FACULTY OF ELECTRICAL
ENGINEERING
UNIVERSITY OF WEST BOHEMIA

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

ESF II – project of the University of West Bohemia
reg. no. CZ.02.2.69/0.0/0.0/18_056/0013239

Theory of Electric Machines 2

KEV/TS2

Karel Hruska / r20201121

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Characteristics of Permanent Magnet Synchronous Machines

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- Two types of characteristics are commonly used
  - Torque angle characteristics
  - Traction characteristics

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  - Traction characteristics
- Torque angle characteristics describe the output power of the machine in dependency on torque angle
- Traction characteristics describe the output power of the machine as it depends on its speed

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Traction Characteristics

Traction Characteristics

- Generally valid for any type of machine
- They describe torque and power, which it may provide in dependency on ots speed
- Very variable usage:
  - Some manufacturers provide traction characteristics of maximal output power
  - Some manufacturers provide traction characteristics of rated output power

Traction Characteristics

- Important point of traction characteristics is its rated point

Traction Characteristics

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  - It is possible to determine rated output power/torque – P_n, M_n
  - It is possible to determine rated maximal power/torque – P_{max}, M_{max}
  - It is a border, from which the flux-weakening must be applied

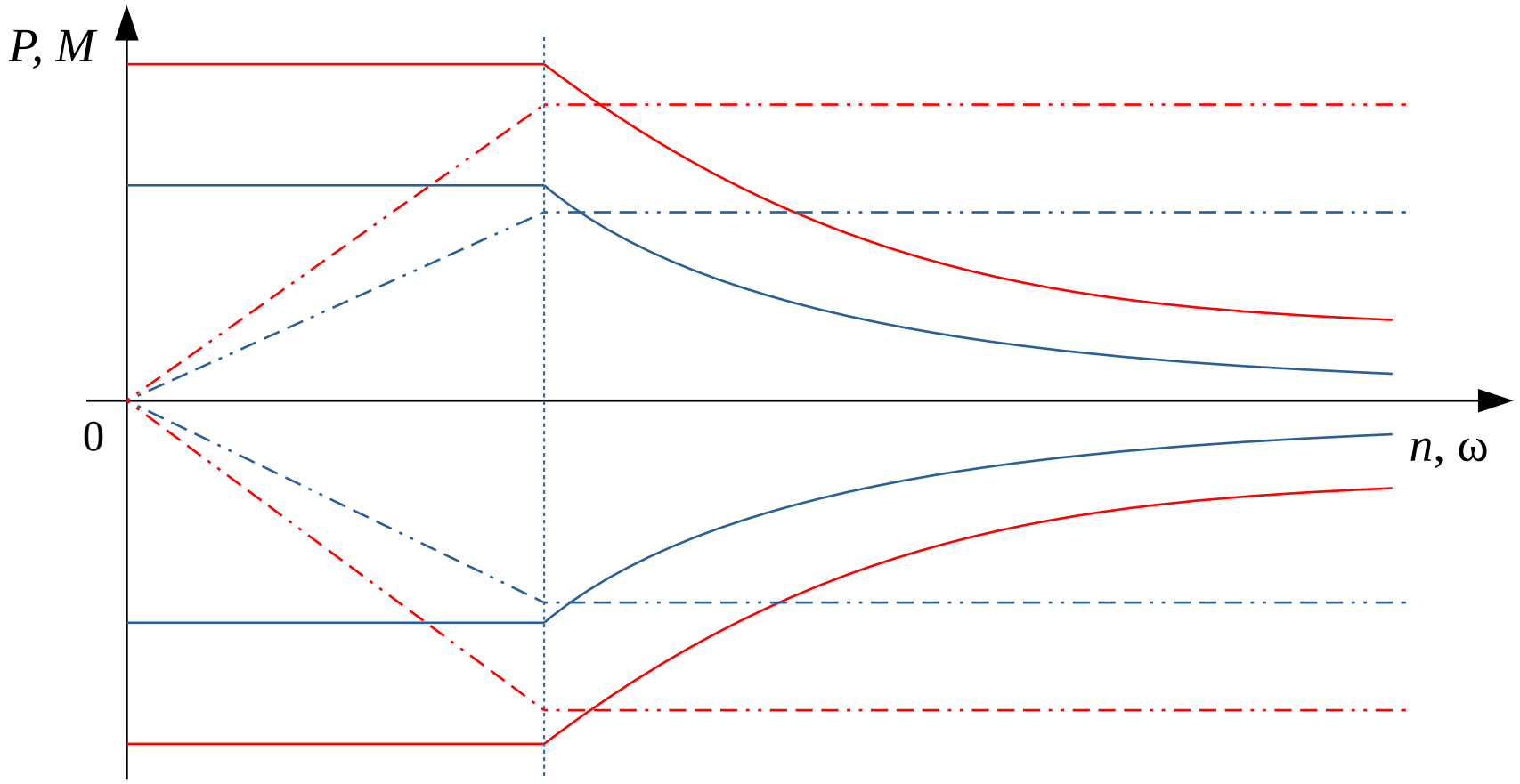
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- The power changes as per $P = M\,\omega$

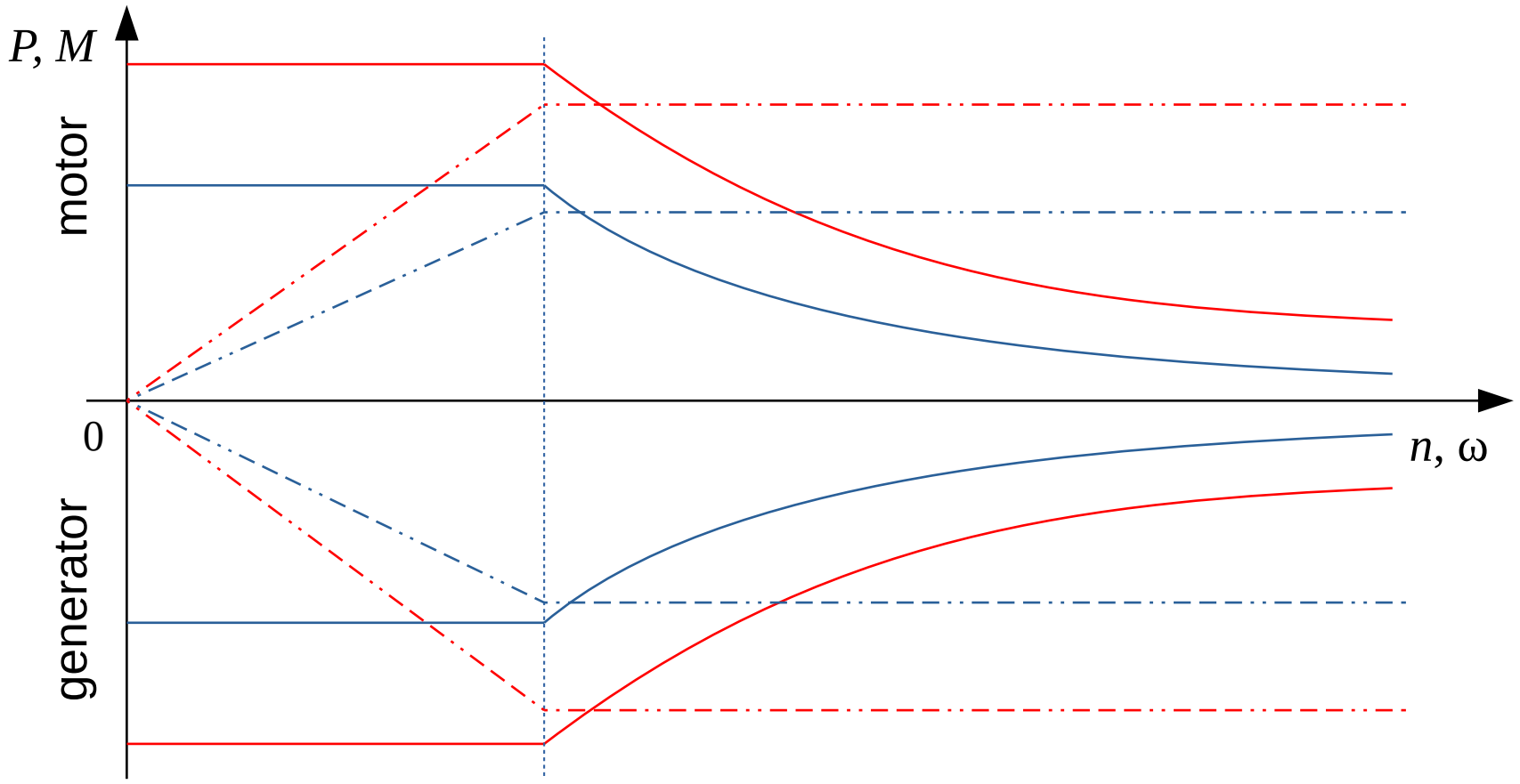
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- Below the rated point it is possible to keep rated/maximal torque (limitation by current applies)
- The power changes as per $P = M\,\omega$
- Over the rated point it is possible to keep the rated/maximal power (limitation by voltage applies)
- The torque changes as per $M = \frac{P}{\omega}$

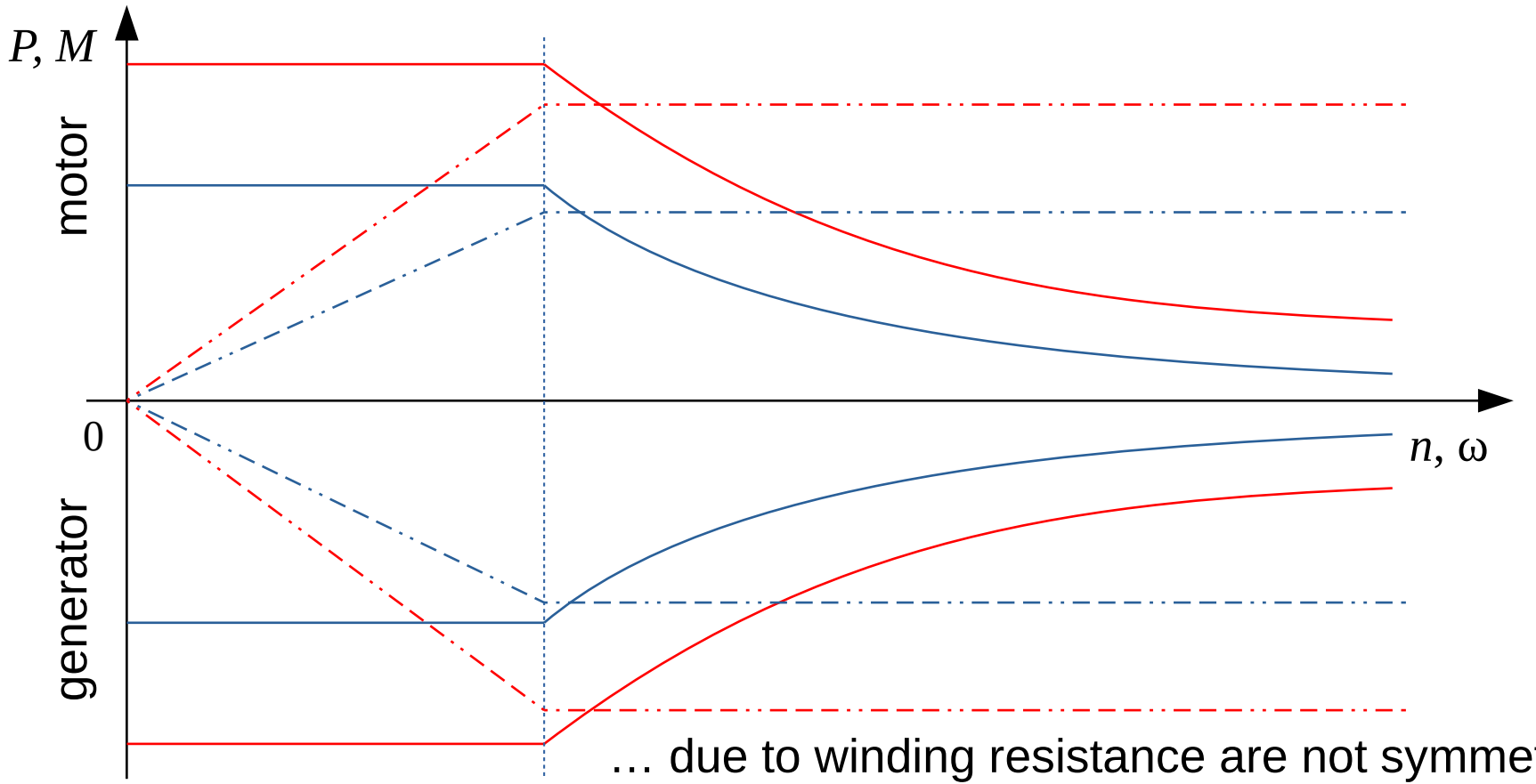
Traction Characteristics



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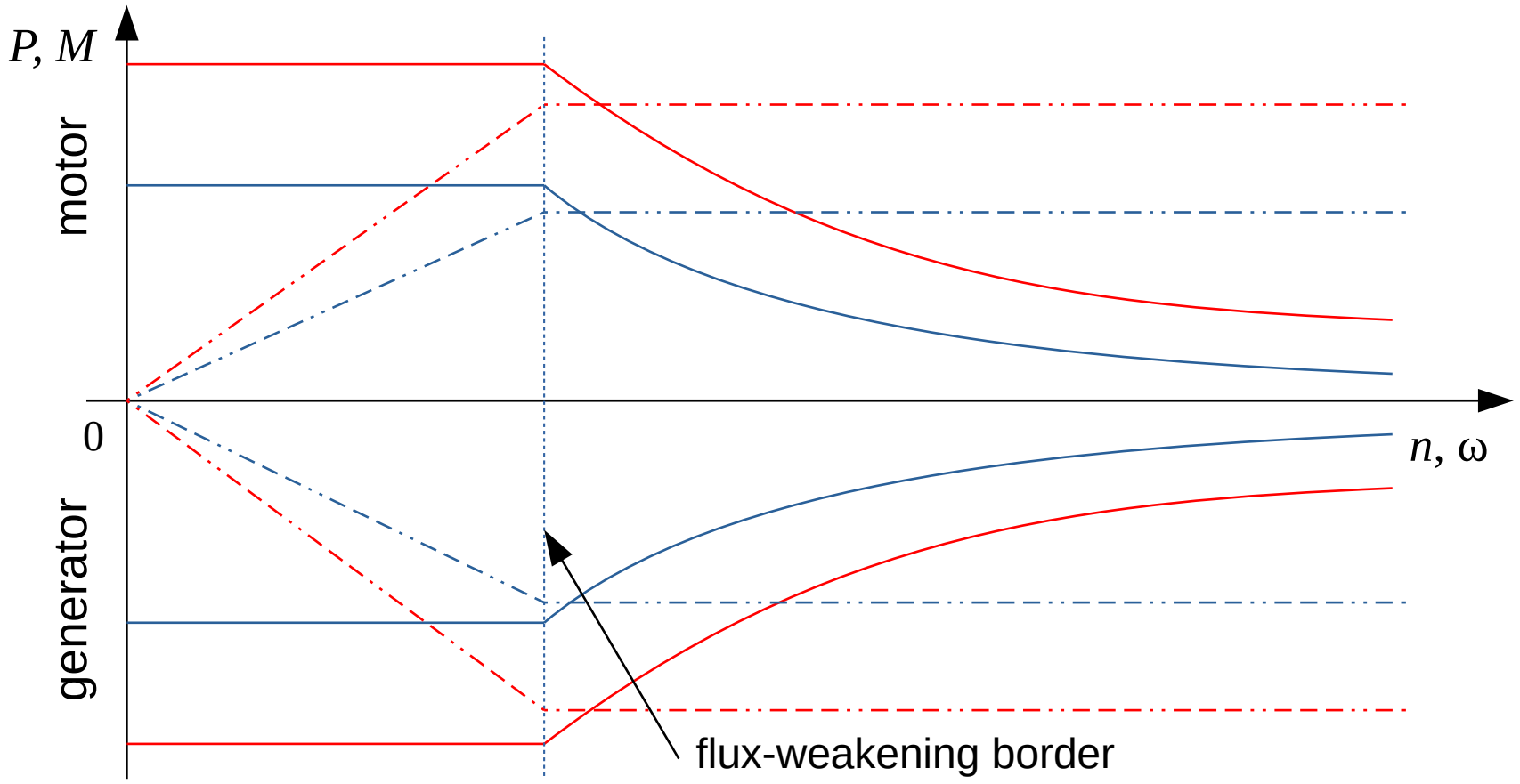


Traction Characteristics

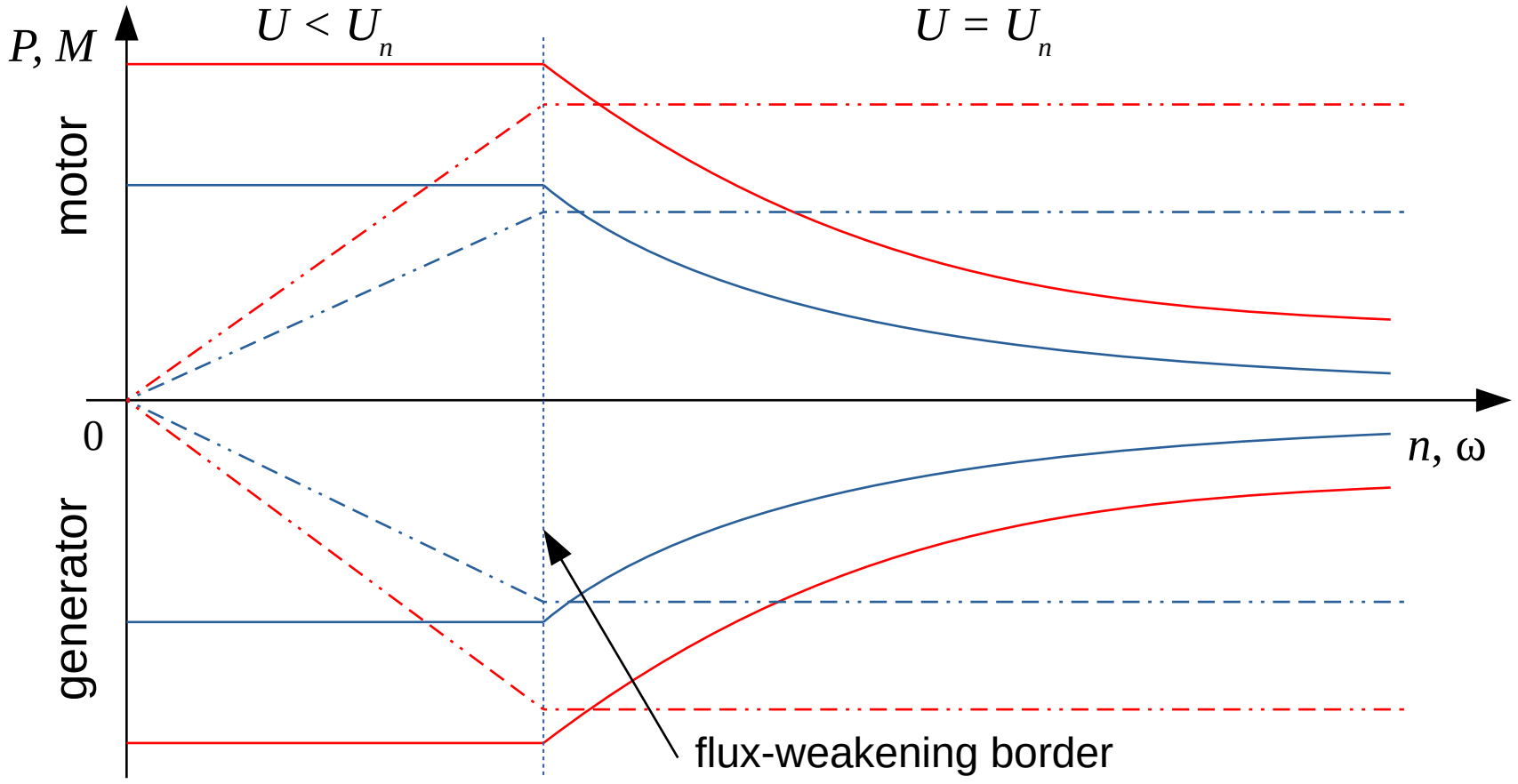


… due to winding resistance are not symmetric

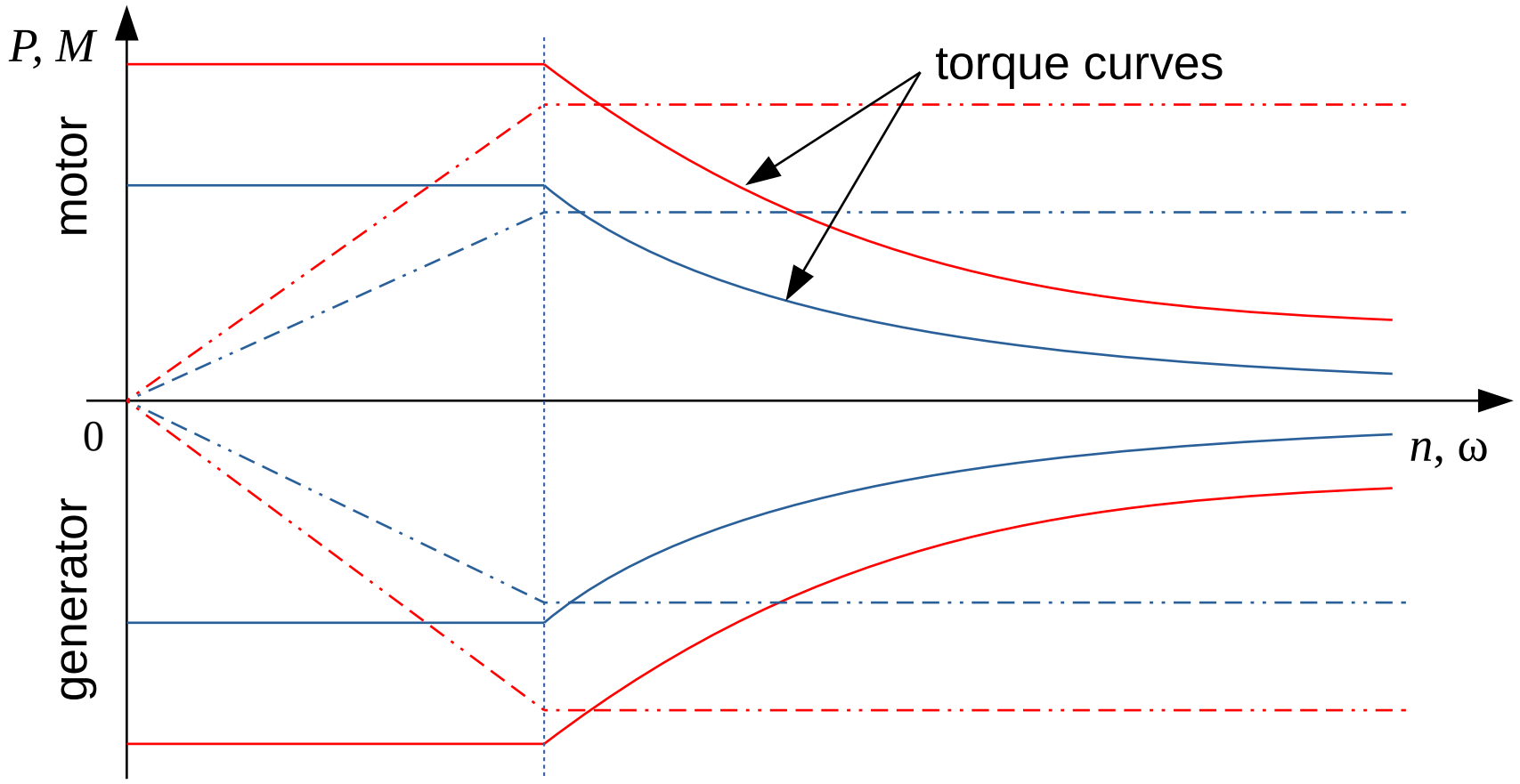
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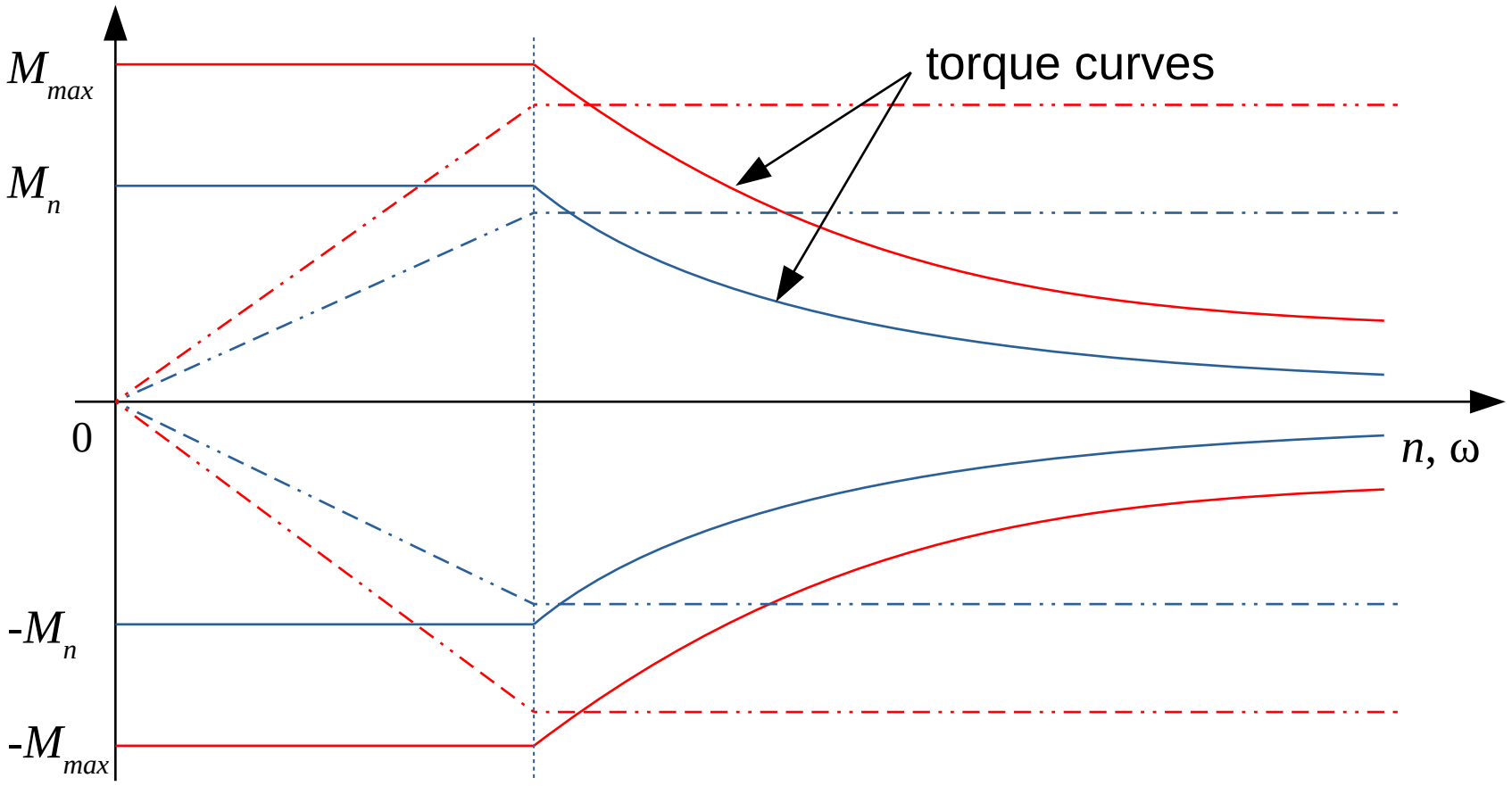
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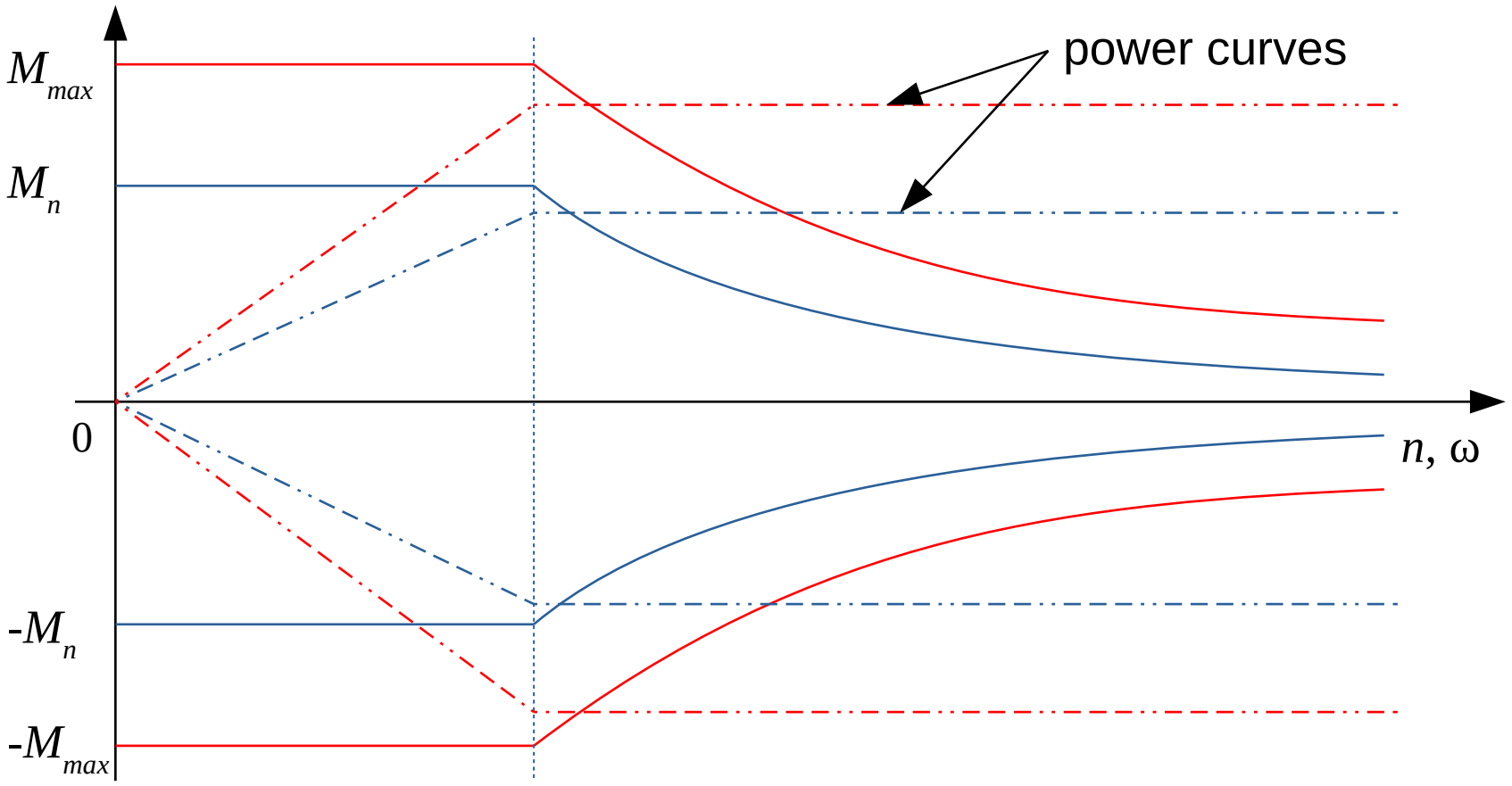
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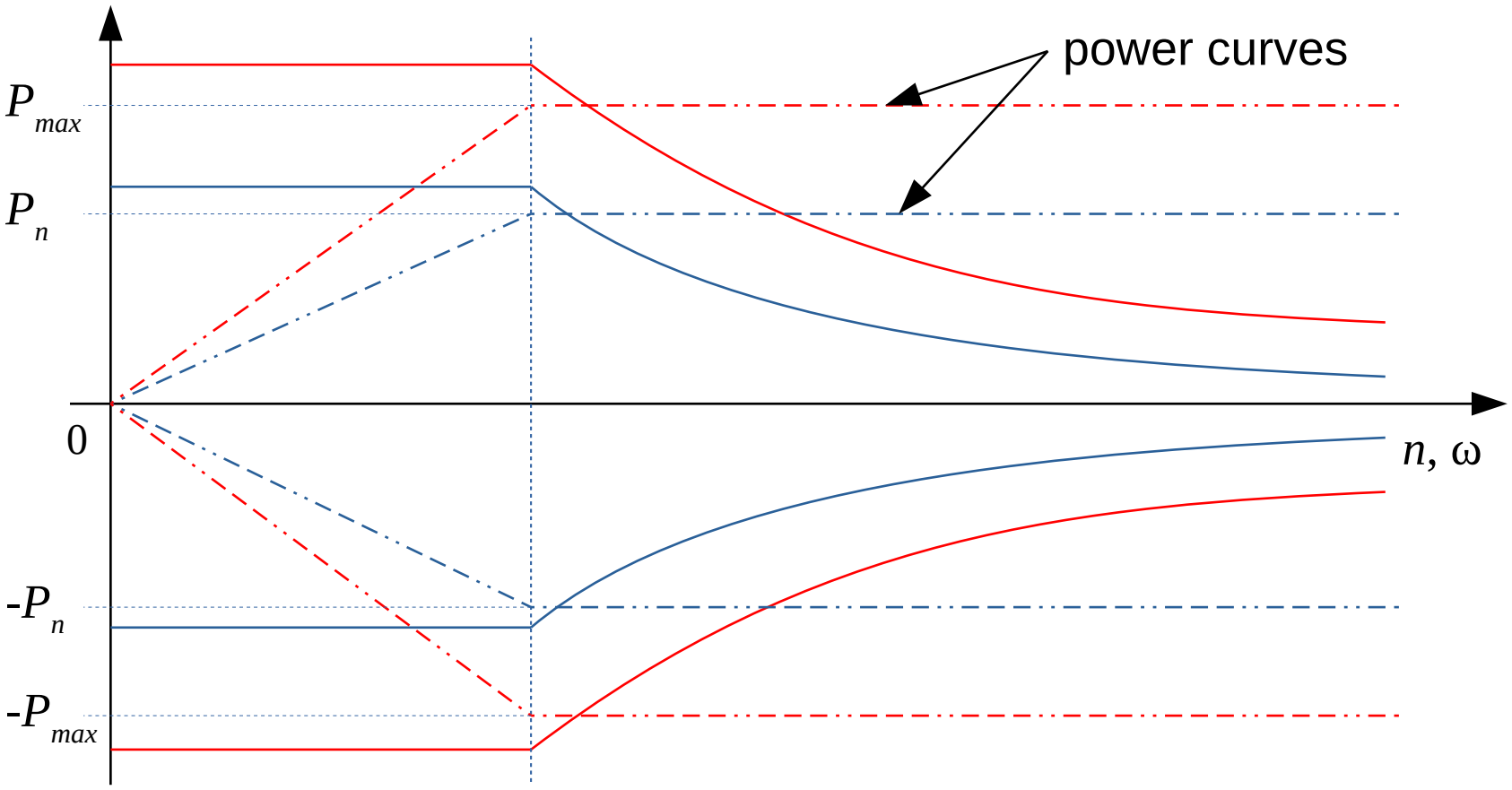
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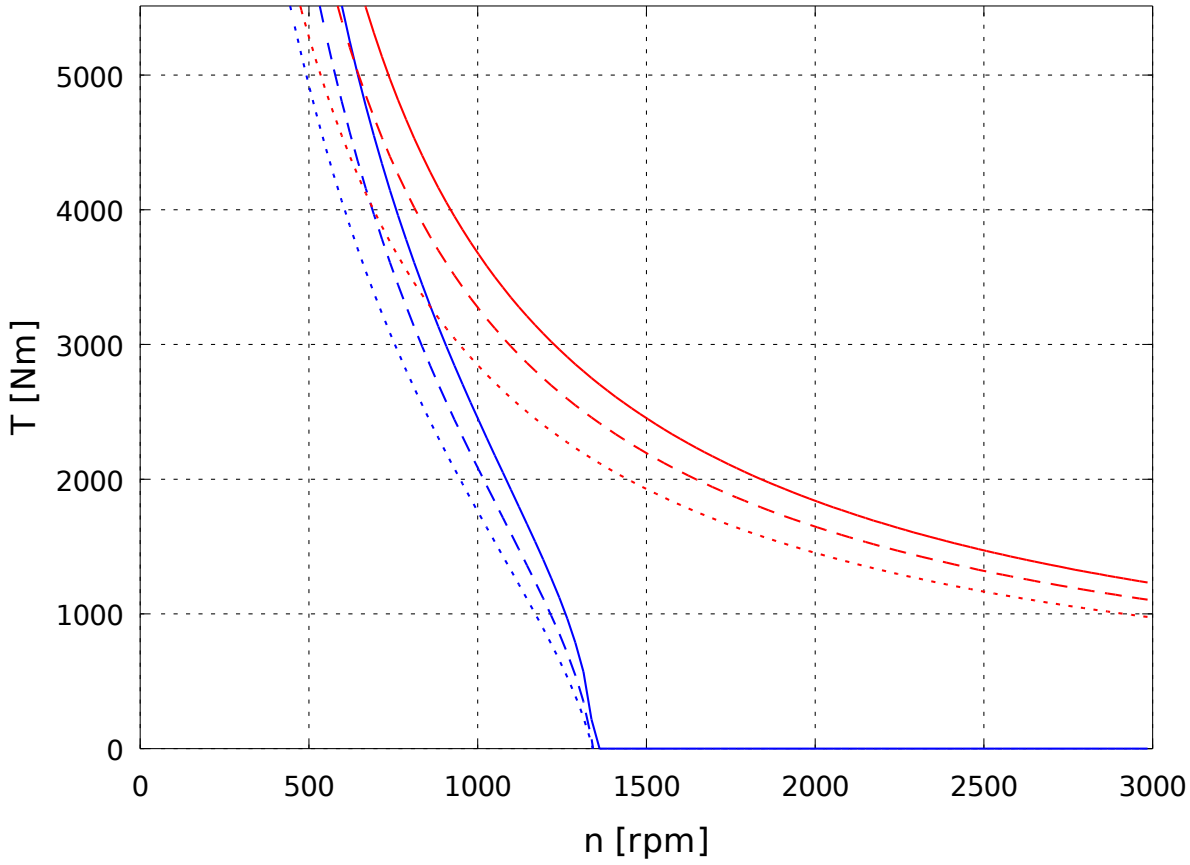
Traction Characteristics



Traction Characteristics

- Notes:
  - From traction characteristic the type of machine cannot be determined
  - Torque angle characteristics must have a certain reserve in comparison to traction characteristics
  - The flux-weakening border is not absolutely straight (it is a curve given by equivalent circuit parameters)

Traction Characteristics



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Torque Angle Dependencies

Torque Angle Dependencies

- Commonly used by electric machine engineers
- Dependencied of torque and power on torque angle
- Neglecting armature resistance the power of the machine is

$$P = m\,U\left[\frac{U_{ib}}{X_d}\sin\beta + \frac{U}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\beta\right]$$

and its torque is

$$M = \frac{p\,m}{\omega_1}\,U\left[\frac{U_{ib}}{X_d}\sin\beta + \frac{U}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\beta\right]$$

Torque Angle Dependencies

- From both dependencies it is possible to read two components:

$$M = \frac{p\,m}{\omega_1} U \left[\frac{U_{ib}}{X_d} \sin\beta + \frac{U}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\beta \right]$$

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Electromagnetic torque given by interaction of fields of stator and rotor.

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Electromagnetic torque given by interaction of fields of stator and rotor.

Reluctance torque given by different inductances in axes d and q.

- It is obvious, that the characteristic is influenced by multiple parameters
  - Existence of excitation
  - Existence and relation between inductances in axes d and q

Torque Angle Dependencies

- In case that the machine is not excited:

$$M = \frac{p\,m}{\omega_1} U \left[\frac{U_{ib}}{X_d} \sin\beta + \frac{U}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\beta \right]$$

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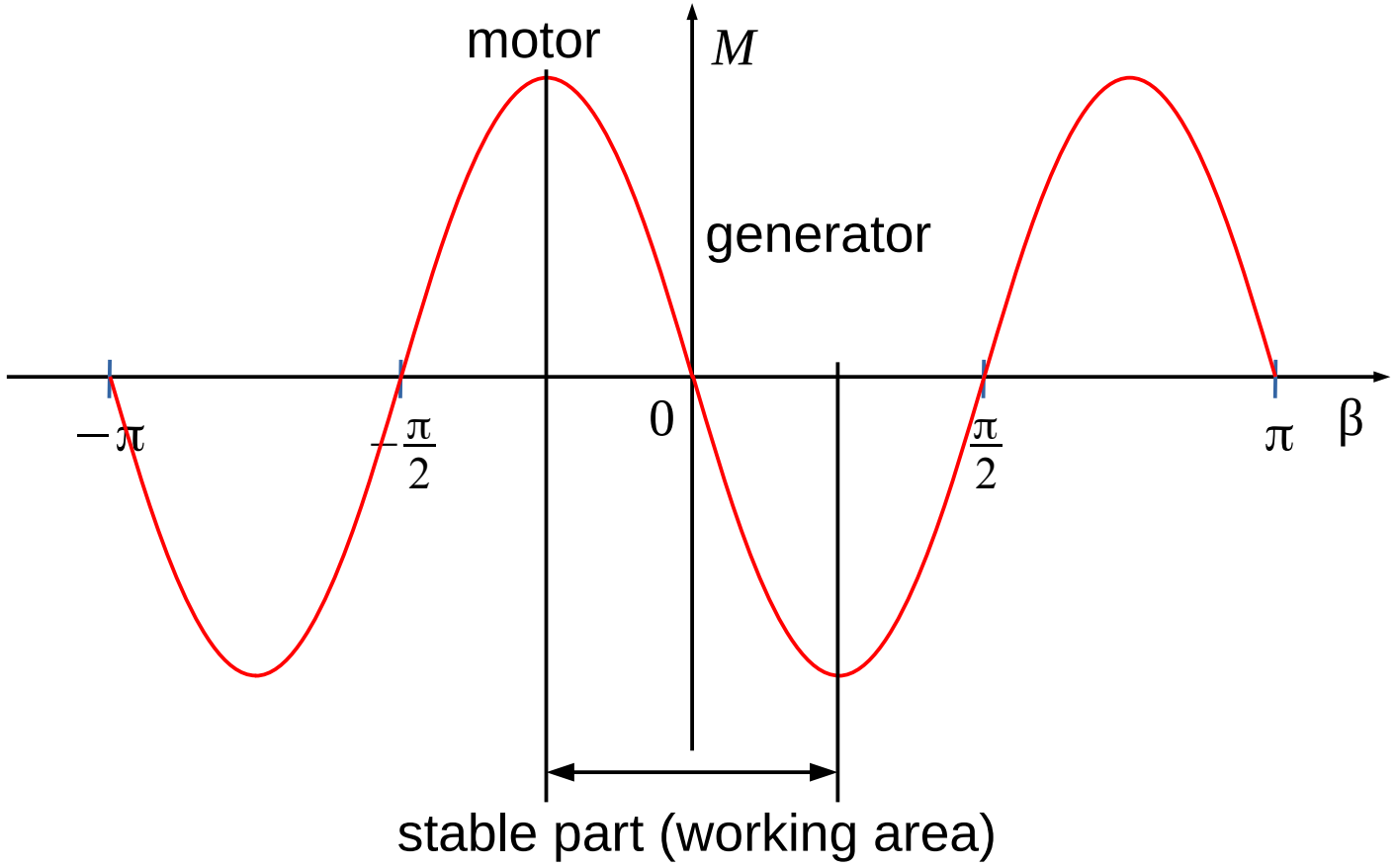
$U_{ib} = 0$

=>

$$M = \frac{p\,m}{\omega_1} U \left[\frac{U}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\beta \right]$$

… the torque of the machine will be larger than zero only if there is a difference if inductances L_d and L_q => reluctance machines

Torque Angle Dependencies



Torque Angle Dependencies

- In case that the inductances in both axes are equal:

$$M = \frac{p\,m}{\omega_1} U \left[\frac{U_{ib}}{X_d} \sin\beta + \frac{U}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\beta \right]$$

Torque Angle Dependencies

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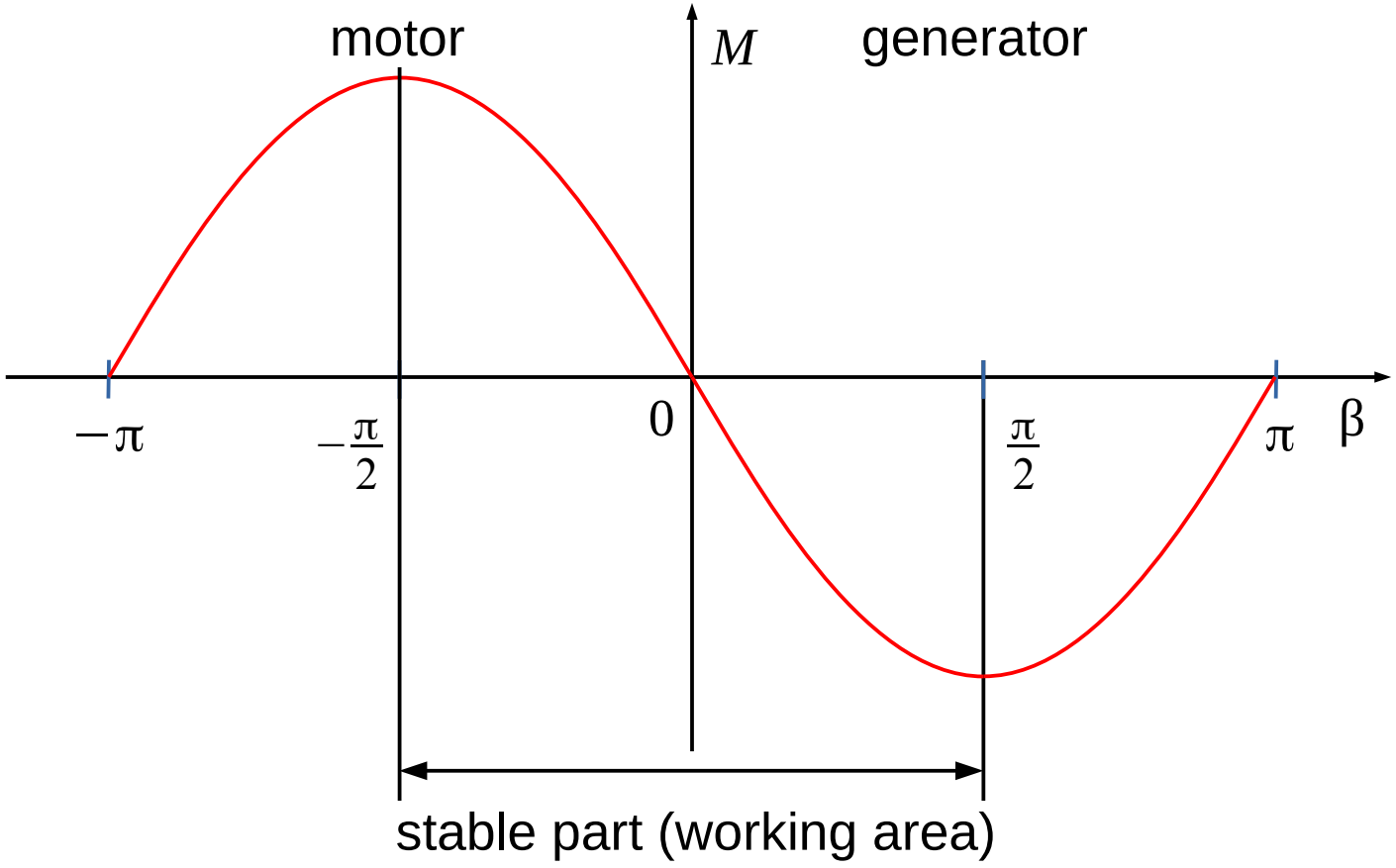
$$M = \frac{p\,m}{\omega_1} U \left[\frac{U_{ib}}{X_d} \sin\beta + \frac{U}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\beta \right]$$

$L_d = L_q \Rightarrow X_d = X_q$

$$\Rightarrow \qquad M = \frac{p\,m}{\omega_1} U \left[\frac{U_{ib}}{X_d} \sin\beta \right]$$

… the torque characteristics is a simple sine wave => cylindrical rotor synchronous machine

Torque Angle Dependencies



Torque Angle Dependencies

- In case that the inductances are different:

$$M = \frac{p\,m}{\omega_1} U \left[\frac{U_{ib}}{X_d} \sin\beta + \frac{U}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\beta \right]$$

Torque Angle Dependencies

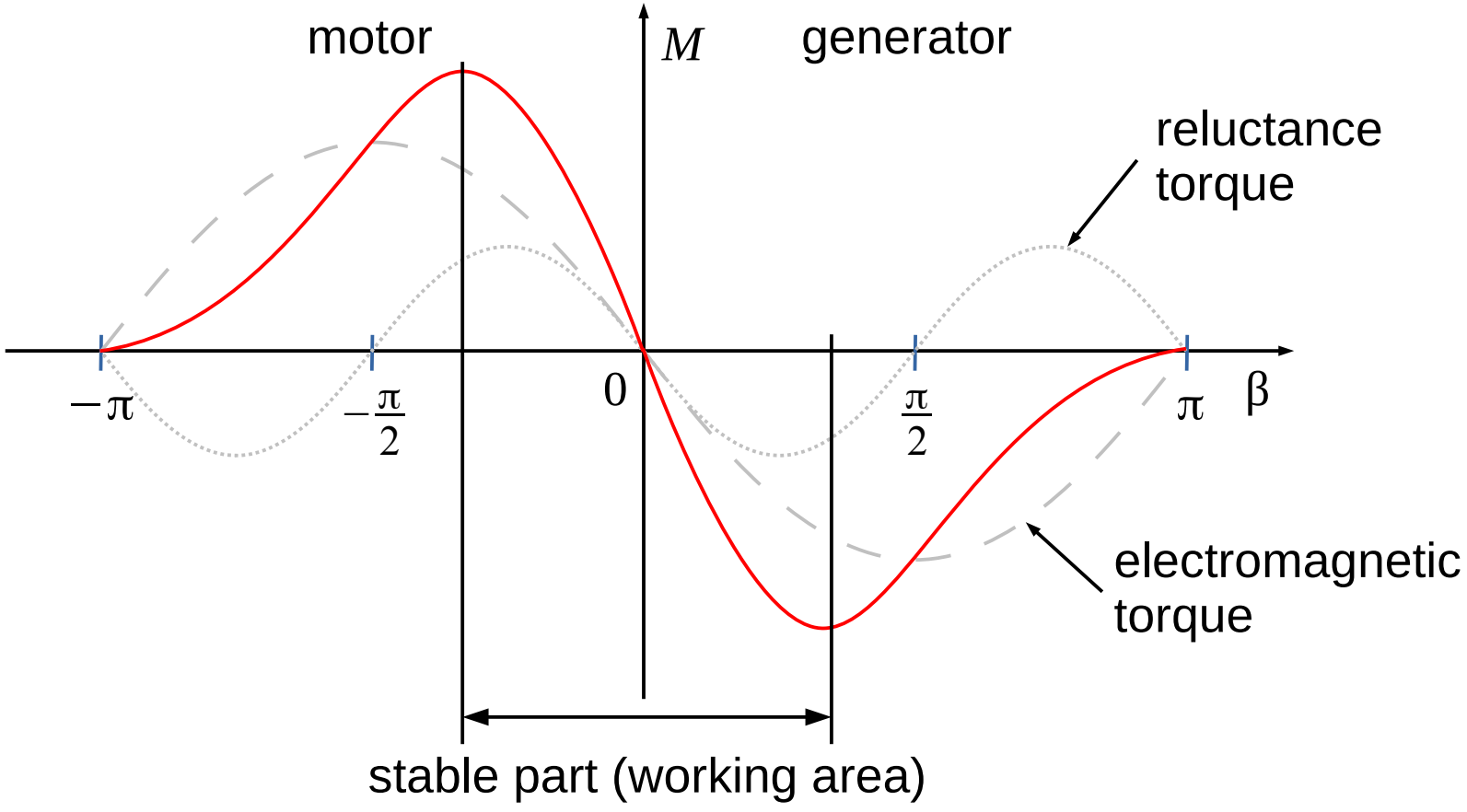
- In case that the inductances are different:

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1) $L_d > L_q \Rightarrow X_d > X_q$

  - The right part in the brackets does not change its mark – both sine waves are in-phase
  - Common in case of salient poles electrically excited synchronous machines, in case of PMSM more or less exception

Torque Angle Dependencies



Torque Angle Dependencies

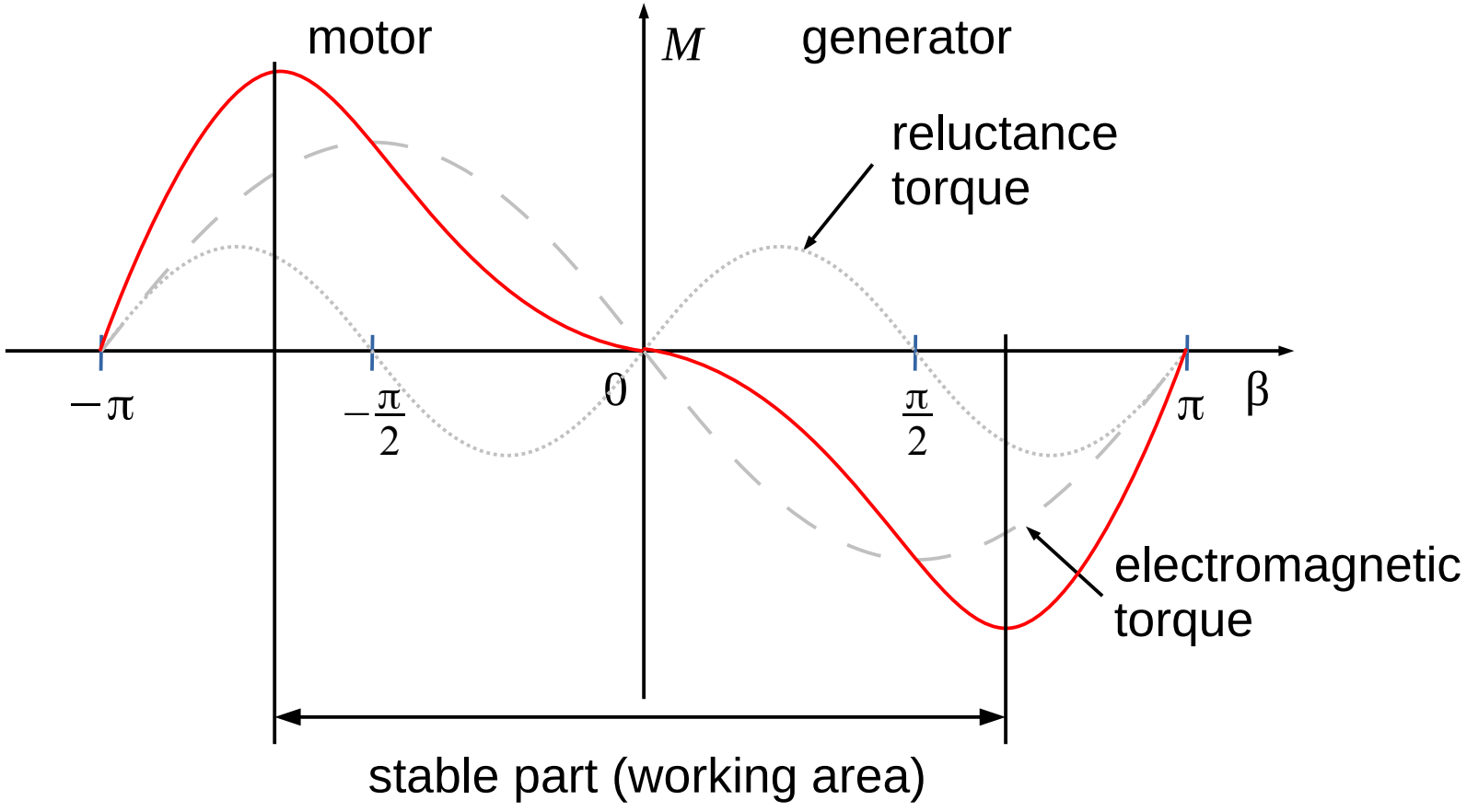
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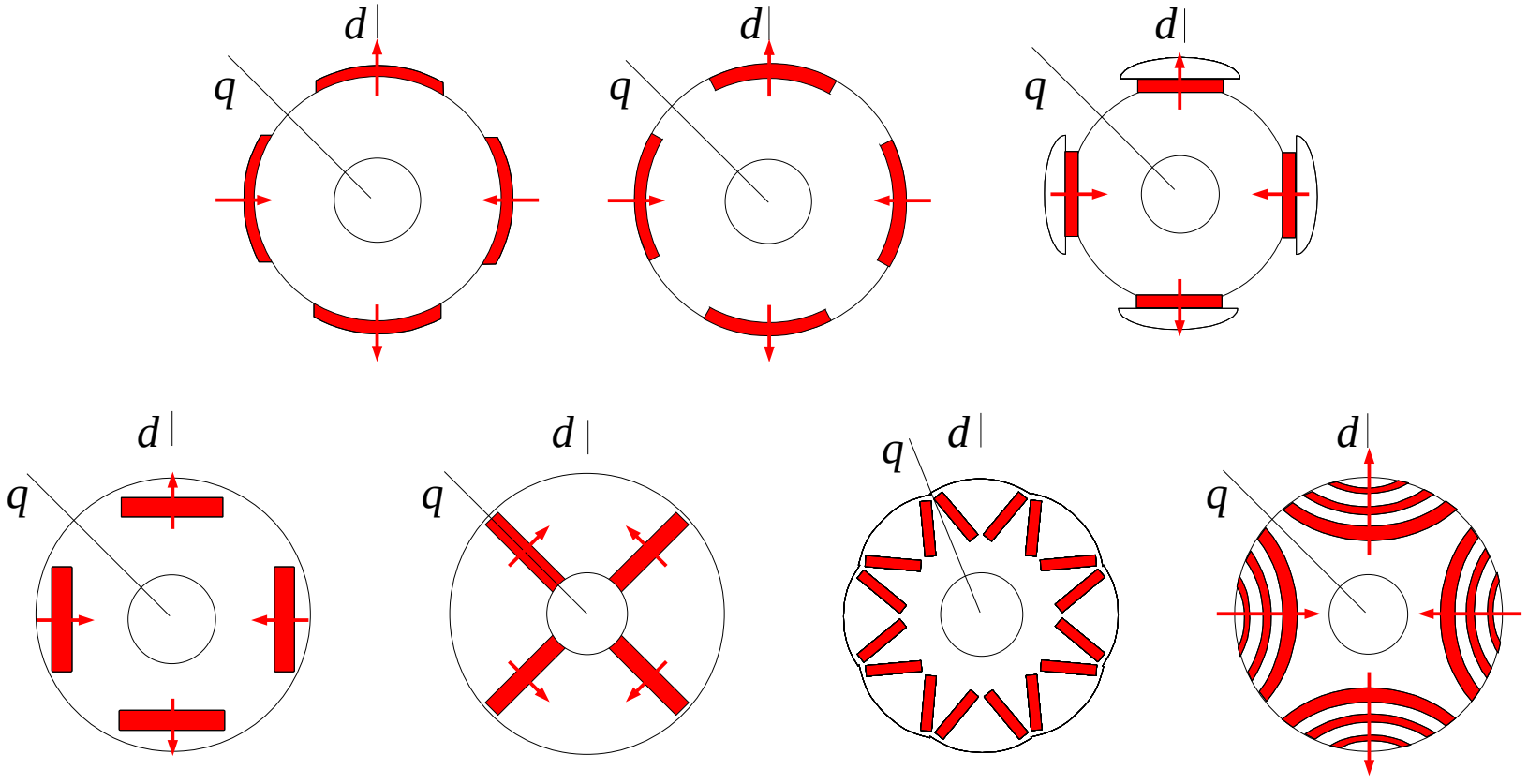
2) $L_d < L_q \Rightarrow X_d < X_q$

  - The right part in the brackets does not change its mark – both sine waves are in-phase
  - Most of interior permanent magnet synchronous machines

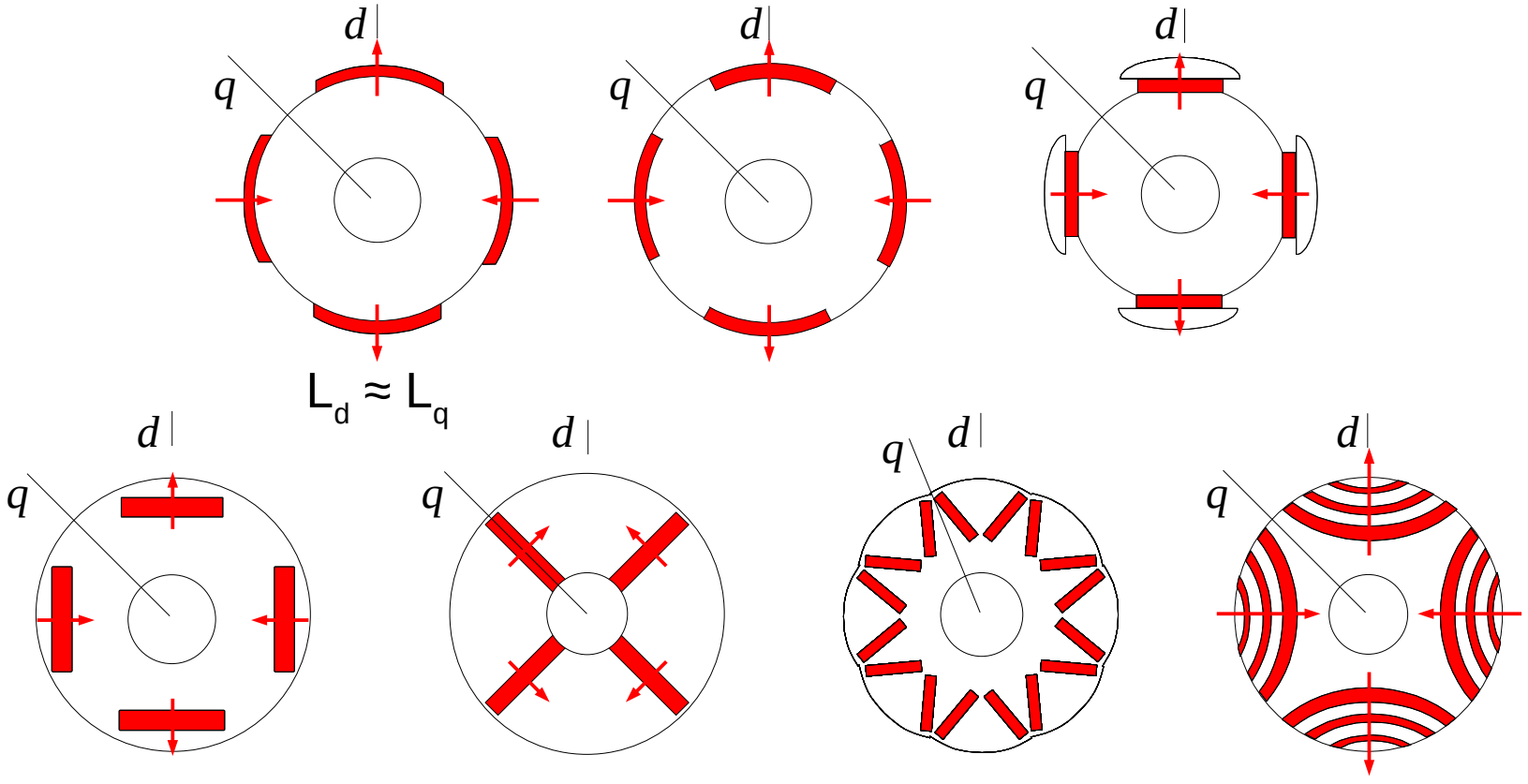
Torque Angle Dependencies



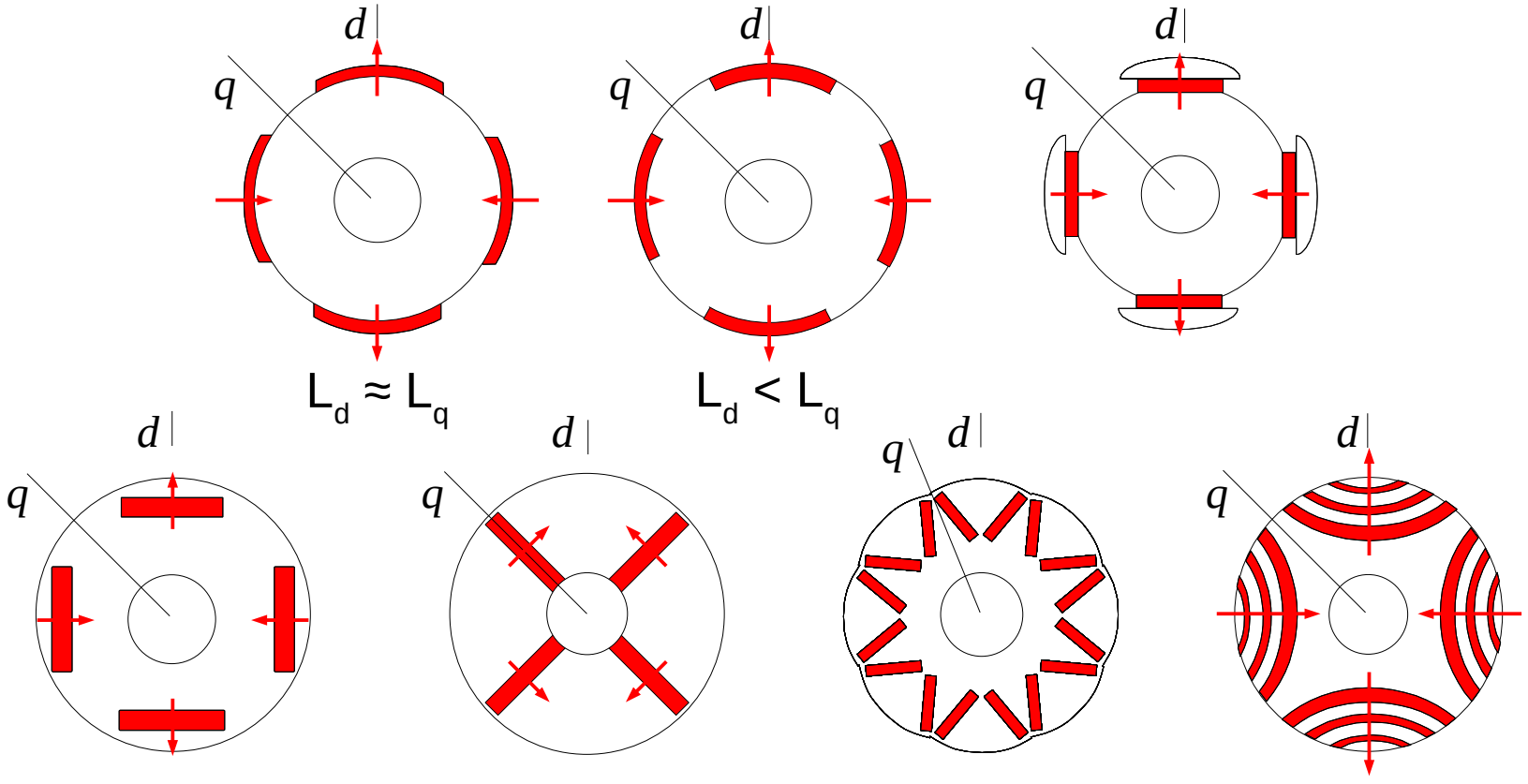
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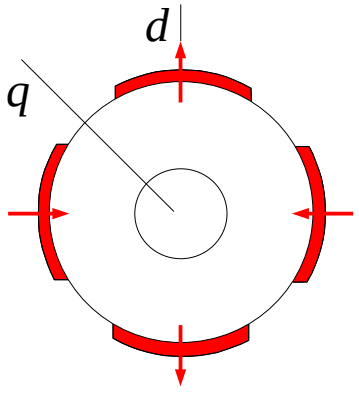
Torque Angle Dependencies



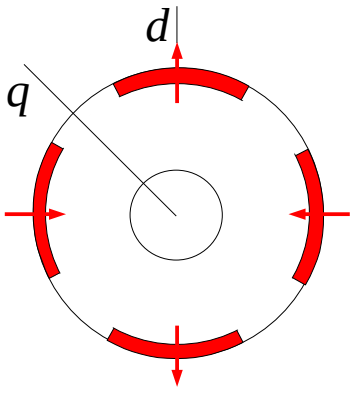
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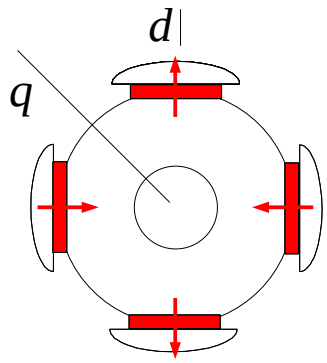
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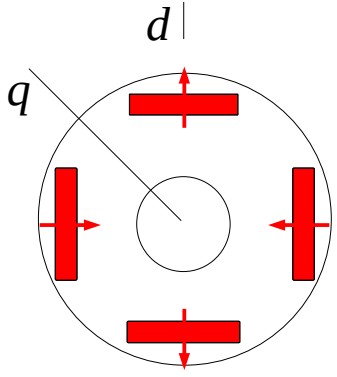
$L_d \approx L_q$

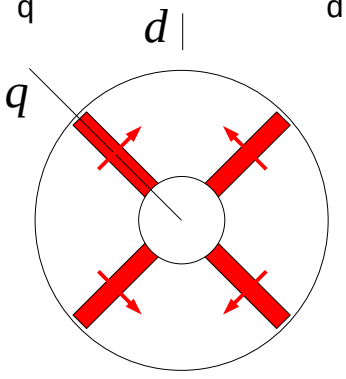


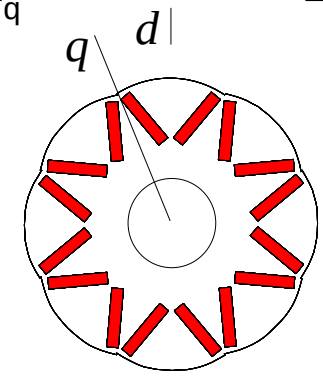
$L_d < L_q$

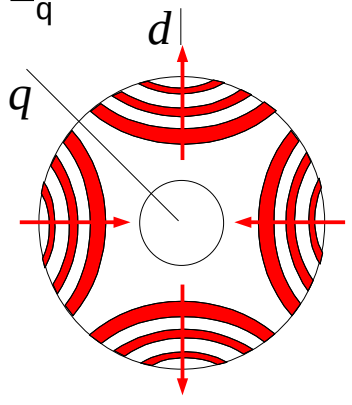


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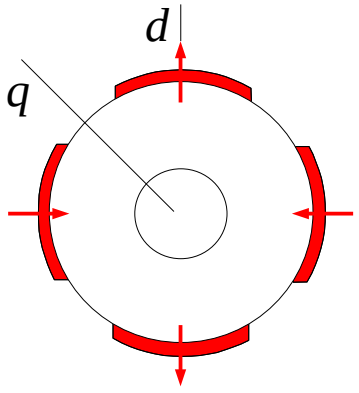




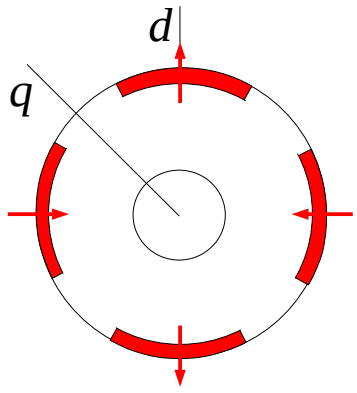




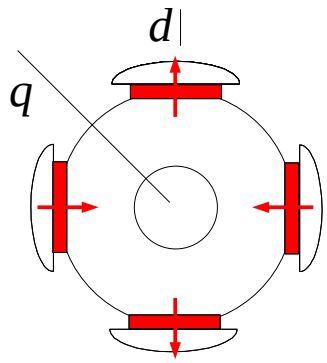
Torque Angle Dependencies



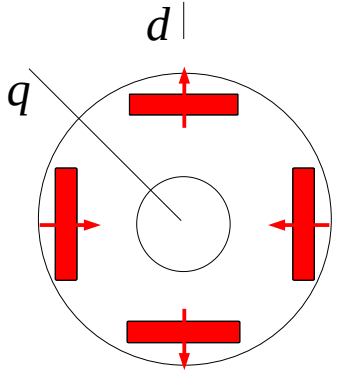
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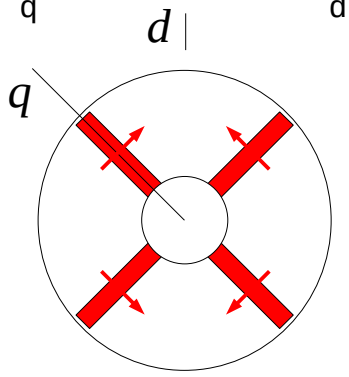
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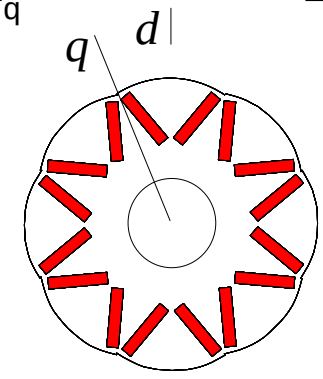


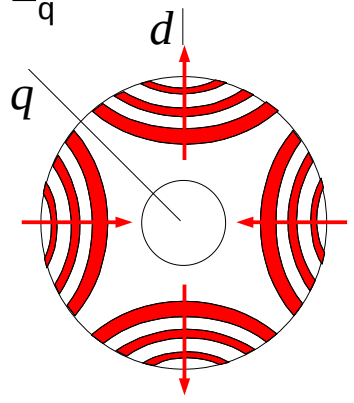
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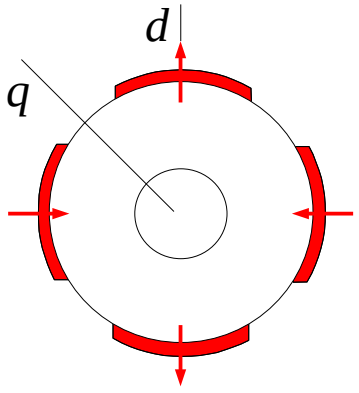
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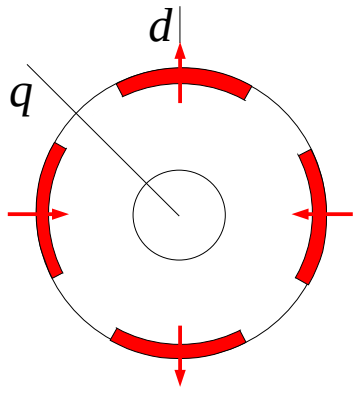




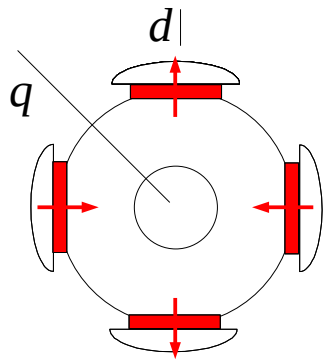
Torque Angle Dependencies



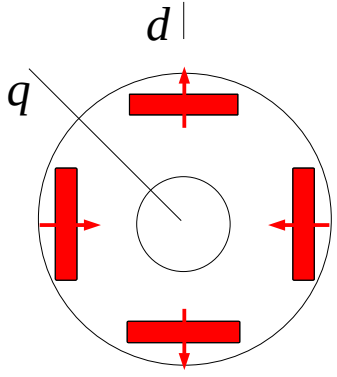
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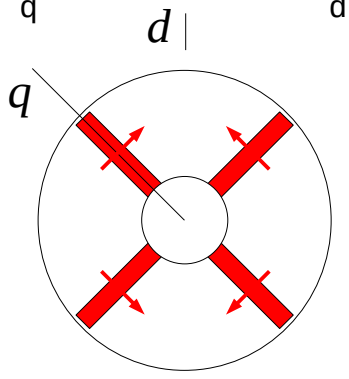
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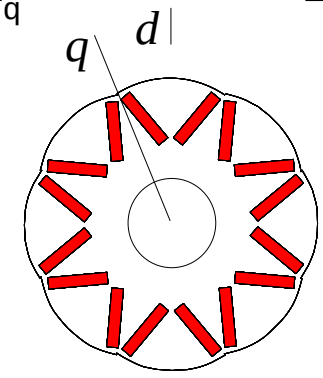
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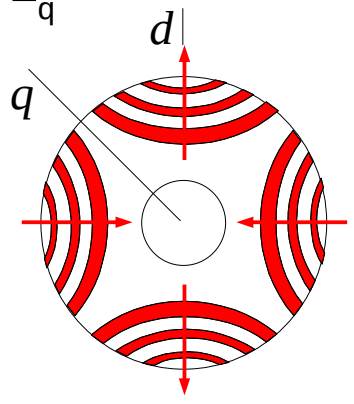


$L_d < L_q$

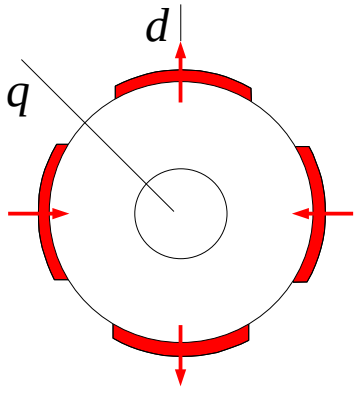


$L_d < L_q$

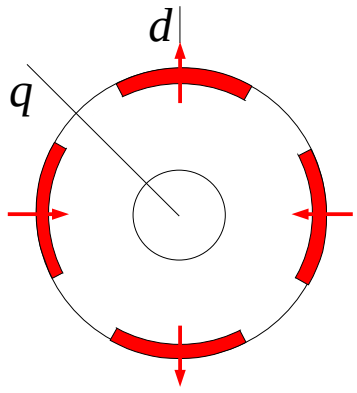




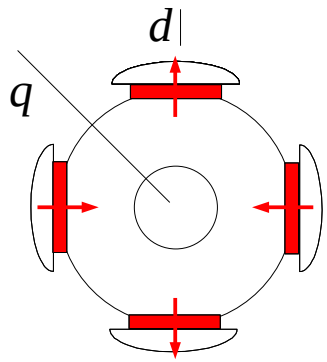
Torque Angle Dependencies



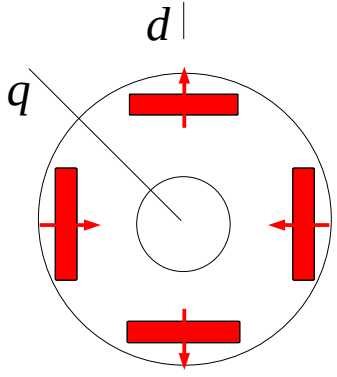
$L_d \approx L_q$



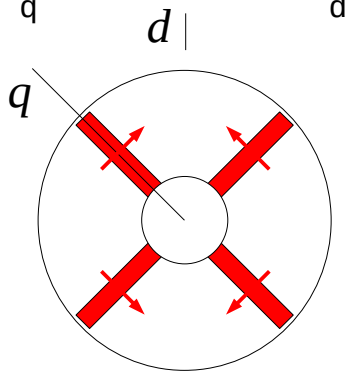
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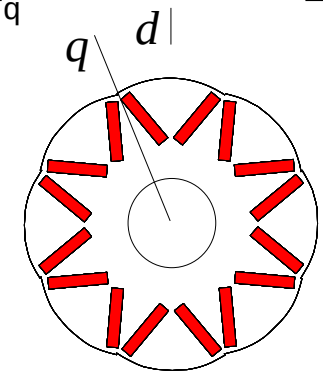
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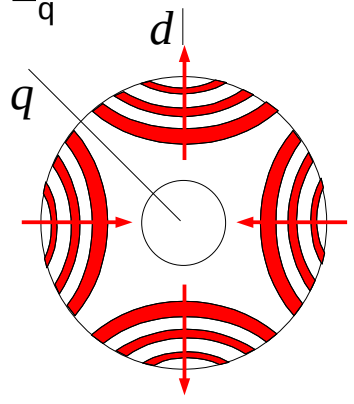
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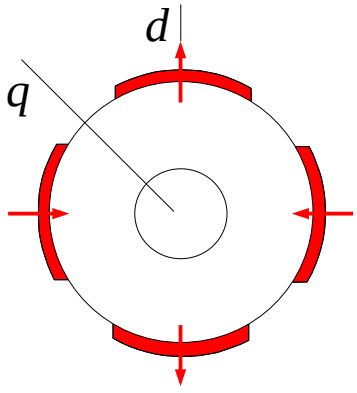
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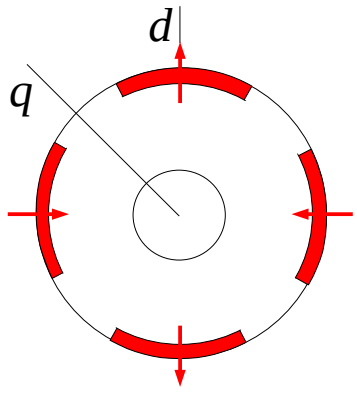
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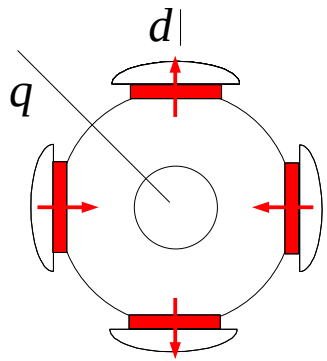
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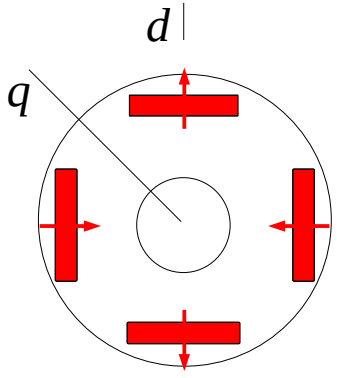
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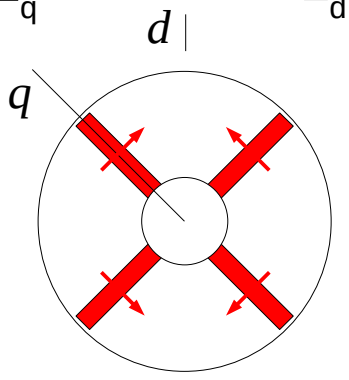
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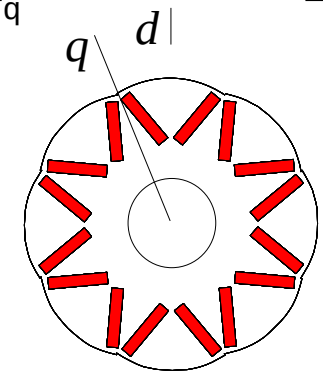
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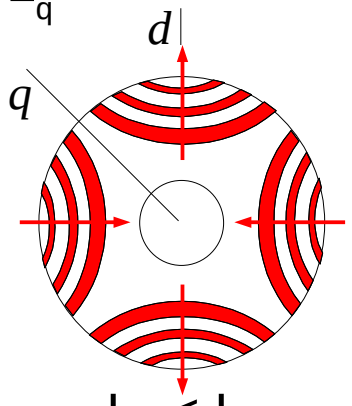
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Optimal Control of Permanent Magnet Synchronous Machines

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- By optimal control of the permanent magnet synchronous machine the maximal torque while keeping minimal losses is understood

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- By optimal control of the permanent magnet synchronous machine the maximal torque while keeping minimal losses is understood
- All regulation algorithms ise current components i_d and i_q, thus it is necessary to use the torque equation in d/q system
  - Commonly used by power electronics engineers

Optimal Control of Permanent Magnet Synchronous Machines

- The torque equation in *d/q* system may be derived from traditional from

$$M = \frac{p\,m}{\omega_1} U \left[\frac{U_{ib}}{X_d} \sin\beta + \frac{U}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\beta \right]$$

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in which we introduce voltage magnitudes

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  a sinus of double torque angle may be substituted by

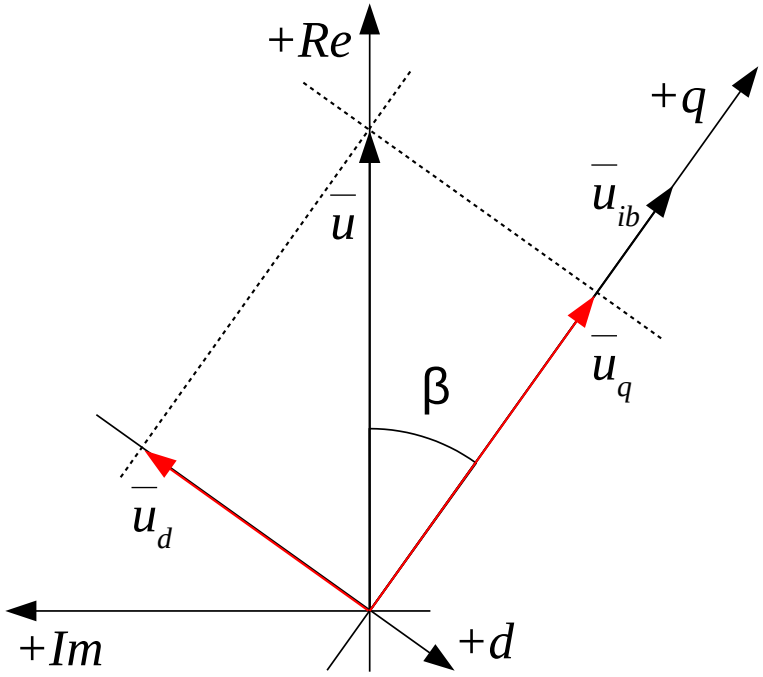
$$\sin 2\beta = 2 \sin\beta \cos\beta$$

- After modification we obtain

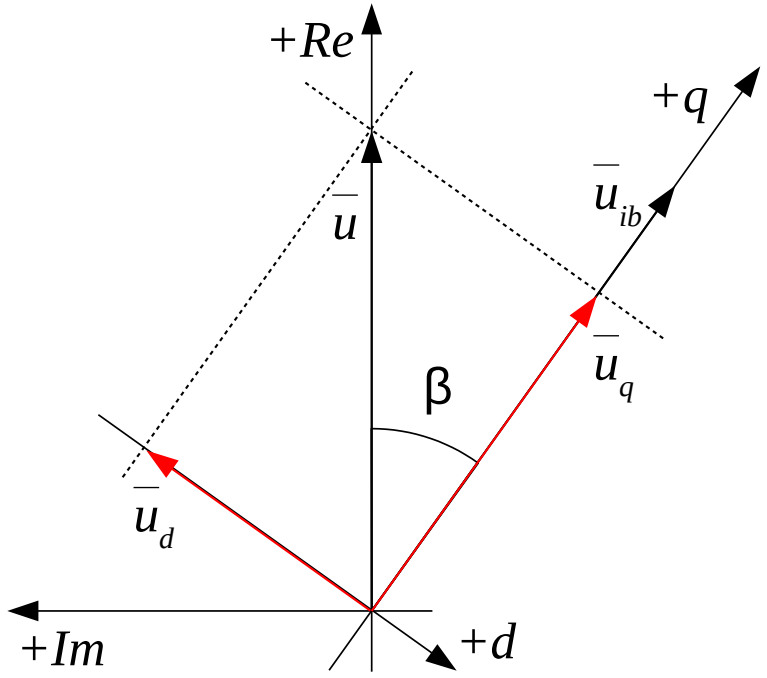
$$M = \frac{pm}{2\omega_1}\left[\frac{u\,\omega_1\,\psi}{X_d}\sin\beta + \frac{u^2}{2}\left(\frac{X_d - X_q}{X_d\,X_q}\right)2\sin\beta\cos\beta\right]$$

Optimal Control of Permanent Magnet Synchronous Machines

- From the phasor diagram it results

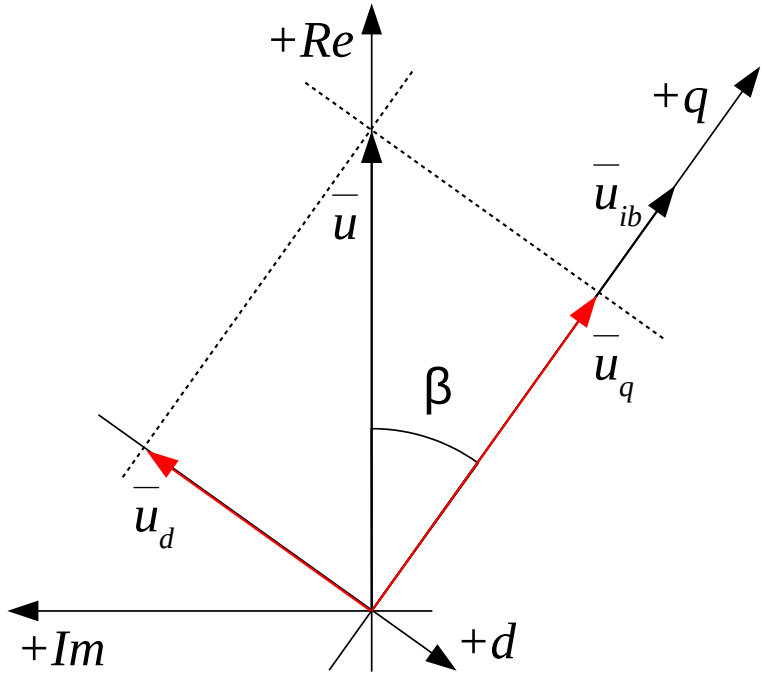


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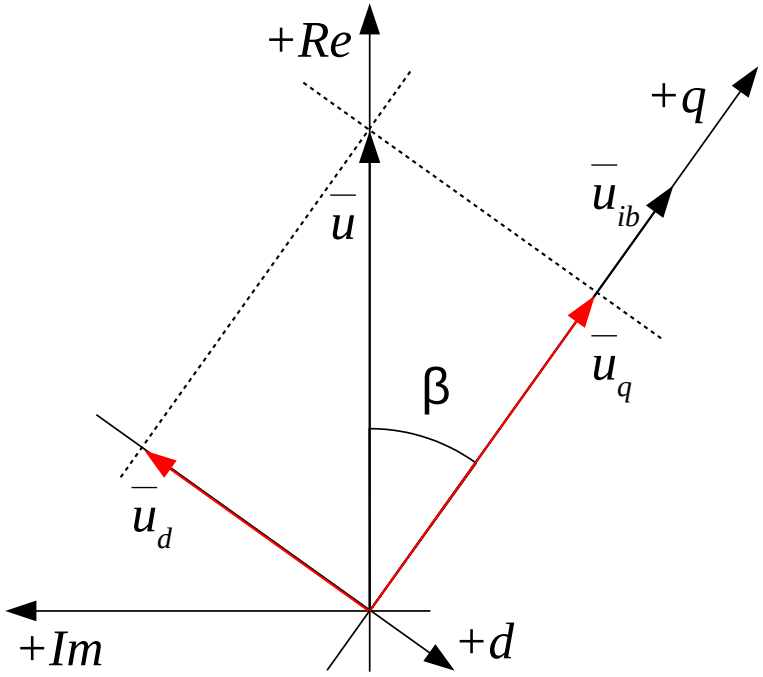
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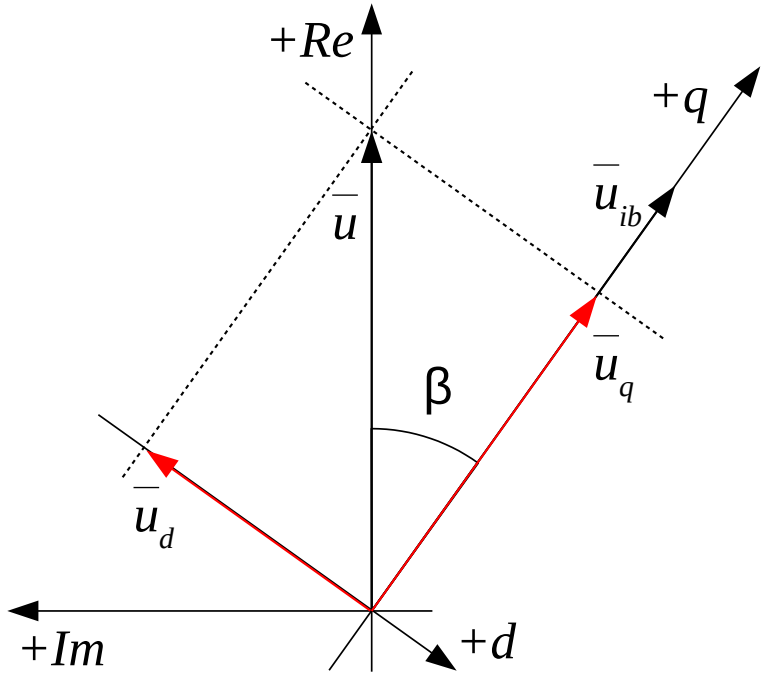
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Optimal Control of Permanent Magnet Synchronous Machines

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- ...thus it is possible to use related currents i_d/i_q instead of terminal voltage, torque angle functions and reactances:

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(1) considers terminal voltage

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in (2) the view into the function of the machine disappears – the torque angle is missing

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Optimal Control of a PMSM with Equal Inductances L_d and L_q

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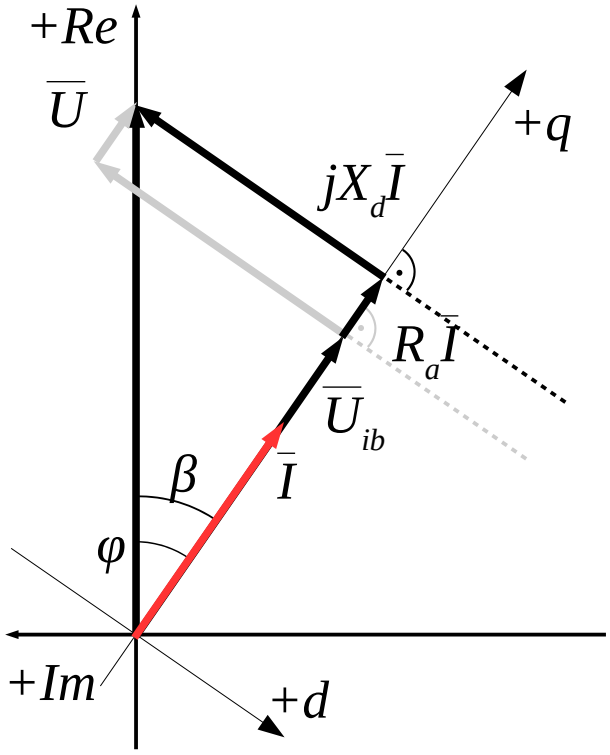
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- If there is no need for flux-weakening, the optimal control is with $i_d = 0$

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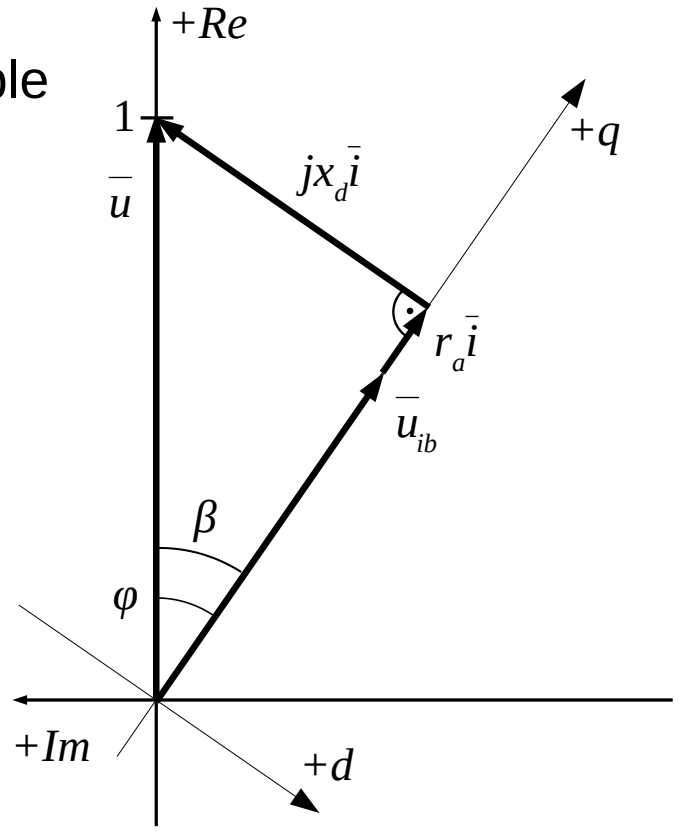
- The phasor diagram of such a machine is a right triangle



Optimal Control of a PMSM with Equal Inductances L_d and L_q

- After transcription to the per unit system it is possible to write a general relation between x_q and u_{ib}:

$$x_d = \sqrt{1 - \left(u_{ib} + r_a\right)^2}$$



Optimal Control of a PMSM with Equal Inductances L_d and L_q

- If the machine has to be optimally controllable for given voltage, it must have:

Optimal Control of a PMSM with Equal Inductances L_d and L_q

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  - U_{ib} lower than terminal voltage
    - u_{ib} in per units lower than 1

Optimal Control of a PMSM with Equal Inductances L_d and L_q

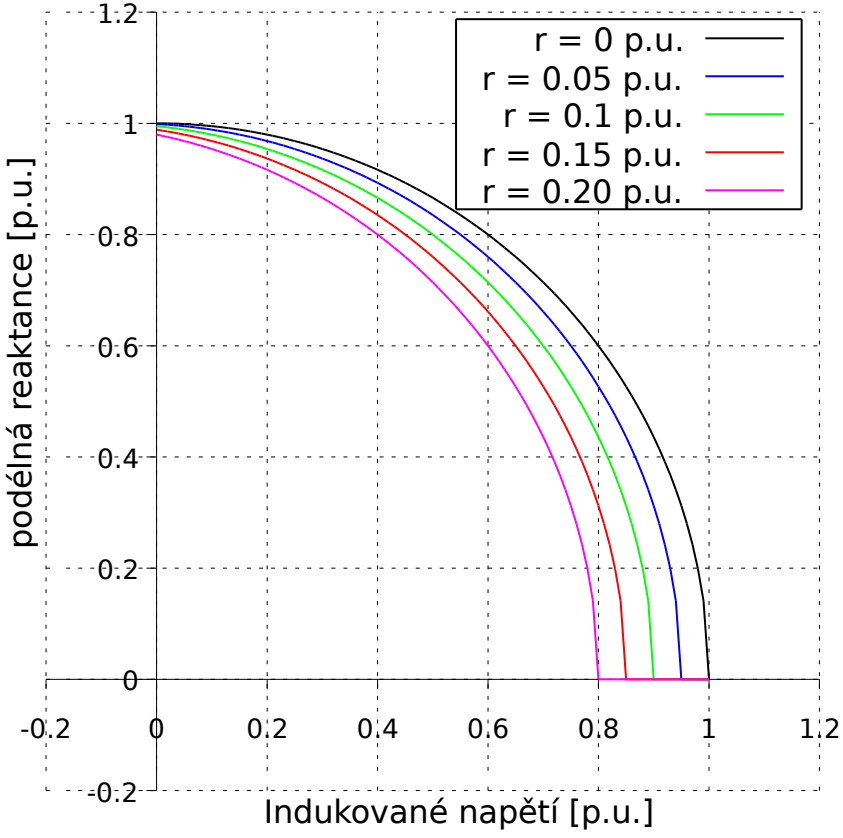
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  - The voltage drop on direct reactance lower than terminal voltage
    - x_d in per units lower than 1
  - Thus we obtain a relation between u_{ib} and x_d

$$x_d = \sqrt{1 - \left(u_{ib} + r_a\right)^2}$$

Optimal Control of a PMSM with Equal Inductances L_d and L_q



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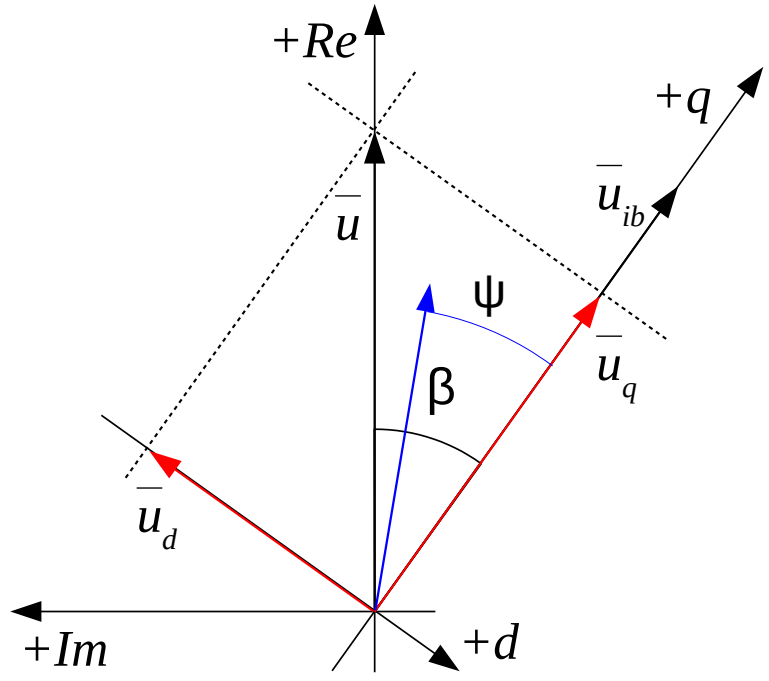
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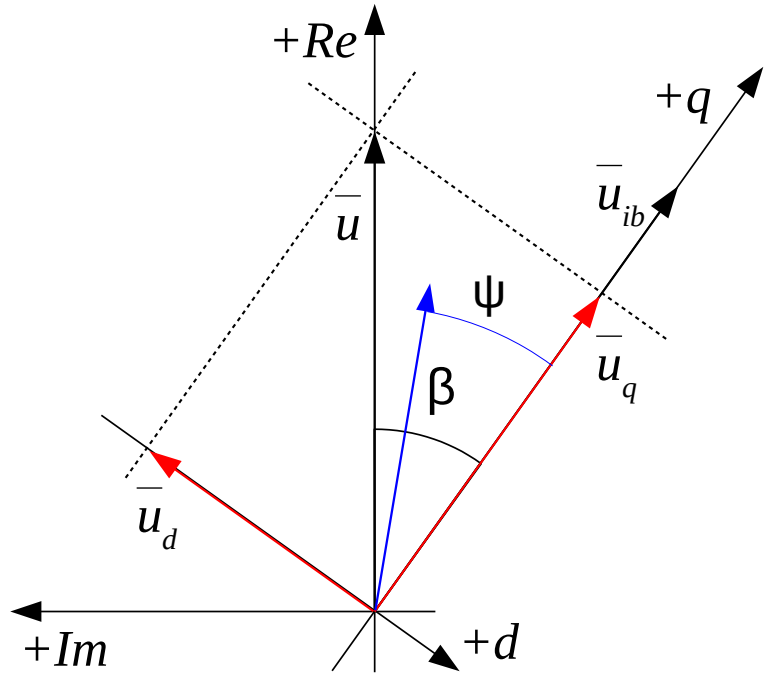


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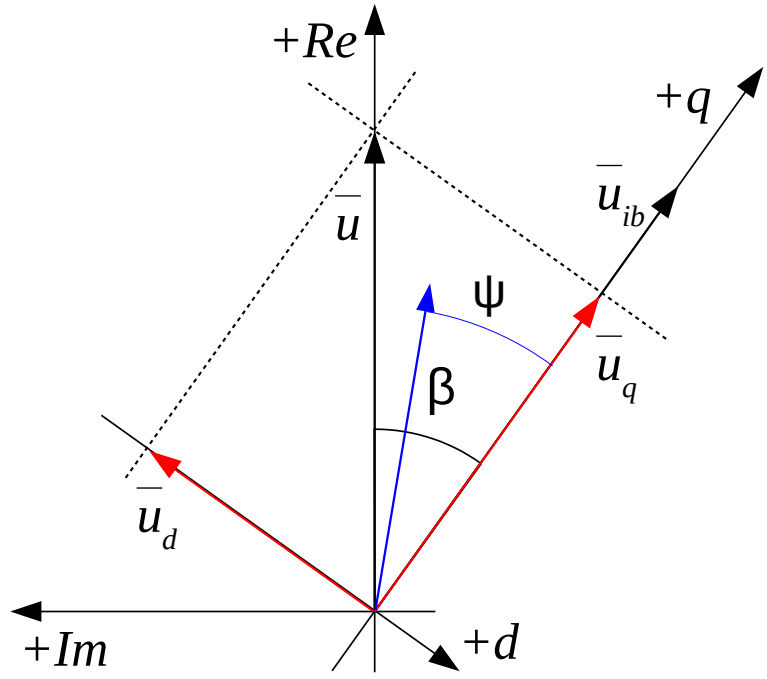
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to the torque equation

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Optimal Control of a PMSM with Different Inductances L_d and L_q

- Then the equation verges into

$$M = \frac{1}{2} mp \left(\psi_{PM} i \cos\psi - \left(L_d - L_q \right) i^2 \sin\psi \cos\psi \right)$$

Optimal Control of a PMSM with Different Inductances L_d and L_q

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$$M = \frac{1}{2} mp \left(\psi_{PM} i \cos\psi - \left(L_d - L_q \right) i^2 \sin\psi \cos\psi \right)$$

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- For the torque maximum we have a condition

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- Even in this case it is possible to tune the parameters of the machine for its work in terms of MTPA

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  - Both solutions are valid only until the maximal voltage is reached, then it is necessary to use the MTPV criterion (Maximum Torque per Volt)

Optimal Control of a PMSM with Different Inductances L_d and L_q

- Notes:
  - The MTPV (Maximum Torque per Volt) criterion is keeping the phasor diagram of the machine

$$i_q = \frac{1}{L_q}\sqrt{\left(\frac{\zeta u}{\omega_1}\right)^2 - \left(L_d i_d + \psi_{PM}\right)^2}$$

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Thank You for Your Attention!

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