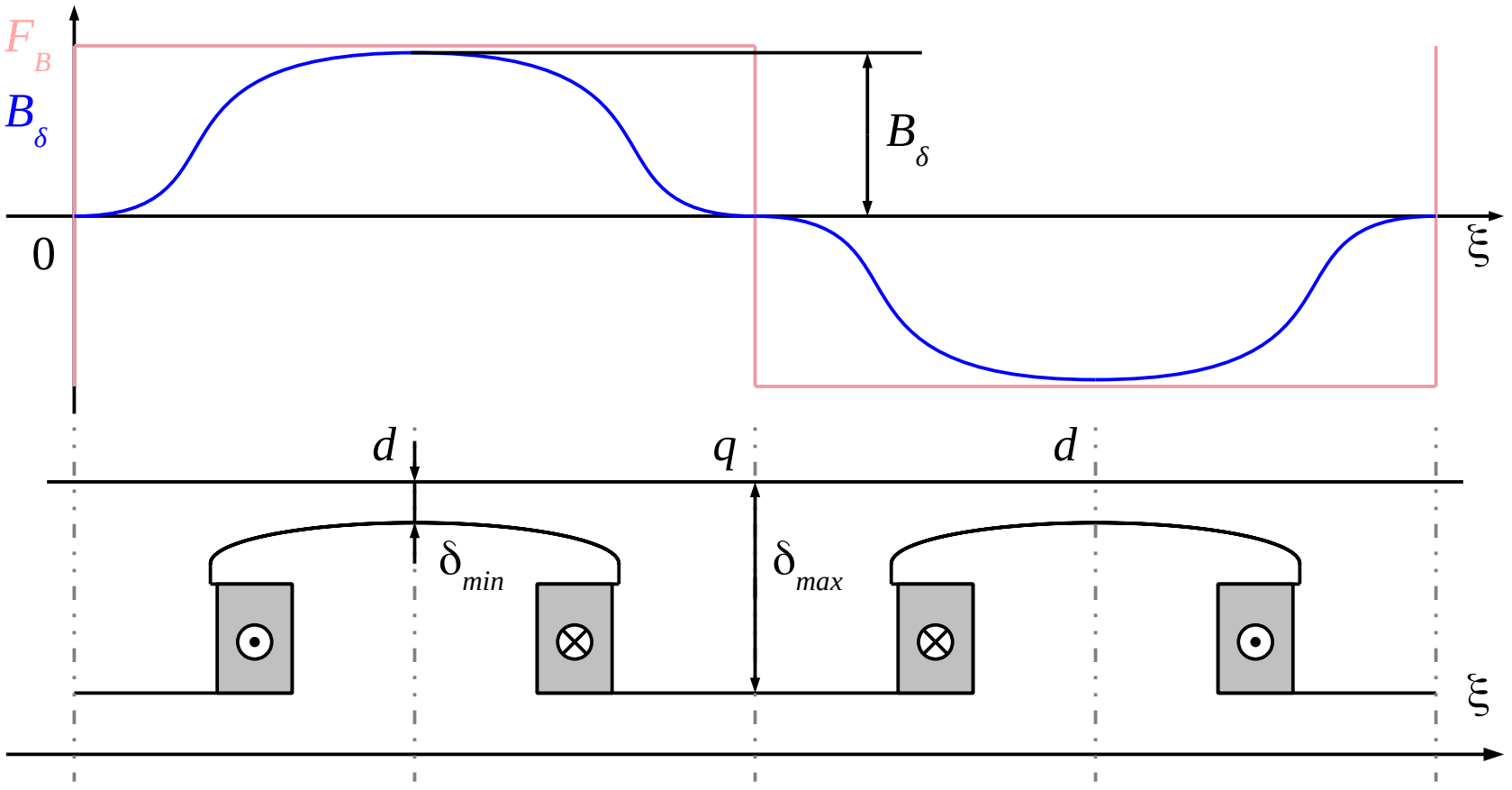
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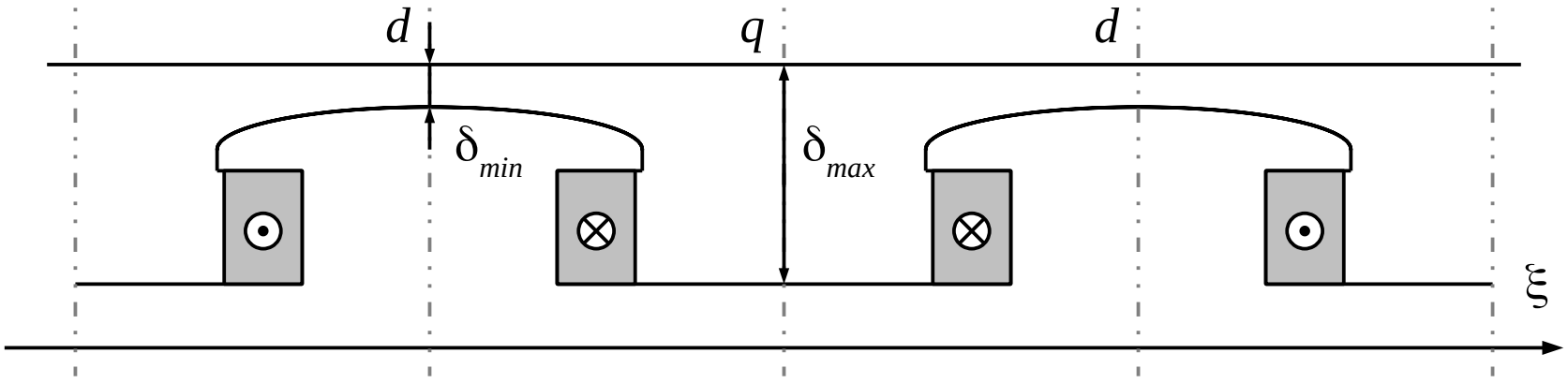


Excitation Field of Salient Pole Synchronous Machines



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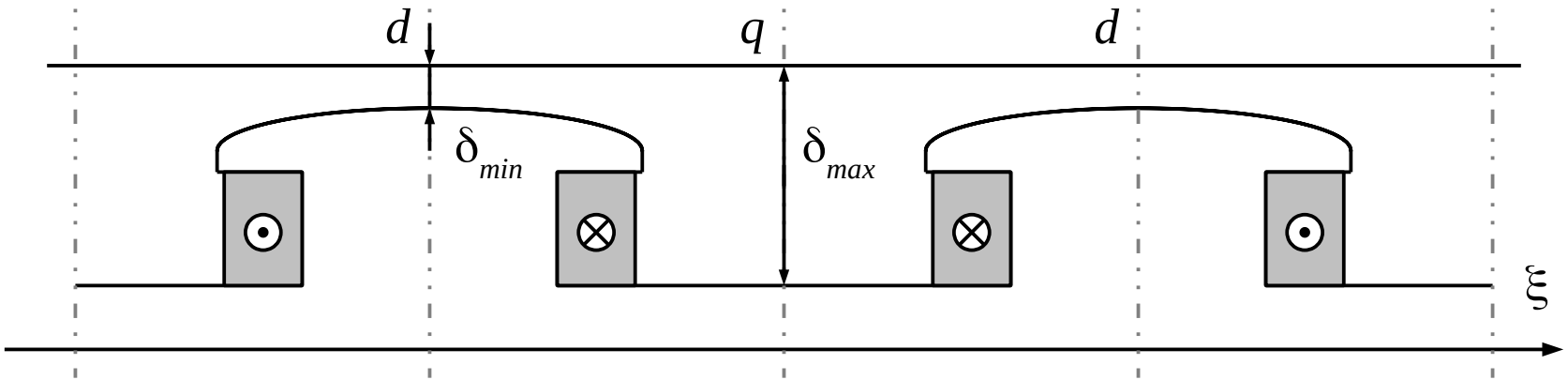
- Due to the shape of the air gap of the salient pole synchronous machine, the air gap magnetic flux density is deformed



Excitation Field of Salient Pole Synchronous Machines

- Due to the shape of the air gap of the salient pole synchronous machine, the air gap magnetic flux density is deformed
- The shape of the air gap may be described only as a general function

$$f(\delta)=\delta(\xi)$$

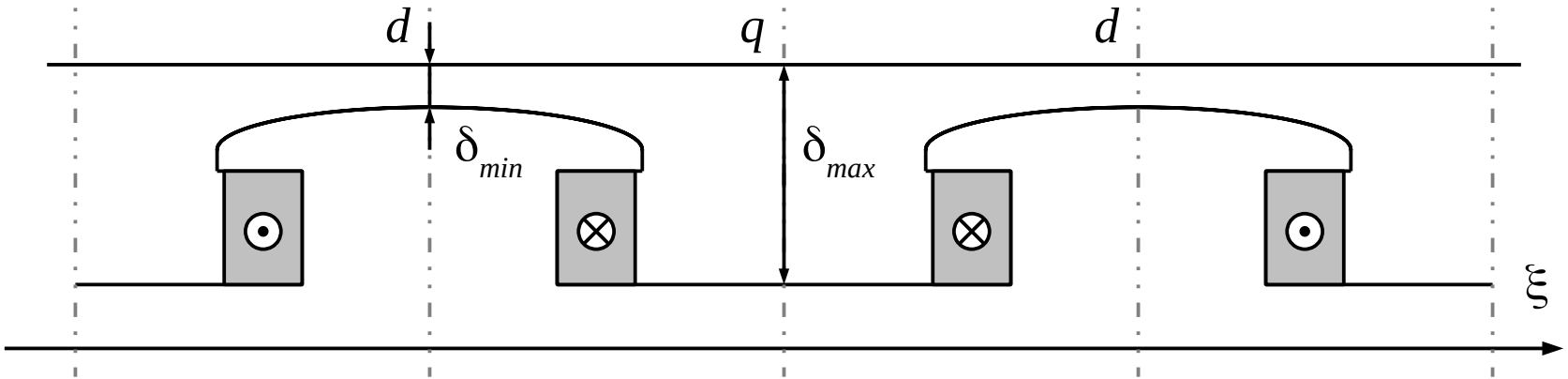


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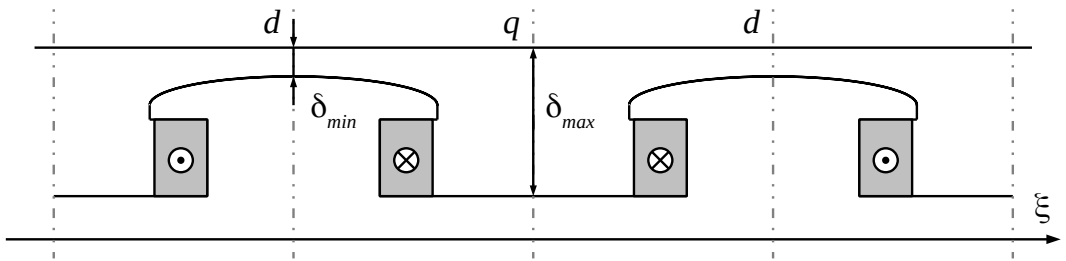
- Whereas $\delta(d)=\delta_{min}$ and $\delta(q)=\delta_{max}$



Excitation Field of Salient Pole Synchronous Machines

- For the air gap magnetic flux density the following relation is valid

$$B_\delta(\xi) = \mu_0 H_\delta(\xi)$$



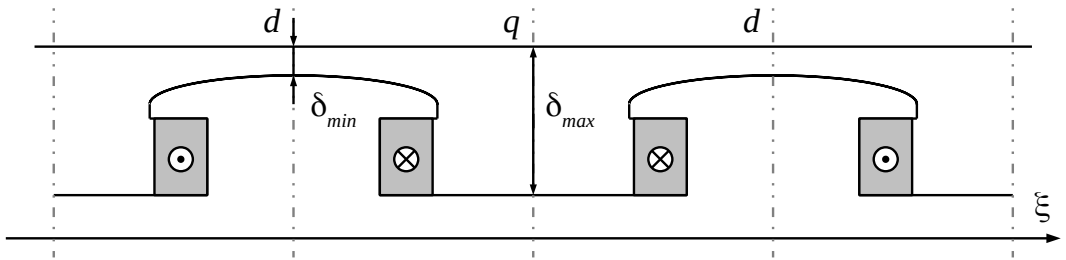
Excitation Field of Salient Pole Synchronous Machines

- For the air gap magnetic flux density the following relation is valid

$$B_\delta(\xi)=\mu_0 H_\delta(\xi)$$

where the magnetic field strength may be determined from rotor magnetomotive force

$$F_B=\int_{\delta(\xi)} H_\delta(\xi)\,dy$$



Excitation Field of Salient Pole Synchronous Machines

- Pro magnetické pole ve vzduchové mezeře platí

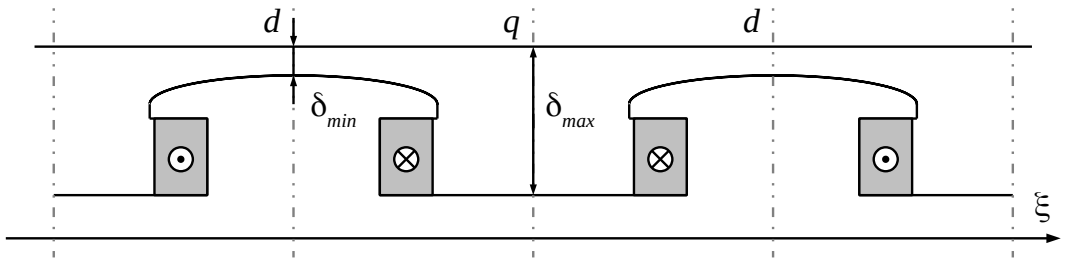
$$B_\delta(\xi) = \mu_0 H_\delta(\xi)$$

kde intenzitu magnetického pole lze určit z magnetického napětí rotoru

$$F_B = \int_{\delta(\xi)} H_\delta(\xi)\, dy$$

...které lze zapsat obecně pouze jako

$$F_B = H_\delta(\xi)\,\delta(\xi)$$



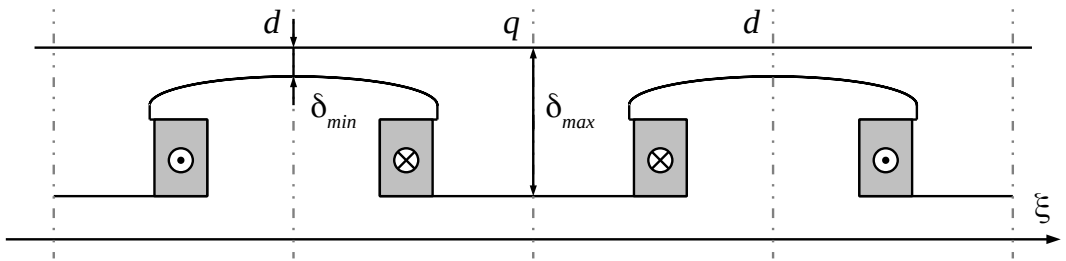
Excitation Field of Salient Pole Synchronous Machines

- After substitution of

$$H_\delta(\xi) = \frac{F_B}{\delta(\xi)}$$

we obtain

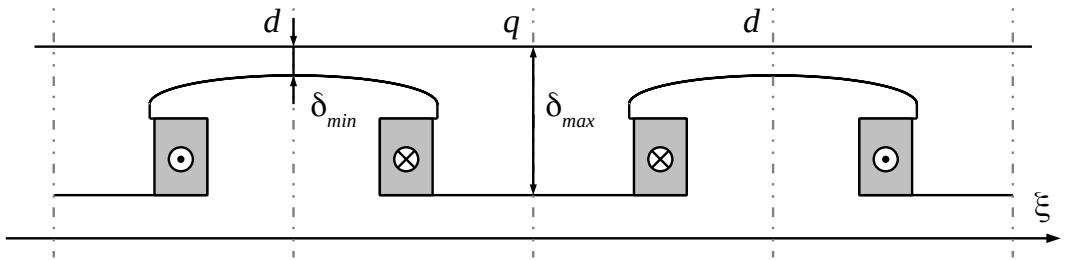
$$B_\delta(\xi) = \mu_0 \frac{F_B}{\delta(\xi)}$$



Excitation Field of Salient Pole Synchronous Machines

- From thhe electromagnetic design of the machine the magnitude of the air gap magnetic flux density B_δ is known, which corresponds to

$$B_\delta = \mu_0 \frac{F_B}{\delta_{min}}$$



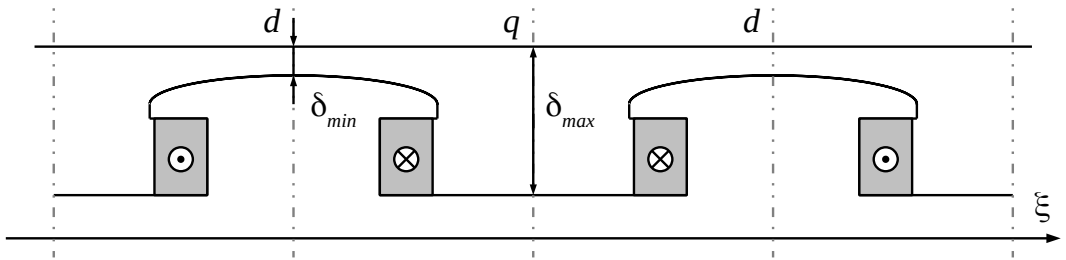
Excitation Field of Salient Pole Synchronous Machines

- From thhe electromagnetic design of the machine the magnitude of the air gap magnetic flux density B_δ is known, which corresponds to

$$B_\delta = \mu_0 \frac{F_B}{\delta_{min}}$$

- Thus there is a possibility to substitute to magnetomotive force from the machine design

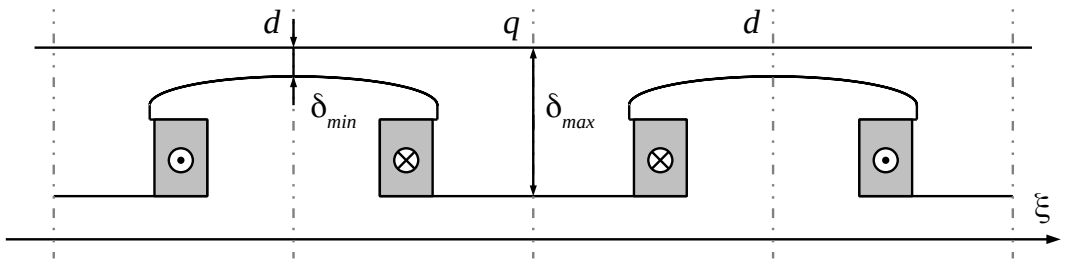
$$F_B = \frac{B_\delta}{\mu_0} \delta_{min}$$



Excitation Field of Salient Pole Synchronous Machines

- The spatially variable function of the air gap magnetic flux density may be then written as

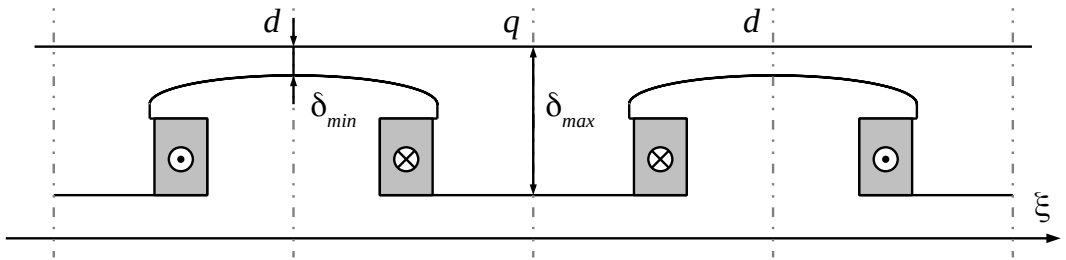
$$B_\delta(\xi) = \mu_0 \frac{\frac{B_\delta}{\mu_0}\delta_{min}}{\delta(\xi)}$$



Excitation Field of Salient Pole Synchronous Machines

- The spatially variable function of the air gap magnetic flux density may be then written as

$$B_\delta(\xi) = \mu_0 \frac{\frac{B_\delta}{\mu_0}\,\delta_{min}}{\delta(\xi)} = B_\delta \frac{\delta_{min}}{\delta(\xi)}$$



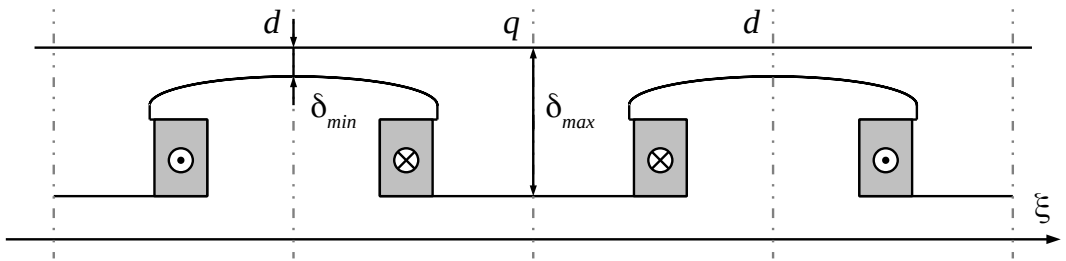
Excitation Field of Salient Pole Synchronous Machines

- The spatially variable function of the air gap magnetic flux density may be then written as

$$B_\delta(\xi) = \mu_0 \frac{\frac{B_\delta}{\mu_0}\delta_{min}}{\delta(\xi)} = B_\delta \frac{\delta_{min}}{\delta(\xi)}$$

- Its fundamental harmonic is

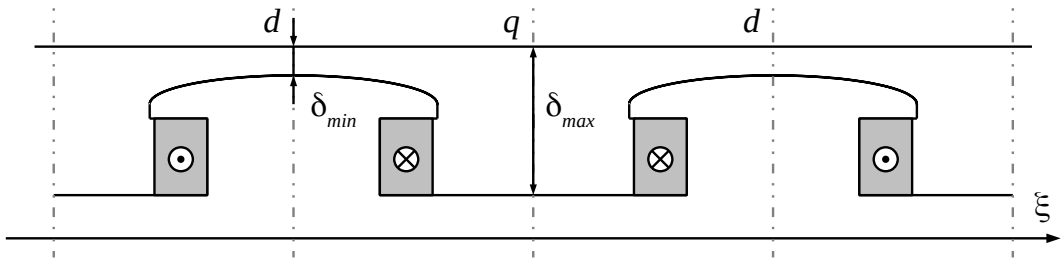
$$B_1 = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} B_\delta(\xi)\sin\xi\, d\xi$$



Excitation Field of Salient Pole Synchronous Machines

- While integrating in the interval between axes d and q, the period is $T=\frac{\pi}{2}$ and the resulting form of the calculated integral is

$$B_1=\frac{4}{\pi}\int_0^{\frac{\pi}{2}} B_\delta \frac{\delta_{min}}{\delta(\xi)}\sin\xi\, d\,\xi$$

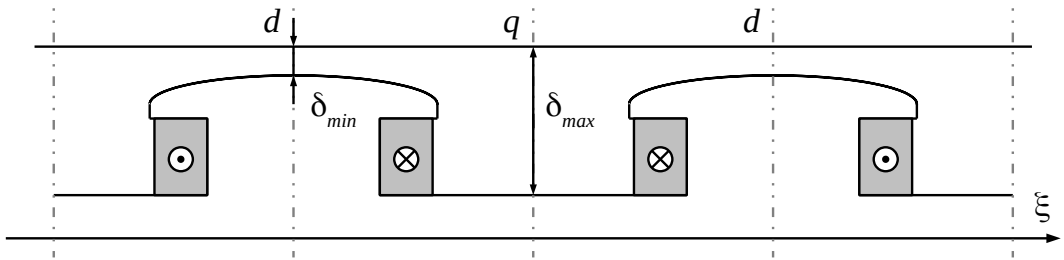


Excitation Field of Salient Pole Synchronous Machines

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$$B_1=\frac{4}{\pi}\int_0^{\frac{\pi}{2}} B_\delta \frac{\delta_{min}}{\delta(\xi)}\sin\xi\, d\,\xi$$

… which may be, considering complicated shape of the air gap, solved only using numeric software (GNU/Octave, Matlab ...)



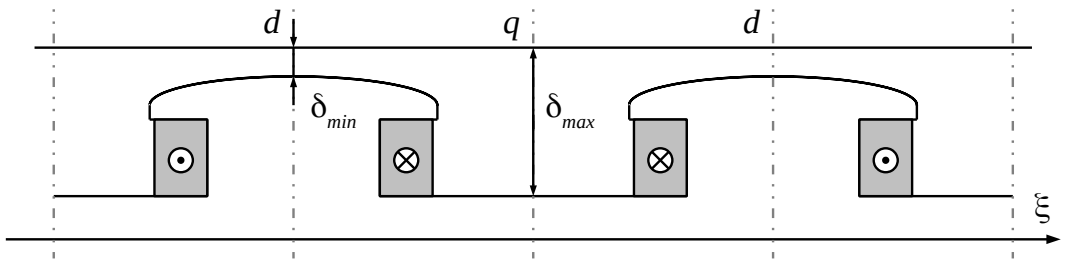
Excitation Field of Salient Pole Synchronous Machines

- The result is, similarly to previous case

$$B_1 = C_{b1} B_\delta$$

where the excitation magnetic field shape factor is dependent on the geometry of the rotor (pole shoe width, their additional shaping etc.):

$$C_{b1} = f\left(\delta(\xi)\right)$$



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MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

ESF II – project of the University of West Bohemia
reg. no. CZ.02.2.69/0.0/0.0/18_056/0013239

Armature Field of Salient Pole Synchronous Machines

Armature Field of Salient Pole Synchronous Machines

- The armature winding generates a magnetic field with magnetomotive force of nearly sinusoidal shape
- This field is moved against rotor by the load angle β
- The result is a general magnetic field with shape depenent on the shape of the air gap – $B_\delta(\xi) = f(\delta(\xi))$
- This magnetic field may be mathematically dissolved to two fields, one in the d-axis and second in the q-axis

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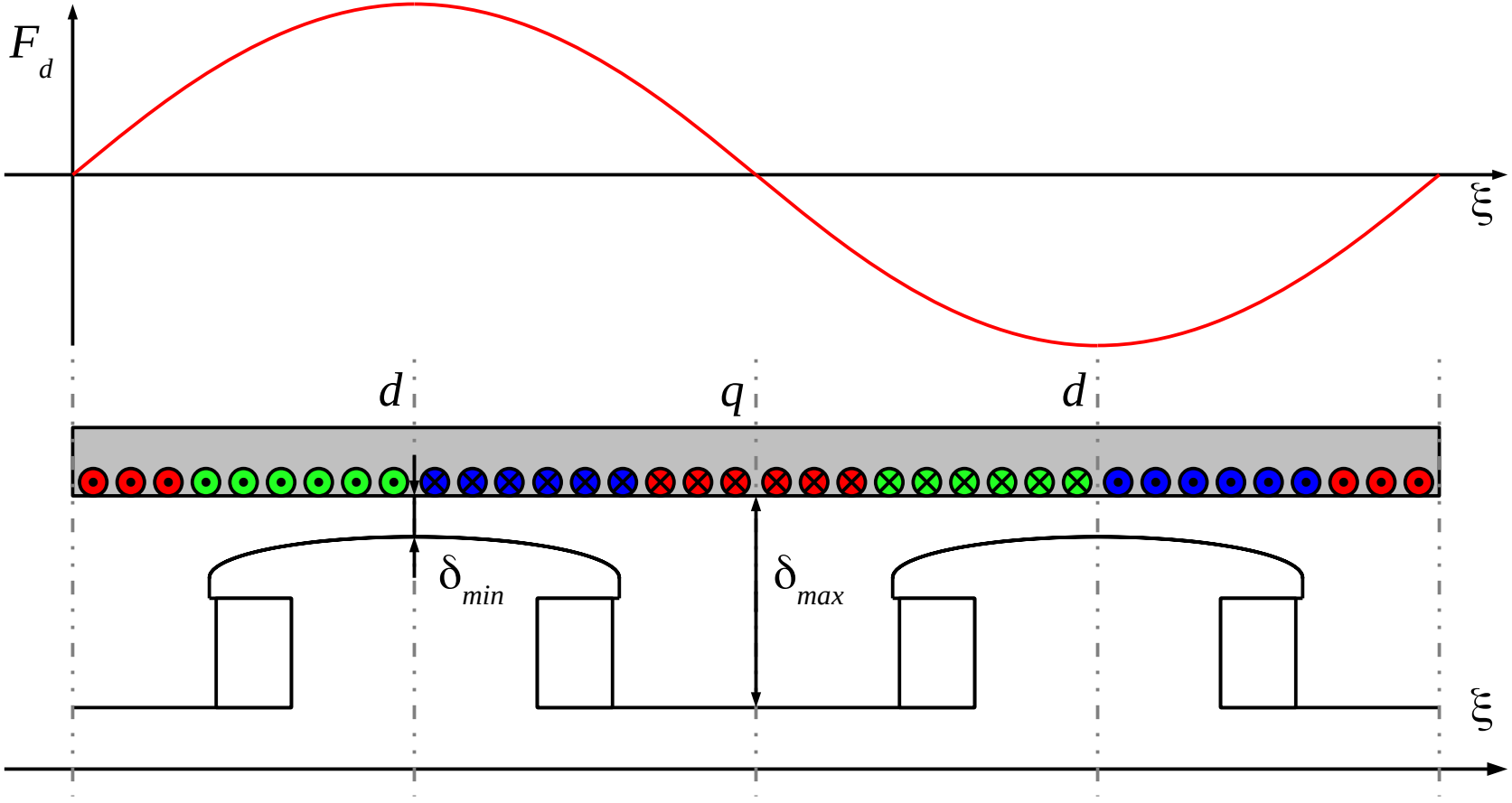
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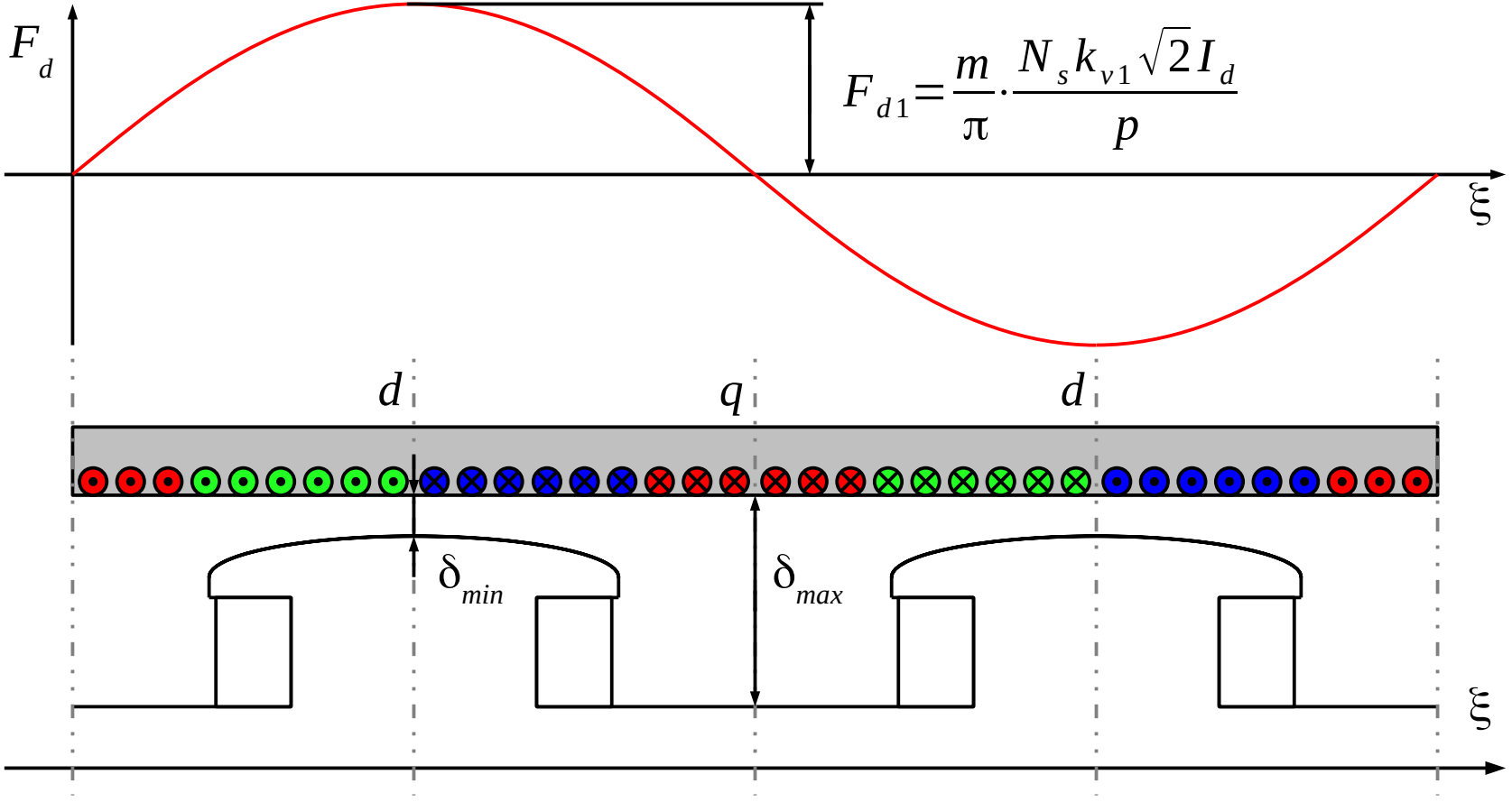
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Armature Magnetic Field of Salient Pole Synchronous Machines in the d-Axis

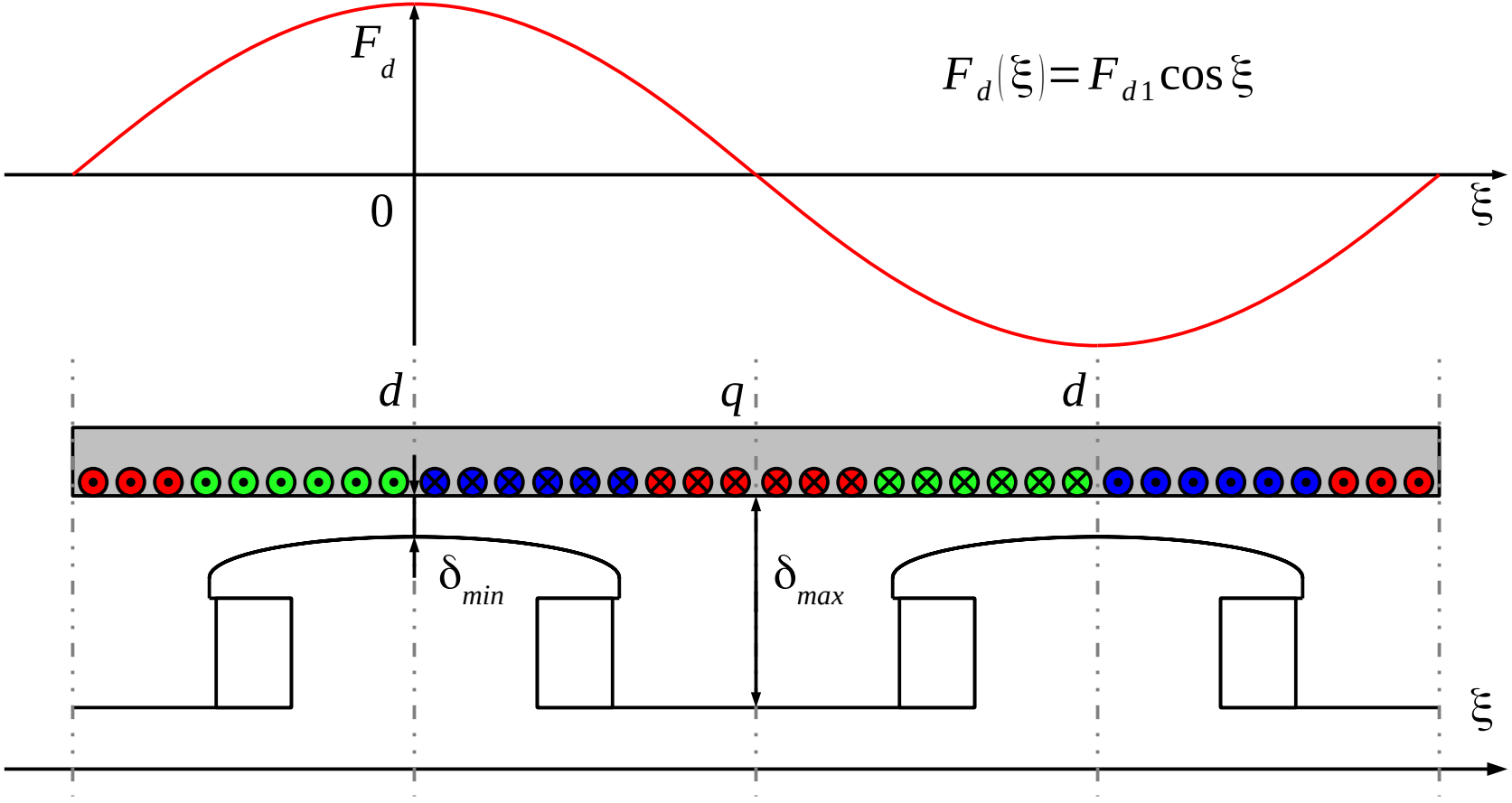
Armature Field of Salient Pole Machines in the *d*-Axis



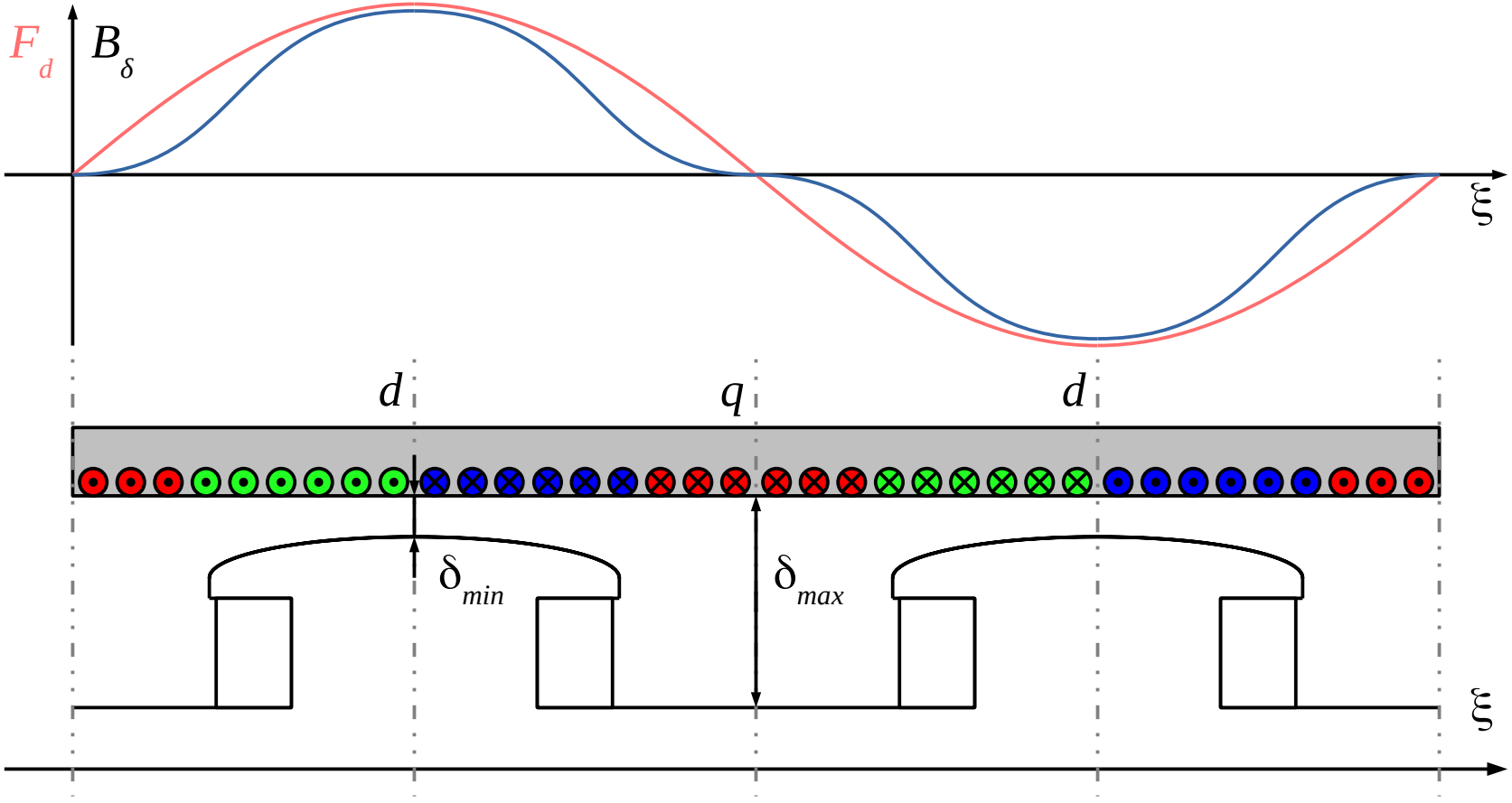
Armature Field of Salient Pole Machines in the *d*-Axis



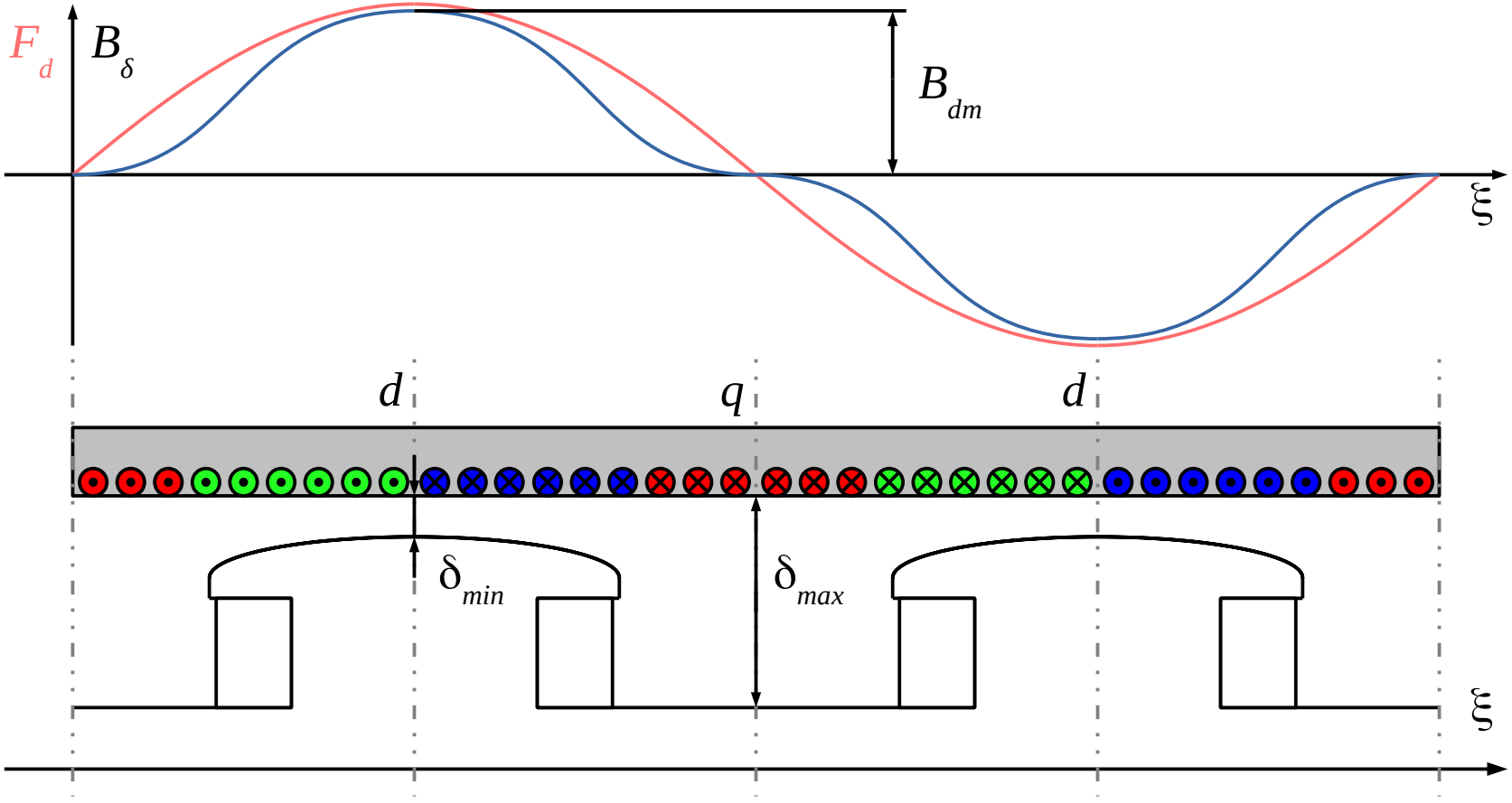
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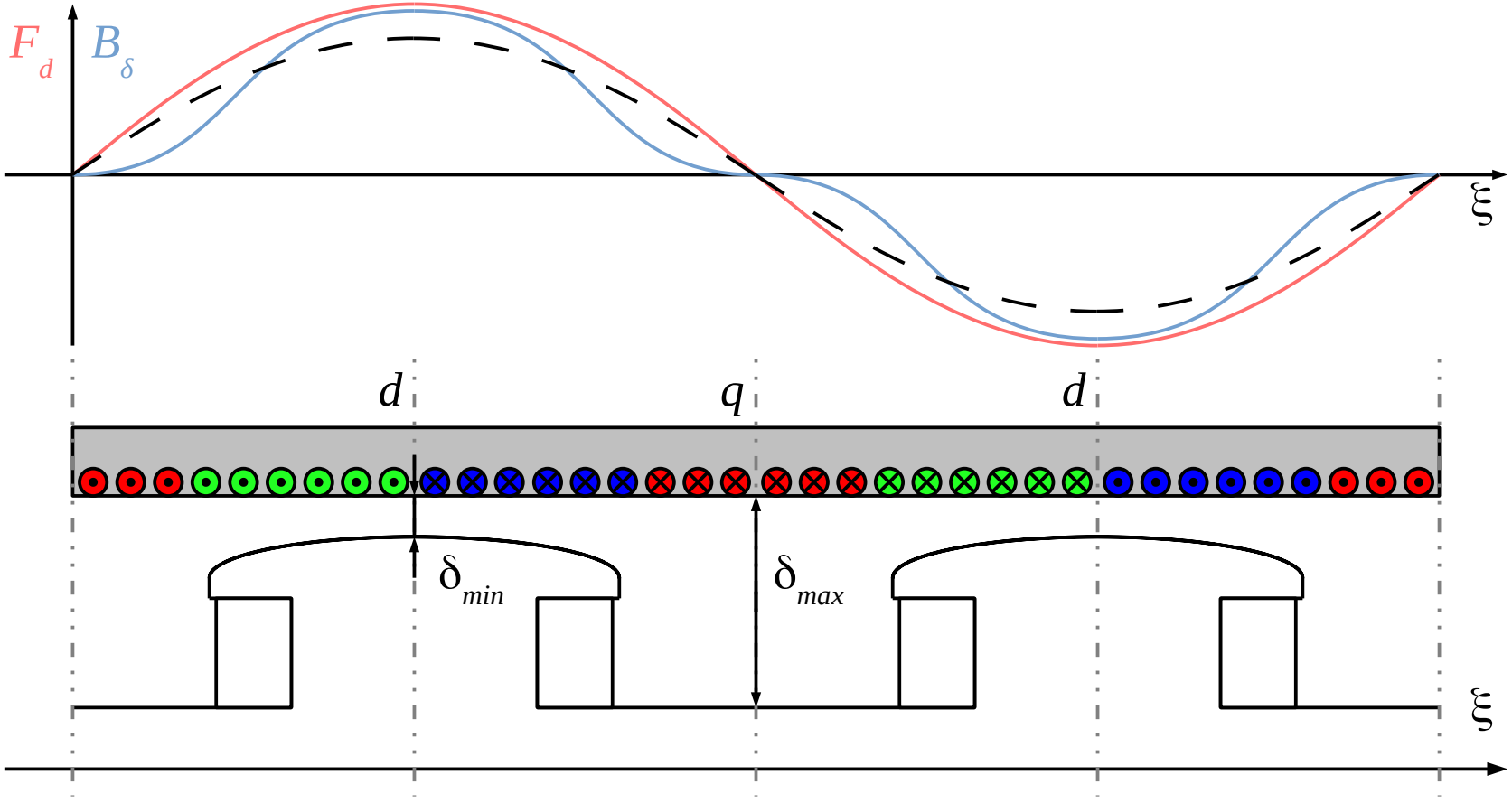
Armature Field of Salient Pole Machines in the *d*-Axis



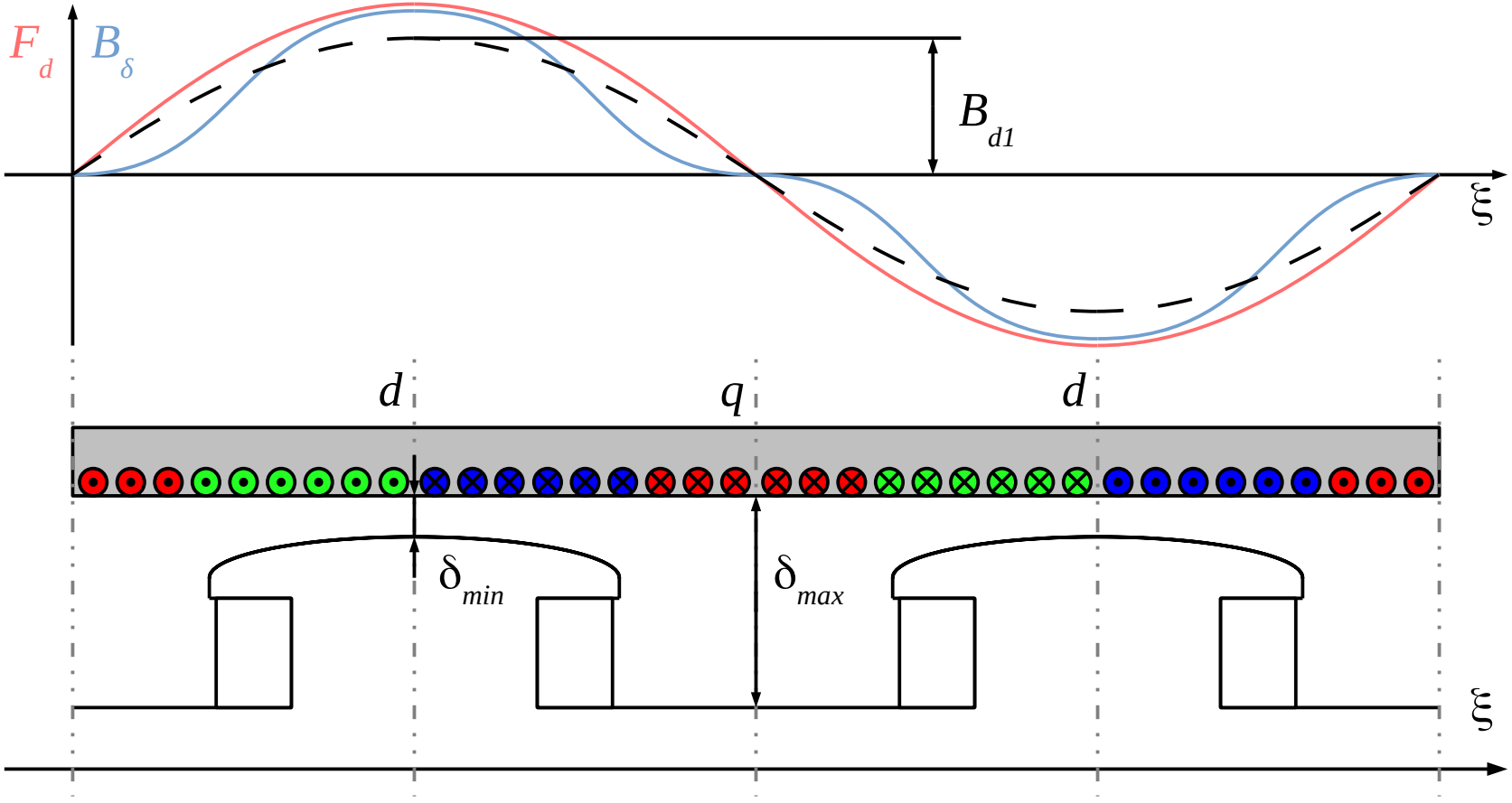
Armature Field of Salient Pole Machines in the *d*-Axis



Armature Field of Salient Pole Machines in the *d*-Axis



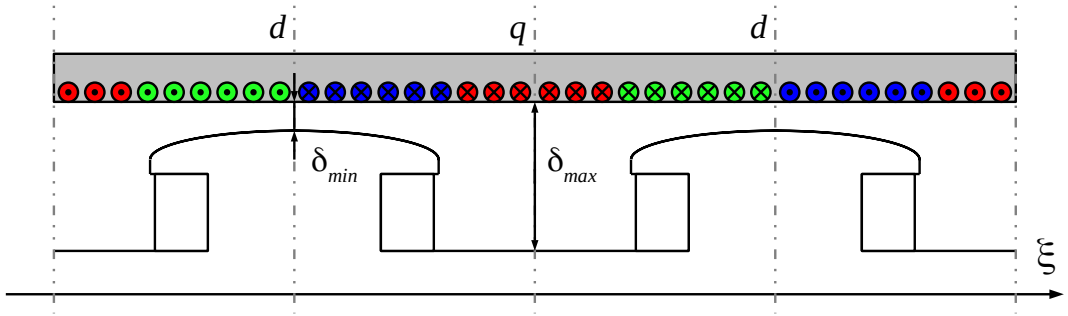
Armature Field of Salient Pole Machines in the *d*-Axis



Armature Field of Salient Pole Machines in the *d*-Axis

- The magnetomotive force of the stator has a (co)sine shape

$$F_d(\xi) = F_{d1}\cos\xi$$



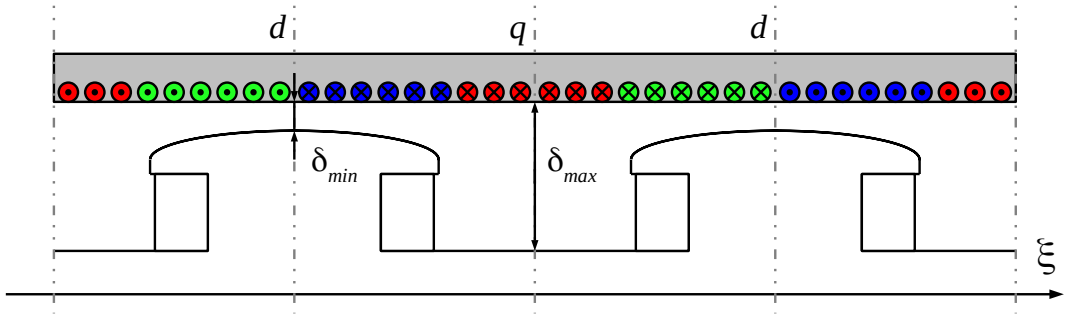
Armature Field of Salient Pole Machines in the *d*-Axis

- The magnetomotive force of the stator has a (co)sine shape

$$F_d(\xi) = F_{d1}\cos\xi$$

and the magnetic field strength in the air gap results from

$$F_d(\xi) = \int_{\delta(\xi)} H_d(\xi)\,dy$$



Armature Field of Salient Pole Machines in the *d*-Axis

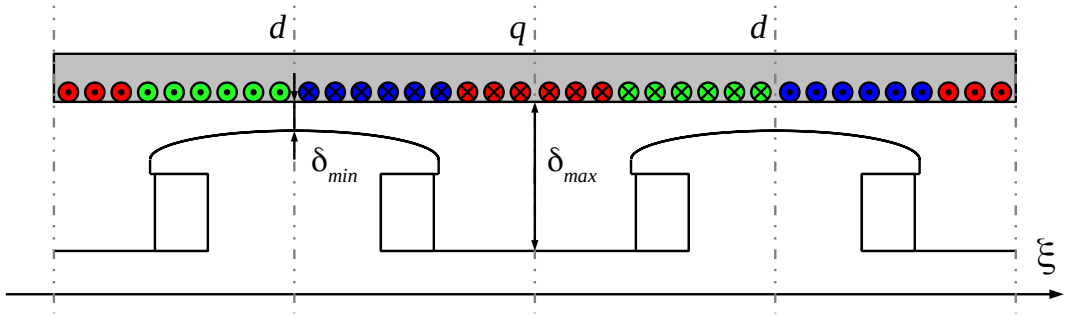
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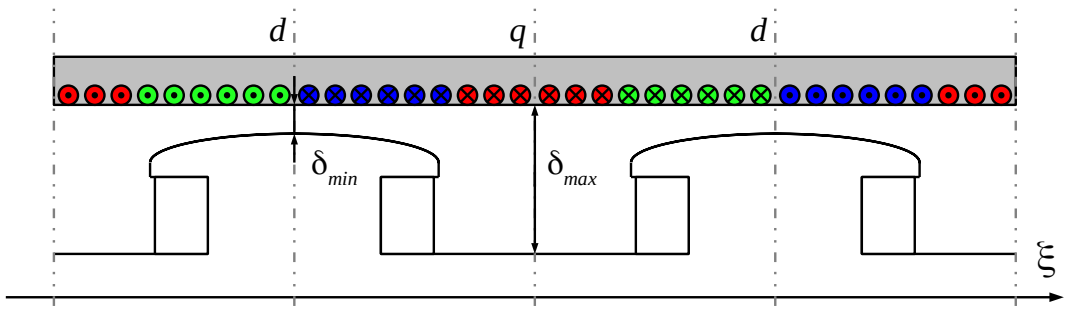
… which gives, like in previous case $F_d(\xi) = H_d(\xi)\,\delta(\xi)$



Armature Field of Salient Pole Machines in the *d*-Axis

- Thus the air gap magnetic field strength

$$H_d(\xi) = \frac{F_{d1}}{\delta(\xi)} \cos \xi$$



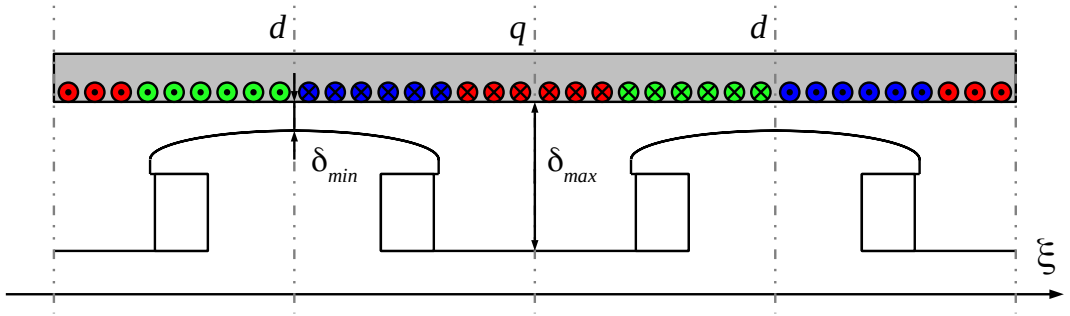
Armature Field of Salient Pole Machines in the *d*-Axis

- Thus the air gap magnetic field strength

$$H_d(\xi) = \frac{F_{d1}}{\delta(\xi)} \cos\xi$$

where we can substitute the magnetomotive force by magnetic field strength from

$$F_{d1} = H_{dm}\,\delta_{min}$$



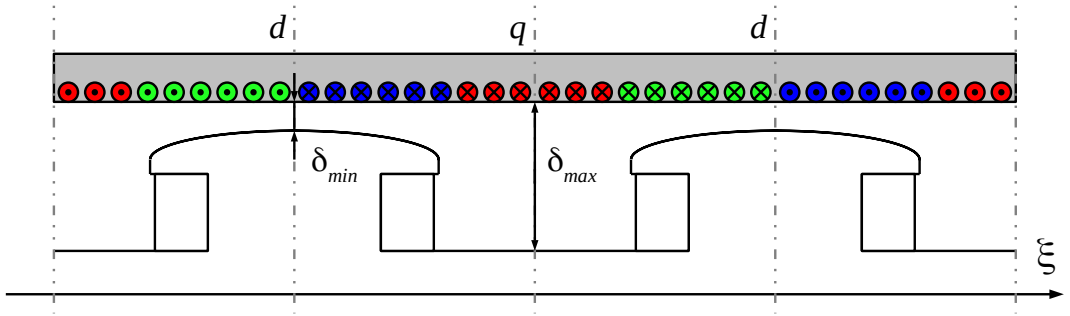
Armature Field of Salient Pole Machines in the *d*-Axis

- The resulting relation between magnetic field strengths

$$H_d(\xi) = H_{dm}\frac{\delta_{min}}{\delta(\xi)}\cos\xi$$

may be transformed to magnetic flux densities

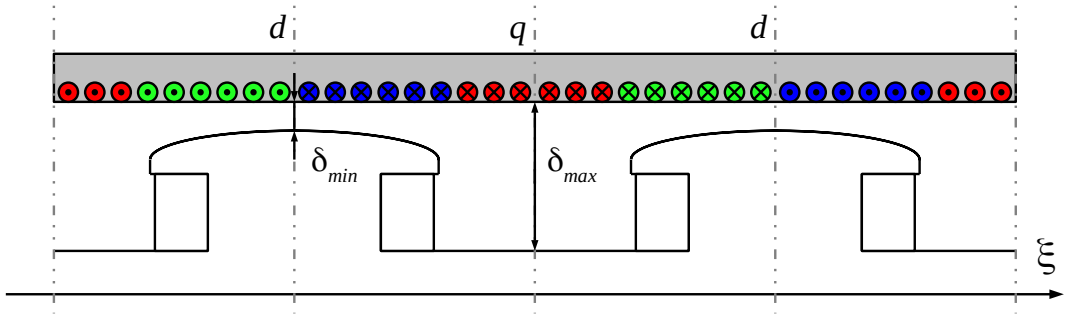
$$B_d(\xi) = B_{dm}\frac{\delta_{min}}{\delta(\xi)}\cos\xi$$



Armature Field of Salient Pole Machines in the *d*-Axis

- Similarly to previous case the fundamental harmonic component results from the Fourier analysis

$$B_{d1} = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} B_d(\xi) \cos \xi \, d\xi$$



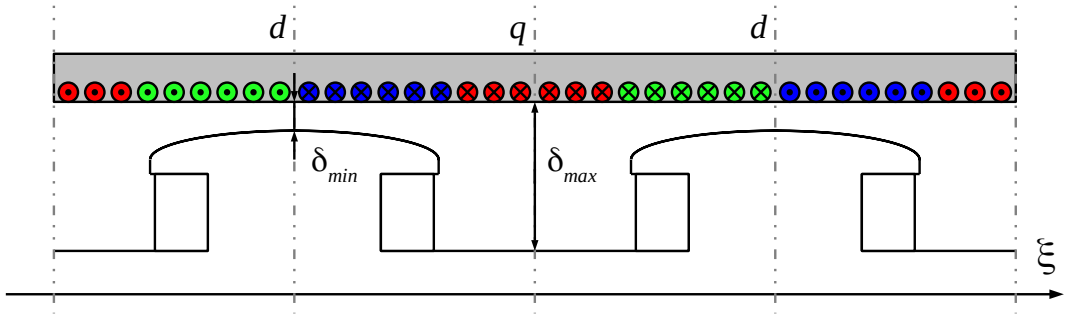
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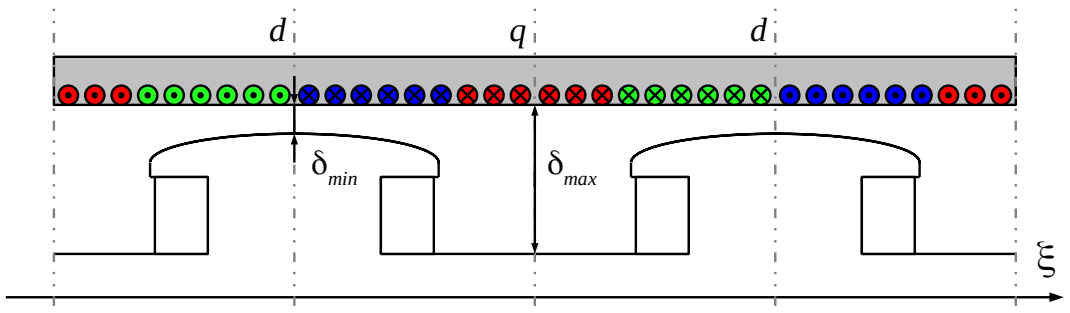
which is after substitution

$$B_{d1} = \frac{4}{\pi} \int_{0}^{\frac{\pi}{2}} B_{dm} \frac{\delta_{min}}{\delta(\xi)} \cos^2 \xi \, d\xi$$



Armature Field of Salient Pole Machines in the *d*-Axis

- The solution leads again to use of numerical software
- The result is then the stator *d*-axis armature magnetic field shape factor defined by $$B_{d1} = C_{d1} B_{dm}$$
- By using the *d*-axis armature magnetic field shape factor it is possible to calculate the current transformation ration of a synchronous machine with salient poles



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Salient Pole Synchronous Machines
Current Transformation Ratio

Salient Pole Synchronous Machines Current Transformation Ratio

- In this case it is calculated from the comparison of air gap magnetic flux densities

Salient Pole Synchronous Machines Current Transformation Ratio

- For the stator the d-axis magnetic flux density is

$$B_{d1} = C_{d1} B_{dm} = C_{d1} \mu_0 H_{dm}$$

Salient Pole Synchronous Machines Current Transformation Ratio

- For the stator the *d*-axis magnetic flux density is

$$B_{d1} = C_{d1} B_{dm} = C_{d1} \mu_0 H_{dm}$$

where the magnetic field strength is

$$H_{dm} = \frac{F_{d1}}{\delta_{min}}$$

Salient Pole Synchronous Machines Current Transformation Ratio

- For the stator the *d*-axis magnetic flux density is

$$B_{d1} = C_{d1} B_{dm} = C_{d1} \mu_0 H_{dm}$$

where the magnetic field strength is

$$H_{dm} = \frac{F_{d1}}{\delta_{min}}$$

and the fundamental harmonic component of the stator magnetomotive force is

$$F_{d1} = \frac{m}{\pi} \cdot \frac{N_s k_{v1} \sqrt{2} I_d}{p}$$

Salient Pole Synchronous Machines Current Transformation Ratio

- For the stator the *d*-axis magnetic flux density is

$$B_{d1} = C_{d1} B_{dm} = C_{d1} \mu_0 H_{dm}$$

where the magnetic field strength is

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and the fundamental harmonic component of the stator magnetomotive force is

$$F_{d1} = \frac{m}{\pi} \cdot \frac{N_s k_{v1} \sqrt{2} I_d}{p}$$

- Then we obtain

$$B_{d1} = C_{d1} \frac{\mu_0}{\delta_{min}} \cdot \frac{m}{\pi} \cdot \frac{N_s k_{v1} \sqrt{2} I_d}{p}$$

Salient Pole Synchronous Machines Current Transformation Ratio

- Similarly in rotor d-axis

$$B_{\delta 1 m} = C_{b1} B_{\delta m} = C_{b1} \mu_0 H_{\delta m}$$

Salient Pole Synchronous Machines Current Transformation Ratio

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$$B_{\delta 1 m} = C_{b1} B_{\delta m} = C_{b1} \mu_0 H_{\delta m}$$

There the magnetic field strength is

$$H_{\delta m} = \frac{F_B}{\delta_{min}}$$

Salient Pole Synchronous Machines Current Transformation Ratio

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$$B_{\delta 1 m} = C_{b1} B_{\delta m} = C_{b1} \mu_0 H_{\delta m}$$

There the magnetic field strength is

$$H_{\delta m} = \frac{F_B}{\delta_{min}}$$

and the excitation winding magnetomotive force is

$$F_B = \frac{N_b I_b}{2p}$$

Salient Pole Synchronous Machines Current Transformation Ratio

- Similarly in rotor *d*-axis

$$B_{\delta 1 m} = C_{b1} B_{\delta m} = C_{b1} \mu_0 H_{\delta m}$$

There the magnetic field strength is

$$H_{\delta m} = \frac{F_B}{\delta_{min}}$$

and the excitation winding magnetomotive force is

$$F_B = \frac{N_b I_b}{2p}$$

- After calculation we obtain

$$B_{\delta 1 m} = C_{b1} \frac{\mu_0}{\delta_{min}} \cdot \frac{N_b I_b}{2p}$$

Salient Pole Synchronous Machines Current Transformation Ratio

- If we assume $B_{d1} = B_{\delta 1m}$

  then after substitution we have

$$C_{d1} \frac{\mu_0}{\delta_{min}} \cdot \frac{m}{\pi} \cdot \frac{N_s k_{v1} \sqrt{2} I_d}{p} = C_{b1} \frac{\mu_0}{\delta_{min}} \cdot \frac{N_b I_b}{2p}$$

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- Thus the salient pole synchronous machine current transformation ratio is

$$k_d g = \frac{I_b}{I_d} = \frac{C_{d1}}{C_{b1}} \cdot \frac{2\sqrt{2} m N_s k_{v1}}{\pi N_b}$$

Salient Pole Synchronous Machines Current Transformation Ratio

- If we assume $B_{d1} = B_{\delta 1m}$

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$$C_{d1} \frac{\mu_0}{\delta_{min}} \cdot \frac{m}{\pi} \cdot \frac{N_s k_{v1} \sqrt{2} I_d}{p} = C_{b1} \frac{\mu_0}{\delta_{min}} \cdot \frac{N_b I_b}{2p}$$

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$$k_d g = \frac{I_b}{I_d} = \frac{C_{d1}}{C_{b1}} \cdot \frac{2\sqrt{2} m N_s k_{v1}}{\pi N_b}$$

- The coefficient k_d has the meaning of d-axis magnetic field deformation coefficient

$$k_d = \frac{C_{d1}}{C_{b1}}$$

  which arises values of $k_d \in (0{,}785\,;0{,}85)$

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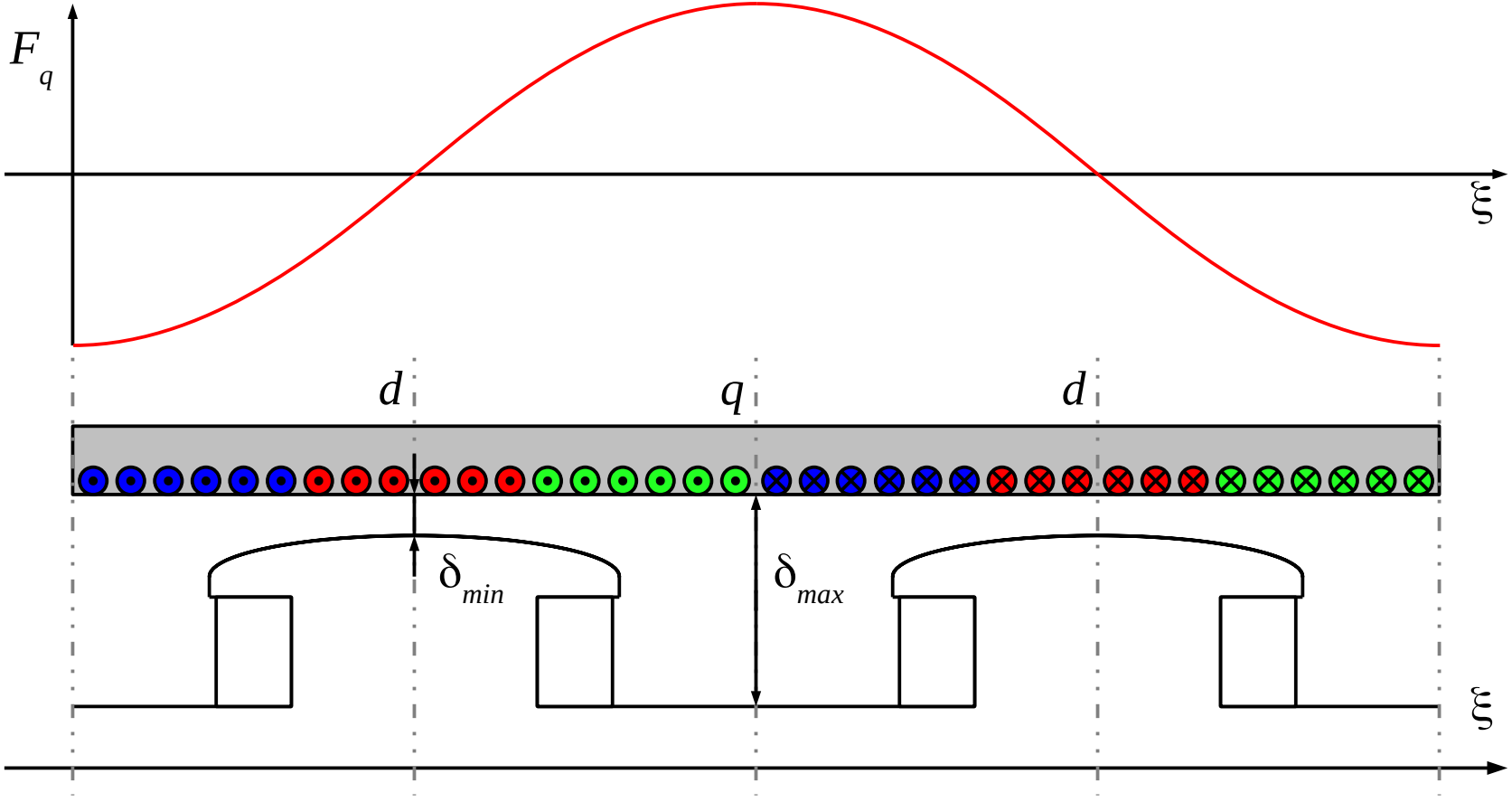
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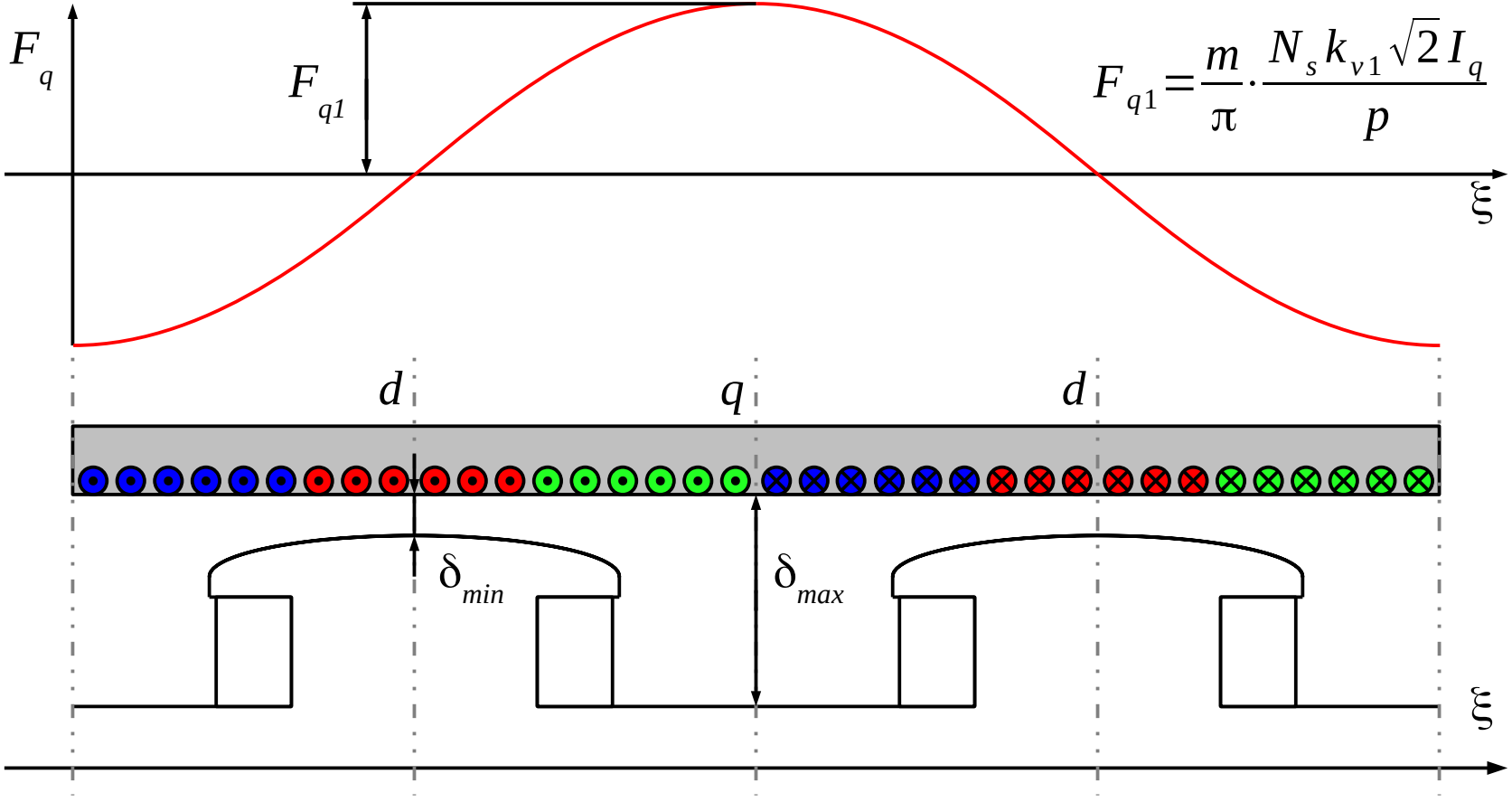
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Armature Magnetic Field of Salient Pole Synchronous Machines in the q-Axis

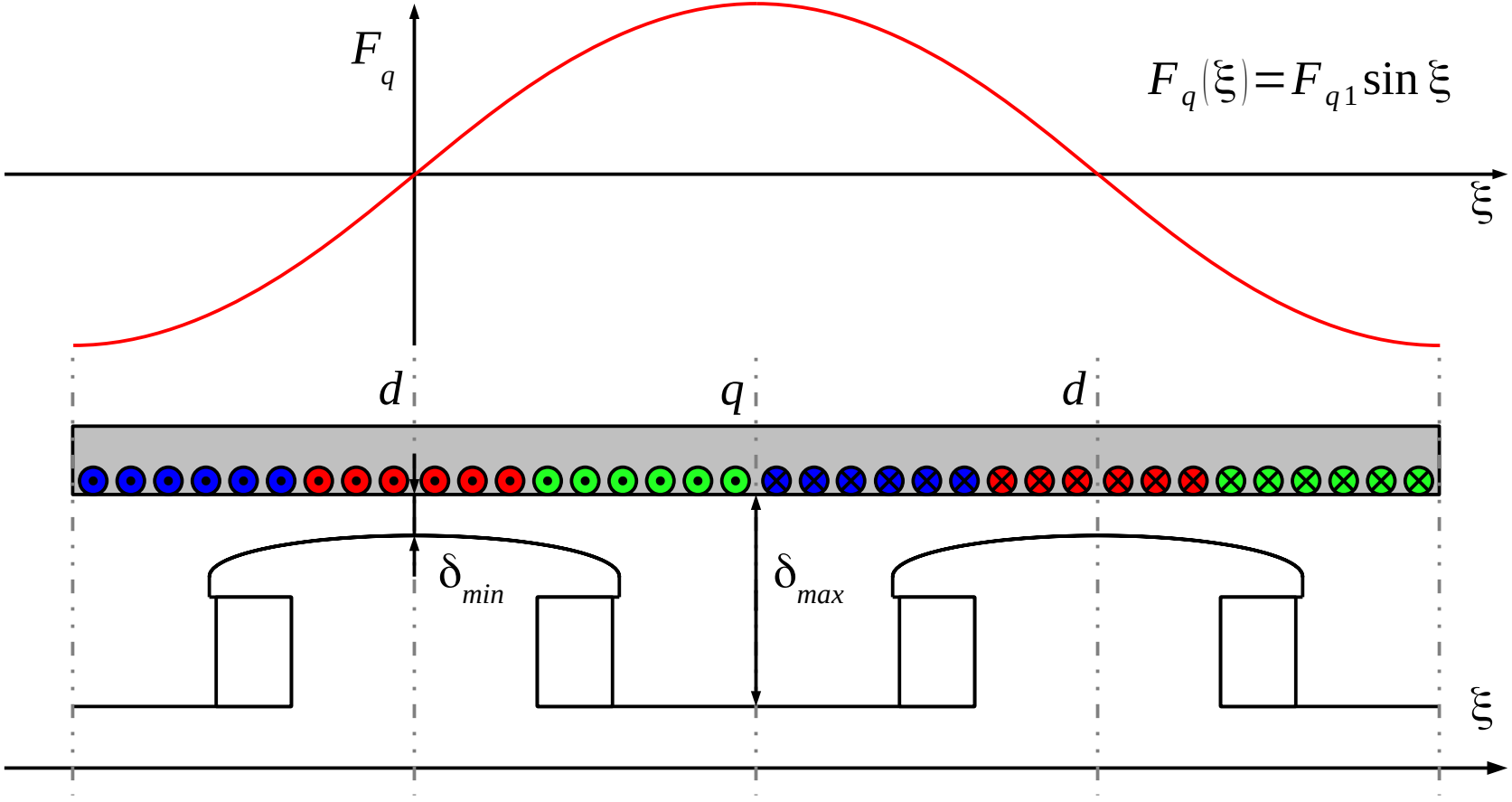
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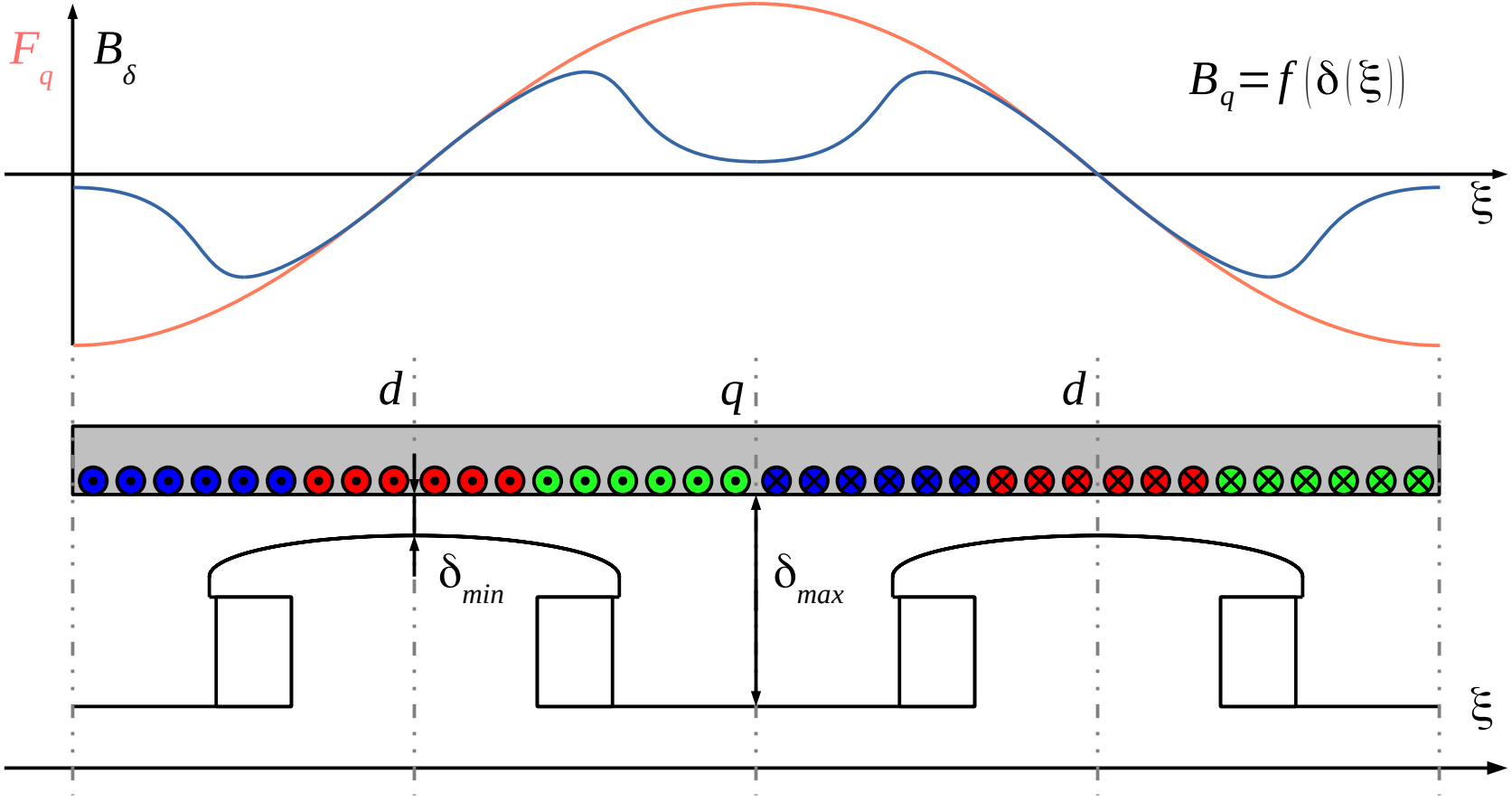
Armature Field of Salient Pole Machines in the *q*-Axis



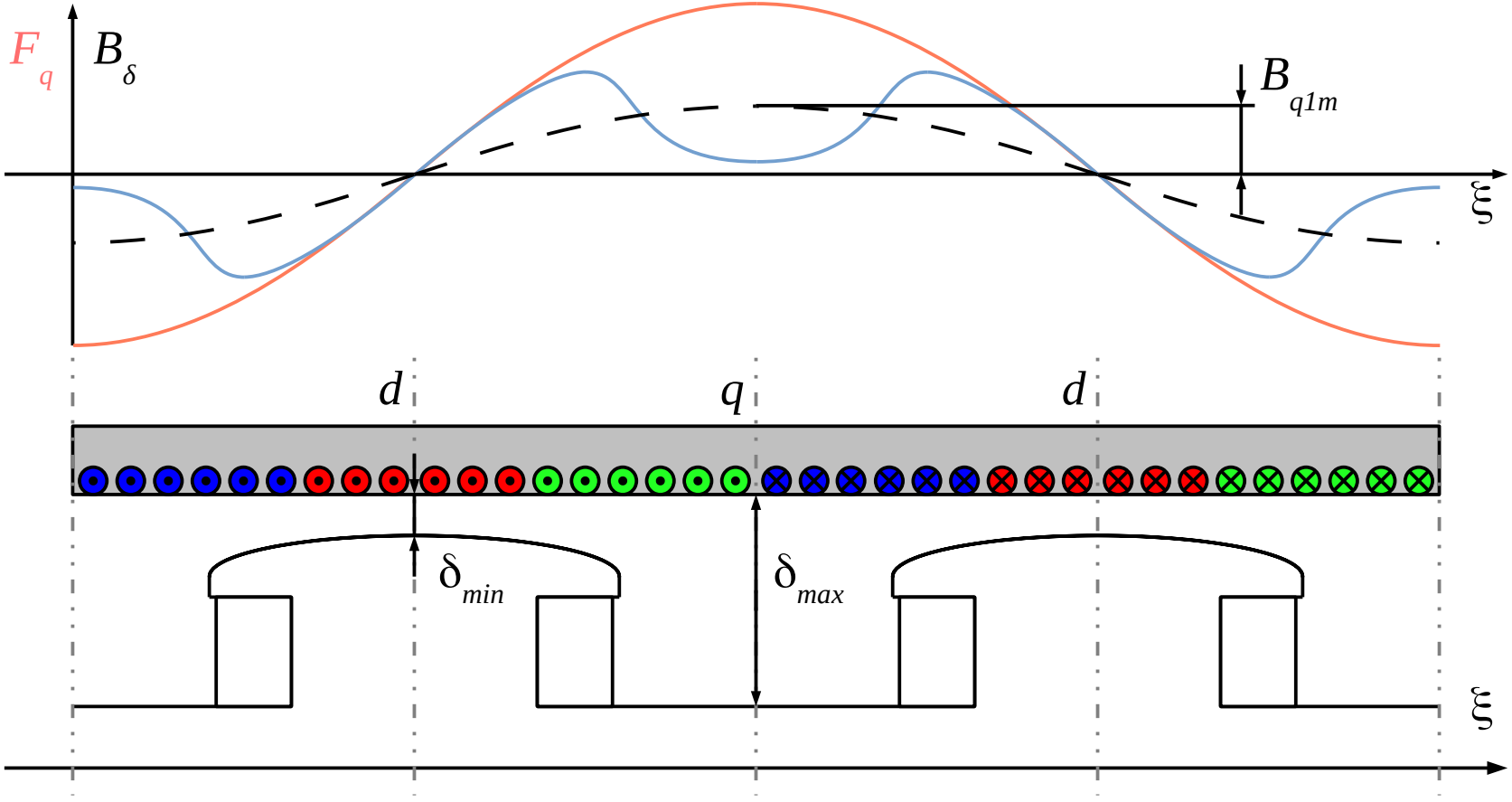
Armature Field of Salient Pole Machines in the *q*-Axis



Armature Field of Salient Pole Machines in the *q*-Axis



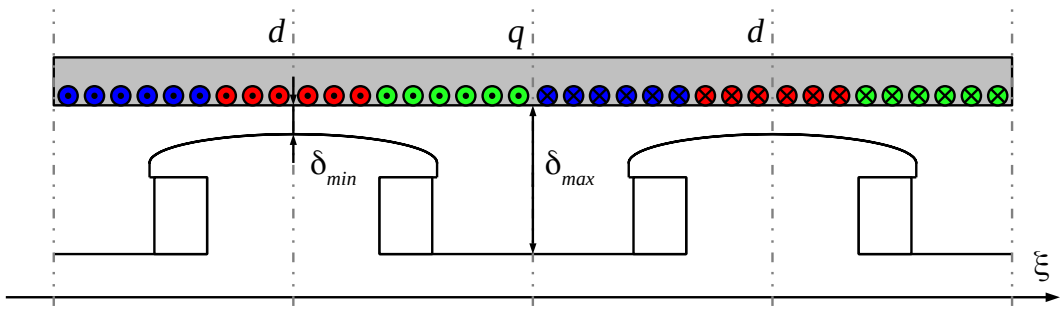
Armature Field of Salient Pole Machines in the *q*-Axis



Armature Field of Salient Pole Machines in the q-Axis

- Using alternative approach like in similar case

$$B_q(\xi) = B_{qm} \frac{\delta_{min}}{\delta(\xi)} \sin \xi$$



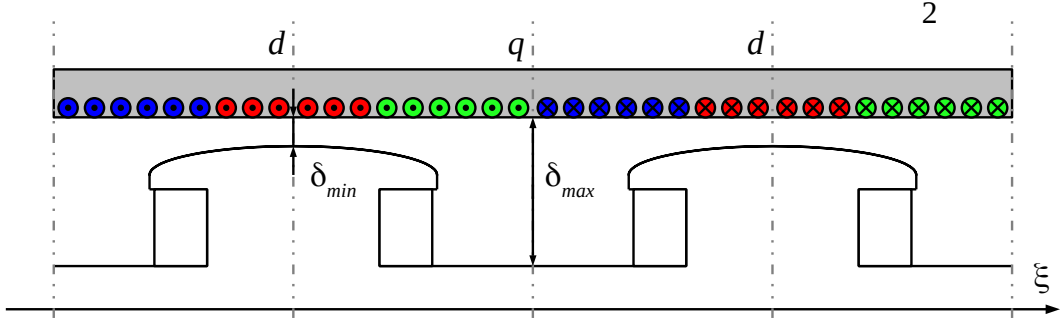
Armature Field of Salient Pole Machines in the q-Axis

- Using alternative approach like in similar case

$$B_q(\xi) = B_{qm} \frac{\delta_{min}}{\delta(\xi)} \sin \xi$$

through Fourier analysis of $B_q(\xi)$ we obtain its fundamental harmonic component

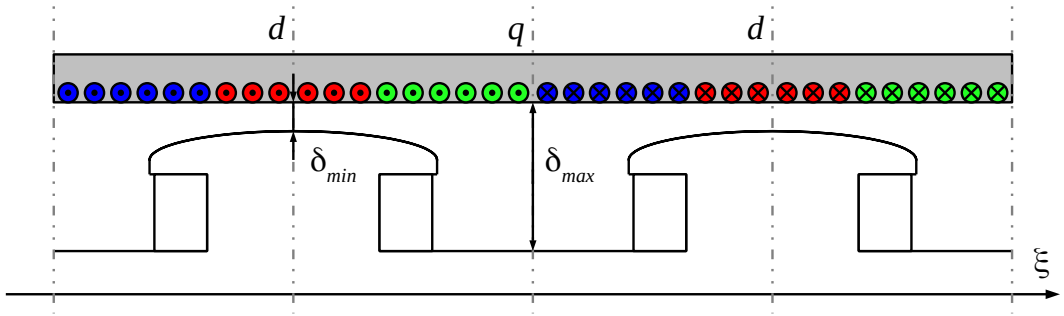
$$B_{q1} = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} B_q(\xi) \sin \xi \, d\xi$$



Armature Field of Salient Pole Machines in the *q*-Axis

- After substitution and after setting limits

$$B_{q1} = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} B_{qm} \frac{\delta_{min}}{\delta(\xi)} \sin^2 \xi \, d\xi$$



Armature Field of Salient Pole Machines in the q-Axis

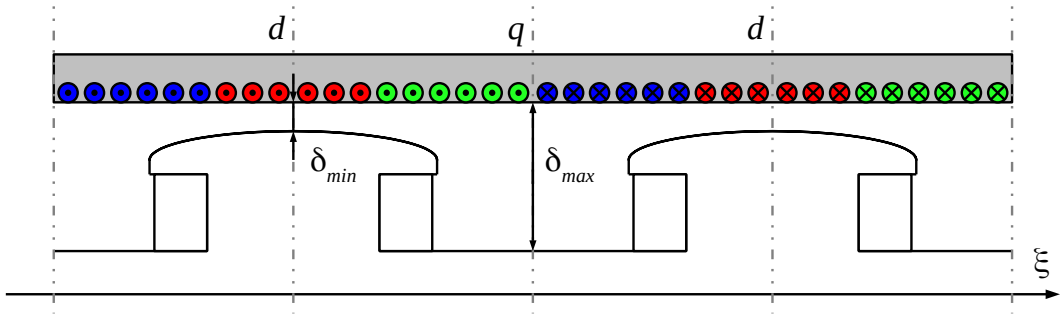
- After substitution and after setting limits

$$B_{q1} = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} B_{qm} \frac{\delta_{min}}{\delta(\xi)} \sin^2 \xi \, d\xi$$

- Similarly to previous cases the q-axis armature field shape factor is introduced:

$$C_{q1} = \frac{B_{q1}}{B_{qm}}$$

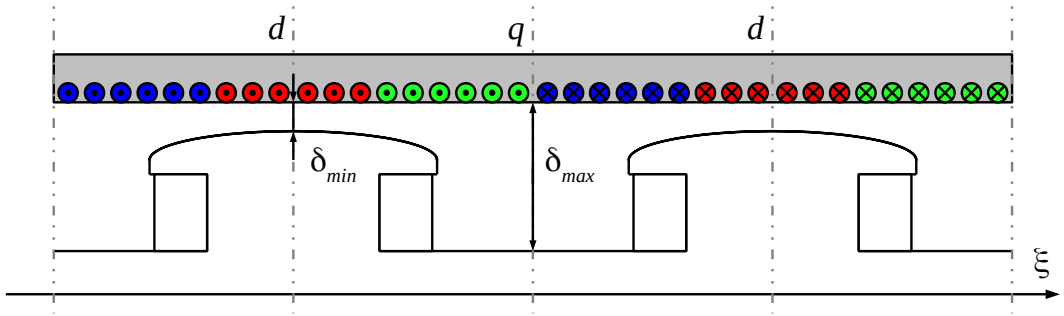
resulting in q-axis magnetic field deformation factor:



$$k_q = \frac{C_{q1}}{C_{b1}}$$

Armature Field of Salient Pole Machines in the q-Axis

- To have results between axes d and q axes comparable, the magnitude B_{qm} is derived from the magnitude of air gap magnetic flux density of a machine with cylindrical rotor
- As a result the values of B_{qm} and B_{dm} merge and the difference is only in shape of the air gap
- Both coefficients C_{d1} and C_{q1} are therefore comparable to each other



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The Influence of C_{d1} and C_{q1} Coefficients on Inductances of Synchronous Machines

The Influence of C_{d1} and C_{q1} Coefficients on Inductances

- Except comparison of magnitudes both magnetic field shape coefficients in axes d and q also respect the weakening of the magnetic field by its deformation and therefore weakening of the mutual boundary between stator and rotor
- As an after effect of this phenomena, the coefficients C_{d1} and C_{q1} project to values of inductances in axes d and q
- The salient pole synchronous machine appear on the outside as machines with inductances L_d for the d-axis or L_q for the q-axis respectively
- Both inductances then arise from the so-called magnetizing inductance

The Influence of C_{d1} and C_{q1} Coefficients on Inductances

- In simplified form the magnetizing inductance may be derived from magnetizing reactance

$$X_{\mu} = \frac{U_i}{I}$$

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- Thus induced voltage $U_i = 2\sqrt{2} N_s k_{v1} B_{\delta} t_p l_e f$

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- The air gap magnetic flux density may be derived from magnetic field strength

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$$F_\delta = \int_{\delta''} H_\delta \, dy = \frac{m}{\pi} \cdot \frac{N_s k_{v1} \sqrt{2} I}{p}$$

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where δ" is the air gap width considering saturation effects by saturation factor k_{sat} and slotting through Carter's factor k_C: $\delta'' = k_{sat} k_C \delta$

The Influence of C_{d1} and C_{q1} Coefficients on Inductances

- After substitution to induced voltage:

$$U_i = 4\,m\,\frac{\mu_0}{\pi}\cdot\frac{\left(N_s k_{v1}\right)^2}{p}\cdot\frac{t_p l_e}{\delta''} f\,I$$

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and magnetizing reactance results

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- Thus magnetizing inductance

$$L_\mu = \frac{X_\mu}{\omega_1} = 2\,m\,\frac{\mu_0}{\pi^2}\cdot\frac{\left(N_s k_{v1}\right)^2}{p}\cdot\frac{t_p l_e}{\delta''}$$

The Influence of C_{d1} and C_{q1} Coefficients on Inductances

- But due to the deformation of the magnetic field this inductance appears to be (in the *d*-axis)

$$L_{ad} = C_{d1} L_{\mu}$$

  and in *q*-axis

$$L_{aq} = C_{q1} L_{\mu}$$

- After consideration of leakage inductances we obtain

$$L_d = L_{ad} + L_{a\sigma}$$

$$L_q = L_{aq} + L_{a\sigma}$$

The Influence of C_{d1} and C_{q1} Coefficients on Inductances

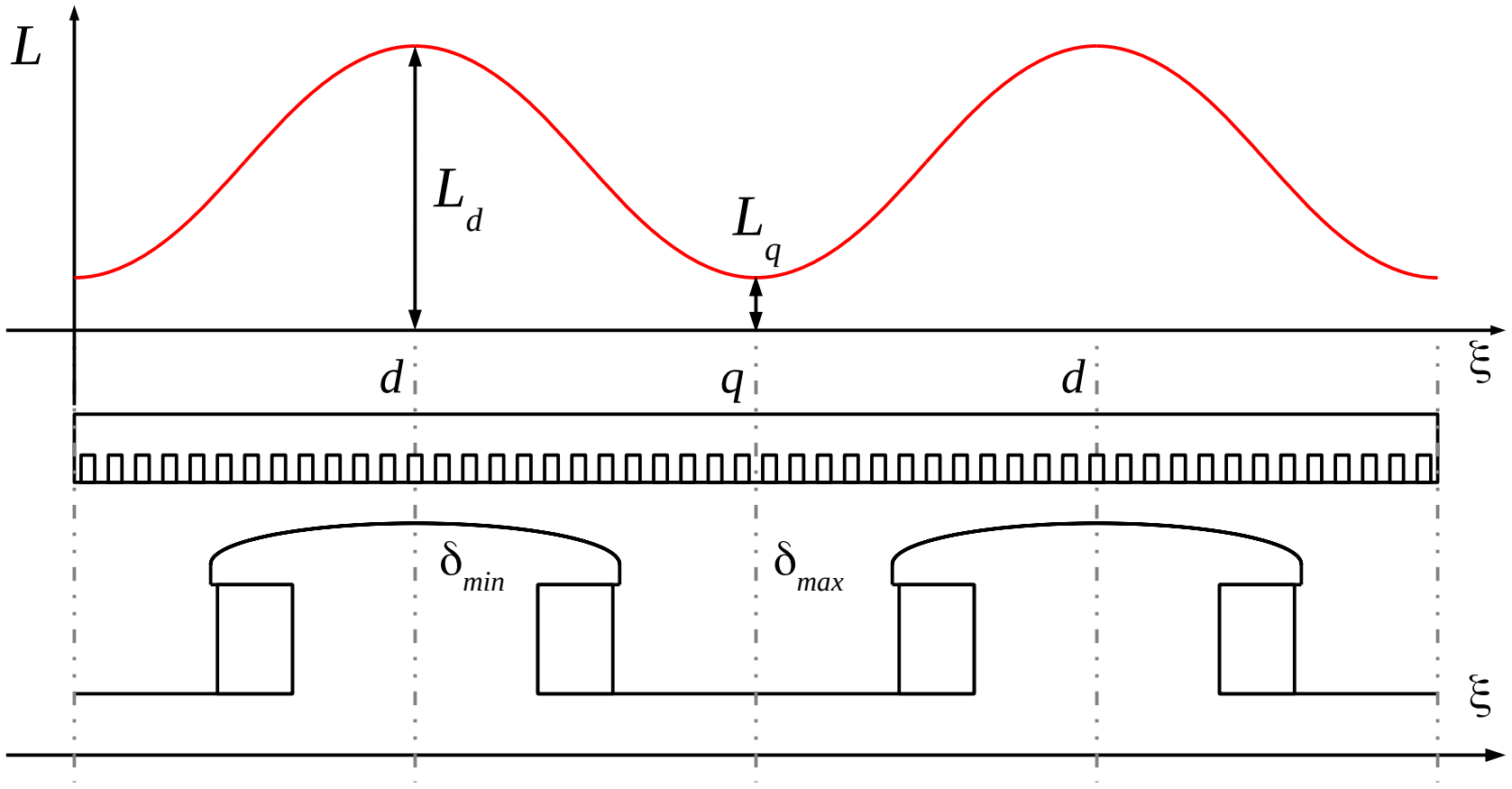
- In case of electrically excited synchronous machines the values of C_{d1} and C_{q1} vary significantly in dependence on geometry of the machine (pole shoes)
- The values of the d-axis armature form factor are commonly

$$C_{d1} \in (0{,}8\,;0{,}92)$$

- while in case of the q-axis armature form factor they are

$$C_{q1} \in (0{,}2\,;0{,}48)$$

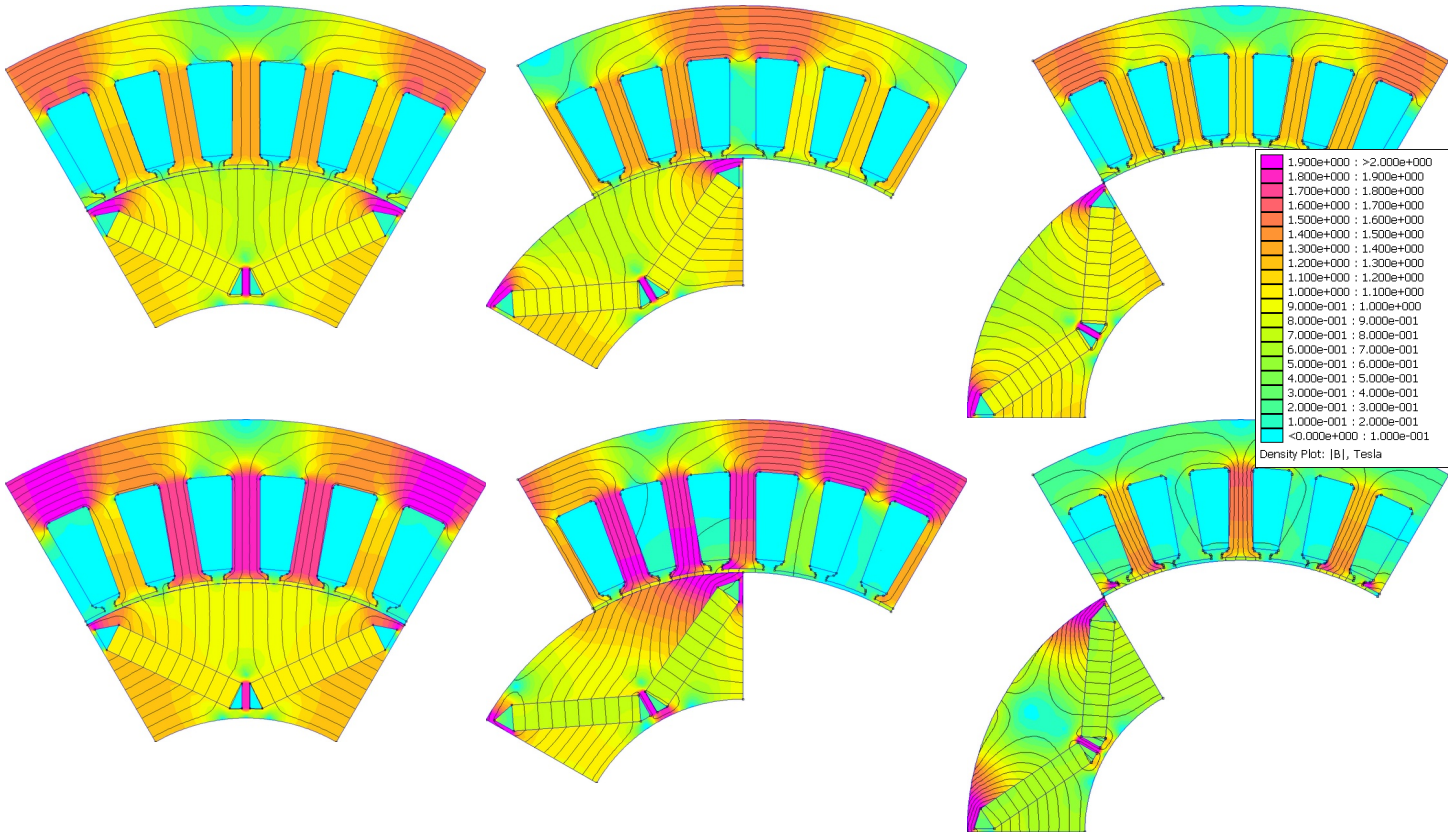
The Influence of C_{d1} and C_{q1} Coefficients on Inductances



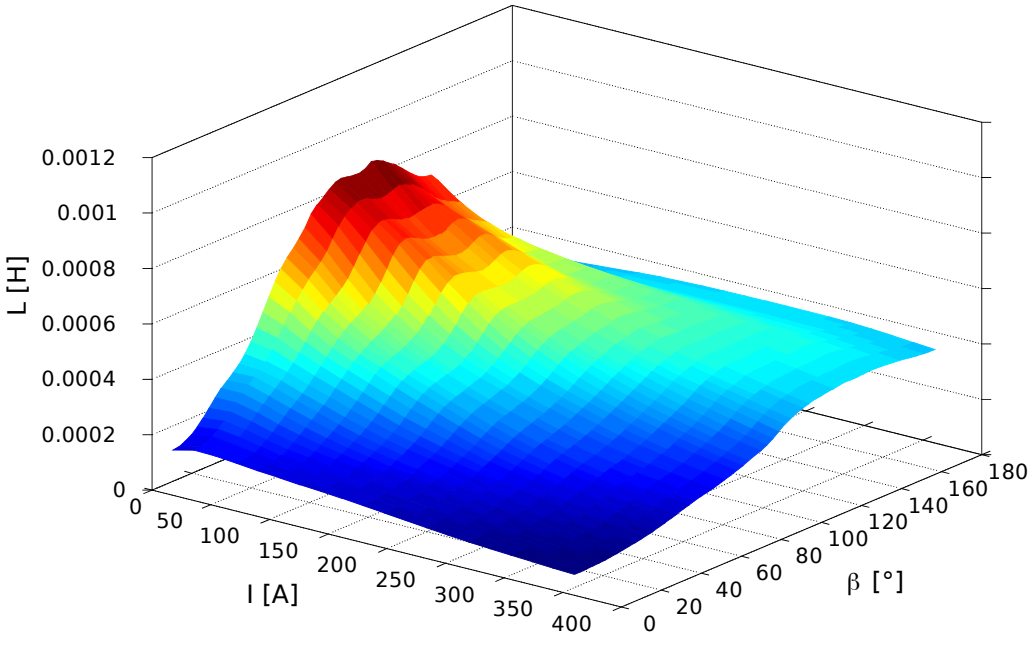
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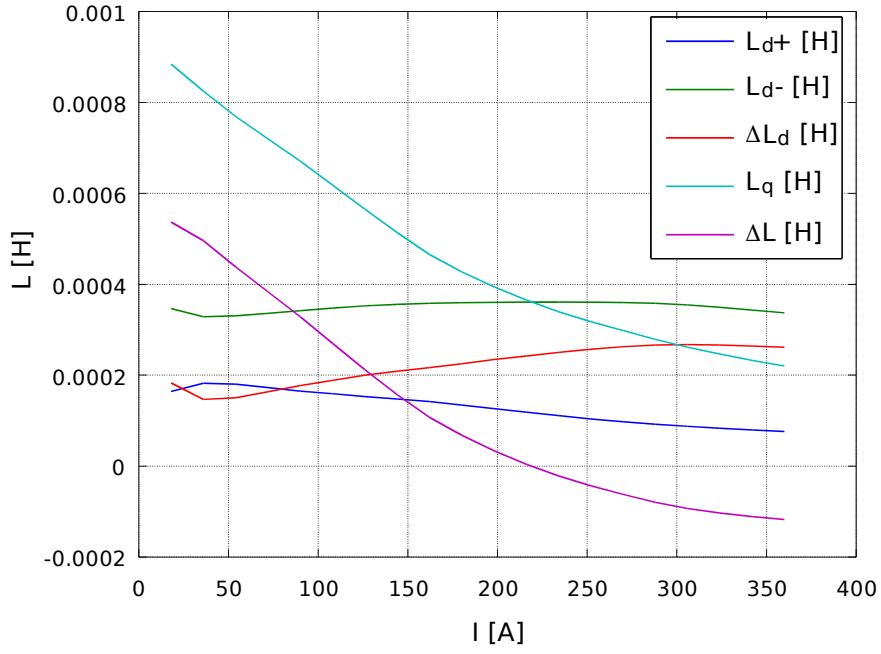
- In case of permanent magnet synchronous machines the difference between inductances of both axes is not usually that significant
- There is an important effect of saturation in higher loads (currents) and changes of load angle
  - The saturation of machine rises and therefore its inductance drops
  - In extreme case there may appear “saliency loss” – the originally different inductances L_d and L_q obtain due to saturation similar values

The Influence of C_{d1} and C_{q1} Coefficients on Inductances



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Thank You for Your Attention!

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