FACULTY OF ELECTRICAL
ENGINEERING
UNIVERSITY OF WEST BOHEMIA

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

ESF II – project of the University of West Bohemia
reg. no. CZ.02.2.69/0.0/0.0/18_056/0013239

Theory of Electric Machines 2

KEV/TS2

Karel Hruska / r20201117

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Harmonic Components in the Air Gap Magnetic Field of an Induction Machine

Harmonics in the Air Gap Magnetic Field of an Induction Machine

- Magnetic field of electric machines includes a whole spectrum of harmonic components
  - From power supply
    - Time harmonic components; source: power electronics
  - From real geometry
    - Spatial harmonic components; source: stator and rotor teeth
- Both types affect the induced voltage of both stator and rotor
- A harmonic component in induced voltage generates current on corresponding frequency, which gnerates also related magnetic field

Harmonics in the Air Gap Magnetic Field of an Induction Machine

- If the torque of the machine is defined by

$$M = \vec{F}_1 \times \vec{F}_2$$

  then higher order harmonic components also transmit from the geometry and the power supply also to its torque
  - The after-effects are in induction machine synchrnous and asynchronous parasitic torque components
- In other types of machines they express themselves in form of additional losses (pulsation losses, surface losses, etc.)
- But targeted introduction of higher order harmonic components may lead to increase of the torque of the machine (e.g. polyphase machines)

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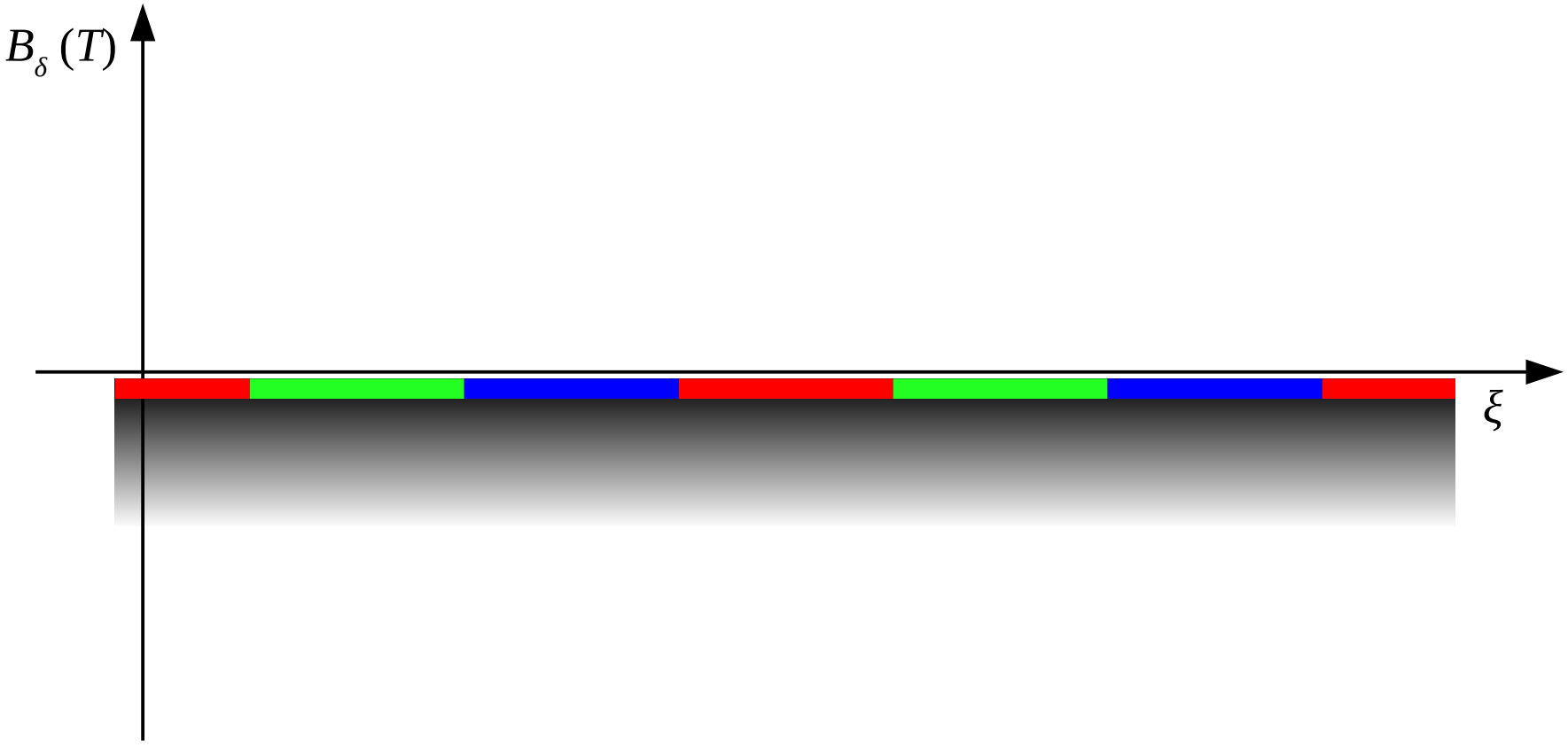
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Spatial Harmonic Components

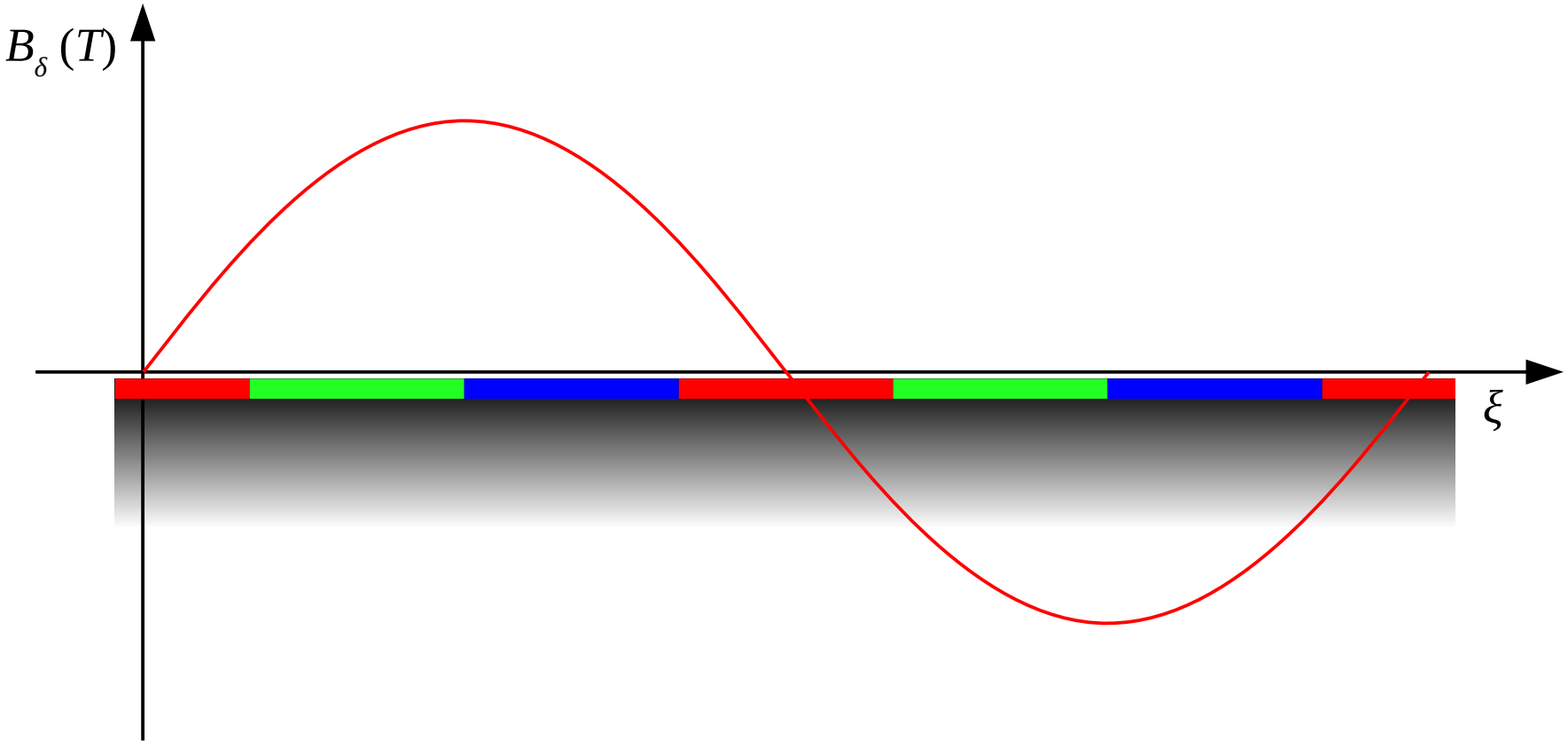
Spatial Harmonic Components

- In an ideal case the winding is laid in an infinite amount of infinitely thin slots
  - The result is sine-wave shape of the air gap magnetic flux density
- In the reality the winding is located in a finite amount of slots
  - Each slot brings a discrete increase of magnetomotive force
- These slots have non-zero width
  - The magnetic field is deformed by changes of the air gap reluctance
- The magnetic circuit is made of a real material
  - The magnetic field is flattened due to saturation of the steel

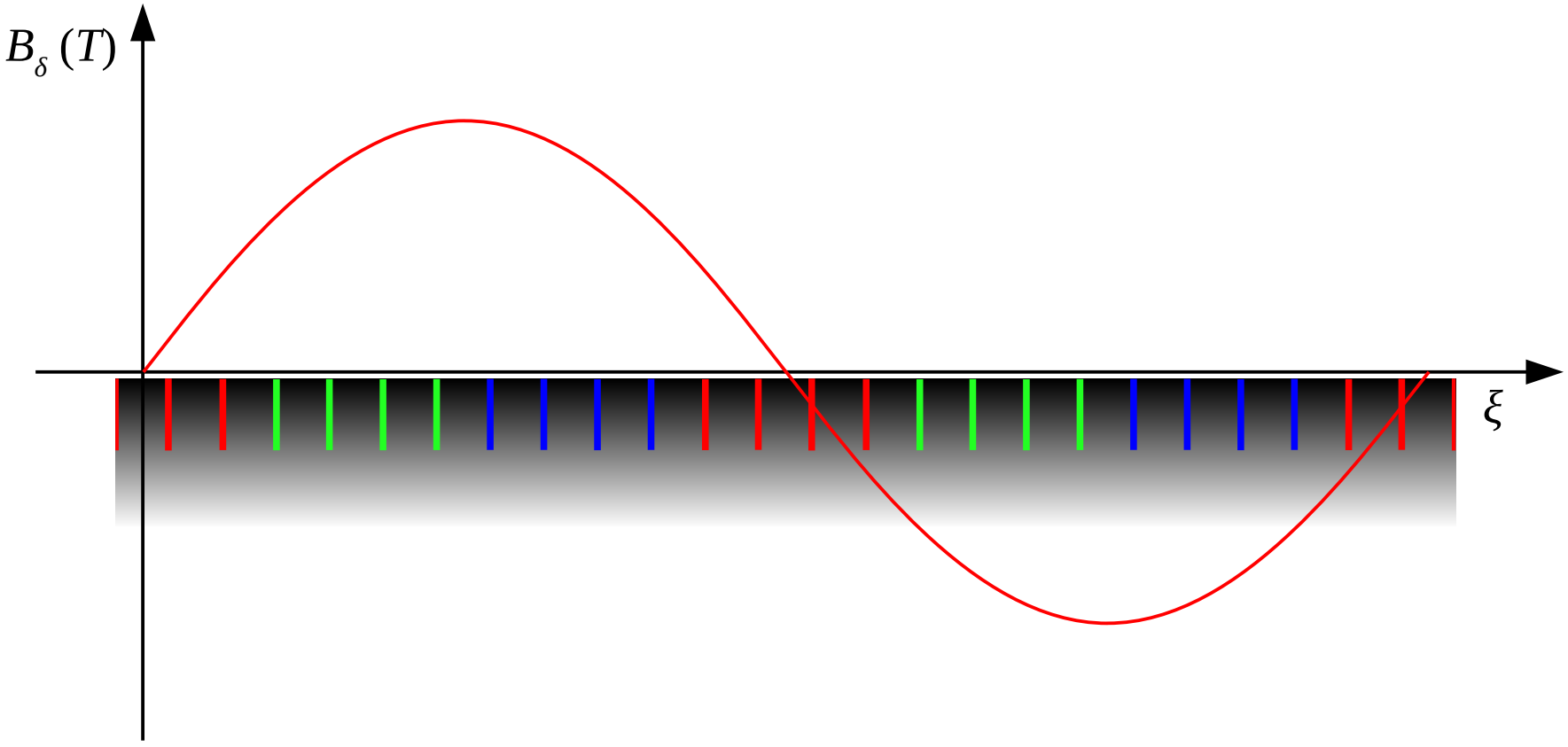
Spatial Harmonic Components



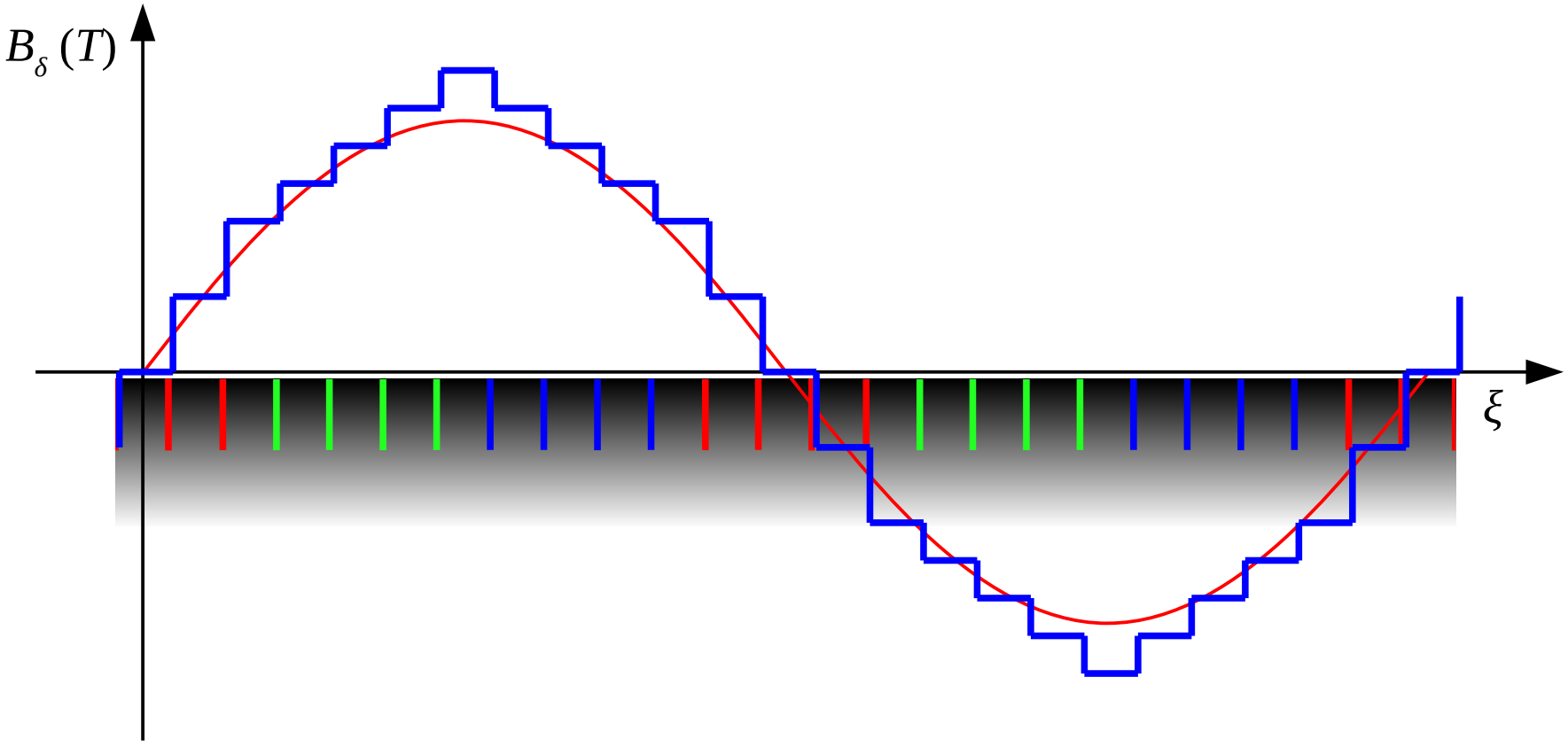
Spatial Harmonic Components



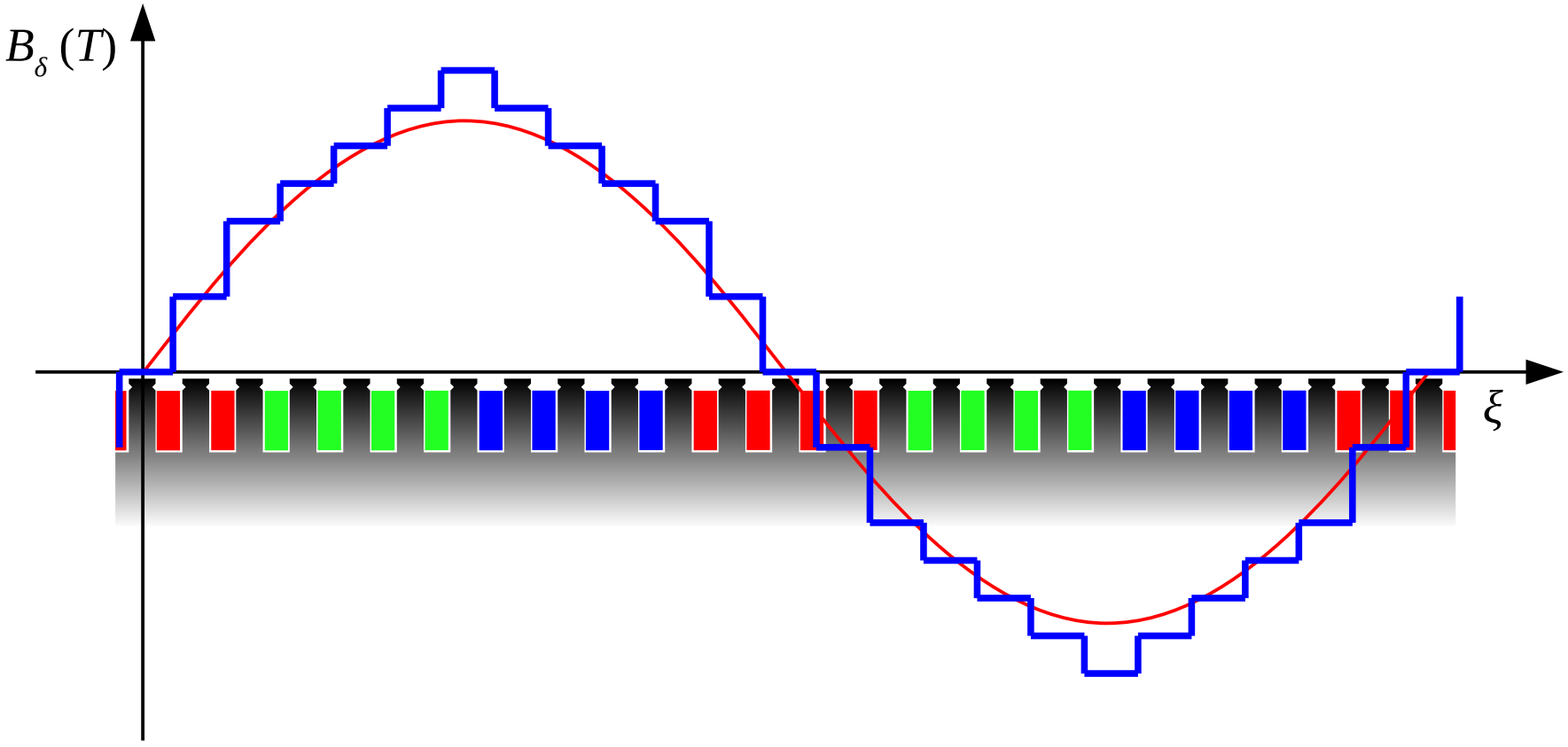
Spatial Harmonic Components



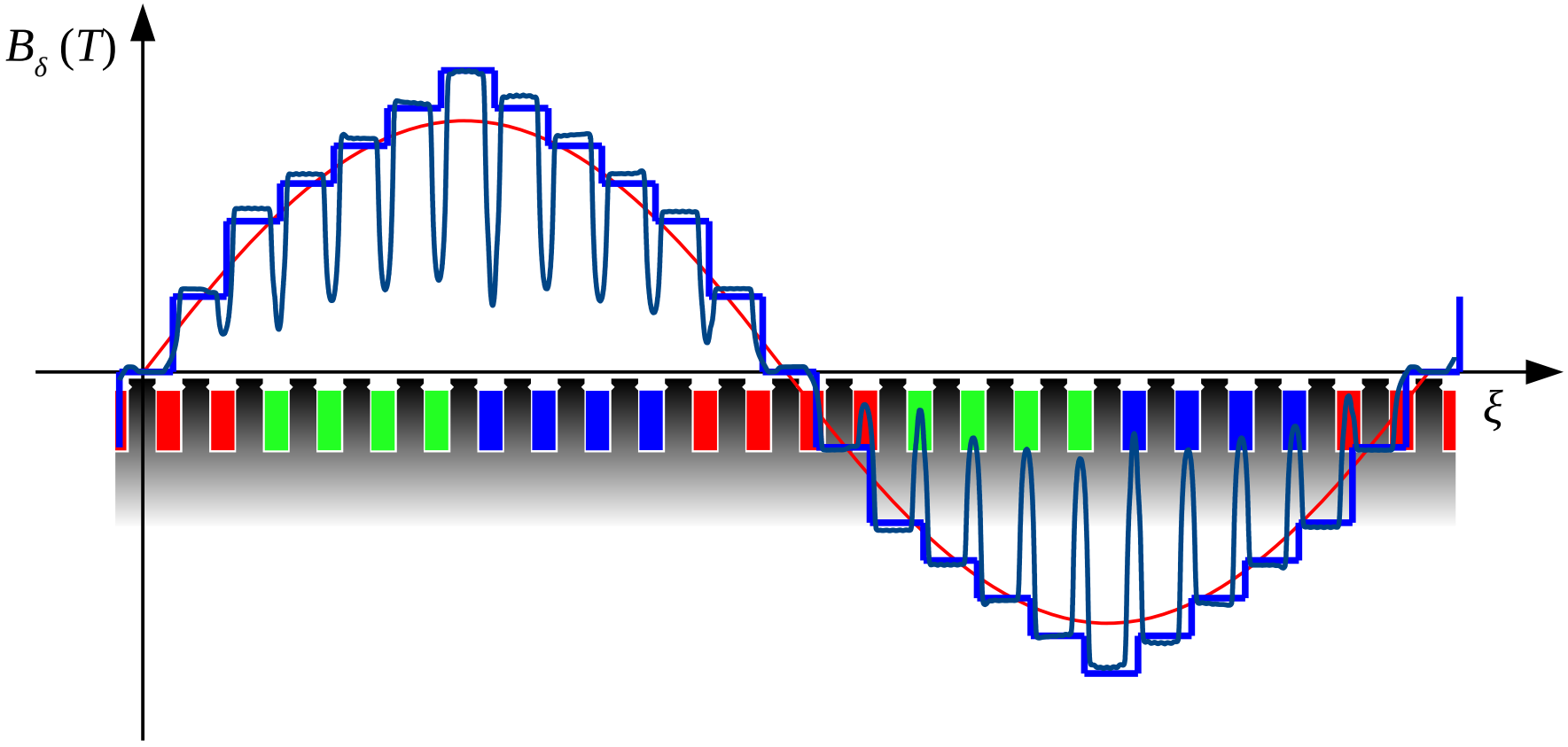
Spatial Harmonic Components



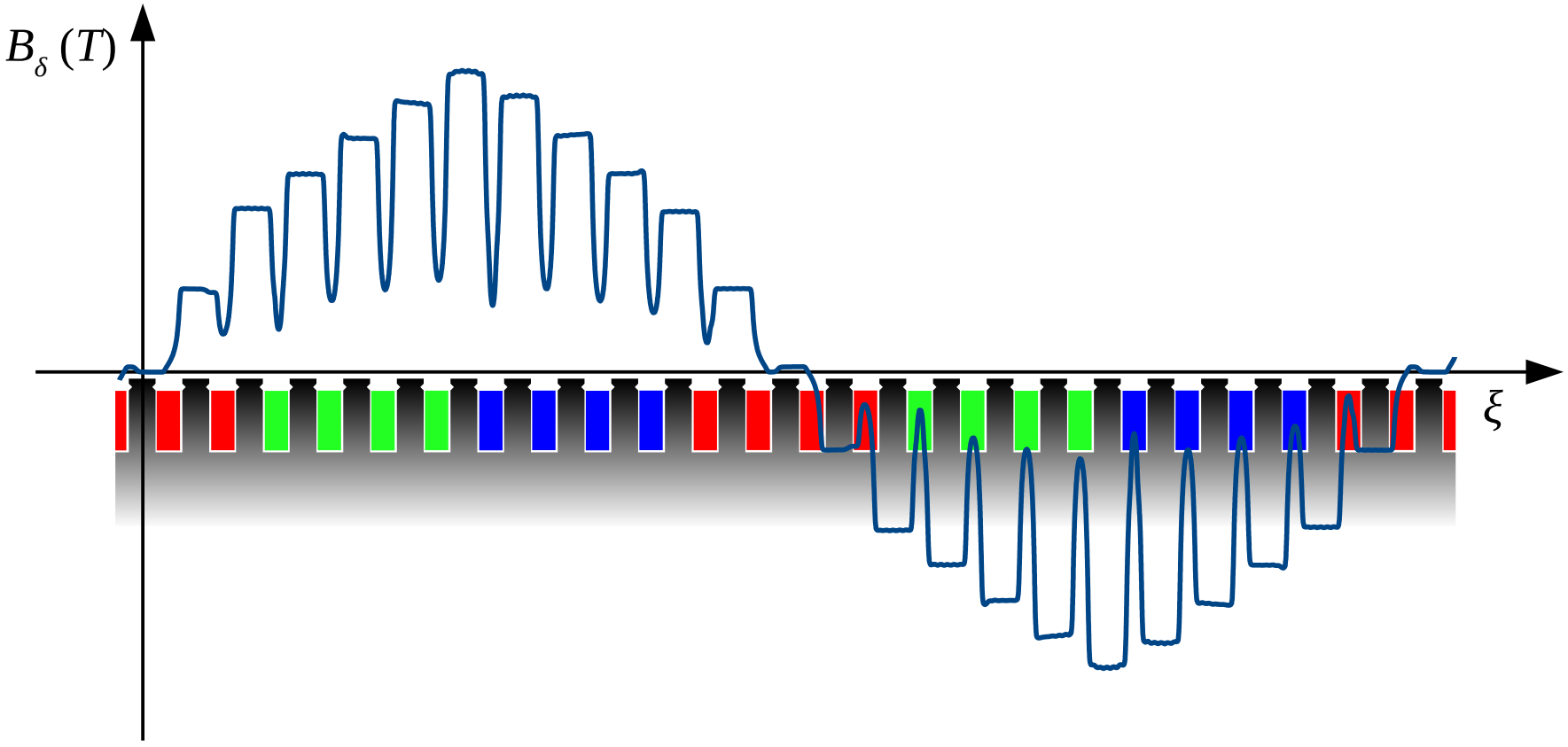
Spatial Harmonic Components



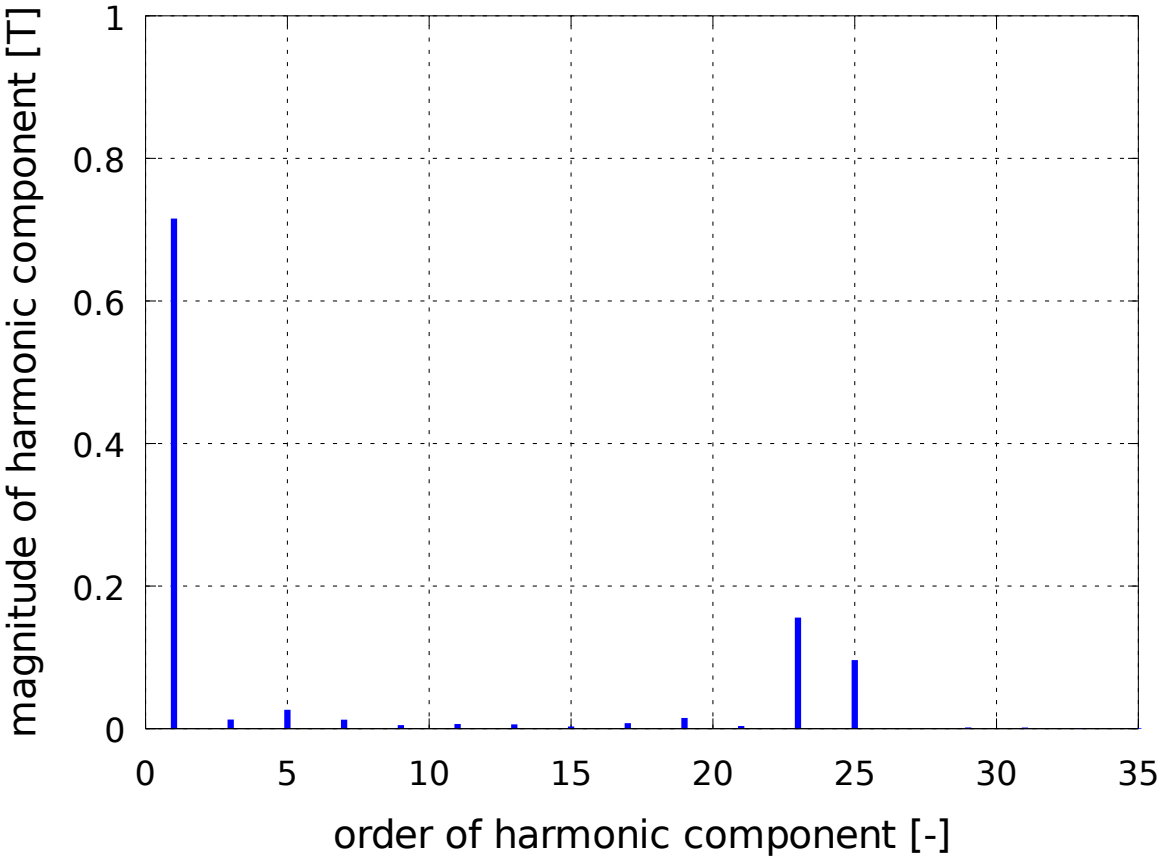
Spatial Harmonic Components



Spatial Harmonic Components

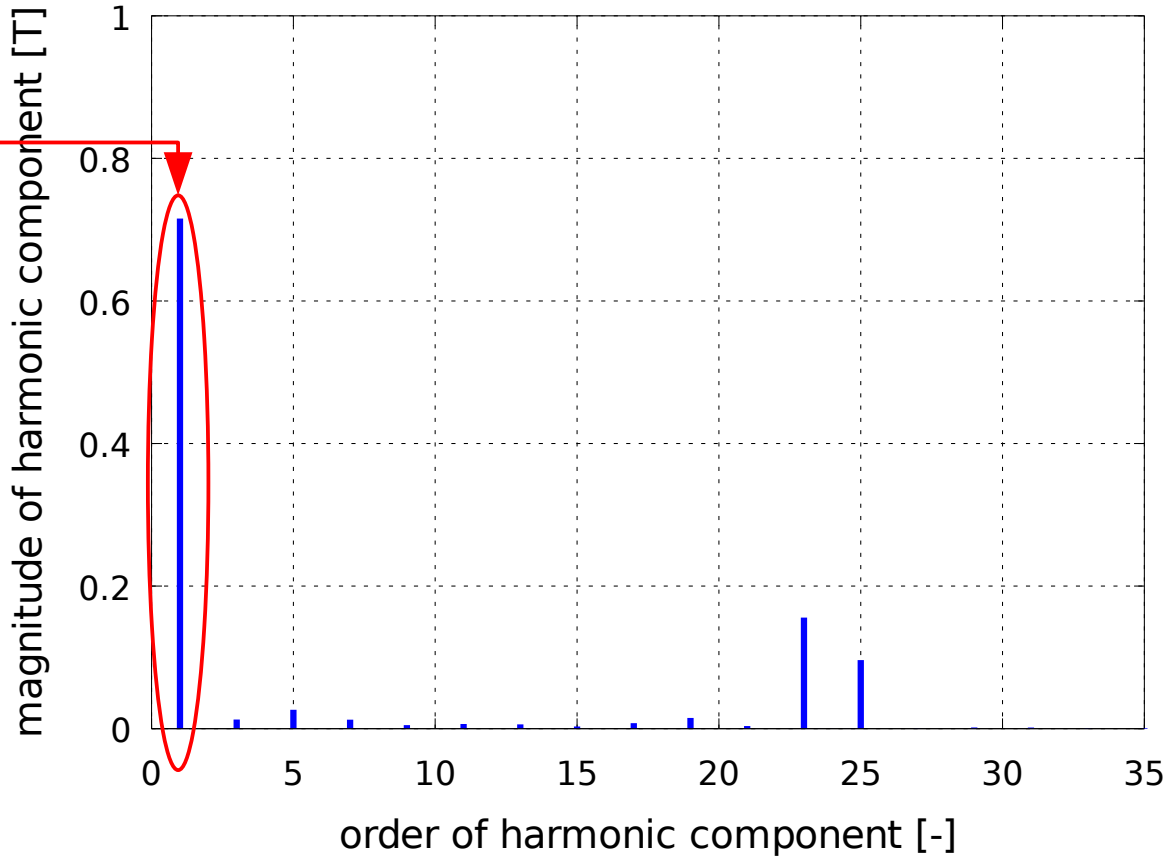


Spatial Harmonic Components

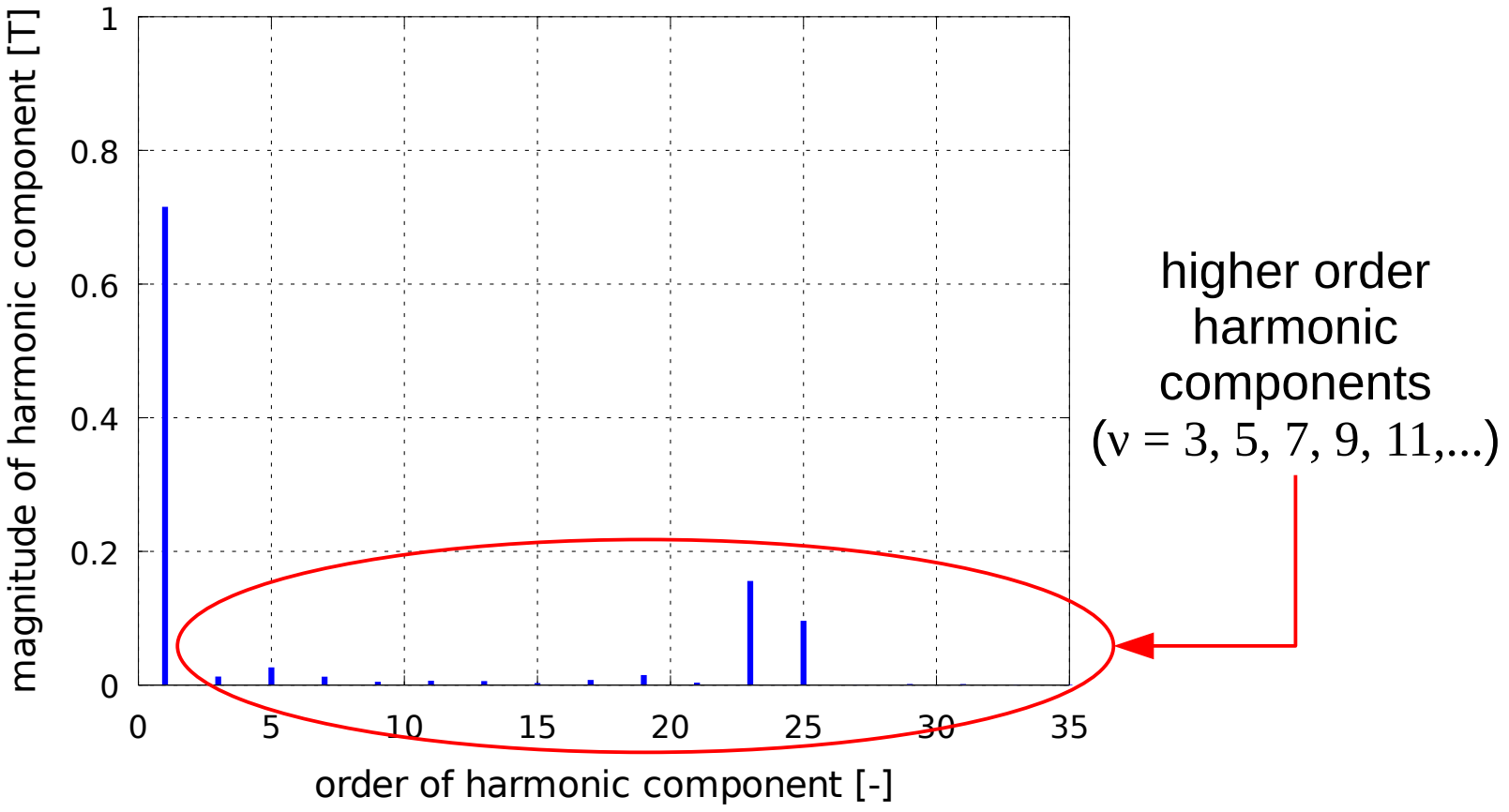


Spatial Harmonic Components

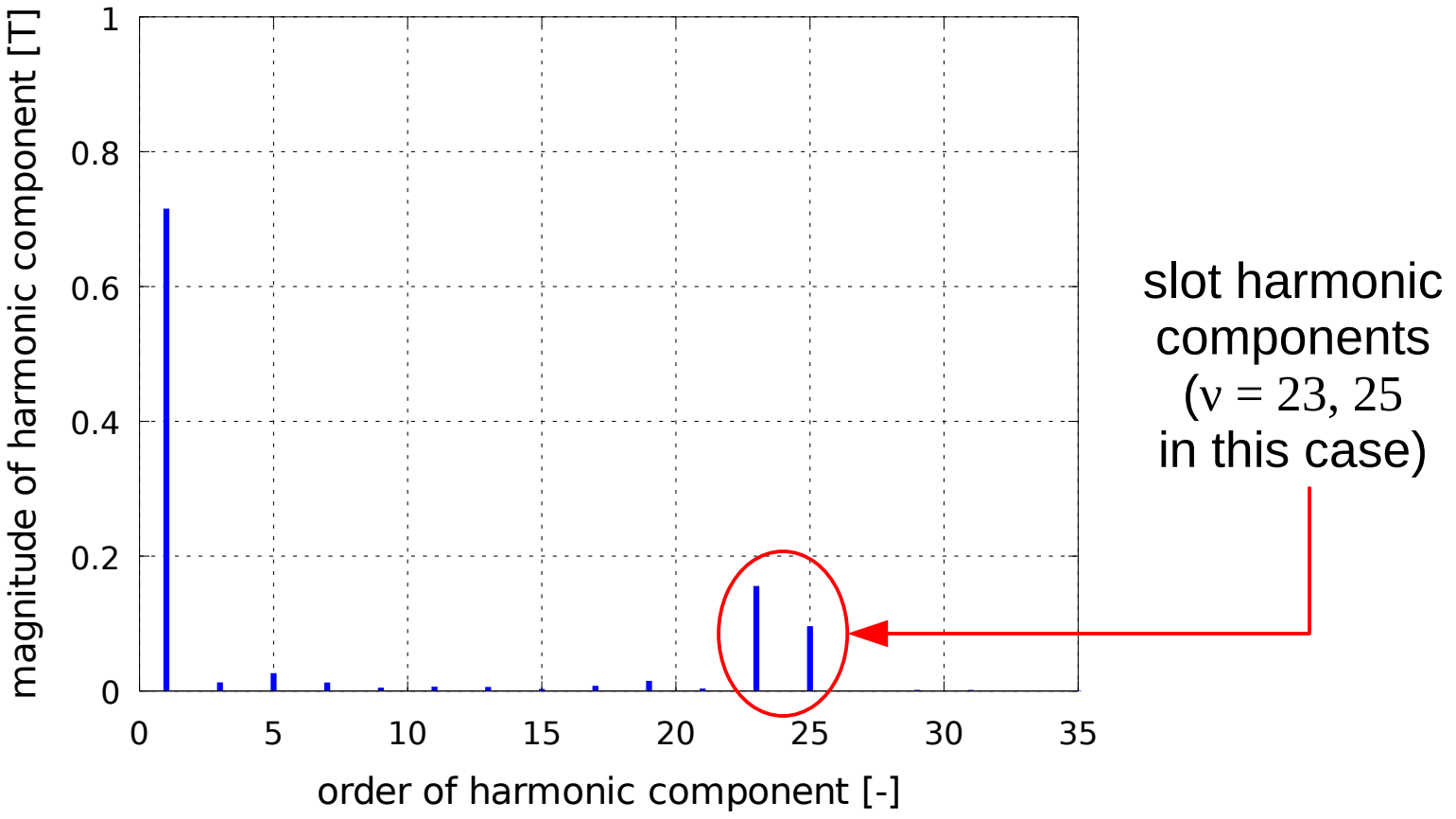
fundamental harmonic component ($\nu = 1$)



Spatial Harmonic Components



Spatial Harmonic Components



Spatial Harmonic Components

- In an ideal case the spectrum should consist of only

$$\nu = 2km \pm 1; \qquad k \in \mathbb{Z}$$

  harmonic components, i.e. in case of $m = 3$ phase machine of $\nu = 5,7,11,13$ etc.
- But, due to saturation effects there are also multiplies of 3rd harmonic component ($\nu = 3, 9, 15, 21$ etc.)
- The whole spectrum is then supplemented by spectral lines corresponding with the amount of slots per pole-pair of the machine – by slot harmonic components

$$\nu_d = \frac{Q}{p} \pm 1$$

  and their multiplies (in this case for a machine with $Q = 48$ slots and $2p = 4$ poles, they result $\nu_d = 23, 25, 47, 49$ etc.)

Spatial Harmonic Components

- From the motor periphery point of view the angle of a given harmonic component is ν – times higher than the angle of fundamental harmonic component: $$\xi_\nu = \nu \xi_1$$
- These harmonic components then rotate ν – times slower* in different directions: $$\omega_\nu = \pm \frac{\omega_1}{\nu}$$
- The rotation direction may be determined from the analysis of the magnetic field

*) because they make one of their wavelength per one period of the power supply

Spatial Harmonic Components

- The fundamental harmonic component of a m phase winding is described by

$$F_{m1} = \frac{m}{\pi} \cdot \frac{N_s k_{v1} I_m}{p} \cos\left(\xi - \omega_1 t\right)$$

- For a single phase and ν – th harmonic component we obtain

$$F_{m1\nu} = \frac{2}{3} \cdot \frac{F_{m1}}{\nu} = \frac{2}{\pi} \cdot \frac{N_s k_{v\nu} I_m}{\nu\, p}$$

- Magnetic field of a ν – th harmonic component may be described by following:

$$F_\nu = F_{m1\nu} \cos\left(\nu\, \xi\right) \cos\left(\omega_1 t\right)$$

… which is an equation of stationary wave

Spatial Harmonic Components

$$F_\nu = F_{m1\nu} \cos(\nu\, \xi) \cos(\omega_1 t)$$

Spatial Harmonic Components

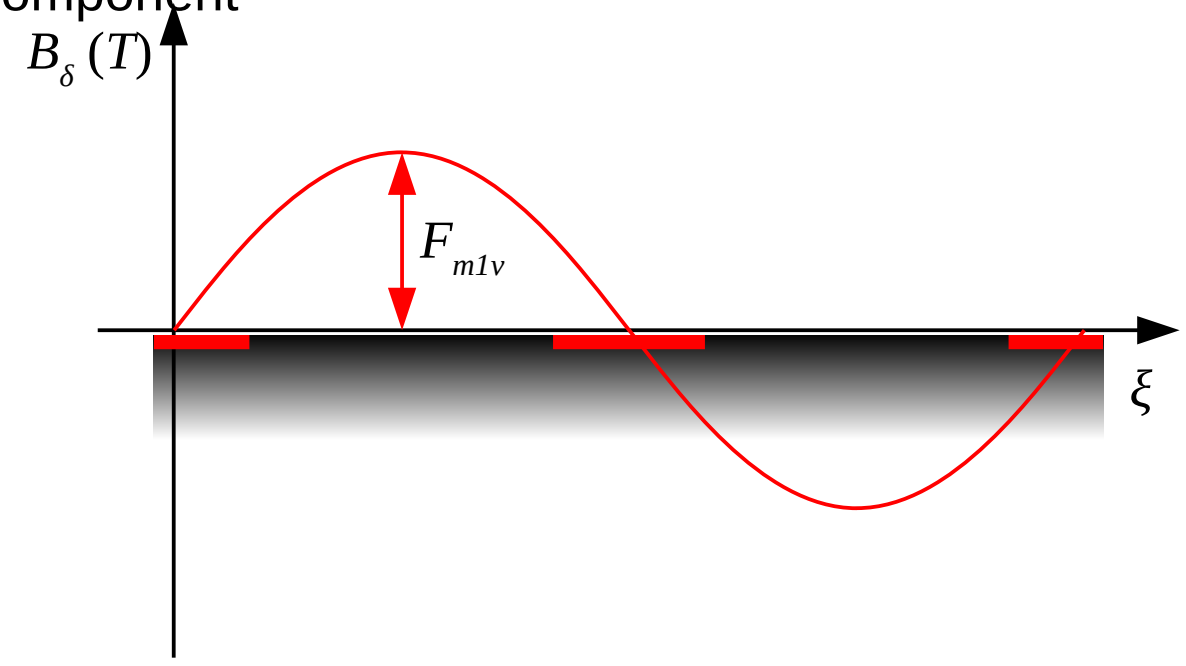
$$F_\nu = F_{m1\nu} \cos(\nu\,\xi) \cos(\omega_1 t)$$

magnitude
of harmonic component

Spatial Harmonic Components

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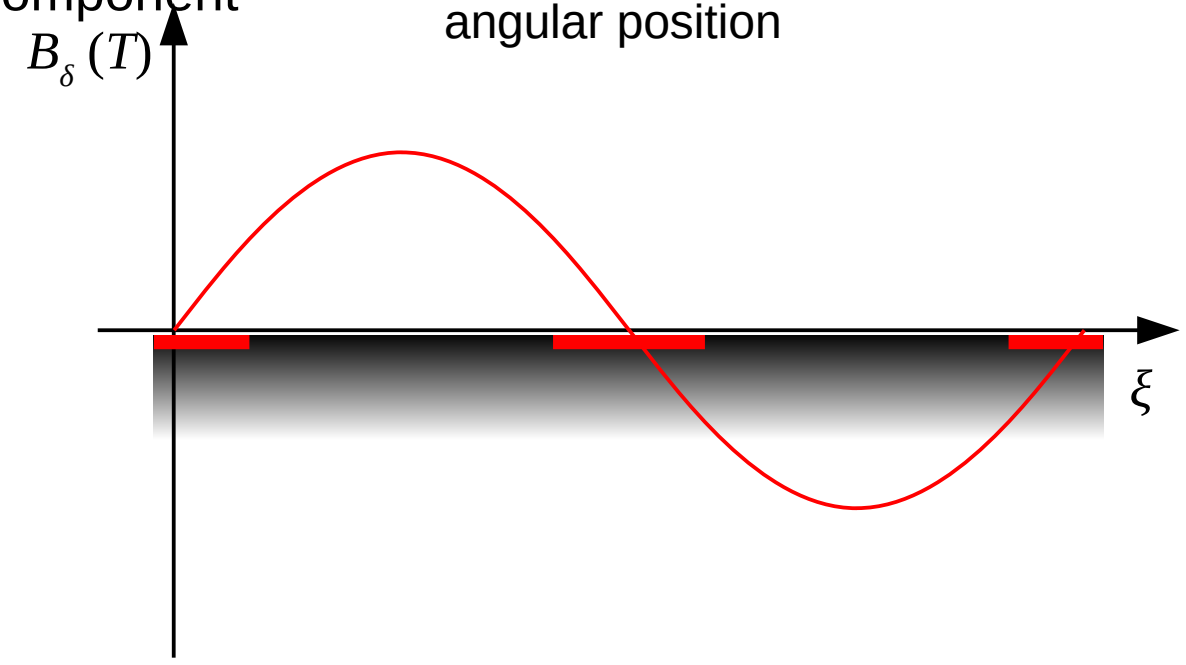


Spatial Harmonic Components

$$F_\nu = F_{m1\nu} \cos(\nu\,\xi) \cos(\omega_1 t)$$

magnitude
of harmonic component

angular position

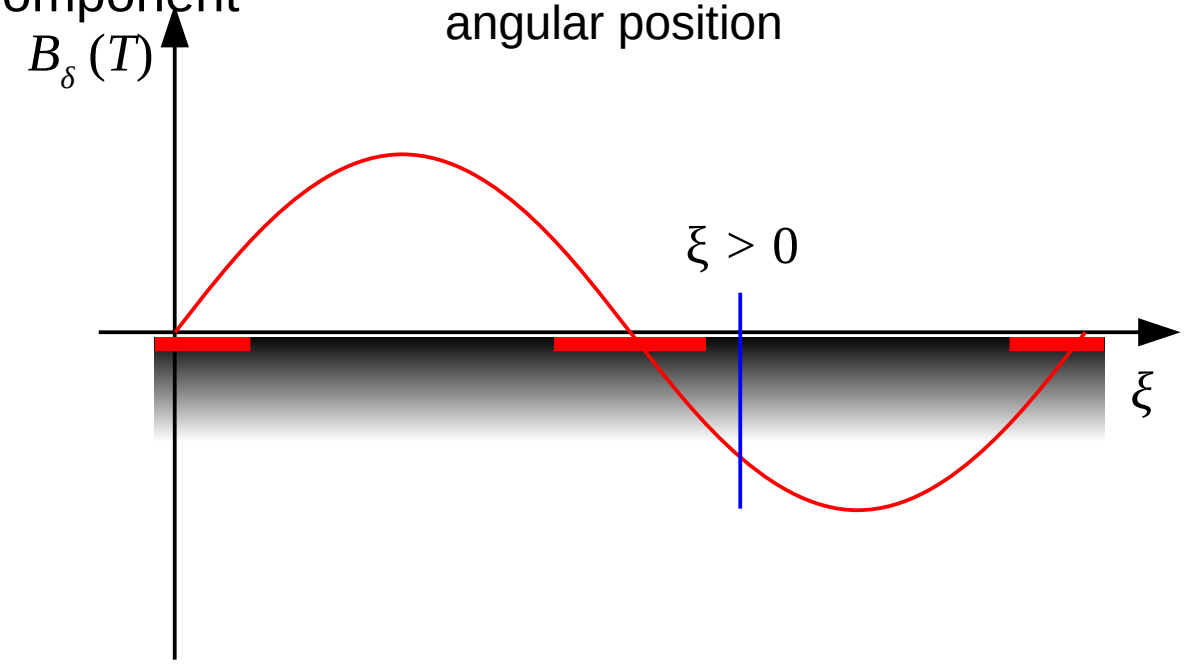


Spatial Harmonic Components

$$F_\nu = F_{m1\nu} \cos(\nu\,\xi) \cos(\omega_1 t)$$

magnitude
of harmonic component

angular position



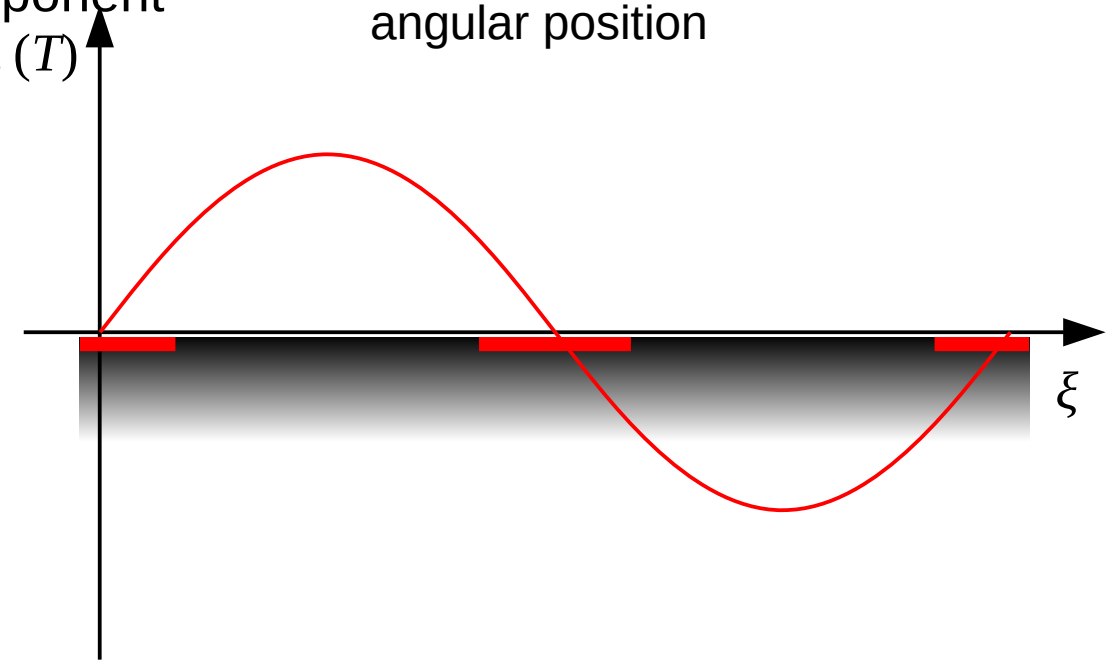
Spatial Harmonic Components

$$F_\nu = F_{m1\nu} \cos(\nu\,\xi) \cos(\omega_1 t)$$

magnitude of harmonic component

angular position

time position



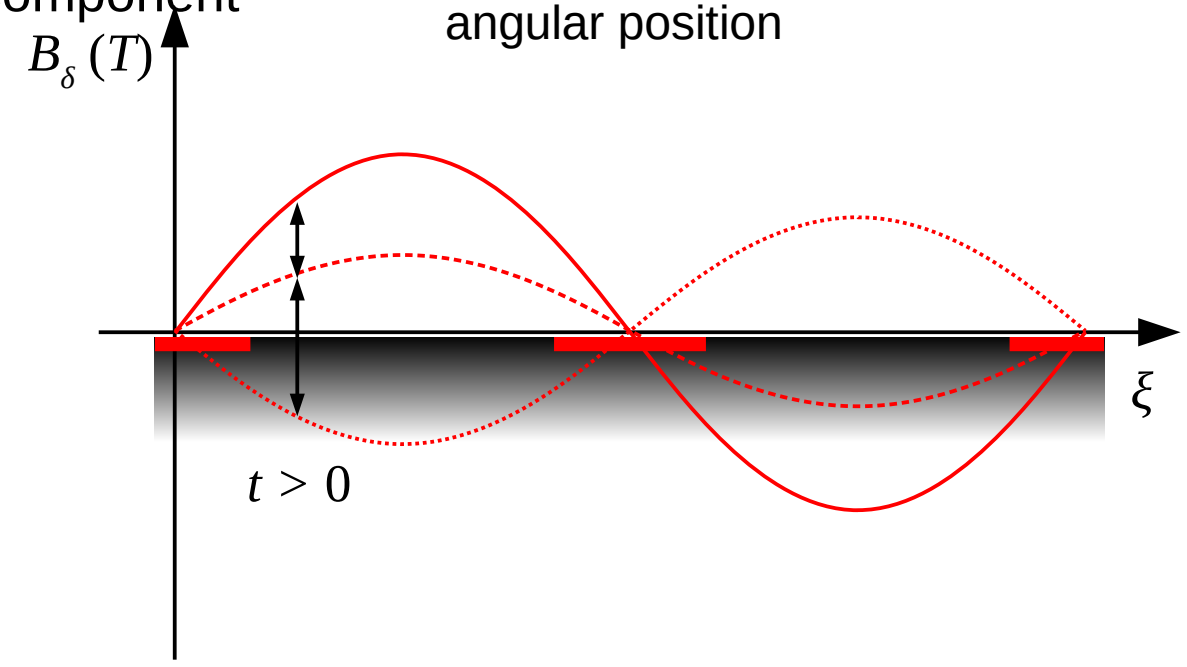
Spatial Harmonic Components

$$F_\nu = F_{m1\nu} \cos(\nu\,\xi) \cos(\omega_1 t)$$

magnitude of harmonic component

angular position

time position



Spatial Harmonic Components

- For three phases: $F_{U\nu} = F_{m1\nu} \cos(\nu\,\xi) \cos(\omega_1 t)$

Spatial Harmonic Components

- For three phases:

$$F_{U\nu} = F_{m1\nu}\cos(\nu\,\xi)\cos(\omega_1 t)$$

$$F_{V\nu} = F_{m1\nu}\cos\left[\nu\left(\xi - \frac{2\pi}{3}\right)\right]\cos\left(\omega_1 t - \frac{2\pi}{3}\right)$$

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$$F_{W\nu} = F_{m1\nu}\cos\left[\nu\left(\xi - \frac{4\pi}{3}\right)\right]\cos\left(\omega_1 t - \frac{4\pi}{3}\right)$$

Spatial Harmonic Components

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$$F_{W\nu} = F_{m1\nu}\cos\left[\nu\left(\xi - \frac{4\pi}{3}\right)\right]\cos\left(\omega_1 t - \frac{4\pi}{3}\right)$$

- After multiplication:

$$F_{U\nu} = F_{m1\nu}\cos(\nu\,\xi)\cos(\omega_1 t)$$

$$F_{V\nu} = F_{m1\nu}\cos\left(\nu\,\xi - \nu\frac{2\pi}{3}\right)\cos\left(\omega_1 t - \frac{2\pi}{3}\right)$$

$$F_{W\nu} = F_{m1\nu}\cos\left(\nu\,\xi - \nu\frac{4\pi}{3}\right)\cos\left(\omega_1 t - \frac{4\pi}{3}\right)$$

Spatial Harmonic Components

- On multiplies of cosines we apply geometric relation

$$\cos x \cos y = \frac{1}{2}\left[\cos\left(x-y\right)+\cos\left(x+y\right)\right]$$

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and we obtain:

$$F_{U\nu} = F_{m1\nu}\cos\left(\nu\,\xi\right)\cos\left(\omega_1 t\right) = \frac{F_{m1\nu}}{2}\left[\cos\left(\nu\,\xi-\omega_1 t\right)+\cos\left(\nu\,\xi+\omega_1 t\right)\right]$$

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$$F_{V\nu} = F_{m1\nu}\cos\left(\nu\,\xi-\nu\frac{2\pi}{3}\right)\cos\left(\omega_1 t-\frac{2\pi}{3}\right) =$$

$$= \frac{F_{m1\nu}}{2}\left[\cos\left(\nu\,\xi-\omega_1 t-\left(\nu-1\right)\frac{2\pi}{3}\right)+\cos\left(\nu\,\xi+\omega_1 t-\left(\nu+1\right)\frac{2\pi}{3}\right)\right]$$

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$$F_{W\nu} = F_{m1\nu}\cos\left(\nu\,\xi-\nu\frac{4\pi}{3}\right)\cos\left(\omega_1 t-\frac{4\pi}{3}\right) =$$

$$= \frac{F_{m1\nu}}{2}\left[\cos\left(\nu\,\xi-\omega_1 t-\left(\nu-1\right)\frac{4\pi}{3}\right)+\cos\left(\nu\,\xi+\omega_1 t-\left(\nu+1\right)\frac{4\pi}{3}\right)\right]$$

Spatial Harmonic Components

- The resulting magnetic field is a sum of magnetic fields of all phases:

$$F_{m\nu} = F_{U\nu} + F_{V\nu} + F_{W\nu}$$

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$$F_{U\nu} = \frac{F_{m1\nu}}{2}\left[\cos\left(\nu\xi - \omega_1 t\right) + \cos\left(\nu\xi + \omega_1 t\right)\right]$$

$$F_{V\nu} = \frac{F_{m1\nu}}{2}\left[\cos\left(\nu\xi - \omega_1 t - \left(\nu-1\right)\frac{2\pi}{3}\right) + \cos\left(\nu\xi + \omega_1 t - \left(\nu+1\right)\frac{2\pi}{3}\right)\right]$$

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$$+ \cos\left(\nu\xi - \omega_1 t - \left(\nu - 1\right)\frac{2\pi}{3}\right) + \cos\left(\nu\xi + \omega_1 t - \left(\nu + 1\right)\frac{2\pi}{3}\right) +$$
$$\left. + \cos\left(\nu\xi - \omega_1 t - \left(\nu - 1\right)\frac{4\pi}{3}\right) + \cos\left(\nu\xi + \omega_1 t - \left(\nu + 1\right)\frac{4\pi}{3}\right)\right]$$

Spatial Harmonic Components

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  1) $\nu = 3$

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$$F_{m3} = \frac{F_{m13}}{2}\left[\cos\left(3\xi - \omega_1 t\right) + \cos\left(3\xi + \omega_1 t\right) + \right.$$
$$+ \cos\left(3\xi - \omega_1 t - \left(3-1\right)\frac{2\pi}{3}\right) + \cos\left(3\xi + \omega_1 t - \left(3+1\right)\frac{2\pi}{3}\right) +$$
$$\left. + \cos\left(3\xi - \omega_1 t - \left(3-1\right)\frac{4\pi}{3}\right) + \cos\left(3\xi + \omega_1 t - \left(3+1\right)\frac{4\pi}{3}\right)\right]$$

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$$+ \cos\left(3\xi - \omega_1 t - \frac{4\pi}{3}\right) + \cos\left(3\xi + \omega_1 t - \frac{8\pi}{3}\right) +$$
$$\left. + \cos\left(3\xi - \omega_1 t - \frac{8\pi}{3}\right) + \cos\left(3\xi + \omega_1 t - \frac{16\pi}{3}\right)\right]$$

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$$\left. + \cos\left(3\xi - \omega_1 t - \frac{8\pi}{3}\right) + \cos\left(3\xi + \omega_1 t - \frac{16\pi}{3}\right)\right]$$

! $\cos\alpha + \cos\left(\alpha - \frac{2\pi}{3}\right) + \cos\left(\alpha - \frac{4\pi}{3}\right) = 0$!

Spatial Harmonic Components

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$$= 0$$

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$$+ \cos\left(3\xi - \omega_1 t - \frac{8\pi}{3}\right) + \cos\left(3\xi + \omega_1 t - \frac{16\pi}{3}\right)\Big]$$

$$= 0 \qquad = 0$$

Spatial Harmonic Components

- For certain harmonic components the result is:

  1) $\nu = 3$

  $$F_{m3} = 0$$

  … a three phase winding does not generate 3rd harmonic component

Spatial Harmonic Components

- For certain harmonic components the result is:
  2) $\nu = 5$

Spatial Harmonic Components

- For certain harmonic components the result is:

  2) $\nu = 5$

$$F_{m5} = \frac{F_{m15}}{2}\left[\cos\left(5\xi - \omega_1 t\right) + \cos\left(5\xi + \omega_1 t\right) + \right.$$
$$+ \cos\left(5\xi - \omega_1 t - (5-1)\frac{2\pi}{3}\right) + \cos\left(5\xi + \omega_1 t - (5+1)\frac{2\pi}{3}\right) +$$
$$\left. + \cos\left(5\xi - \omega_1 t - (5-1)\frac{4\pi}{3}\right) + \cos\left(5\xi + \omega_1 t - (5+1)\frac{4\pi}{3}\right)\right]$$

Spatial Harmonic Components

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$$+ \cos\left(5\xi - \omega_1 t - \frac{8\pi}{3}\right) + \cos\left(5\xi + \omega_1 t - \frac{12\pi}{3}\right) +$$
$$\left. + \cos\left(5\xi - \omega_1 t - \frac{16\pi}{3}\right) + \cos\left(5\xi + \omega_1 t - \frac{24\pi}{3}\right)\right]$$

Spatial Harmonic Components

- For certain harmonic components the result is:

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$$+ \cos\left(5\xi - \omega_1 t - \frac{8\pi}{3}\right) + \cos\left(5\xi + \omega_1 t - \frac{12\pi}{3}\right) +$$
$$\left. + \cos\left(5\xi - \omega_1 t - \frac{16\pi}{3}\right) + \cos\left(5\xi + \omega_1 t - \frac{24\pi}{3}\right)\right]$$
$$= 0$$

Spatial Harmonic Components

- For certain harmonic components the result is:

  2) $\nu = 5$

$$F_{m5} = \frac{3}{2} F_{m15} \cos\left(5\xi + \omega_1 t\right)$$

Spatial Harmonic Components

- For certain harmonic components the result is:

  2) $\nu = 5$

$$F_{m5} = \frac{3}{2} F_{m15} \cos\left(5\xi + \omega_1 t\right)$$

  … the 5th harmonic component creates a rotary magnetic field, which rotates in counter-direction in comparison to the fundamental harmonic component:

$$F_{m1} = \frac{m}{\pi} \cdot \frac{N_s k_{v1} I_m}{p} \cos\left(\xi - \omega_1 t\right)$$

Spatial Harmonic Components

- For certain harmonic components the result is:
  3) $\nu = 7$

Spatial Harmonic Components

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  3) $\nu = 7$

$$F_{m7} = \frac{F_{m17}}{2}\left[\cos(7\xi - \omega_1 t) + \cos(7\xi + \omega_1 t) + \right.$$
$$+ \cos\left(7\xi - \omega_1 t - (7-1)\frac{2\pi}{3}\right) + \cos\left(7\xi + \omega_1 t - (7+1)\frac{2\pi}{3}\right) +$$
$$\left. + \cos\left(7\xi - \omega_1 t - (7-1)\frac{4\pi}{3}\right) + \cos\left(7\xi + \omega_1 t - (7+1)\frac{4\pi}{3}\right)\right]$$

Spatial Harmonic Components

- For certain harmonic components the result is:

  3) $\nu = 7$

$$F_{m7} = \frac{F_{m17}}{2}\left[\cos\left(7\xi - \omega_1 t\right) + \cos\left(7\xi + \omega_1 t\right) + \right.$$
$$+ \cos\left(7\xi - \omega_1 t - \frac{12\pi}{3}\right) + \cos\left(7\xi + \omega_1 t - \frac{16\pi}{3}\right) +$$
$$\left. + \cos\left(7\xi - \omega_1 t - \frac{24\pi}{3}\right) + \cos\left(7\xi + \omega_1 t - \frac{32\pi}{3}\right)\right]$$

Spatial Harmonic Components

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  3) $\nu = 7$

$$F_{m7} = \frac{F_{m17}}{2}\left[\cos(7\xi - \omega_1 t) + \cos(7\xi + \omega_1 t) + \right.$$
$$+ \cos\left(7\xi - \omega_1 t - \frac{12\pi}{3}\right) + \cos\left(7\xi + \omega_1 t - \frac{16\pi}{3}\right) +$$
$$\left. + \cos\left(7\xi - \omega_1 t - \frac{24\pi}{3}\right) + \cos\left(7\xi + \omega_1 t - \frac{32\pi}{3}\right)\right]$$
$$= 0$$

Spatial Harmonic Components

- For certain harmonic components the result is:

  3) $\nu = 7$

$$F_{m7} = \frac{3}{2} F_{m17} \cos\left(7\xi - \omega_1 t\right)$$

Spatial Harmonic Components

- For certain harmonic components the result is:

  3) $\nu = 7$

$$F_{m7} = \frac{3}{2} F_{m17} \cos\left(7\xi - \omega_1 t\right)$$

… the 7th harmonic creates a rotary magnetic field, which rotates in the same direction as the field of fundamental harmonic component

$$F_{m1} = \frac{m}{\pi} \cdot \frac{N_s k_{v1} I_m}{p} \cos\left(\xi - \omega_1 t\right)$$

Spatial Harmonic Components

- By similar approach we can prove, that the magnetic field of a three phase machine will not include waves, which are multiplies of 3rd harmonic component:

$$\nu = 3k\,;k \in \mathbb{Z}$$

Spatial Harmonic Components

- By similar approach we can prove, that the magnetic field of a three phase machine will not include waves, which are multiplies of 3rd harmonic component:

$$\nu = 3k\,;k \in \mathbb{Z}$$

- On the other side we can prove, that it **will include** waves of order

$$\nu = 2km \pm 1\,;k \in \mathbb{Z}$$

whereas waves of order

$$\nu = 2km - 1\,;k \in \mathbb{Z}$$

will form fileds rotating in counter-direction and waves of order

$$\nu = 2km + 1\,;k \in \mathbb{Z}$$

will form fields rotating in direction of the fundamental harmonic component.

Spatial Harmonic Components

- Each of these fields may be written (for $m = 3$ phases) as

$$F_{m\nu}(\xi,t)=\frac{3}{2}F_{m1\nu}\cos(\nu\xi\mp\omega_1 t)$$

- And the resulting magnetic field will be a sum of all elementary fields:

$$F_{m}(\xi,t)=\frac{3}{2}\sum_{\nu=1}^{\infty}F_{m1\nu}\cos(\nu\xi\mp\omega_1 t)$$

- The presence of the third harmonic component may be an after-effect of following phenomena:
  - saturation of magnetic circuit
  - strongly saturated machines have a large volume of 3rd harmonic
    - superposition of magnetic fields of the stator and the rotor

Spatial Harmonic Components

- The presented foundations may be generalized for polyphase machines
  - Here the winding does not generate spatial harmonic components of order lower than the amount of machine phases and also it does not generate magnetic fields with order of multiplies of the amount of phases
  - The field of each harmonic component is described by

$$F_{m\nu}(\xi,t)=\frac{m}{2}F_{m1\nu}\cos(\nu\xi\mp\omega_1 t)$$

  - And the resulting field is then

$$F_{m}(\xi,t)=\frac{m}{2}\sum_{\nu=1}^{\infty}F_{m1\nu}\cos(\nu\xi\mp\omega_1 t)$$

where $\nu=2km\pm1;k\in\mathbb{Z}$

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Asynchronous Parasitic Torque Components of Induction Machines

Asynchronous Parasitic Torque Components of Induction Machines

- The presence of higher order harmonic components in the magnetic field leads several after-effects:
  - Higher order harmonic components from the magnetic filed induce corresponding harmonic components of voltage (and therefore current) into rotor winding
  - Higher order harmonic components of current create higher order harmonic components of magnetic field of the rotor
  - By superposition of harmonic components of the stator and the rotor higher order torque components appear
- Asynchronous torque components are caused by harmonic components rotating with different speeds

Asynchronous Parasitic Torque Components of Induction Machines

- Asynchronous torque components are caused by harmonic components rotating with different speeds
- The important variable is here the slip of the fundamental harmonic component

$$s_1 = \frac{\omega_1 - \omega}{\omega_1}$$

and slips of higher order harmonic components, which are defined in similar way:

$$s_\nu = \frac{\omega_\nu - \omega}{\omega_\nu}$$

or

$$s_\nu = \frac{\frac{\omega_1}{\nu} - \omega}{\frac{\omega_1}{\nu}} = \frac{\omega_1 - \nu\,\omega}{\omega_1}$$

Asynchronous Parasitic Torque Components of Induction Machines

- If the rotor frequency of the fundamental harmonic component is defined using slip as

$$f_2 = s f_1$$

  then there will be a similar definition of the rotor frequency of the ν–th harmonic component: $f_{2\nu} = s_\nu f_1$
- Let us presume, that the torque of each harmonic component may be represented by a virtual induction machine with its own torque characteristic
- This torque characteristic will cross the zero when:

$$s_\nu = \frac{\omega_1 - \nu\omega}{\omega_1} = 0$$

Asynchronous Parasitic Torque Components of Induction Machines

- This corresponds to position: $\omega_1 - \nu\omega = 0$

$$\omega = \frac{\omega_1}{\nu}$$

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- The exact position of the cross-section of the torque characteristic of a virtual machine with the slip axis will depend on the order of the harmonic component and its direction:

$$\omega = \frac{\omega_1}{\nu} =$$

Asynchronous Parasitic Torque Components of Induction Machines

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- The exact position of the cross-section of the torque characteristic of a virtual machine with the speed axis will depend on the order of the harmonic component and its direction:

$$\omega = \frac{\omega_1}{\nu} = \left\{ \frac{\omega_1}{7}; \frac{\omega_1}{13}; \frac{\omega_1}{19}; \ldots \right.$$

Asynchronous Parasitic Torque Components of Induction Machines

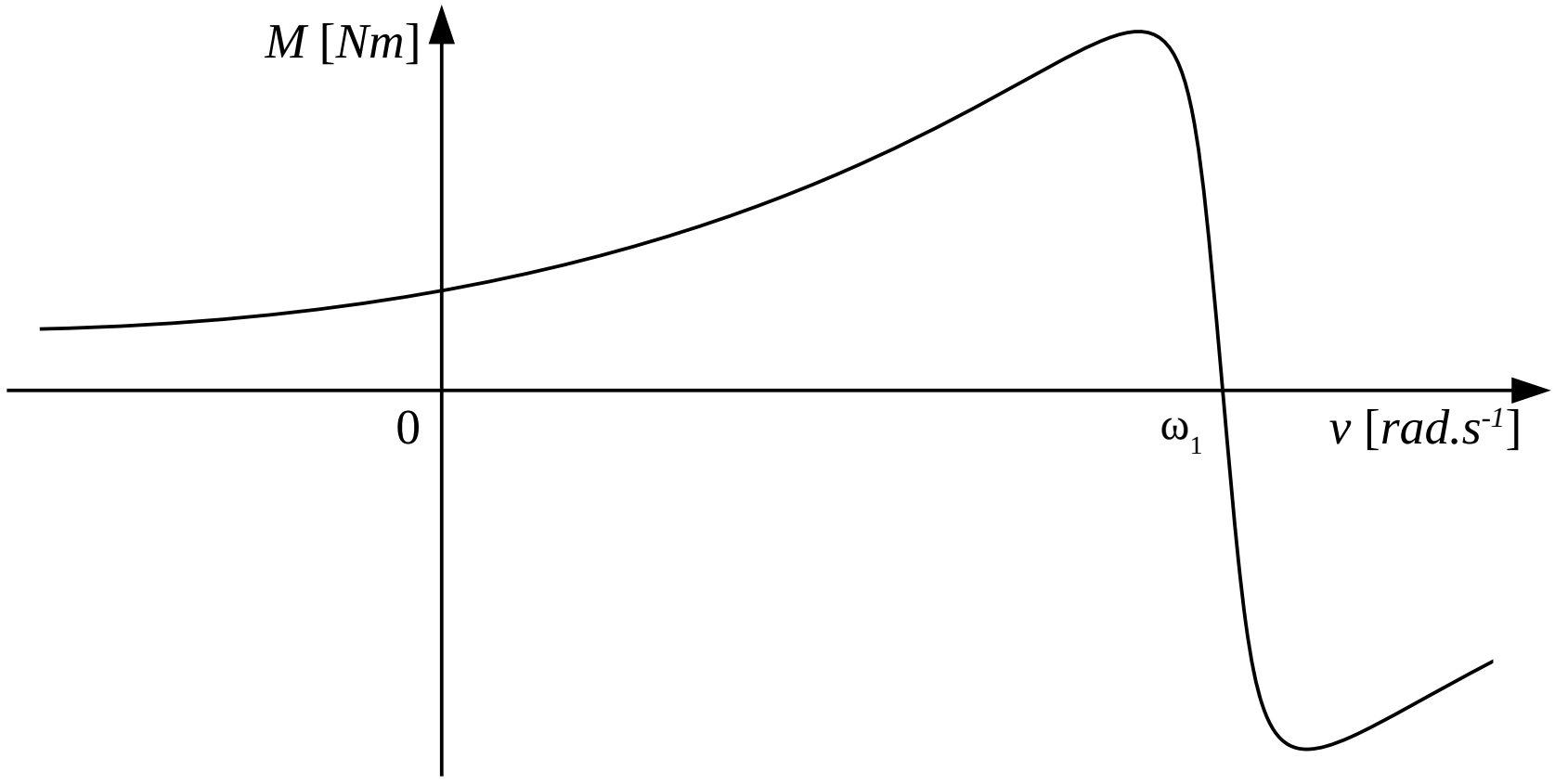
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$$\omega = \frac{\omega_1}{\nu}$$

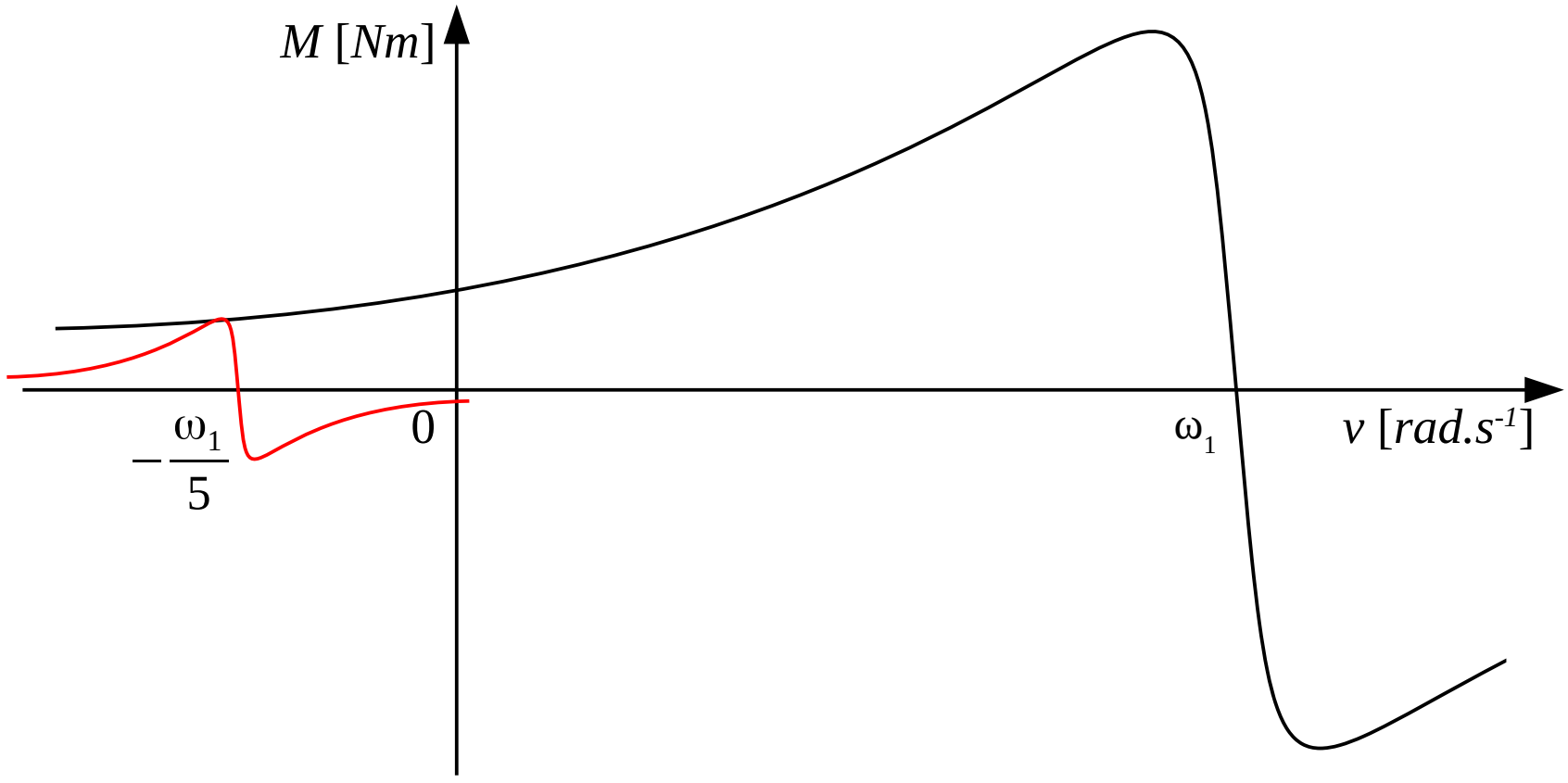
- The exact position of the cross-section of the torque characteristic of a virtual machine with the speed axis will depend on the order of the harmonic component and its direction:

$$\omega = \frac{\omega_1}{\nu} = \begin{cases} \frac{\omega_1}{7}; \frac{\omega_1}{13}; \frac{\omega_1}{19}; \dots \\ -\frac{\omega_1}{5}; -\frac{\omega_1}{11}; -\frac{\omega_1}{17}; \dots \end{cases}$$

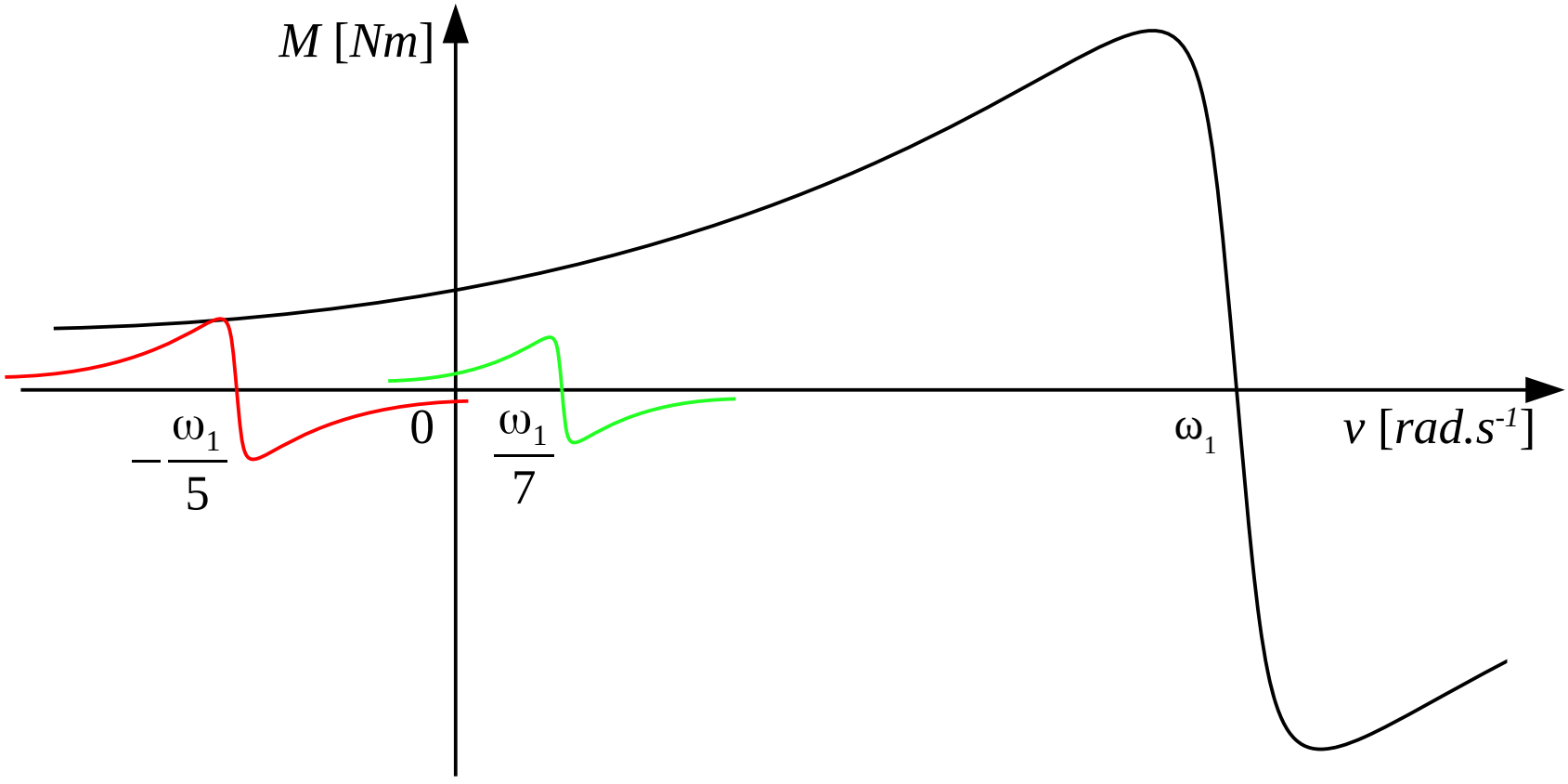
Asynchronous Parasitic Torque Components of Induction Machines



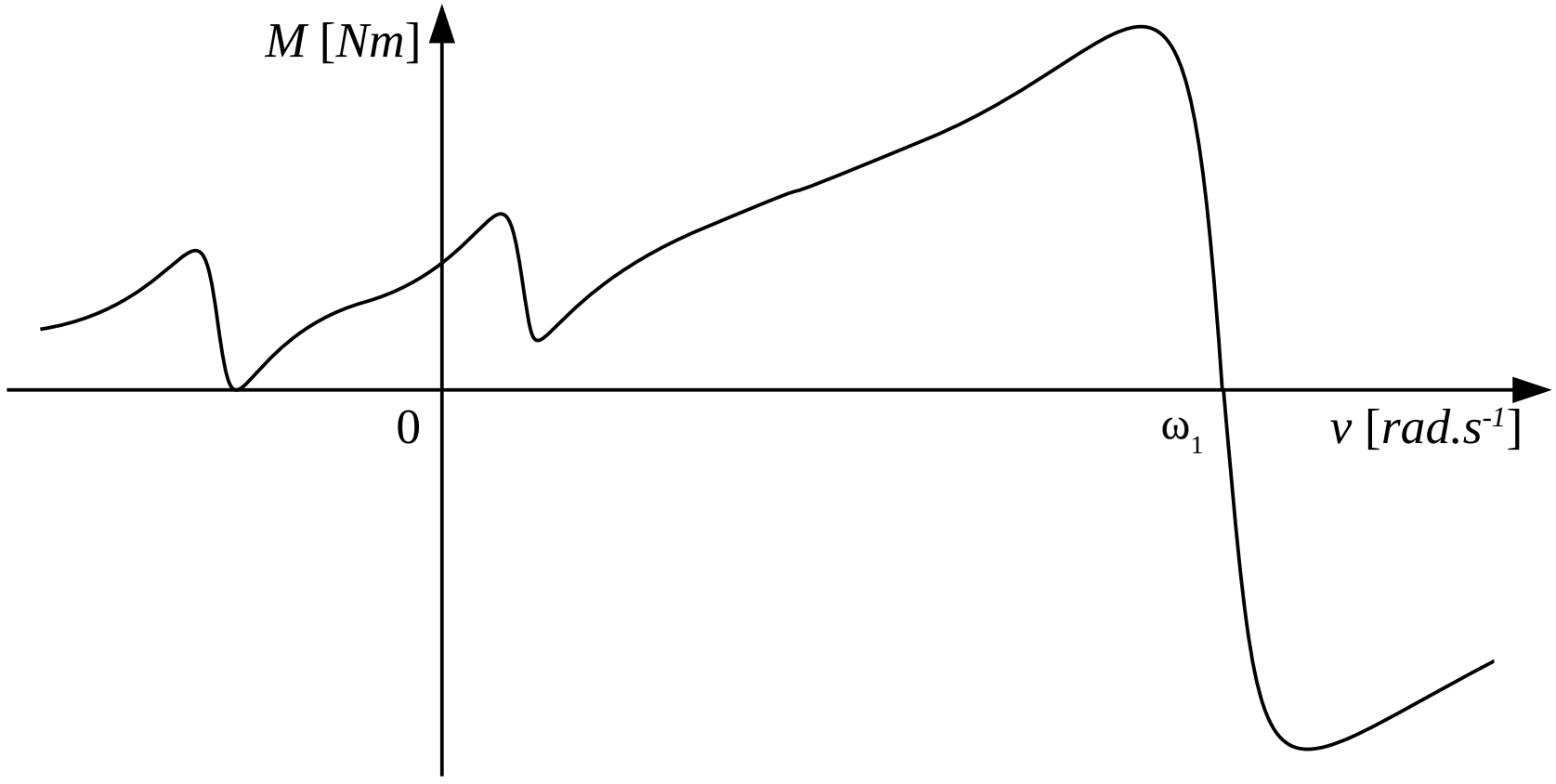
Asynchronous Parasitic Torque Components of Induction Machines



Asynchronous Parasitic Torque Components of Induction Machines



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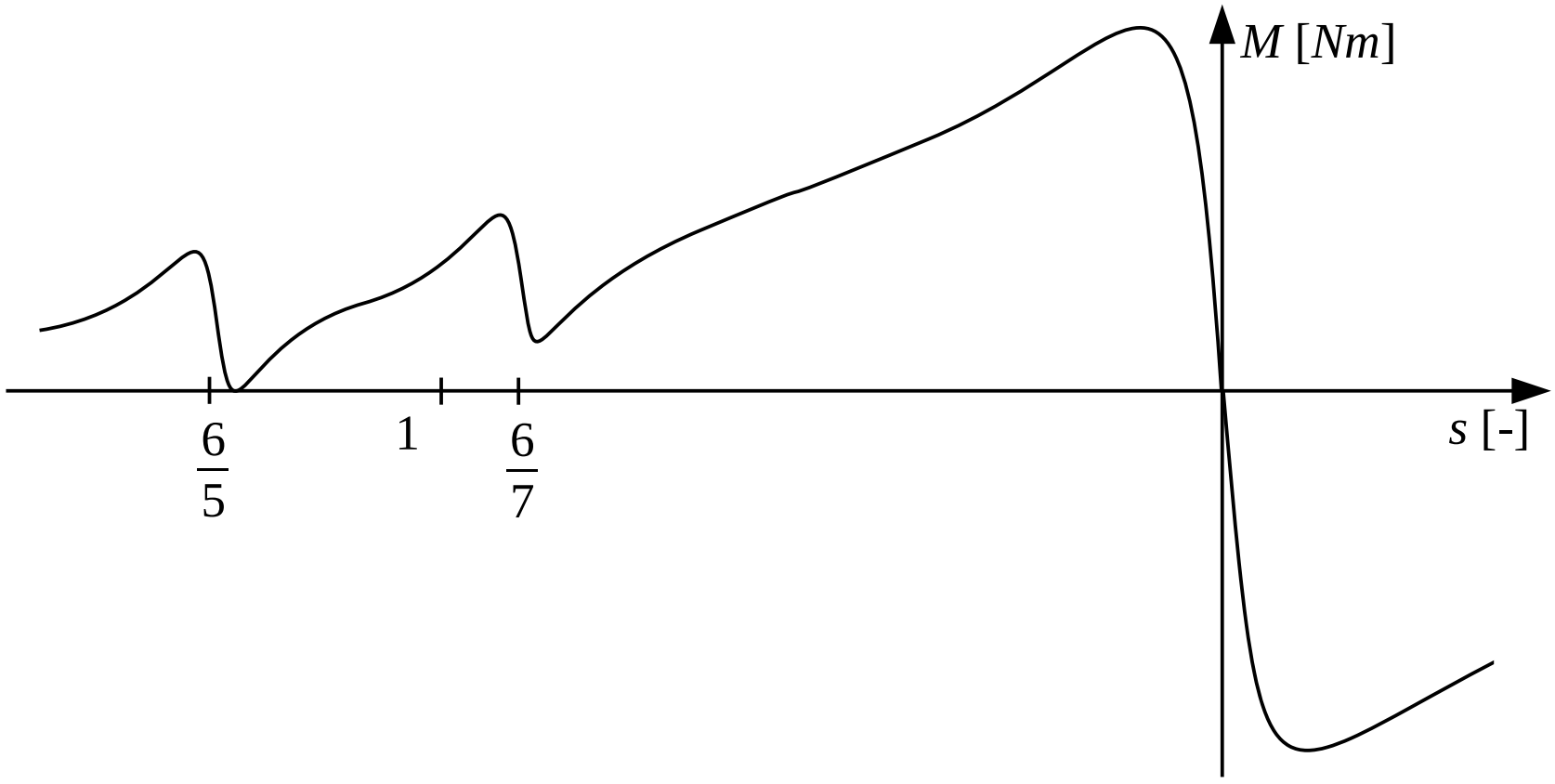


Asynchronous Parasitic Torque Components of Induction Machines

- From the slip point of view their position is given by

$$s_1 = \frac{\omega_1 - \frac{\omega_1}{\nu}}{\omega_1} = \frac{\nu - 1}{\nu}$$

Asynchronous Parasitic Torque Components of Induction Machines



Asynchronous Parasitic Torque Components of Induction Machines

- For the rotor harmonic components the situation is slightly different – their speed against rotor is given by

$$\omega_r = s_1 \omega_1$$

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$$\omega_{r\nu} = \frac{\omega_1 - \omega}{\omega_1} \cdot \frac{\omega_1}{\nu}$$

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$$\omega_{r\nu} = \frac{\omega_1 - \omega}{\nu}$$

Asynchronous Parasitic Torque Components of Induction Machines

- These harmonic components will induce back into the stator harmonic components on speed resulting from the sum of their speed with the rotor speed:

$$\omega_{1\nu} = \omega_{r\nu} + \omega$$

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$$\omega_{1\nu} = \frac{\omega_1 - \omega + \nu\,\omega}{\nu}$$

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$$\omega_{1\nu} = \omega_{r\nu} + \omega$$

$$\omega_{1\nu} = \frac{\omega_1 - \omega}{\nu} + \omega$$

$$\omega_{1\nu} = \frac{\omega_1 - \omega + \nu\,\omega}{\nu}$$

- These harmonic components will again form “torque saddles”

Asynchronous Parasitic Torque Components of Induction Machines

- Torque saddles from rotor harmonics will have zero value at

$$\omega_{1\nu} = \frac{\omega_1 - \omega + \nu\,\omega}{\nu} = 0$$

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$$\omega = \frac{\omega_1}{1 - \nu}$$

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$$\omega = \frac{\omega_1}{1 - \nu}$$

  which corresponds with slip of

$$s_1 = \frac{-\nu}{1 - \nu}$$

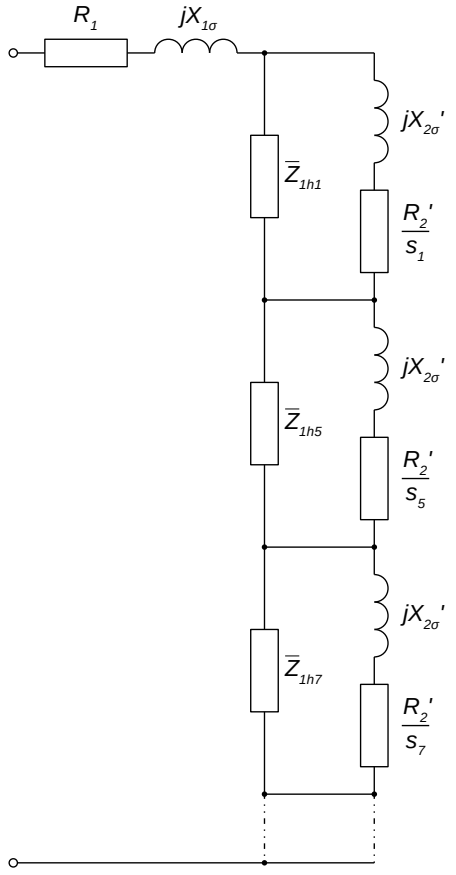
Asynchronous Parasitic Torque Components of Induction Machines

- In form of a table:

	torque characteristic position (slip)	
ν	stator harmonics	rotor harmonics
-5	6/5	5/6
7	6/7	7/6
-11	12/11	11/12
13	12/13	13/12
	$s_1 = \frac{\nu - 1}{\nu}$	$s_1 = \frac{-\nu}{1 - \nu}$

Asynchronous Parasitic Torque Components of Induction Machines

- From the functional point of view the machine appears like it was composed of independent machines, one per each harmonic component
- This may be in certain cases used for obtaining additional torque (especially in combination with injection of time harmonic components)
- The equivalent circuit of the induction machine with inclusion of higher order harmonic components:



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Synchronous Parasitic Torque Components of Induction Machines

Synchronous Parasitic Torque Components of Induction Machines

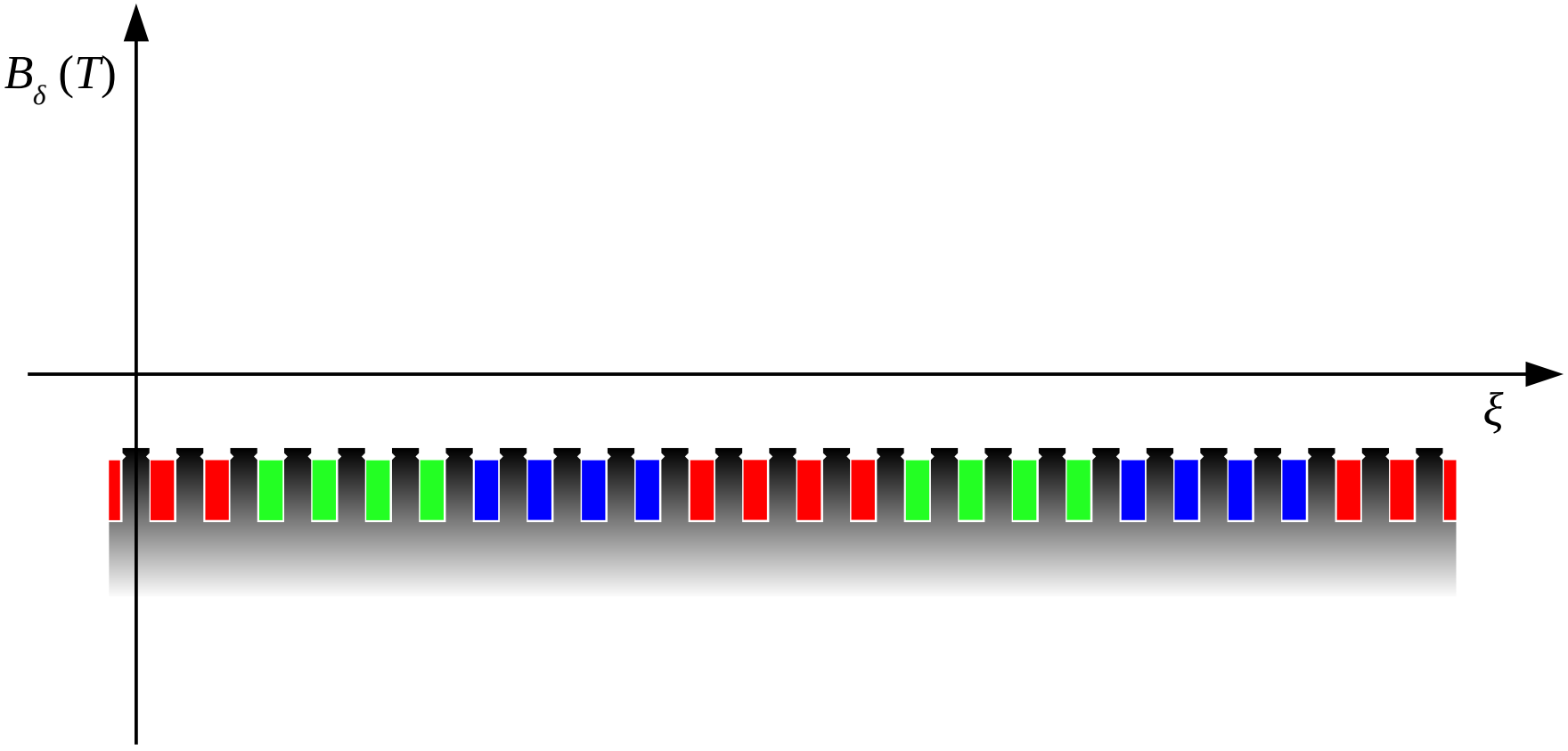
- Synchronous parasitic torque components are caused by harmonic components rotating at identical speeds
- These harmonic components are caused by stator and rotor slots
- They form actually a parasitic reluctance torque between the stator and the rotor
- An extreme example is a machine with identical amount of stator and rotor slots Q_1 and Q_2, which may not start at all
- Thus it is unsuitable to have $Q_1 = Q_2$ while similar effect is achieved with

$$Q_1 \mp p = Q_2 \pm p$$

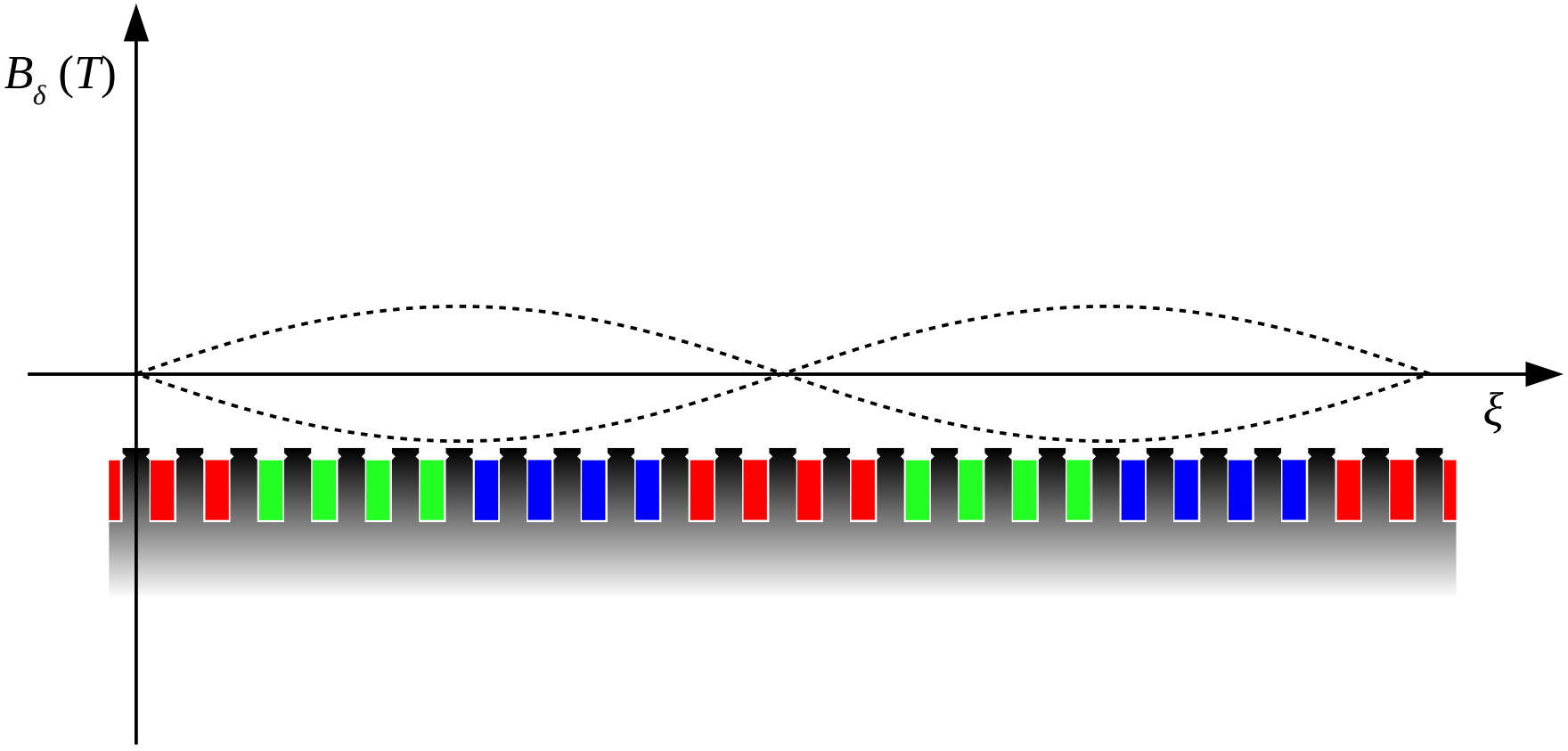
Synchronous Parasitic Torque Components of Induction Machines

- Simply it is possible to imagine the slot harmonic as a presence of a beat with slot harmonic component superimposed to the fundamental harmonic component
  - The magnitude of the beat is equal to the magnitude of the slot harmonic component and it has the wavelength of the fundamental harmonic component
  - Into this the wave of variable magnitude with the wavelength of the slot harmonic is written
  - All of this is superimposed on the wave of the fundamental harmonic component

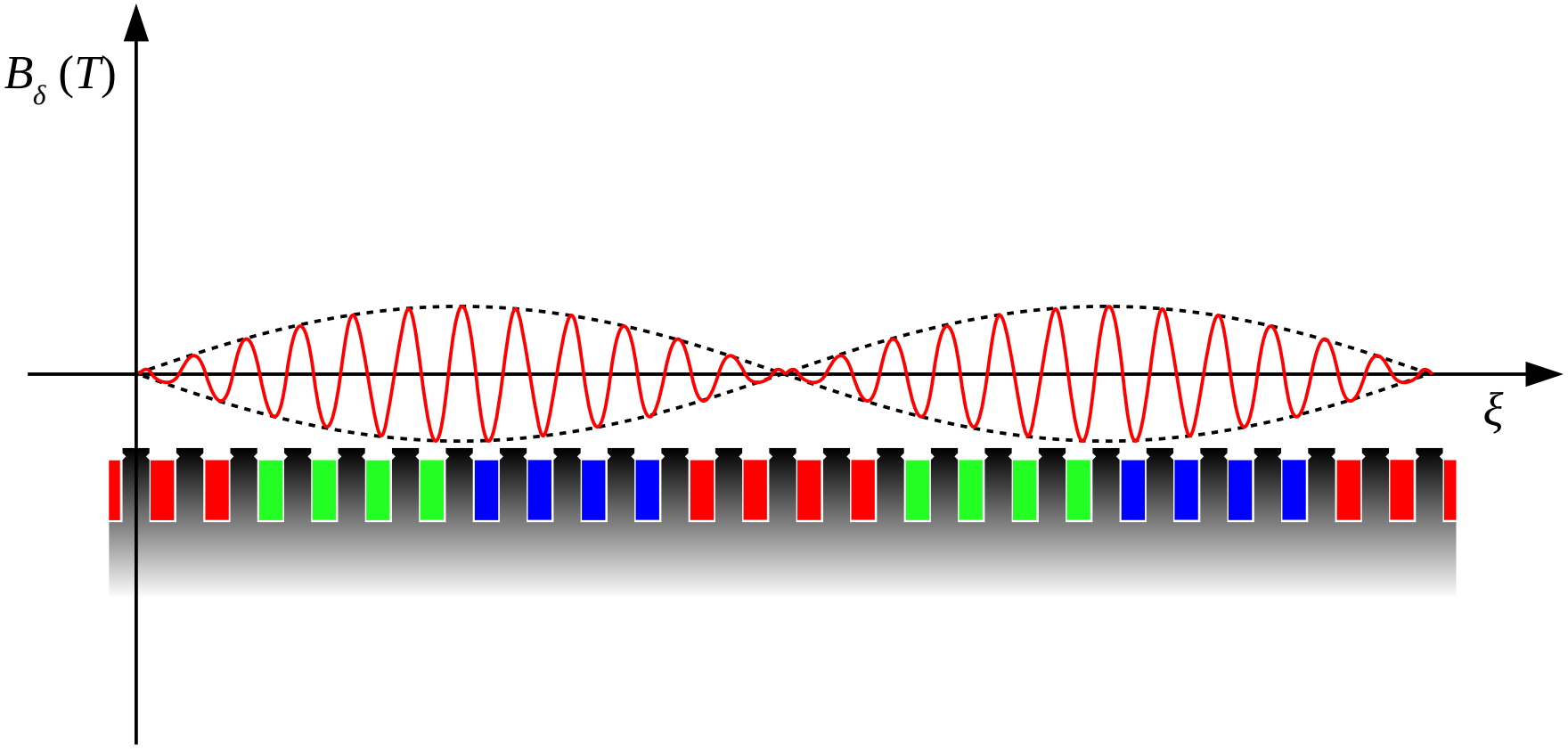
Synchronous Parasitic Torque Components of Induction Machines



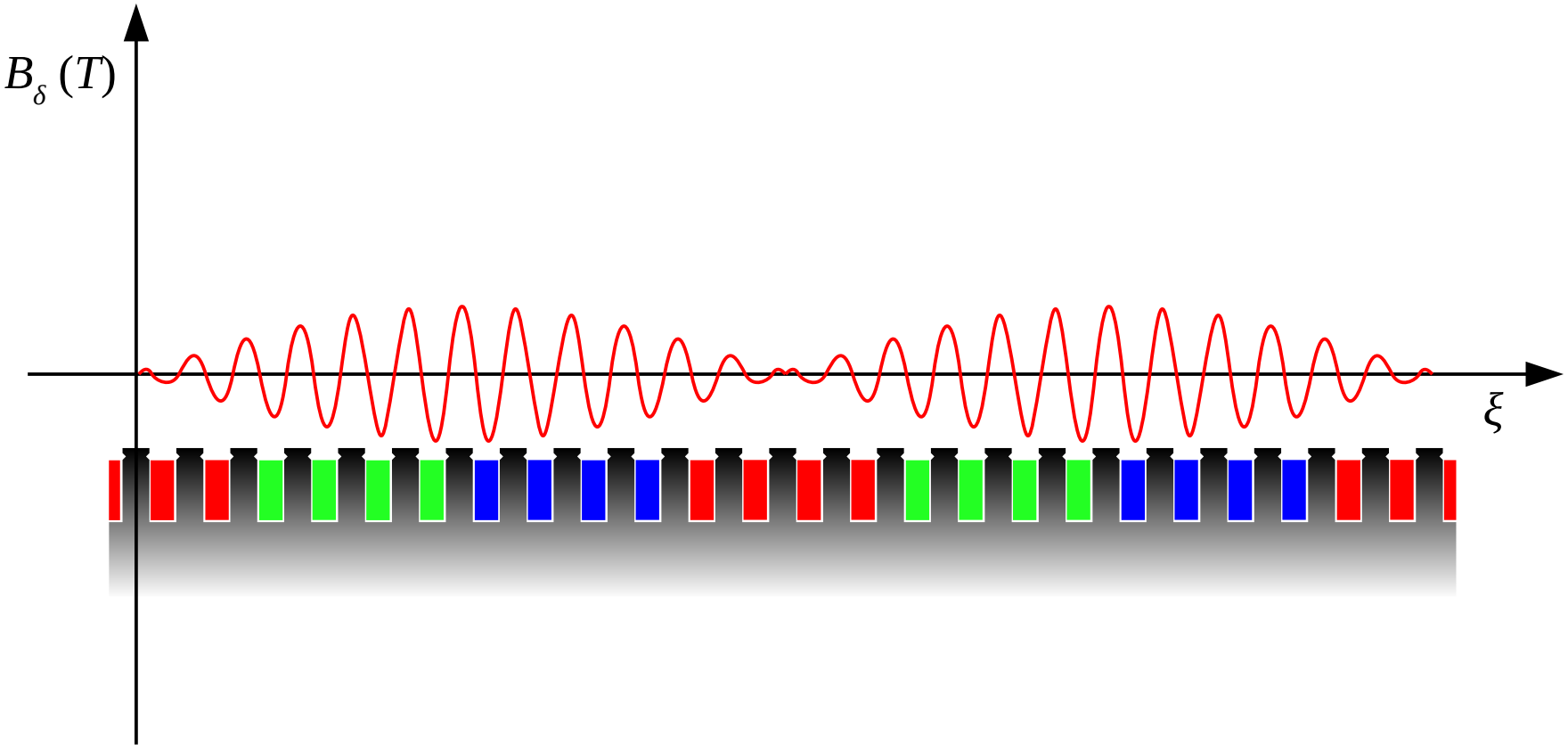
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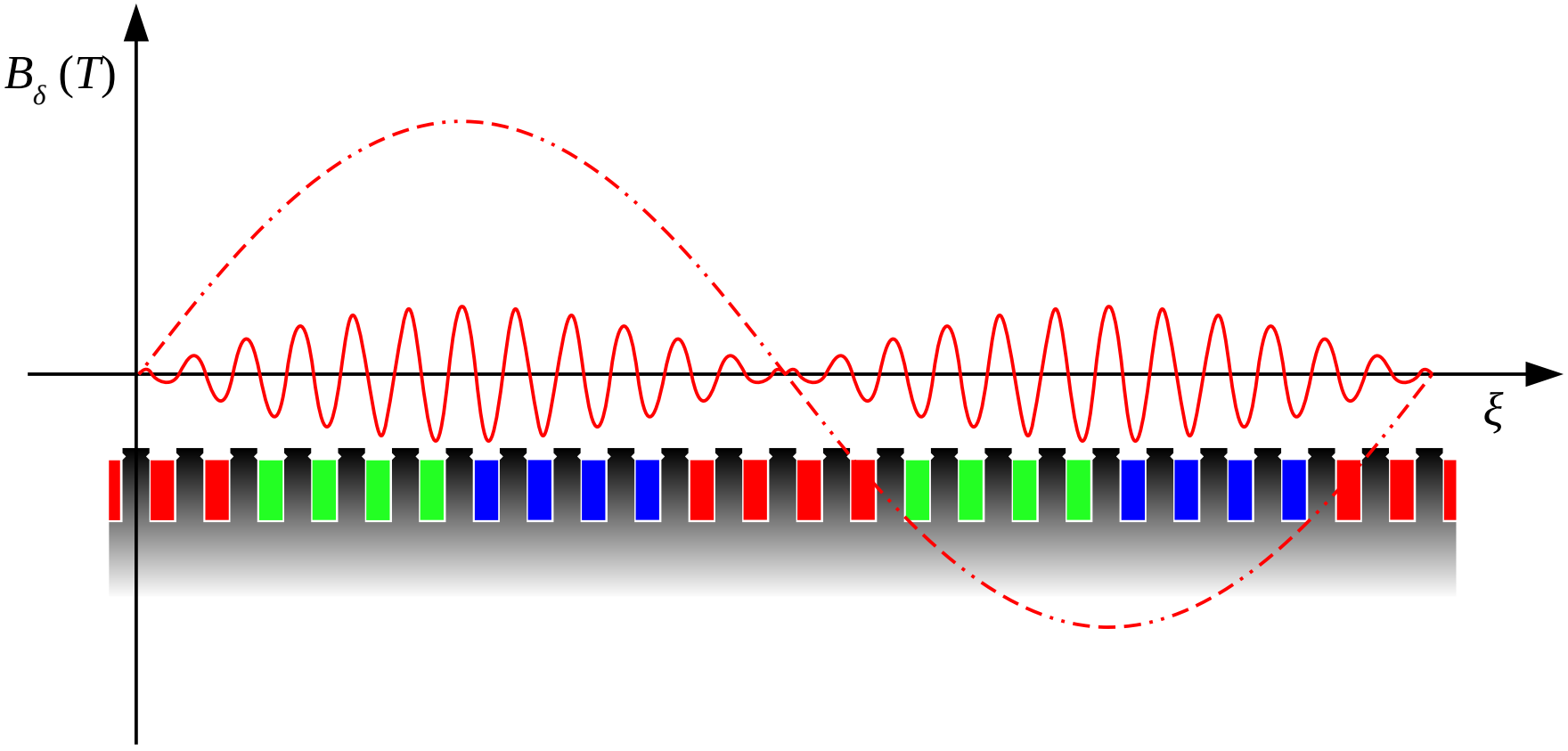
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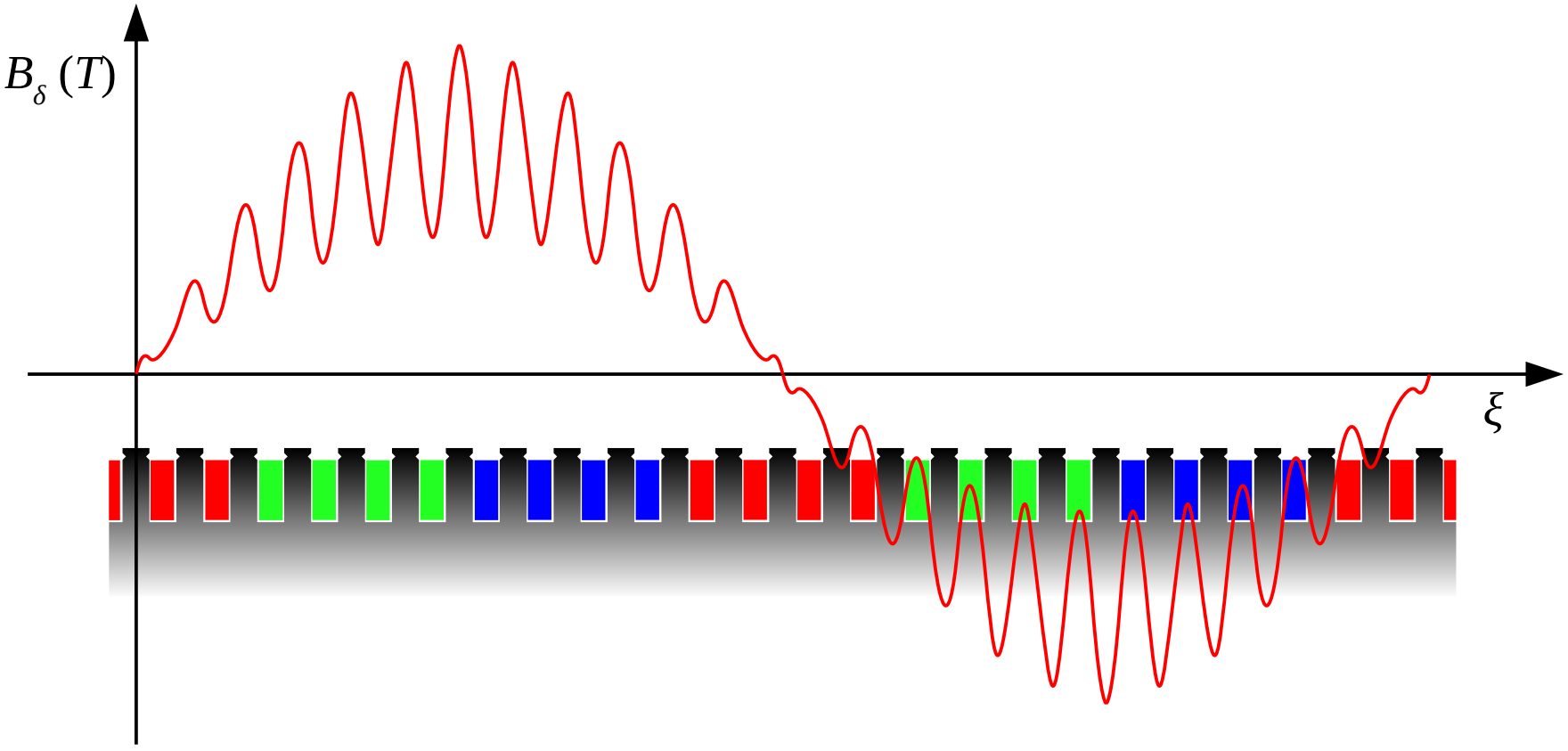
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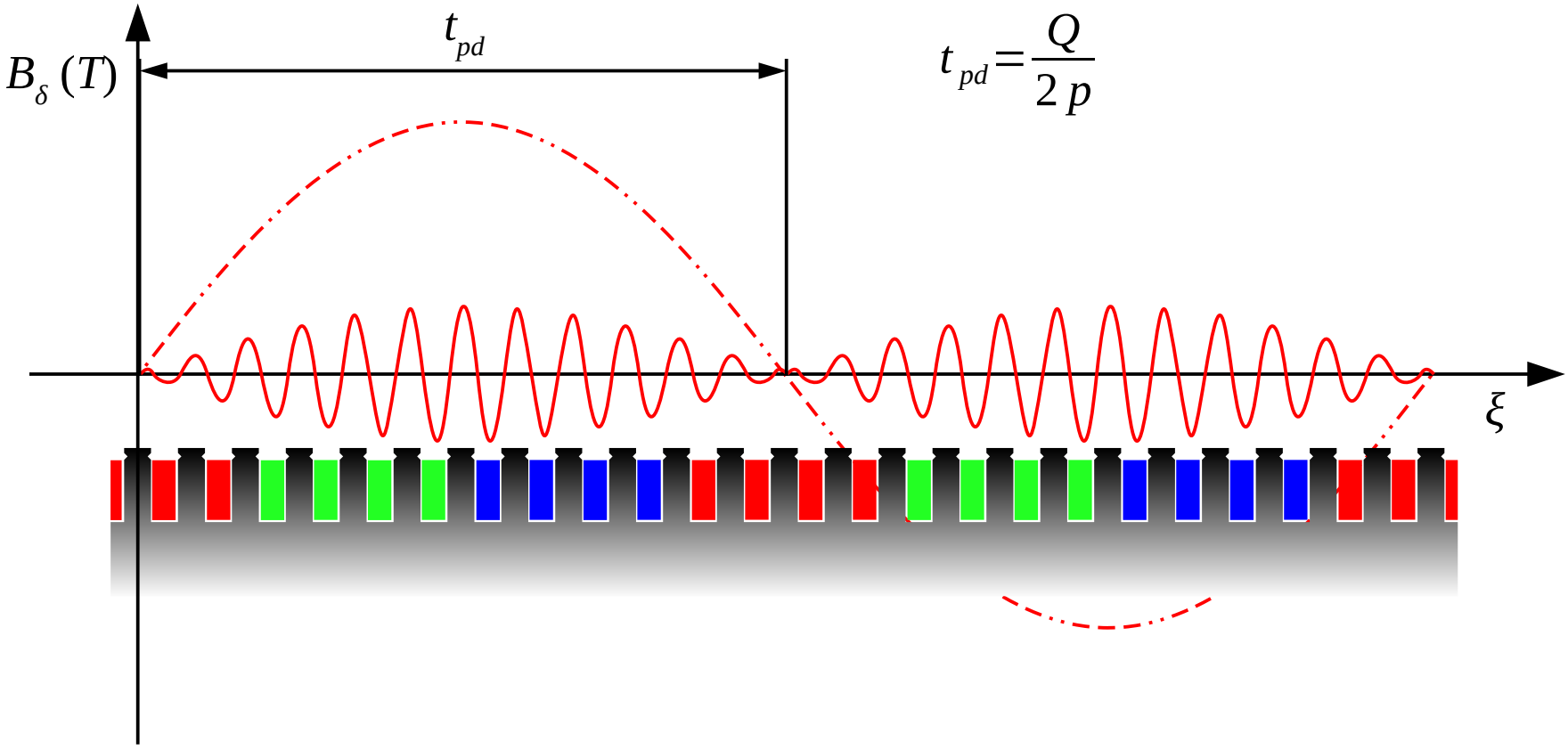
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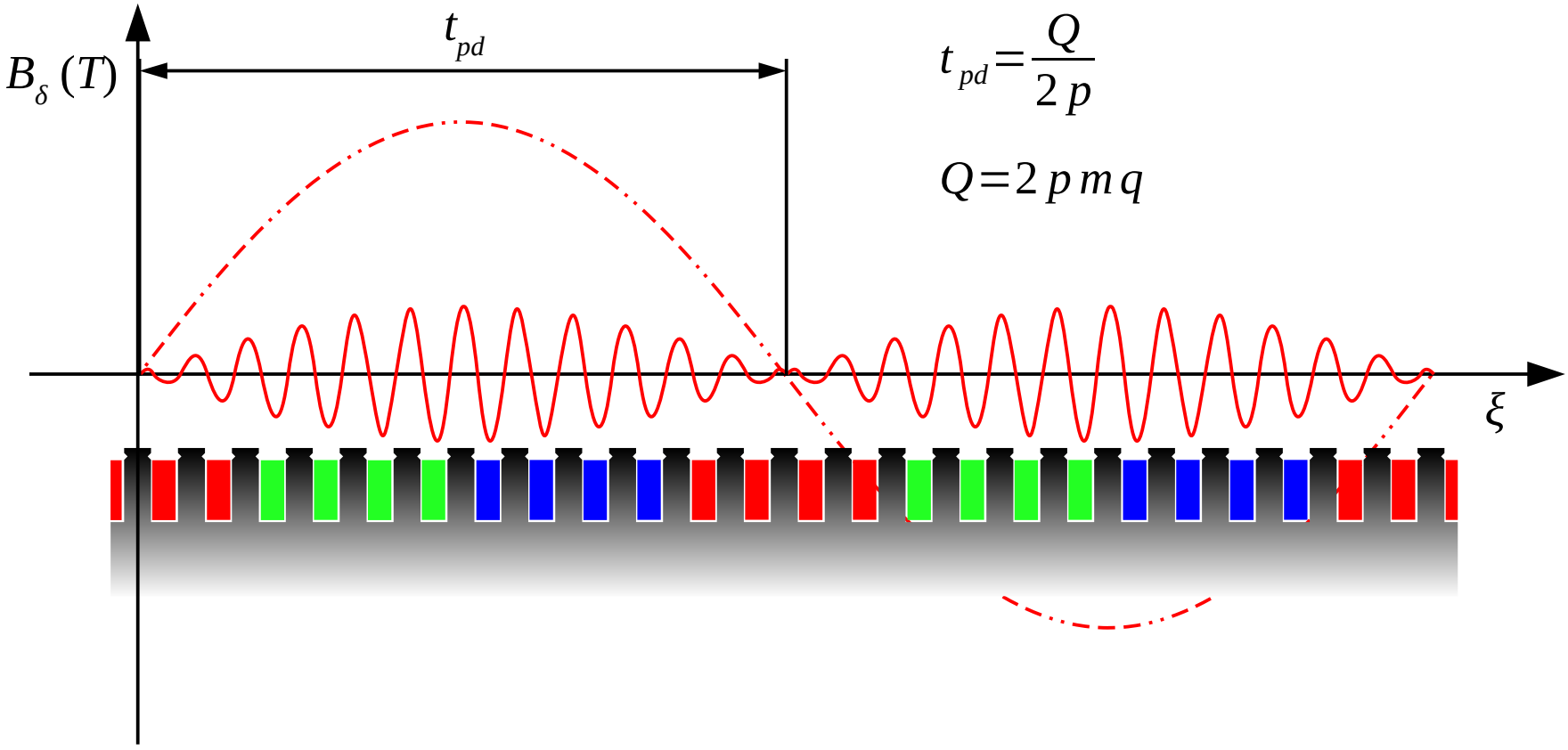
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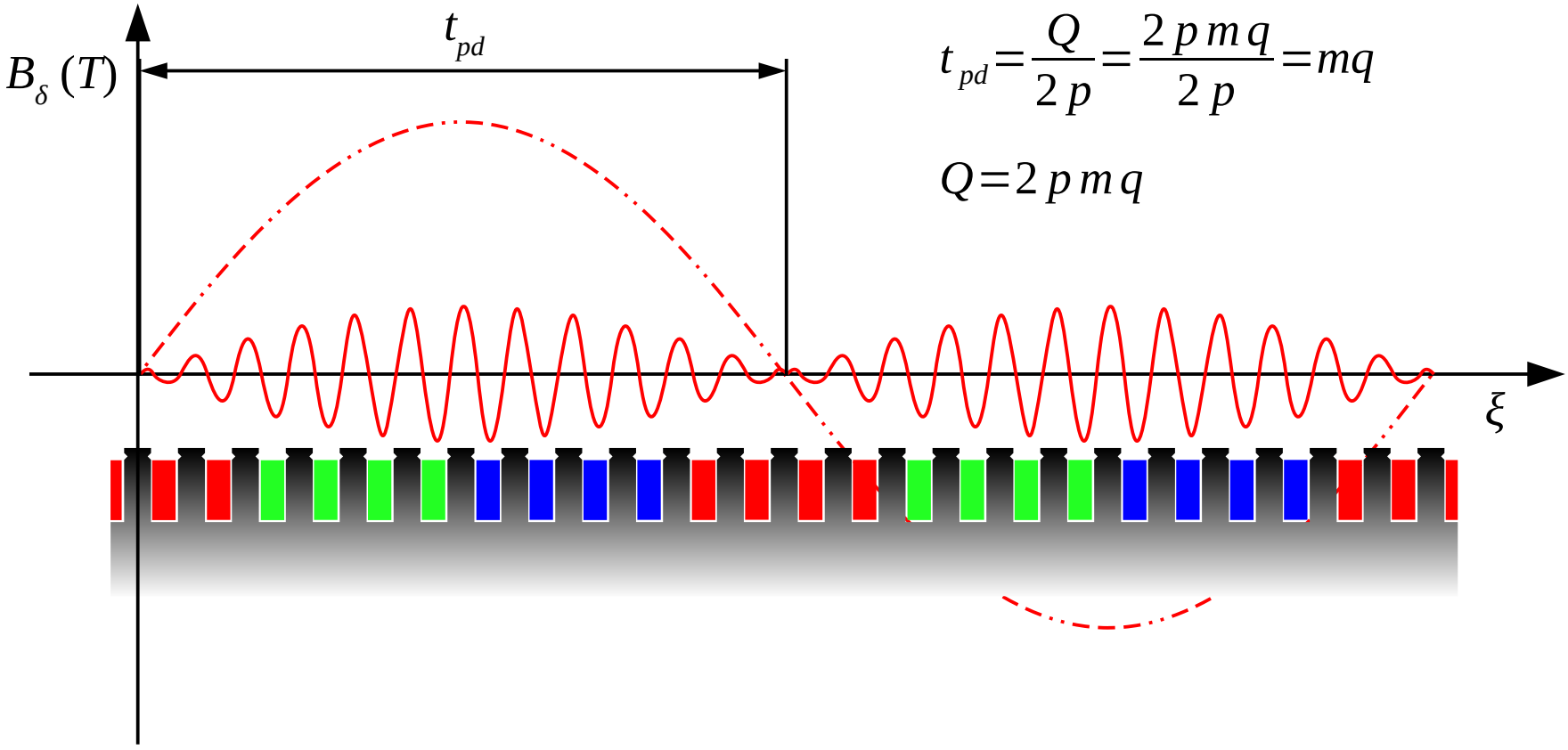
Synchronous Parasitic Torque Components of Induction Machines



$$t_{pd} = \frac{Q}{2p}$$

$$Q = 2pmq$$

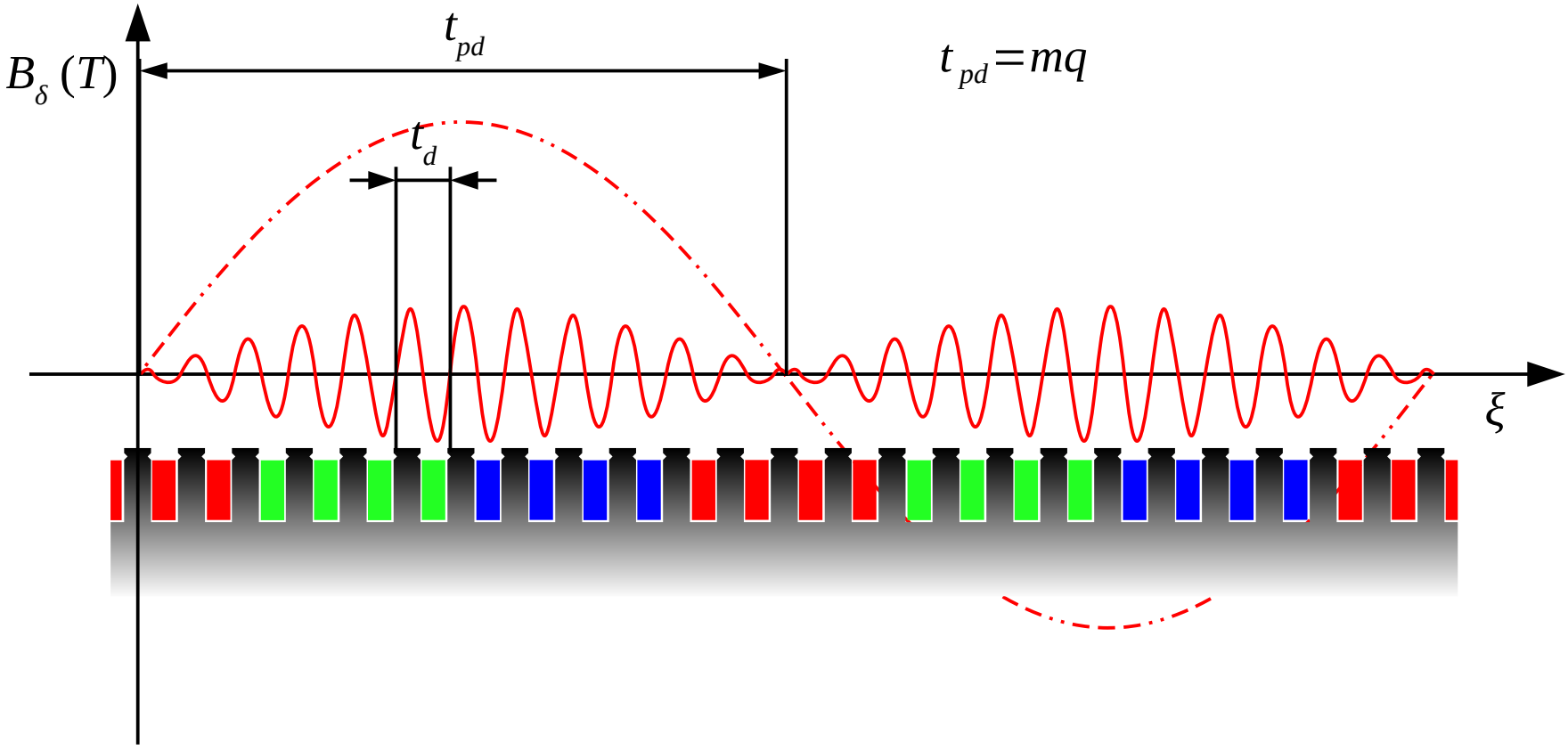
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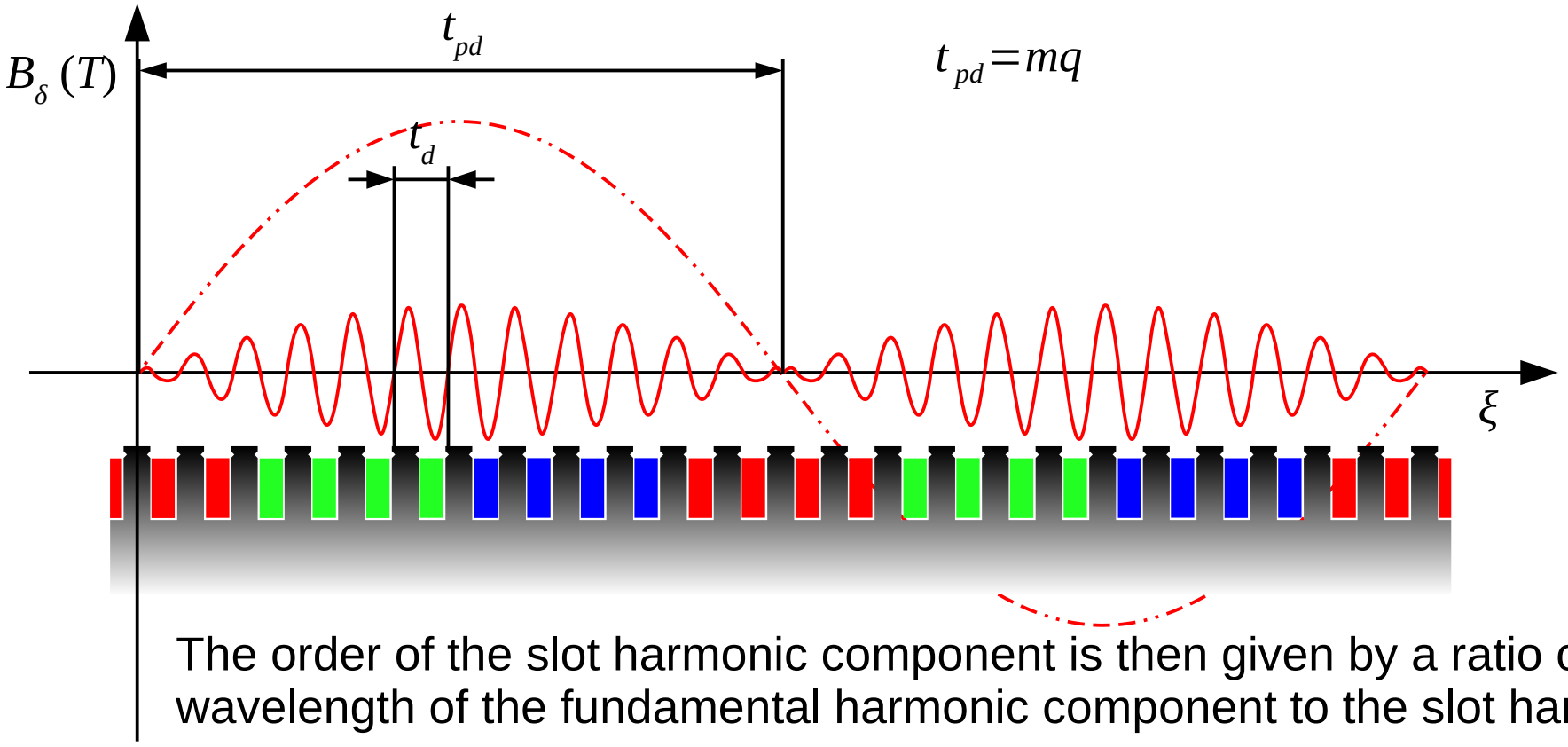
$$t_{pd} = \frac{Q}{2p} = \frac{2pmq}{2p} = mq$$

$$Q = 2pmq$$

Synchronous Parasitic Torque Components of Induction Machines

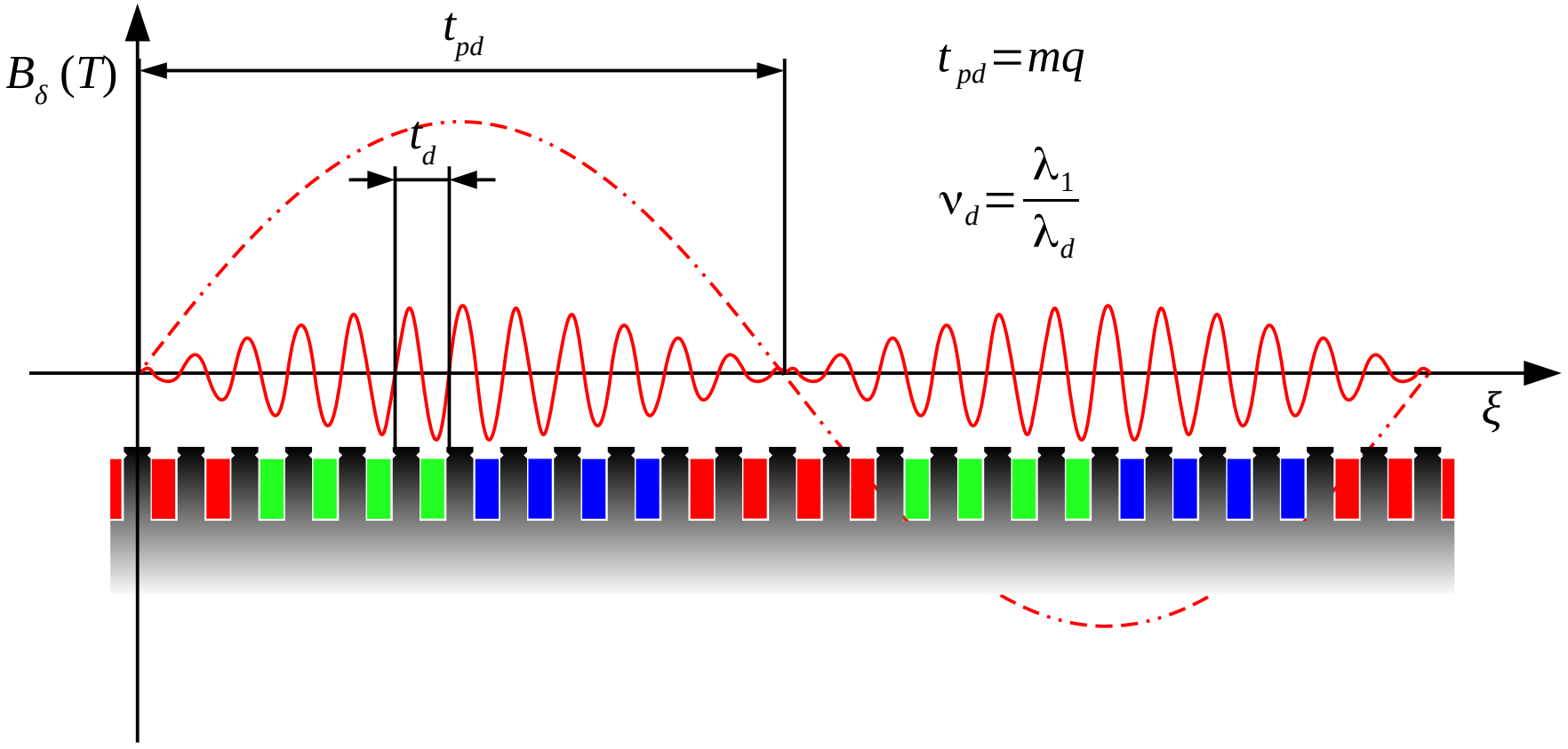


Synchronous Parasitic Torque Components of Induction Machines

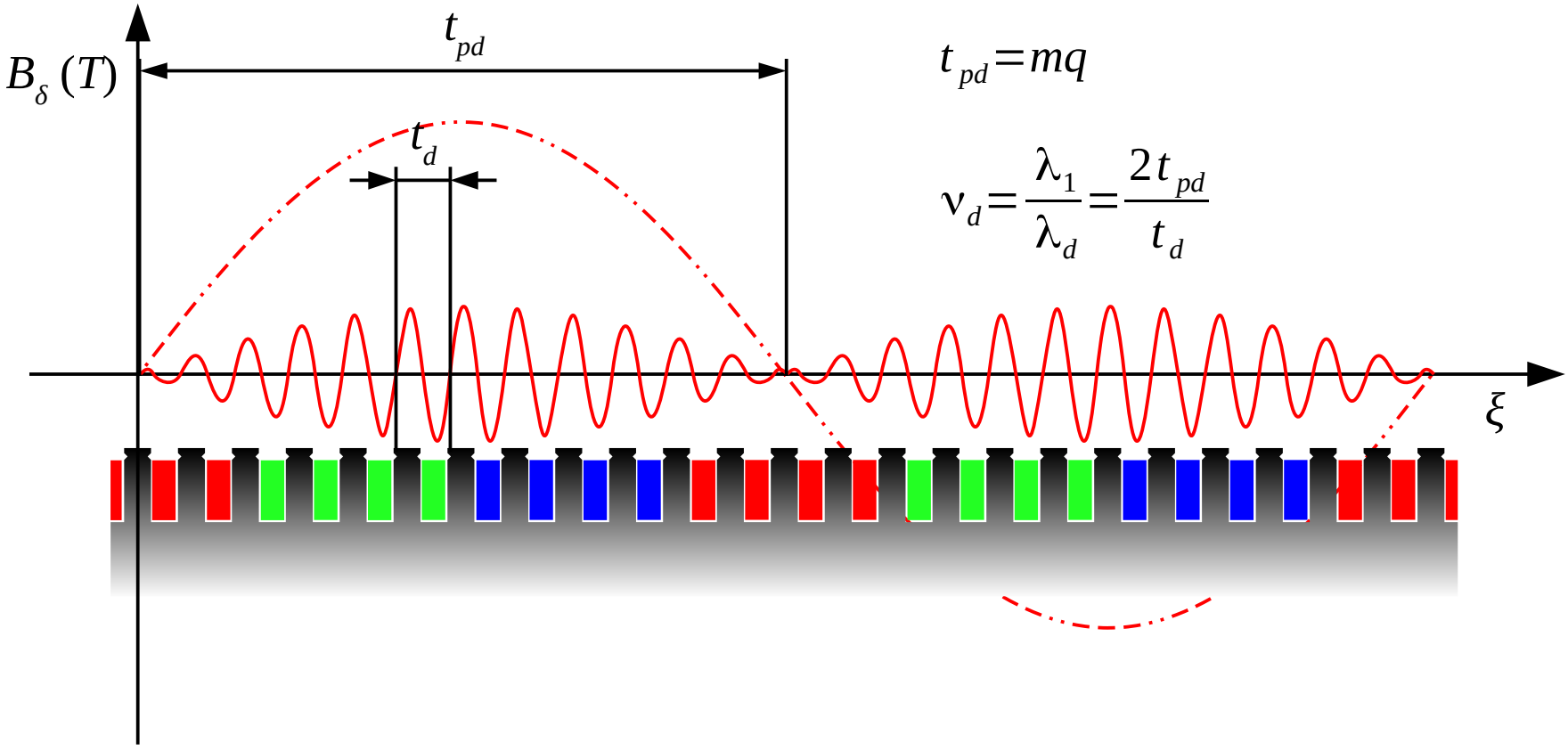


The order of the slot harmonic component is then given by a ratio of the wavelength of the fundamental harmonic component to the slot harmonic...

Synchronous Parasitic Torque Components of Induction Machines



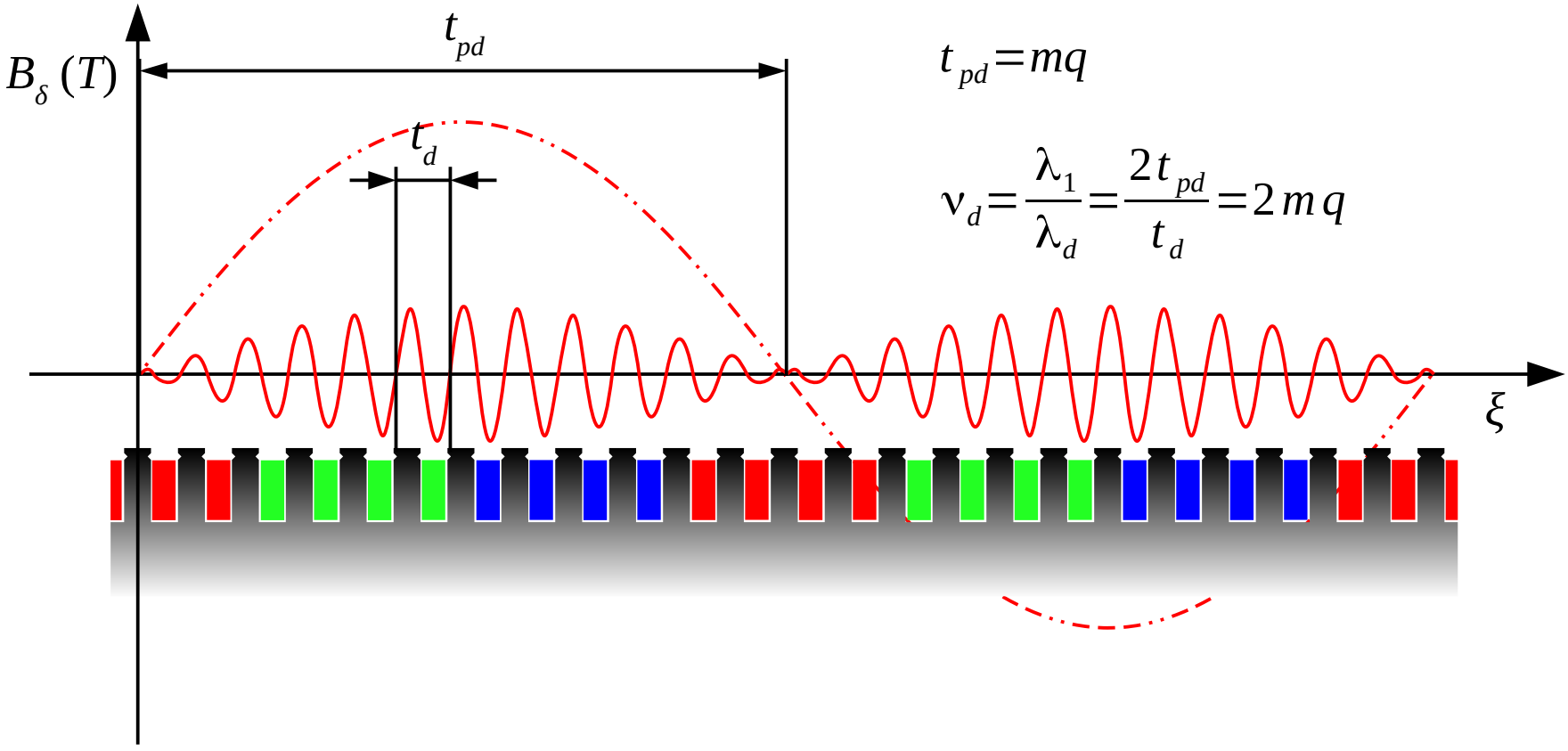
Synchronous Parasitic Torque Components of Induction Machines



$$t_{pd} = mq$$

$$\nu_d = \frac{\lambda_1}{\lambda_d} = \frac{2t_{pd}}{t_d}$$

Synchronous Parasitic Torque Components of Induction Machines



$$t_{pd} = mq$$

$$\nu_d = \frac{\lambda_1}{\lambda_d} = \frac{2t_{pd}}{t_d} = 2mq$$

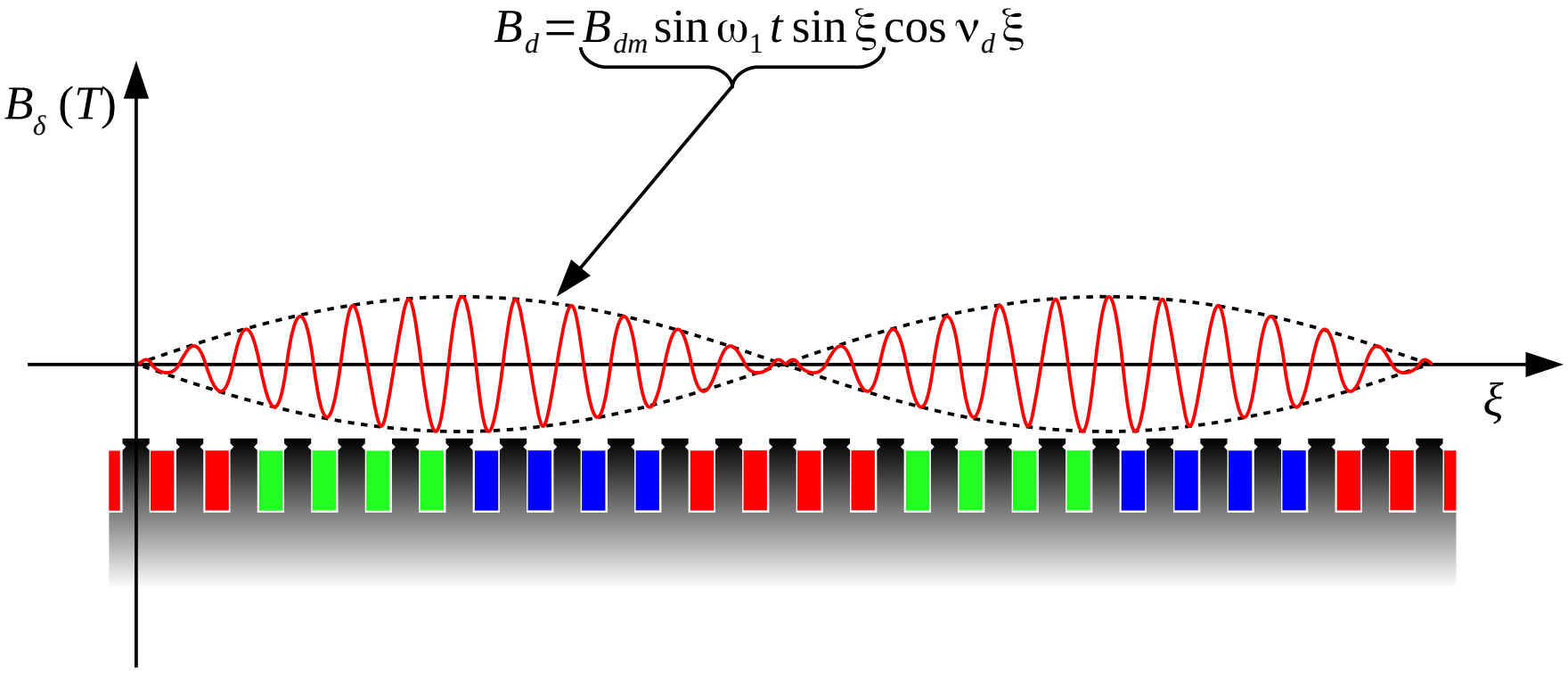
Synchronous Parasitic Torque Components of Induction Machines

- The beats may be described by

$$B_d = B_{dm} \sin \omega_1 t \sin \xi \cos \nu_d \xi$$

Synchronous Parasitic Torque Components of Induction Machines

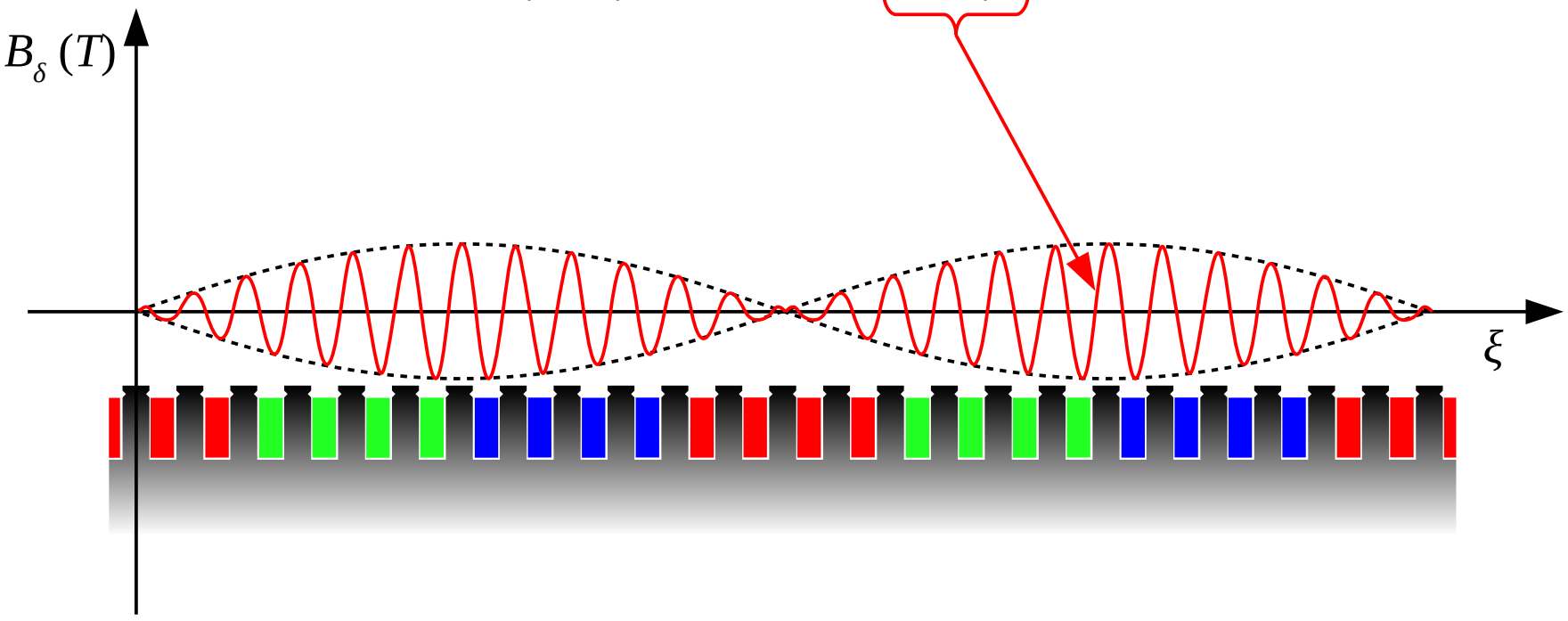
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after modification using sum relation

$$\sin x \cos y = \frac{1}{2}\left[\sin\left(x+y\right)+\sin\left(x-y\right)\right]$$

Synchronous Parasitic Torque Components of Induction Machines

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$$B_d = B_{dm} \sin \omega_1 t \sin \xi \cos \nu_d \xi$$

after modification using sum relation

$$\sin x \cos y = \frac{1}{2}\left[\sin\left(x+y\right)+\sin\left(x-y\right)\right]$$

we obtain

$$B_d = \frac{B_{dm}}{2} \sin \omega_1 t \left[\sin\left(\nu_d+1\right)\xi+\sin\left(\nu_d-1\right)\xi\right]$$

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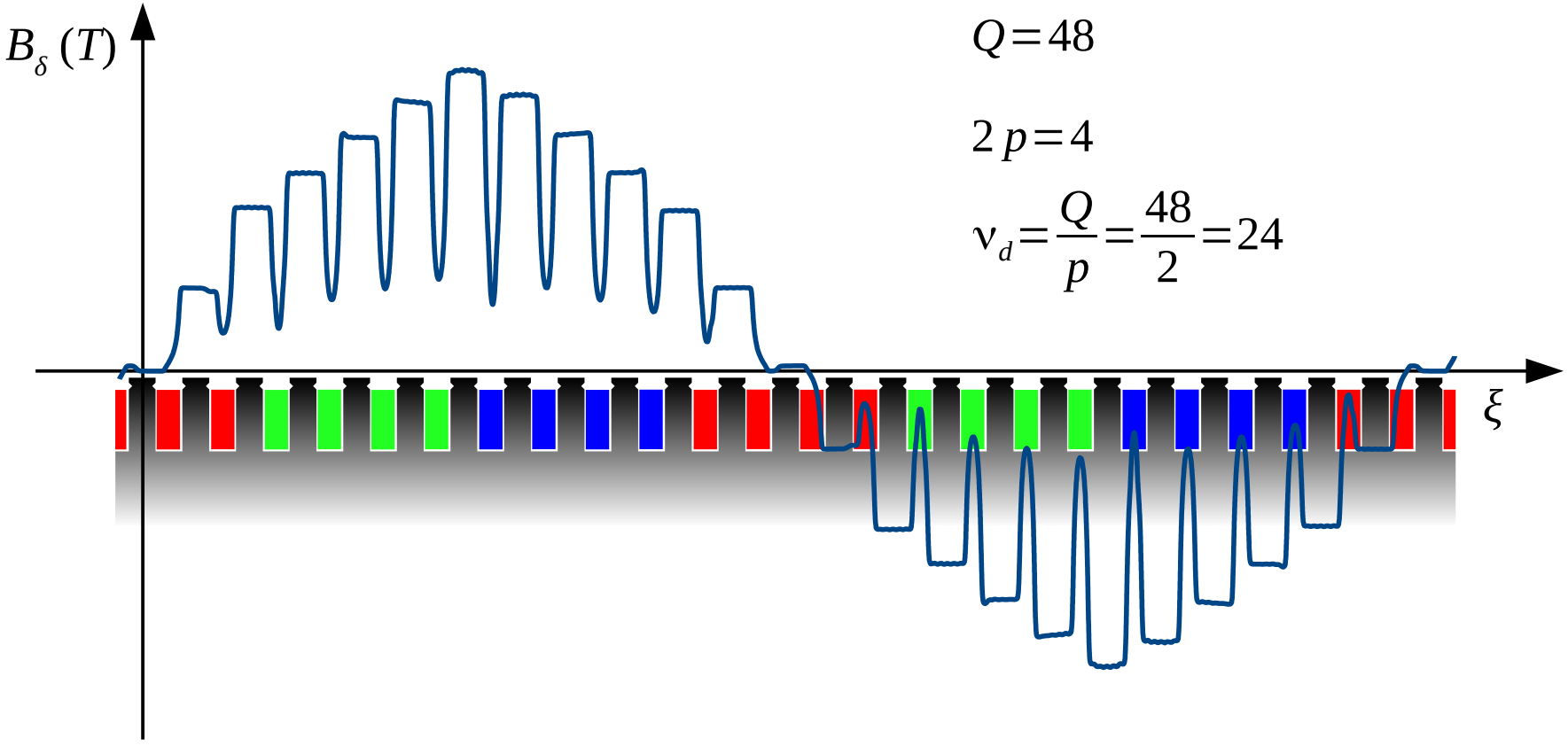
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- Thus it is obvious, that after performing the Fourier analysis harmonic components of order $\nu = \nu_d \pm 1$ appear

Synchronous Parasitic Torque Components of Induction Machines

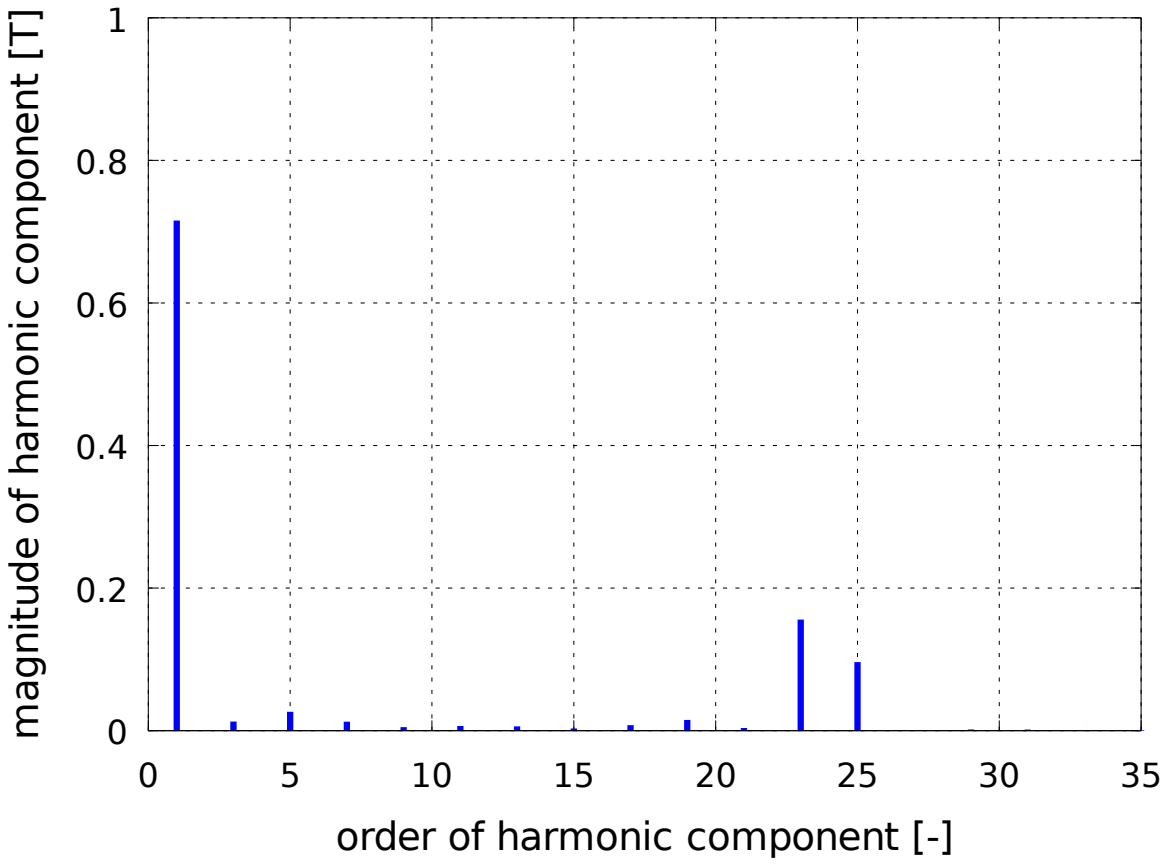


$$Q = 48$$

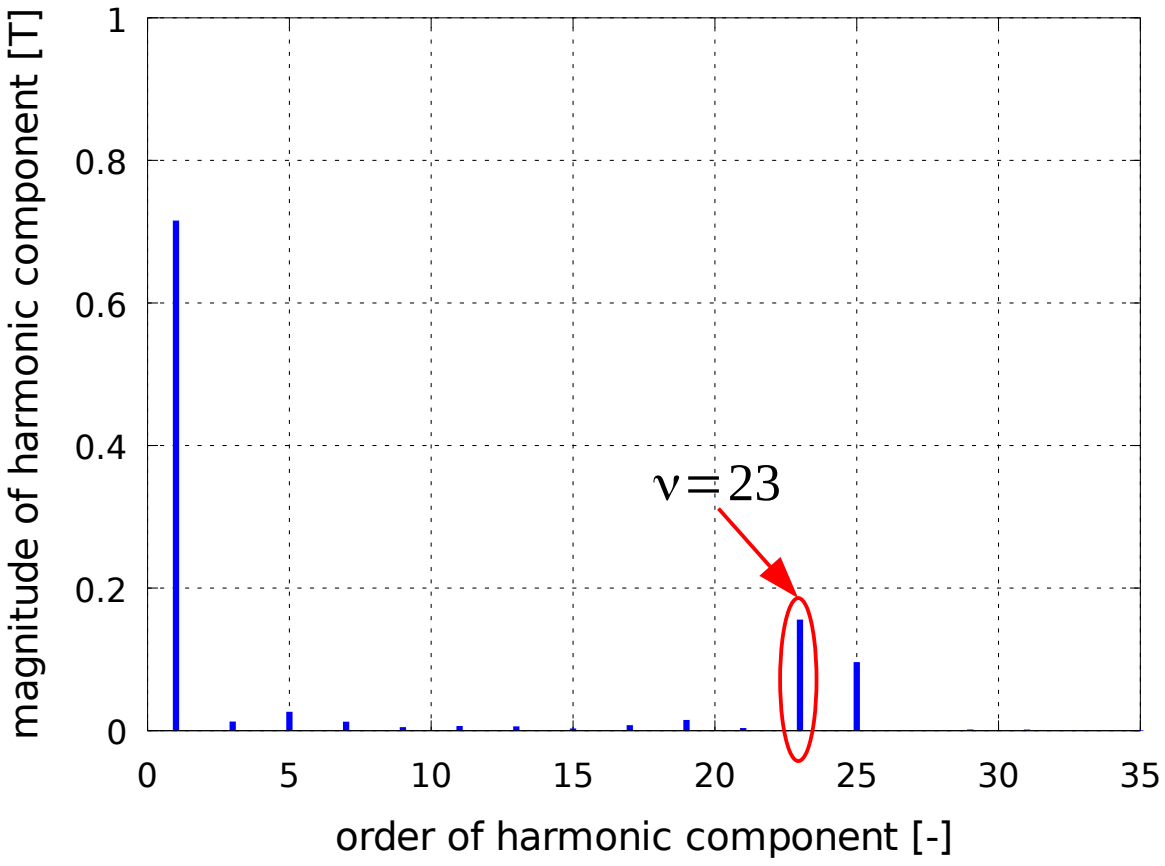
$$2p = 4$$

$$\nu_d = \frac{Q}{p} = \frac{48}{2} = 24$$

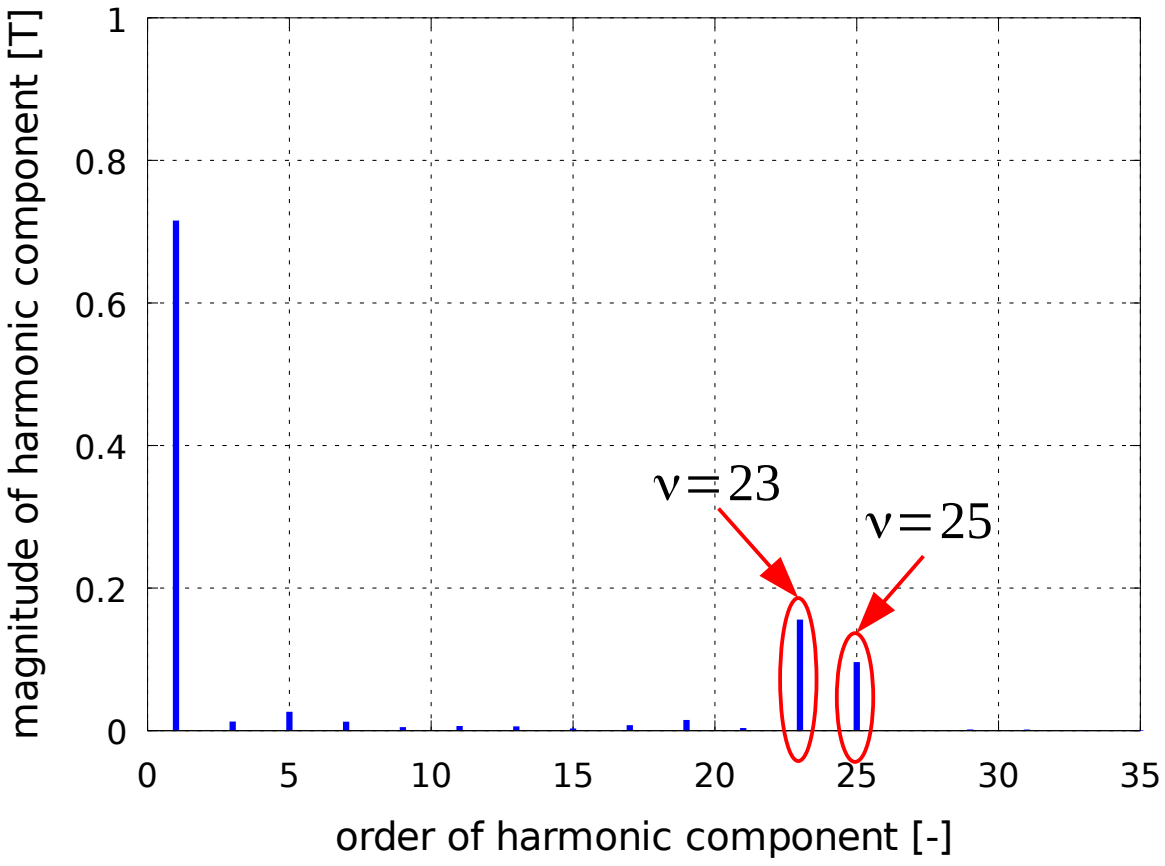
Synchronous Parasitic Torque Components of Induction Machines



Synchronous Parasitic Torque Components of Induction Machines



Synchronous Parasitic Torque Components of Induction Machines



Synchronous Parasitic Torque Components of Induction Machines

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- Their wavelengths are

$$\lambda_d = \frac{2\,t_p}{\nu}$$

Synchronous Parasitic Torque Components of Induction Machines

- Synchronous parasitic moments appear, when the wavelength of harmonic components from the stator nad the rotor are equal

$$\lambda_{d1} = \lambda_{d2}$$

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$$Q_1 \pm p = Q_2 \pm p$$

Synchronous Parasitic Torque Components of Induction Machines

- If we have to avoid the appearance of parasitic synchronous torque components, we must have $$Q_1 \pm p \neq Q_2 \pm p$$
- A significant reduction of parasitic torque components may be achieved by skewing of stator/rotor slots by one slot pitch
- But, parasitic synchronous torques may be practically used by design of a machine with strongly expressed slot harmonic components
  - The result is then a reluctance machine with a conventional three phase winding and a possibility of asynchronous start-up

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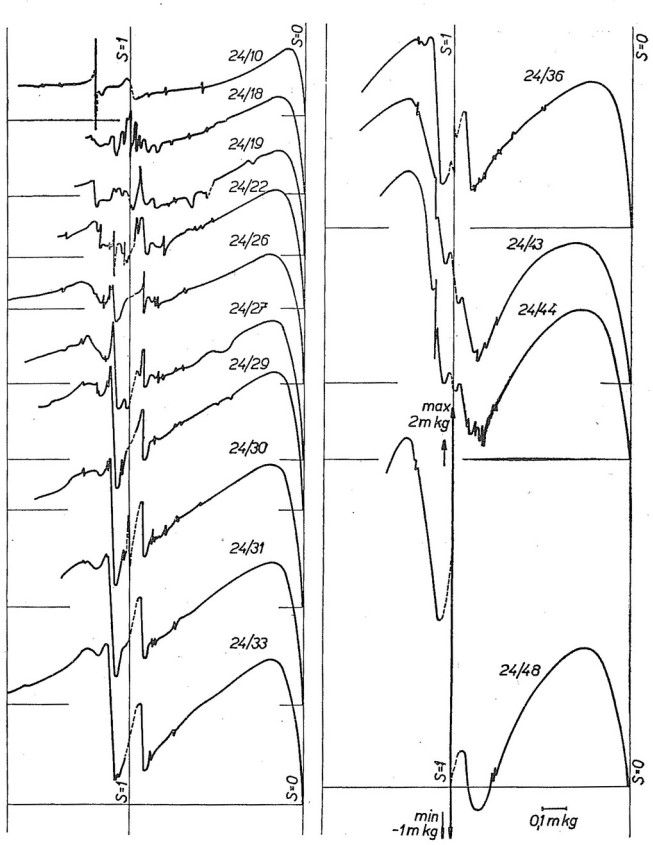
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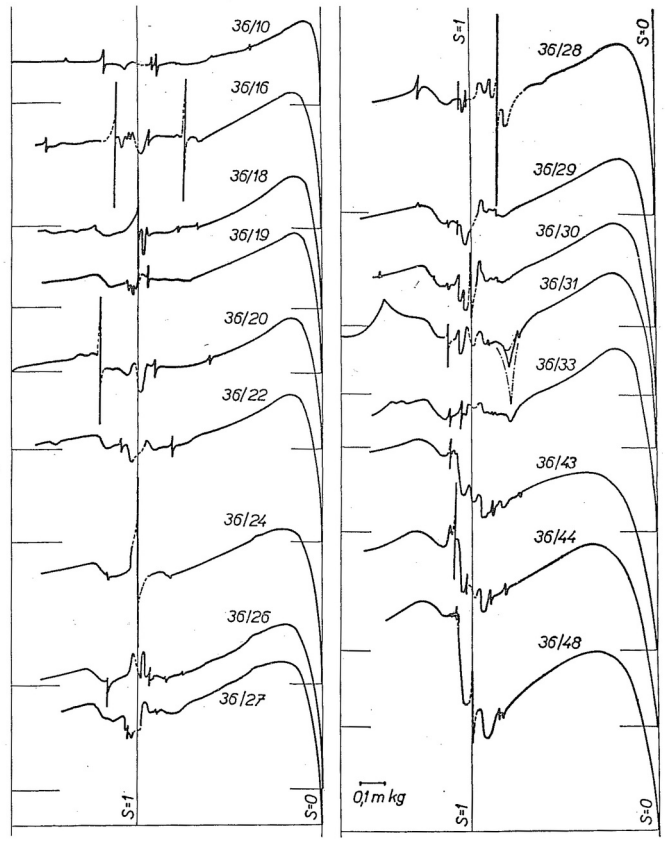
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Spatial Harmonic Components After-Effects in Torque Characteristics

Spatial Harmonics After-Effects in Torque Characteristics

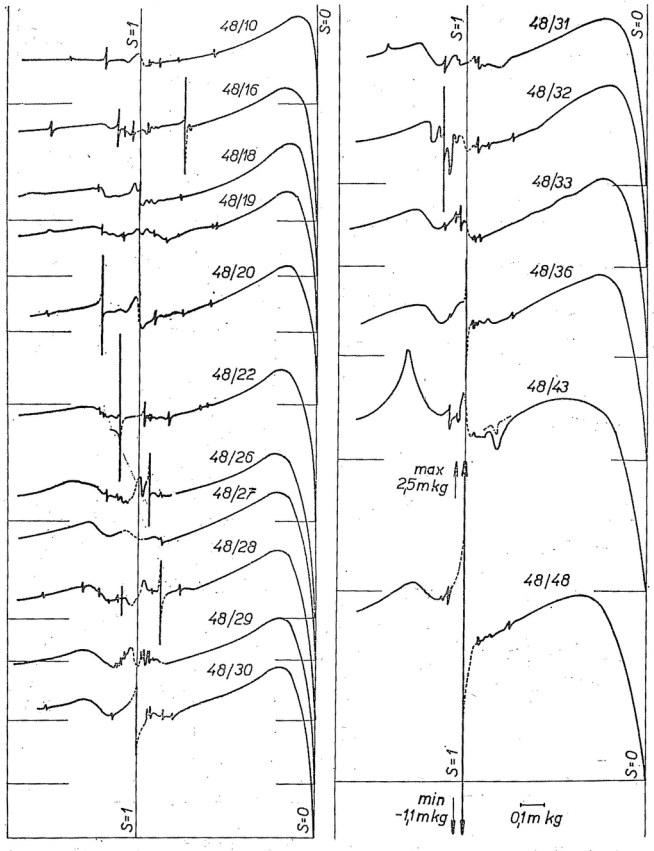


Obr. 50. Průběh momentu asynchronního motoru $P = 1{,}1$ kW, $2p = 4$, $Z_1 = 24$ s rotory o různém počtu drážek Z_2.

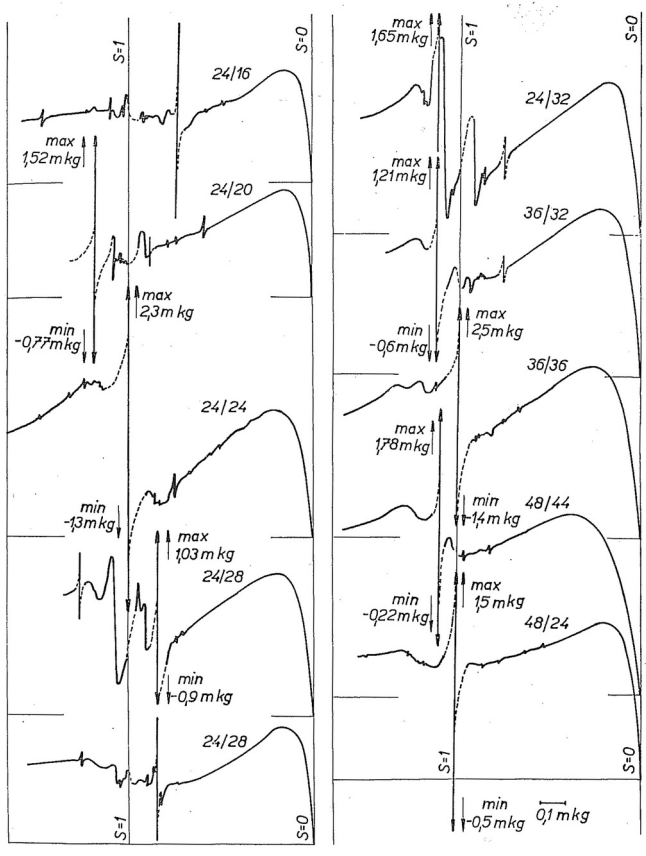


Obr. 51. Průběh momentu asynchronního motoru $P = 1{,}1$ kW, $2p = 4$, $Z_1 = 36$ s rotory o různém počtu drážek Z_2.

Spatial Harmonics After-Effects in Torque Characteristics



Obr. 52. Průběh momentu asynchronního motoru $P = 1{,}1$ kW, $2p = 4$, $Z_1 = 48$ s rotory o různém počtu drážek Z_2.



Obr. 53. Průběh momentu u asynchronního motoru $P = 1{,}1$ kW, $2p = 4$ při nevhodné kombinaci počtu statorových a rotorových drážek.

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Time Harmonic Components

Time Harmonic Components

- The time harmonic componets are injected into a machine through its power supply
- The source is most commonly the power electronics
- The volume of time harmonic components changes depending of machine control strategy
- In general the speed of time harmonic components is μ-times higher than the speed of the fundamental harmonic component:

$$\omega_\mu = \pm \mu \, \omega_1$$

- For their directions and speeds the same rules as for spatial harmonics are valis

Time Harmonic Components

- For a couple of two μ–th time and ν–th spatial harmonic componet of the same order we have:

$$\mp \nu \xi \pm \mu \omega_1 t = 0$$

thus

$$\xi = \pm \frac{\mu}{\nu} \omega_1 t$$

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$$s_\mu=\frac{\omega_\mu-\omega}{\omega_\mu}$$

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$$s_\mu = \frac{\omega_\mu - \omega}{\omega_\mu} = \frac{\mu \omega_1 - \omega}{\mu \omega_1}$$

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- Since μ is usually a large number, the induction machine works from the time harmonic components point of view near to the short circuit state

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- The harmonic components induced into the rotor winding have then speed

$$\omega_{2\mu} = \omega_{\mu} - \omega$$

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- The currents corresponding with these harmonic components then generate mainly additional losses in rotor bars
- At the same time parasitic torques of small magnitude arise
- By interference of waves of different orders further pulsating torques appear

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Thank You for Your Attention!

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