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EUROPEAN UNION
European Structural and Investment Funds
Operational Programme Research,
Development and Education

MINISTRY OF EDUCATION,
YOUTH AND SPORTS

Selected Topics in Electrical Machines (KEV/VPS)

Magnetic forces in electrical machines

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EUROPEAN UNION
European Structural and Investment Funds
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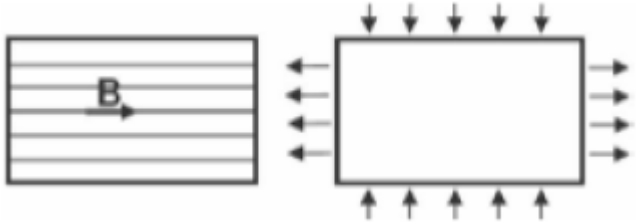
MINISTRY OF EDUCATION, YOUTH AND SPORTS

Magnetic forces in electrical machines

- **Magnetic forces origin**
- Magnetostriction
  - Inside ferromagnetic materials, usually not important for rotating electrical machines e-NVH
  - Force proportional to $\boldsymbol{B^2}$
- Lorentz force
  - Acting on conductors in slot
  - Force proportional to $\boldsymbol{I}$, $\boldsymbol{B}$
- Maxwell force (from Maxwell Stress Tensor)
  - Acting at core – air interface
  - Force proportional to $\boldsymbol{B^2}$

Magnetic forces in electrical machines

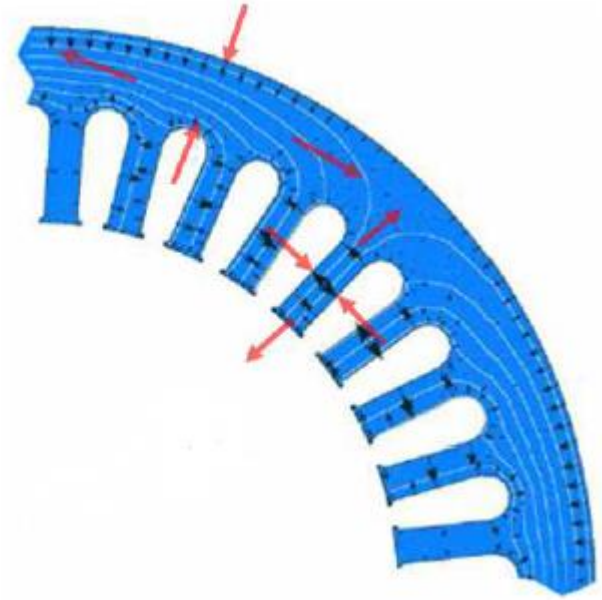
- **Magnetic forces origin – Magnetostriction**
- Change of shape and volume of ferromagnetic materials in case of AC magnetization
  - Dimension change in μm

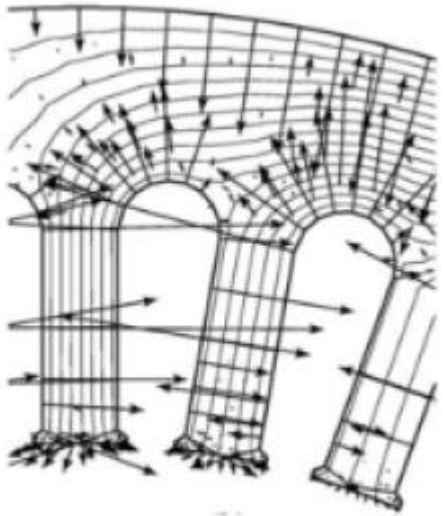


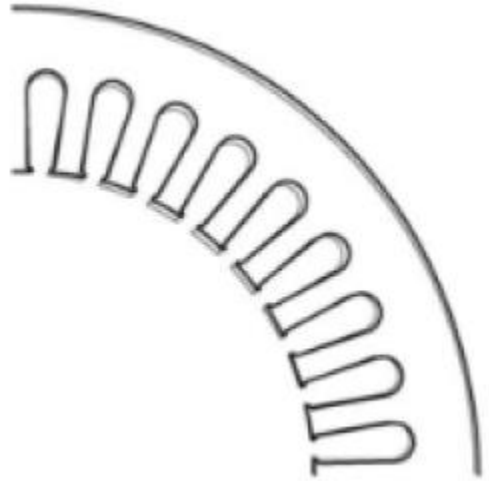
- Causes yoke radial vibration and airborne noise
- Force at twice line frequency ***2L***

Magnetic forces in electrical machines

- **Magnetic forces origin – Magnetostriction**
- Magnetostrictive forces act locally depending on magnetic field fluxlines







Source: EOMYS Engineering

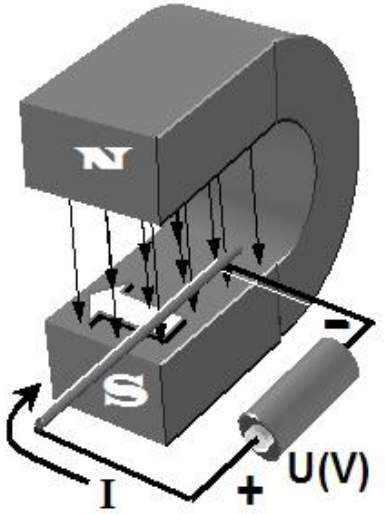
Magnetic forces in electrical machines

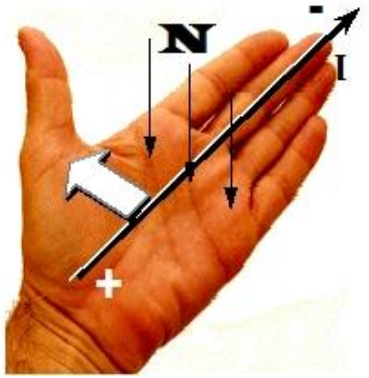
- **Magnetic forces origin - Lorentz force**
- Assumption of $\vec{B}$ and $\vec{I}$ perpendicularity for electrical machines

$$F = BIl_{Fe}$$

- Flux density in slot is low
  - => forces can be usually neglected

- Important in case of endwinding vibration analysis in large machines





Zdroj: https://www.wikiskripta.eu/w/Lorentzova_s%C3%ADla

Magnetic forces in electrical machines

- **Magnetic forces origin - Maxwell force**
- Origin at interface of two materials having different permeability => acting in diretion of lower $\boldsymbol{\mu}$

- Magnetic field energy accumulates in air gap
  - Energy can‘t be stored without stress => energy density relevant to pressure (stress)

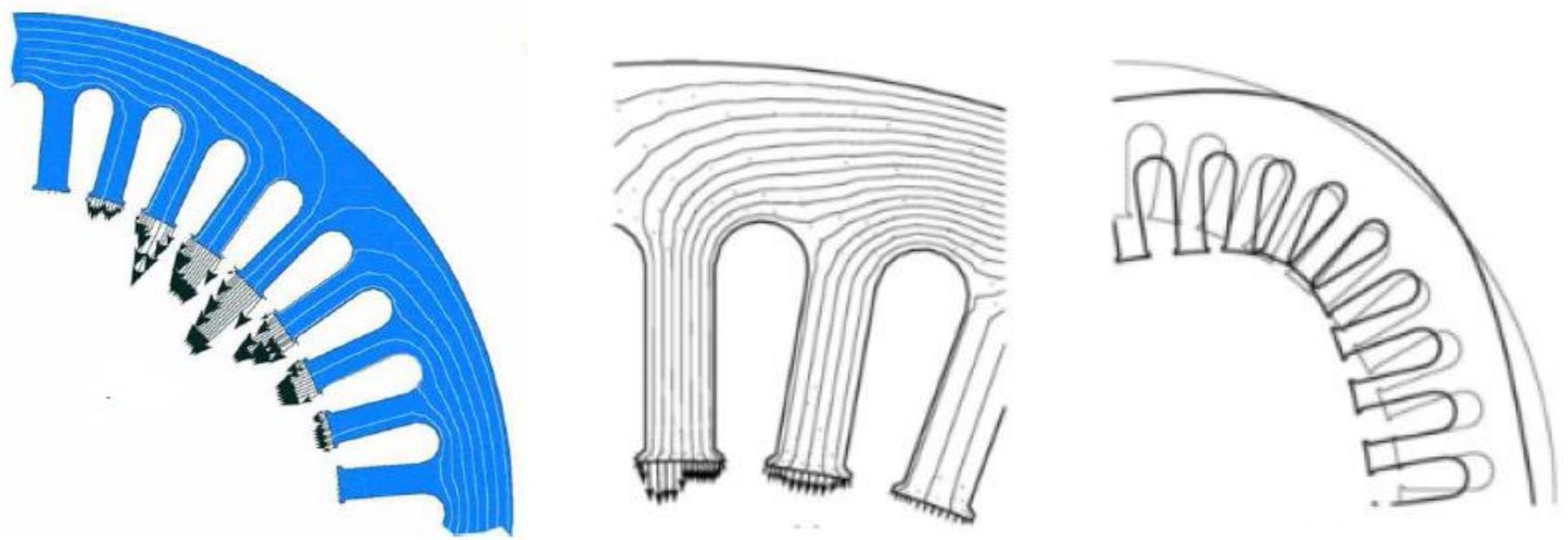
$$N \cdot m^{-2} = Pa$$

- Maxwell force integration over the rotor = contrary to integration over the stator
  - Origin of opposite oriented torque of stator and rotor

- Radial and tangential forces have both the same harmonic content

Magnetic forces in electrical machines

- **Magnetic forces origin - Maxwell force**
- Interface of two materials having different μ => acting in diretion of lower μ



Zdroj: EOMYS Engineering

Magnetic forces in electrical machines

- **Magnetic forces origin - Maxwell force**
- Calculation using Maxwell Stress Tensor

$$\boldsymbol{T} = \frac{1}{\mu}\begin{bmatrix} B_x^2 - \frac{1}{2}|\boldsymbol{B}|^2 & B_x B_y & B_x B_z \\ B_x B_y & B_y^2 - \frac{1}{2}|\boldsymbol{B}|^2 & B_y B_z \\ B_x B_z & B_y B_z & B_z^2 - \frac{1}{2}|\boldsymbol{B}|^2 \end{bmatrix} \qquad \boldsymbol{B} = (B_x, B_y, B_z)$$

Magnetic forces in electrical machines

- **Magnetic forces origin - Maxwell force**
- Volume integral utilization
- Volumetric force density

$$\Delta \boldsymbol{f} = div \boldsymbol{T}$$

- Total force acting on body having volume $\boldsymbol{V}$

$$\boldsymbol{F} = \int_V \Delta \boldsymbol{f} \, dV$$

Magnetic forces in electrical machines

- **Magnetic forces origin - Maxwell force**
- Surface integral utilization
- Surface force density or specific magnetic pressure ($N \cdot m^{-2} = Pa$)

$$\Delta \boldsymbol{p} = \boldsymbol{T} \cdot \boldsymbol{n}$$

- Total force acting on face having surface $\boldsymbol{S}$

$$\boldsymbol{F} = \int_S \Delta \boldsymbol{p}\, dS$$

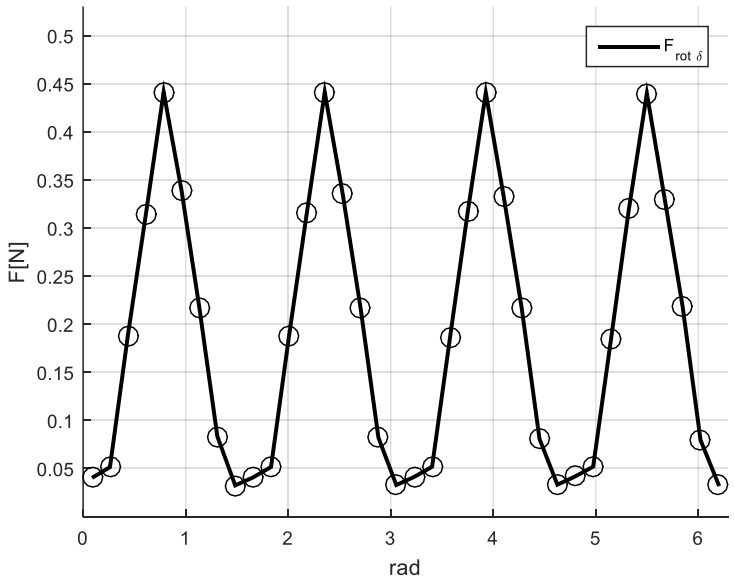
Magnetic forces in electrical machines

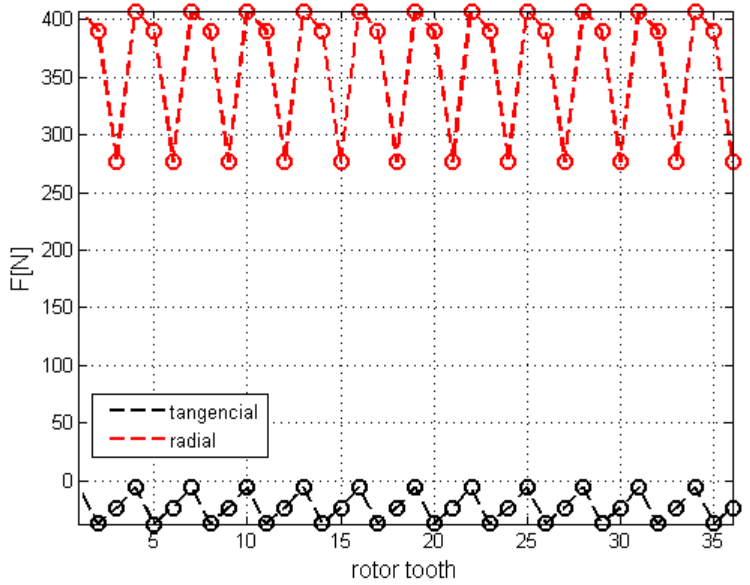
- **Magnetic forces origin - Maxwell force**
- 2D analysis of forces acting at ferromagnetic – air interface in Cartesian coordinate system

$$\boldsymbol{F} = \frac{1}{\mu_0}\int_S \begin{bmatrix} B_x^2 - \frac{1}{2}|B|^2 & B_x B_y \\ B_x B_y & B_y^2 - \frac{1}{2}|B|^2 \end{bmatrix} \begin{Bmatrix} n_x \\ n_y \end{Bmatrix} dS$$

Magnetic forces in electrical machines

- **Magnetic forces origin**
- Comparison of Lorentz force acting on IM rotor bars (in the left) and Maxwell force acting on rotor teeth (in the right)
  - Induction motor 11 kW, Q_2 = 36





Magnetic forces in electrical machines

- Dynamic effects of air gap magnetic field => variable magnetic forces

- Magnetic force directions
  - Radial
  - Tangencial
  - Axial

- Force value derived from surface force density (specific magnetic pressure)

$$p = \frac{BH}{2} = \frac{B^2}{2\mu_0} = \frac{B^2}{8.\pi.10^{-7}} = 4.B^2.10^5\,Pa = 0{,}4.B^2\,MPa$$

Magnetic forces in electrical machines

- Dynamic effects of air gap magnetic field => variable magnetic forces

- Magnetic force directions
  - Radial
  - Tangencial
  - Axial

Act in all cases

Acts in specific cases only

- Force value derived from surface force density (specific magnetic pressure)

$$p = \frac{BH}{2} = \frac{B^2}{2\mu_0} = \frac{B^2}{8.\pi.10^{-7}} = 4.B^2.10^5\,Pa = 0{,}4.B^2 MPa$$

Magnetic forces in electrical machines

- **Radial magnetic forces**
- Equal flux density distribution for any pole-pair
  - Radial force equally distributed
  - No shaft bending stresses
  - No additional stress on bearings

- Unequal flux density distribution (e.g. eccentricity, bent shaft)
  - Unbalanced magnetic pull origin (***UMP***)
  - Additional radial stress on bearings
  - Bending moment acting on shaft

Magnetic forces in electrical machines

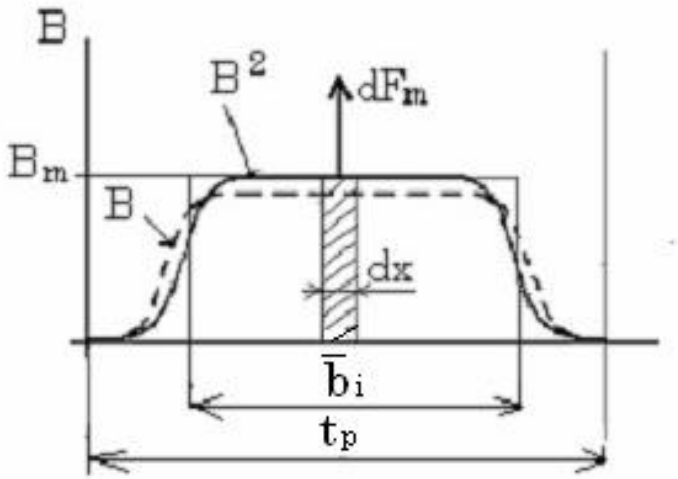
- **Radial magnetic forces – symmetrical magnetic field distribution**
- Uniform magnetic pull around air gap

- Elementary force acting on dx element in B^2 waveform

$$dF_m = p.dS = p.dx.l_i$$

- Total force acting on pole pitch

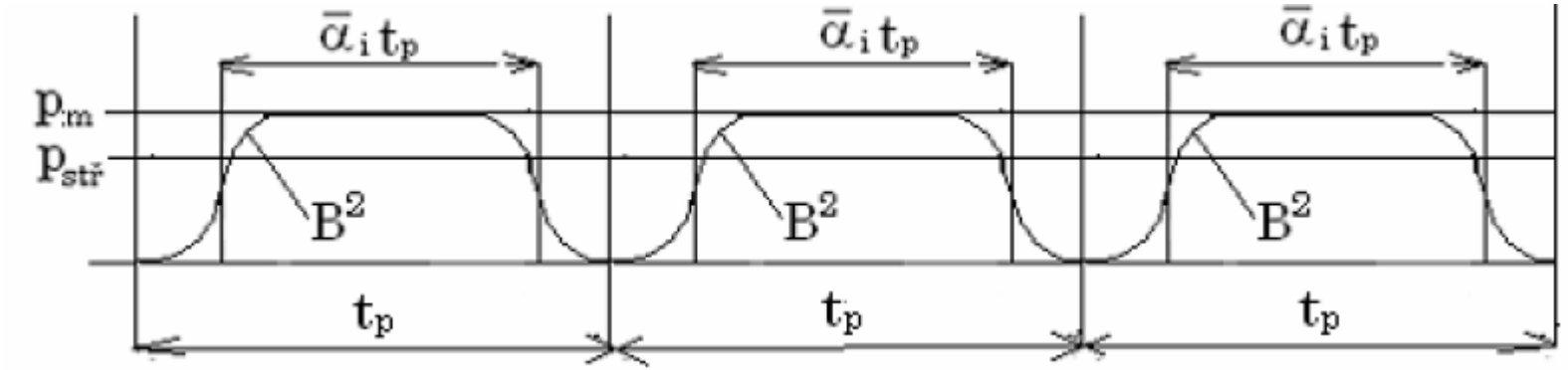
$$F_m = \int_0^{t_p} dF_m = \int_0^{t_p} p l_i dx = \int_0^{t_p} \frac{B^2}{2\mu_0} l_i dx = \frac{l_i}{2\mu_0} \int_0^{t_p} B^2 dx = \frac{1}{2\mu_0} B_m^2 \overline{\alpha}_i t_p l_i$$



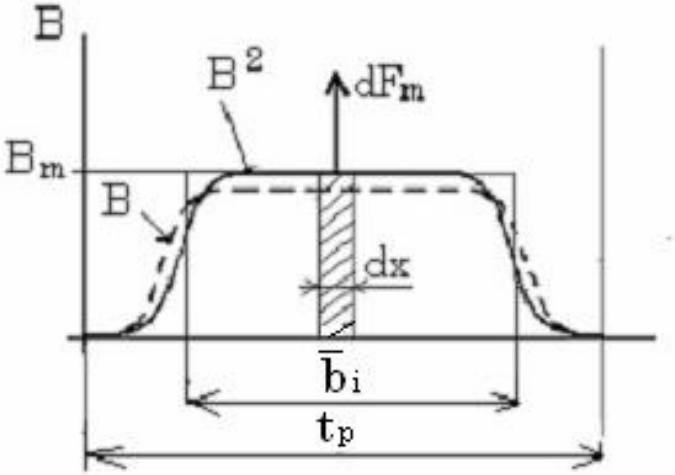
Magnetic pull on pole pitch

Magnetic forces in electrical machines

- **Radial magnetic forces – symmetrical magnetic field distribution**
- Value of $\int_0^{t_p} B^2 dx$ = rectangular surface B_m^2 x $\bar{b}_i$ $\quad (\bar{b}_i = \bar{\alpha}_i t_p)$
- For $p_m = \frac{1}{2\mu_0} B_m^2$ => $F_m = p_m \bar{\alpha}_i t_p l_i$



Average value of specific magnetic pull

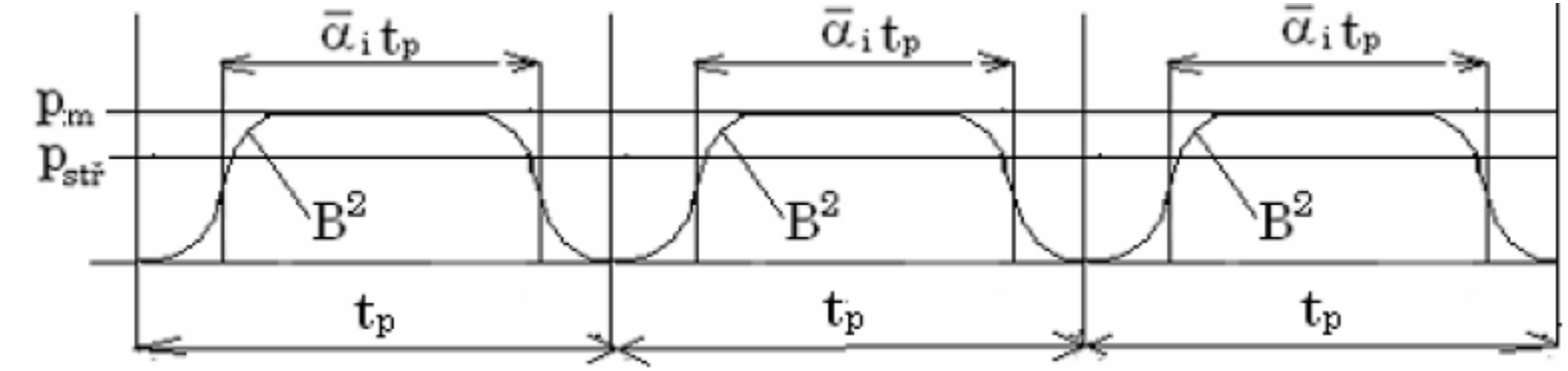


Magnetic pull on pole pitch

Magnetic forces in electrical machines

- **Radial magnetic forces – symmetrical magnetic field distribution**
- Average value of specific magnetic pull $p_{stř}$
- Replacing waveform B^2 by $p_{stř}\ x\ t_p$ rectagle

$$F_m = p_m \overline{\alpha}_i t_p l_i = p_{stř} t_p l_i \qquad p_{stř} = \overline{\alpha} . p_m$$



Average value of specific magnetic pull

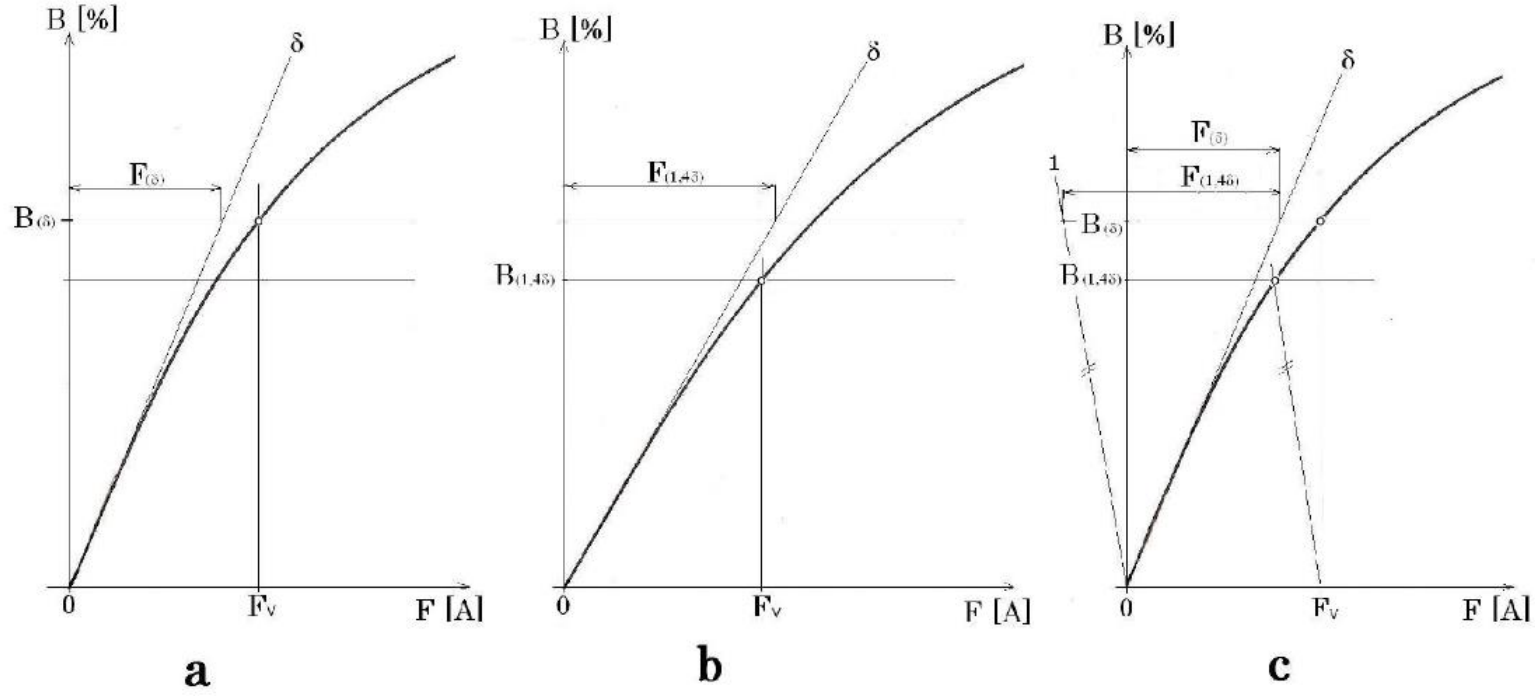
Magnetic forces in electrical machines

- **Radial magnetic forces – unbalanced magnetic pull**
- Unsymmetrical air gap flux density distribution => uneven radial force distribution under individual pole pitches => unbalanced magnetic pull (UMP)

- Assumptions for UMP value calculation
  - Maximal eccentricity $e_{max} = 0{,}1\delta$
  - Total MMF for any pole pitch is constant (Fv = const.)
  - For concentric rotor is air gap length δ and magnetic flux density B
  - At least four-pole machine (2p ≥ 4)

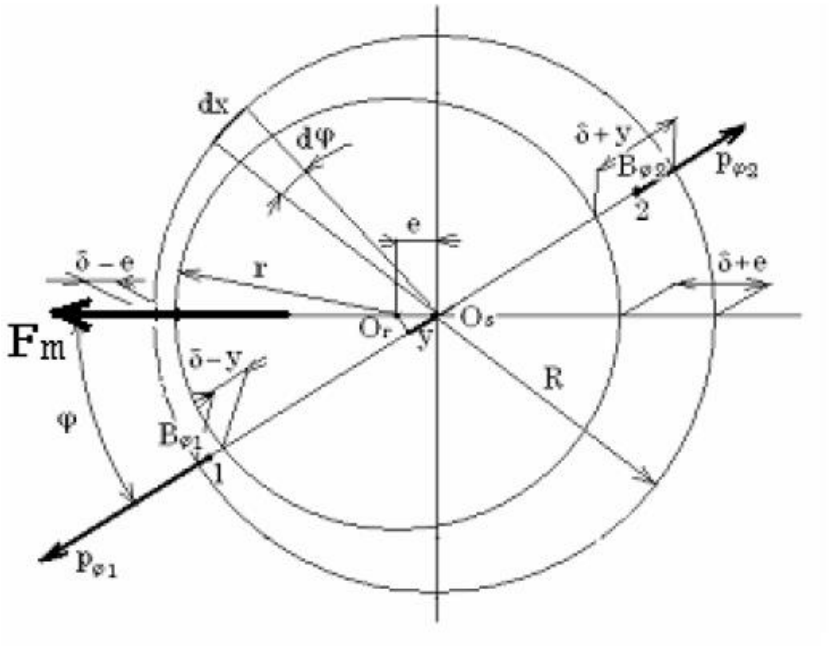
Magnetic forces in electrical machines

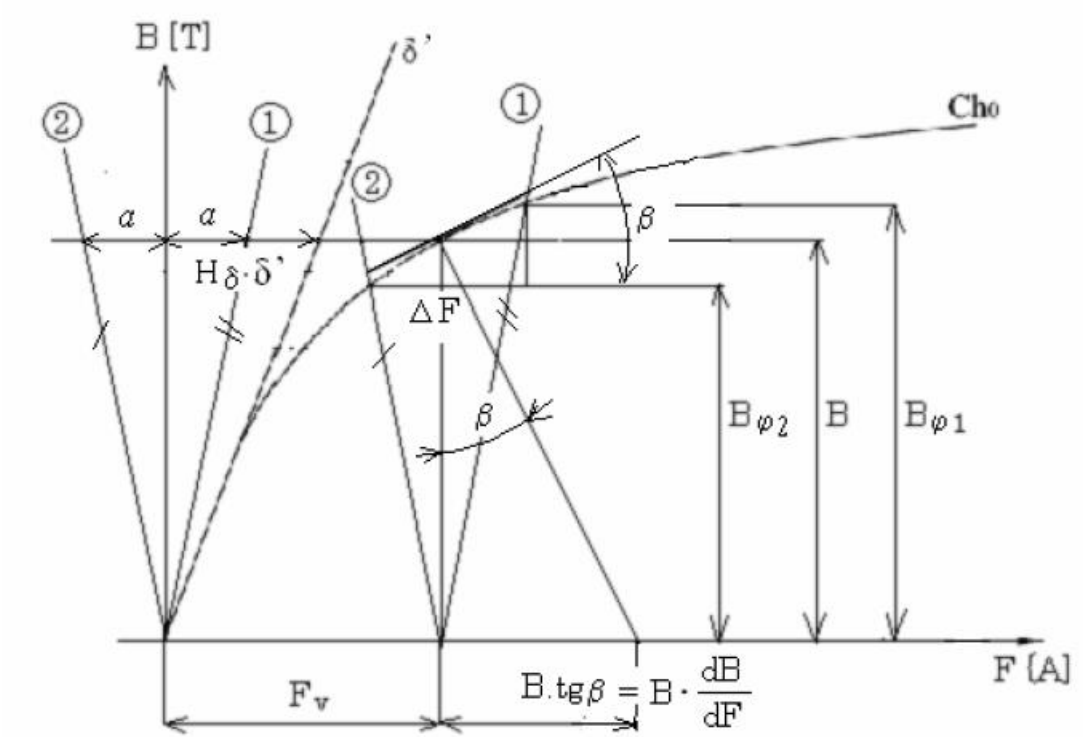
- **Radial magnetic forces – unbalanced magnetic pull**
- Flux density change in case of 40% change of air gap length



Magnetic forces in electrical machines

- **Radial magnetic forces – unbalanced magnetic pull**
- Similar principle applies in case of air gap length change due to eccentricity





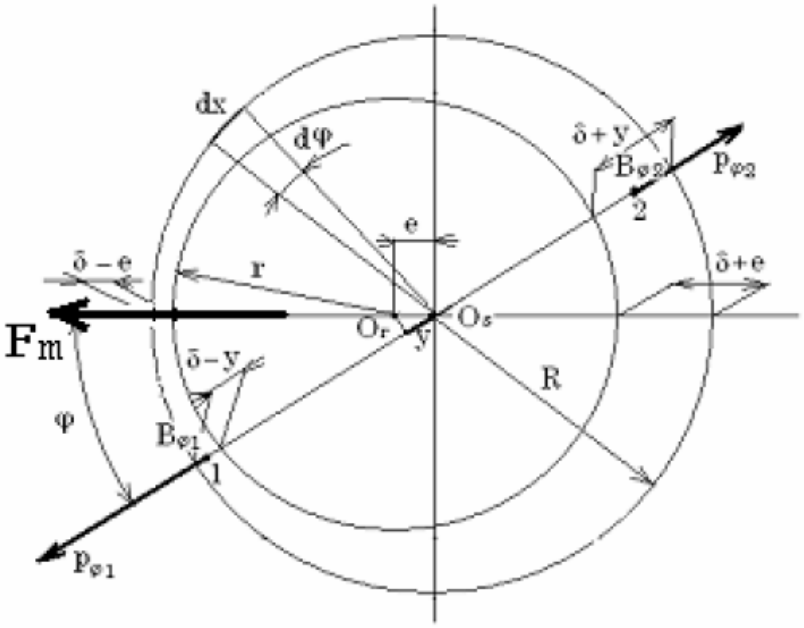
Magnetic forces in electrical machines

- **Radial magnetic forces – unbalanced magnetic pull**
- Eccentricity in horizontal direction
- For any position of axial line under φ angle
  - Eccentricity projection in the line direction is y
  - Air gap length: spot 1 **($\delta - y$)**, spot 2 **($\delta + y$)**

$$y = e.\cos\varphi\,, \qquad \delta = R - r\,, \qquad dx = r.d\varphi$$

- Specific magnetic pressures

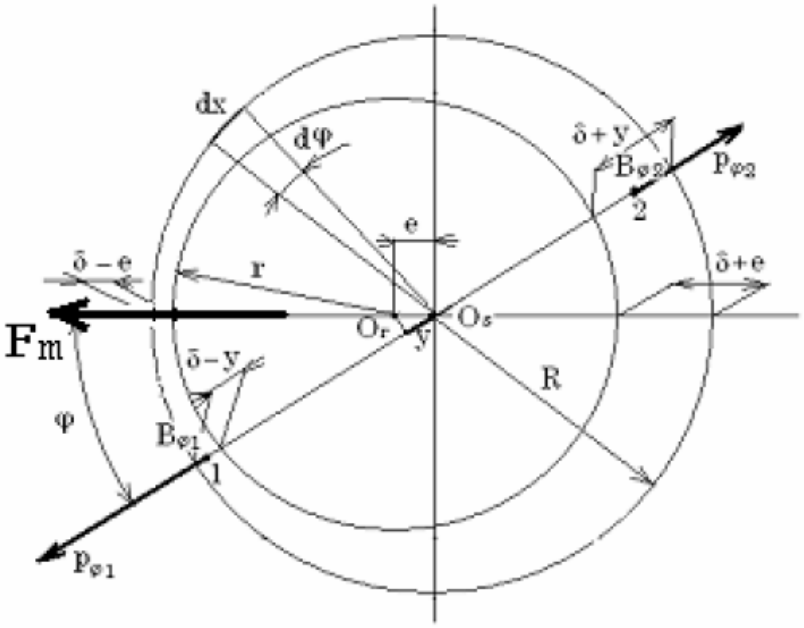
$$p_{\varphi 1} = \frac{B_{\varphi 1}^2}{2\mu_0} \qquad p_{\varphi 2} = \frac{B_{\varphi 2}^2}{2\mu_0}$$



Magnetic forces in electrical machines

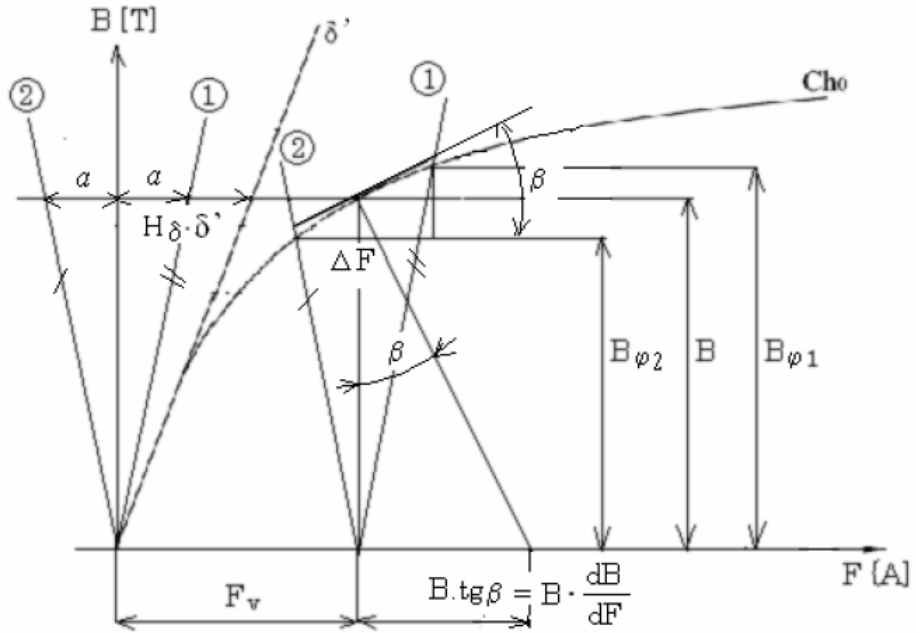
- **Radial magnetic forces – unbalanced magnetic pull**
- Spot 1 – smaller air gap
  - Smaller reluctance => larger flux density =>
    => larger magnetic pull
  - Differential magnetic pull acts in smaller
    air gap direction

$$\Delta p_{\varphi} = p_{\varphi 1} - p_{\varphi 2} = \frac{B_{\varphi 1}^2 - B_{\varphi 2}^2}{2\mu_0} = \frac{(B_{\varphi 1} + B_{\varphi 2})(B_{\varphi 1} - B_{\varphi 2})}{2\mu_0}$$



Magnetic forces in electrical machines

- **Radial magnetic forces – unbalanced magnetic pull**
- If $e \leq \delta$ => very small MMF for eccentricity => section between lines 1 a 2 at the curve Ch_o can be assumed linear



- => right triangle have legs ΔF a ΔB

$$\Delta F = 2a = 2F_\delta = 2H_\delta y$$

Magnetic forces in electrical machines

- **Radial magnetic forces – unbalanced magnetic pull**
- One can introduce

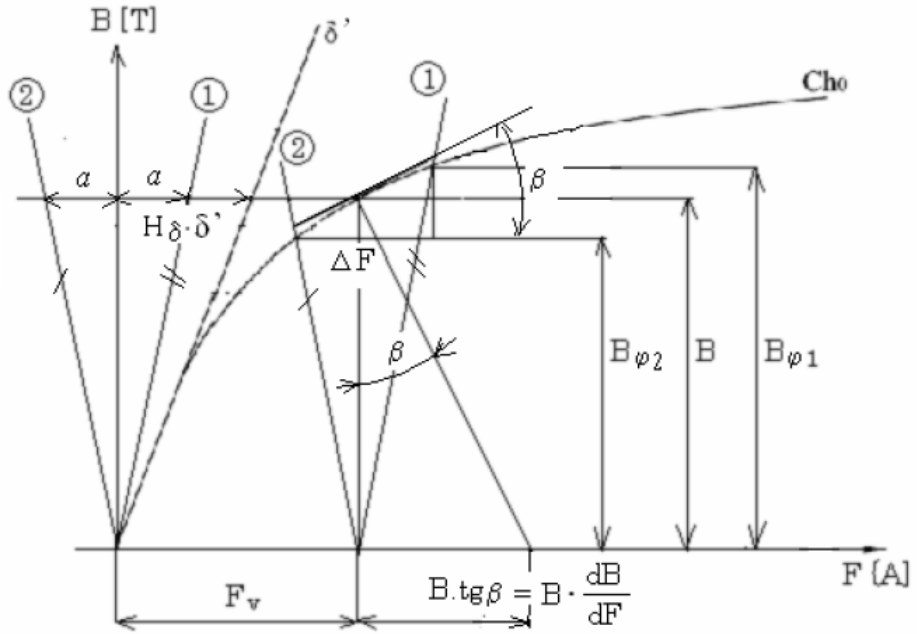
$$B_{\varphi 1} + B_{\varphi 2} = 2B$$

$$tg\beta = \frac{\Delta B}{\Delta F} \qquad B_{\varphi 1} - B_{\varphi 2} = \Delta B = \Delta F.tg\beta = 2H_\delta y.tg\beta$$

- Then,

$$\Delta p_\varphi = \frac{2B.2H_\delta y.tg\beta}{2\mu_0} = \frac{2B^2}{\mu_0^2} y.tg\beta = \frac{2B^2}{\mu_0^2} e\frac{dB}{dF}cos\varphi$$

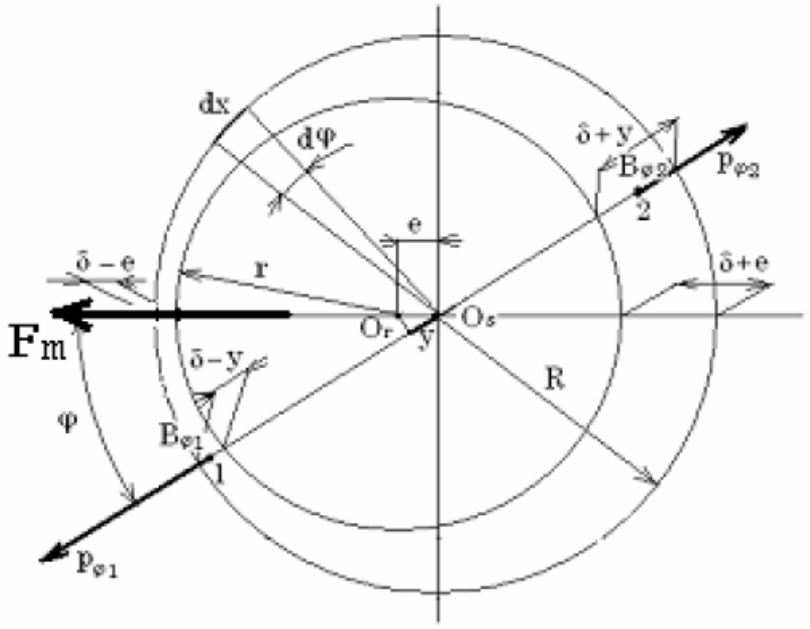
where $H_\delta = \frac{B}{\mu_0}$, $tg\beta = \frac{dB}{dF}$ a $y = e.cos\varphi$



Magnetic forces in electrical machines

- **Radial magnetic forces – unbalanced magnetic pull**
- Magnetic pull at the element dx

$$dF_m = \Delta p_\varphi . dx . l_i$$



- Total magnetic pull around air gap circumference
  - Integration between $\varphi = 0$ and $\varphi = \pi$
  - For integration up to $\varphi = 2\pi$ => $F_m = 0$

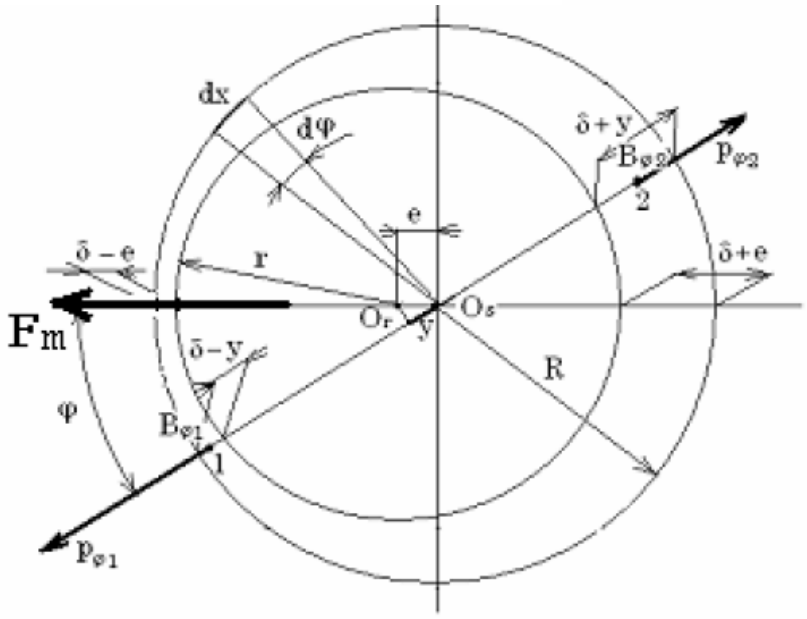
$$F_m = \int_{\varphi=0}^{\pi} \overline{\alpha} . dF_m \cos\varphi = \int_{\varphi=0}^{\pi} \overline{\alpha} l_i \Delta p_\varphi r \cos\varphi . d\varphi = l_i \overline{\alpha} r \frac{2B^2}{\mu_0^2} e \frac{dB}{dF} \int_{\varphi=0}^{\pi} \cos^2\varphi . d\varphi = \frac{\overline{\alpha} \pi D_1 l_i e}{2\mu_0^2} B^2 \frac{dB}{dF}$$

Magnetic forces in electrical machines

- **Radial magnetic forces – unbalanced magnetic pull**
- Total magnetic pull around air gap circumference

$$F_m = \int_{\varphi=0}^{\pi} \overline{\alpha}.dF_m \cos\varphi = \int_{\varphi=0}^{\pi} \overline{\alpha} l_i \Delta p_\varphi r \cos\varphi.d\varphi = l_i \overline{\alpha} r \frac{2B^2}{\mu_0^2} e \frac{dB}{dF} \int_{\varphi=0}^{\pi} \cos^2\varphi.d\varphi = \frac{\overline{\alpha}\pi D_1 l_i e}{2\mu_0^2} B^2 \frac{dB}{dF}$$

- r is bore radius of the core
- $D_1 = 2r$ a $\int_{\varphi=0}^{\pi} \cos^2\varphi.d\varphi = \frac{\pi}{2}$



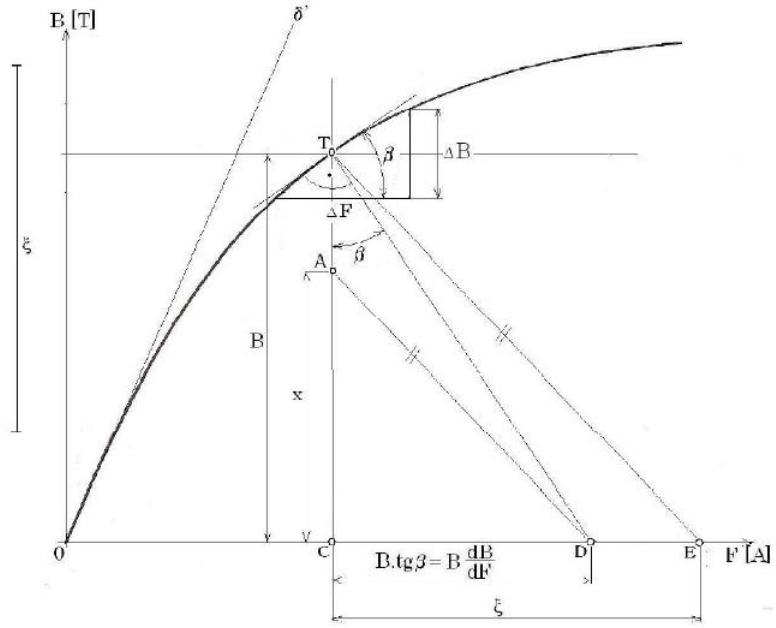
Magnetic forces in electrical machines

- **Radial magnetic forces – unbalanced magnetic pull**
- Total magnetic pull around air gap circumference

$$F_m = \int_{\varphi=0}^{\pi} \overline{\alpha}.dF_m \cos\varphi = \int_{\varphi=0}^{\pi} \overline{\alpha} l_i \Delta p_\varphi r \cos\varphi.d\varphi = l_i \overline{\alpha} r \frac{2B^2}{\mu_0^2} e \frac{dB}{dF} \int_{\varphi=0}^{\pi} \cos^2\varphi.d\varphi = \frac{\overline{\alpha}\pi D_1 l_i e}{2\mu_0^2} B^2 \frac{dB}{dF}$$

  - r is bore radius of the core
  - $D_1 = 2r$ a $\int_{\varphi=0}^{\pi} \cos^2\varphi.d\varphi = \frac{\pi}{2}$

- UMP value for given e depends on $B^2 \frac{dB}{dF}$

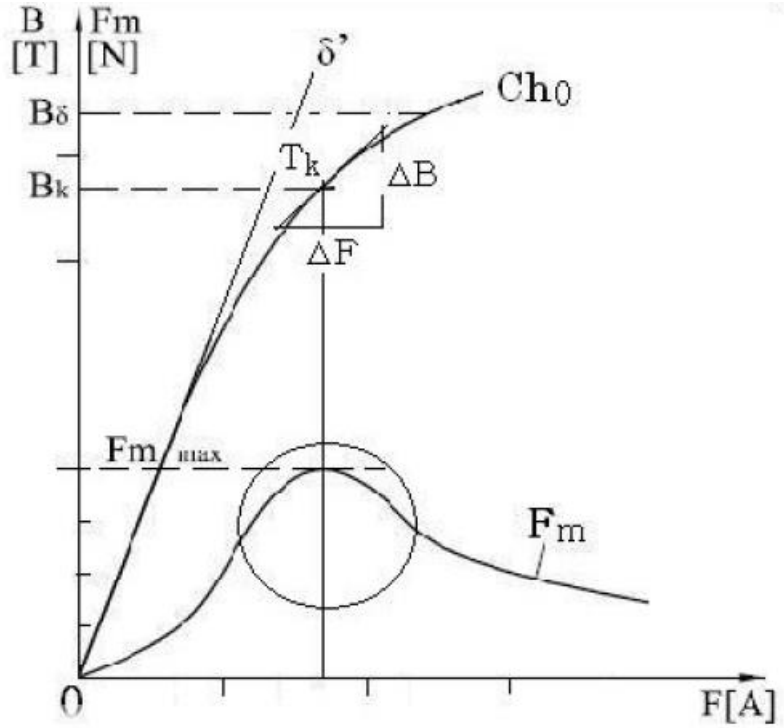


Magnetic forces in electrical machines

- **Radial magnetic forces – unbalanced magnetic pull**
- Critical value B_k and maximal UMP value
  - UMP value for given e depends on $B^2 \frac{dB}{dF}$

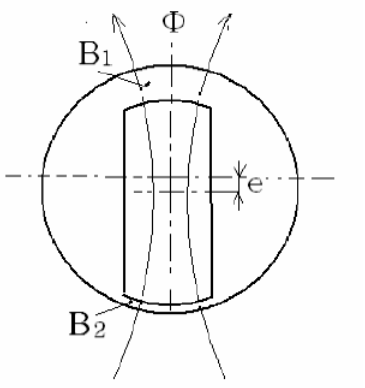
$$F_{m_{max}} = \frac{\overline{\alpha}\pi D_1 l_i e}{2\mu_0^2} B_k^2 \left(\frac{\Delta B}{\Delta F} \right)_{B=B_k}$$

  - sections ΔB a ΔF are legs of any right triangle, its hypotenuse is tangent to no-load curve Ch_0 in a point B_k



Magnetic forces in electrical machines

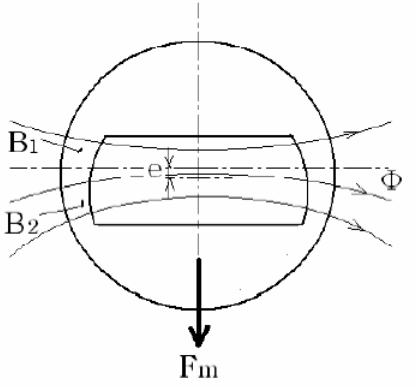
- **Radial magnetic forces – unbalanced magnetic pull**
- Two-pole machines



Eccentricity in pole-axis direction

$$B_1 = B_2$$

$$F_m = 0$$



Eccentricity perpendicular to the pole-axis direction

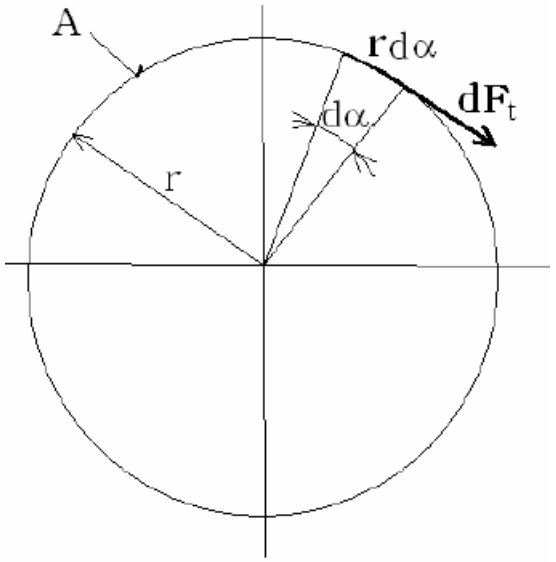
$$B_2 > B_1$$

$$F_{m,2p=2} = (0{,}5 \div 0{,}6) F_{m,2p\to\infty}$$

Magnetic forces in electrical machines

- **Tangencial magnetic forces**
- Act in circumferential direction => create torque (+ vibration)

- Assumptions:
  - Uniform linear current density A around rotor core circumference having radius r and axial length l
  - Magnetic flux Φ due to flux density B goes through current layer
  - Circumferential element $r.d\alpha$ corresponding to the elementary angle $d\alpha$ have a current $A.r.d\alpha$ and is under force $dF_t = B_\delta .A.r.d\alpha.l$



- Total circumferential force $F_t = 2p.A.r.l.\int_0^{\frac{2\pi}{2p}} B.d\alpha$

Magnetic forces in electrical machines

- **Tangencial magnetic forces**
- Air gap flux density waveform is non-sinusoidal => difficult integration

- Total tangential force can be derived from torque and bore diameter

$$F_t = \frac{M}{D_1/2} = \frac{2M}{D_1}$$

Magnetic forces in electrical machines

- **Axial magnetic forces**
- Act in the shaft axis direction

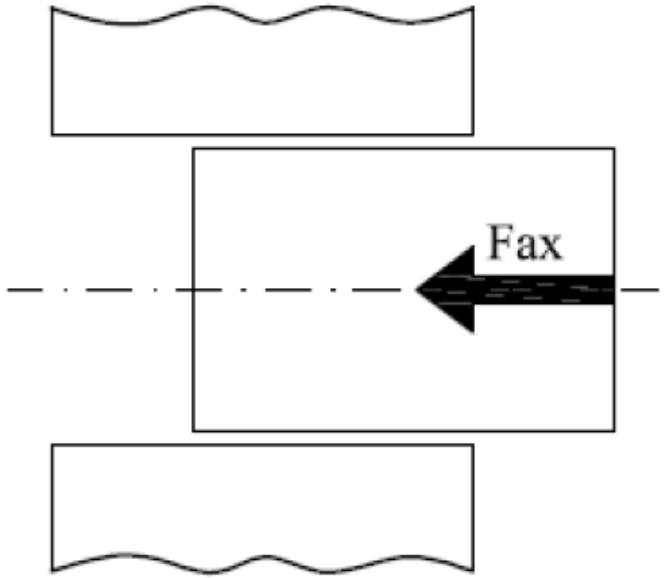
- Additional bearing load in axial direction

- Causes of origin:
  - Rotor pulled out of the stator
  - Conical rotor and stator
  - Skewed slots

Magnetic forces in electrical machines

- **Axial magnetic forces – rotor pulled out of the stator**
- Manufacturing fault

- Rotor is pulled back into the stator bore due to magnetic field
  - Tends to reach equilibrium
  - Analogy to the solenoid having a core pulled out



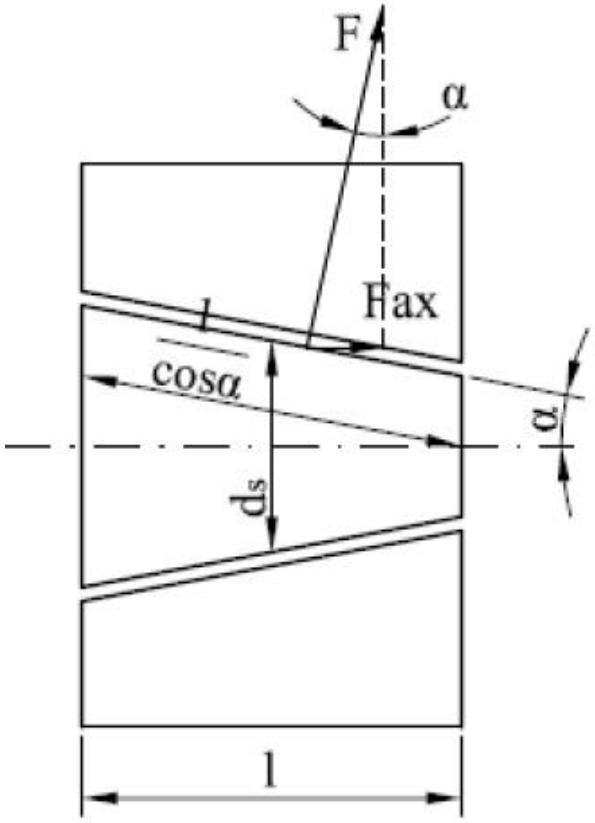
Magnetic forces in electrical machines

- **Axial magnetic forces – conical rotor and stator**
- Axial force component derived from radial force

$$F = \pi . d_s \frac{l}{\cos \alpha} p_{stř}$$

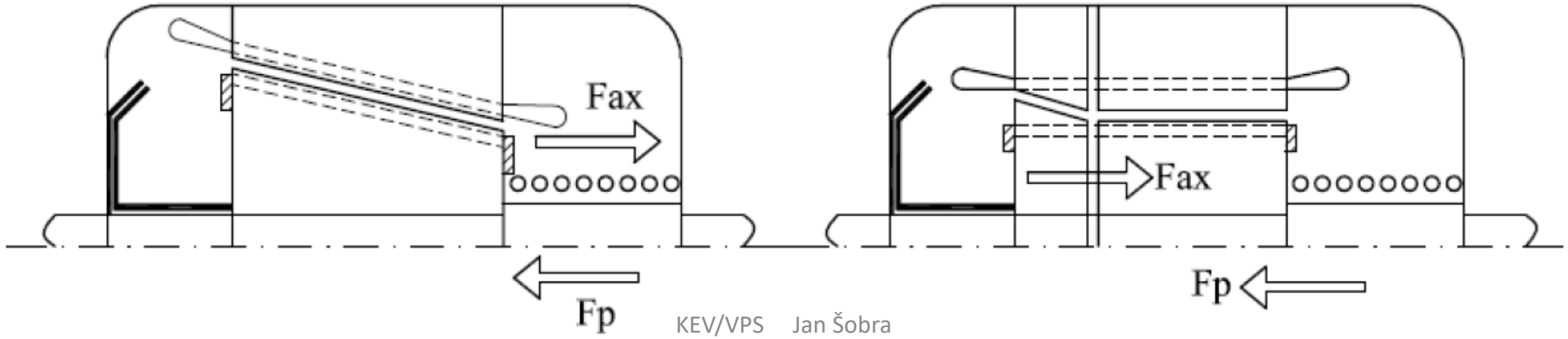
$$F_{ax} = F . \sin \alpha$$

- d_s is average rotor diameter, $p_{stř}$ is average value of specific magnetic pull



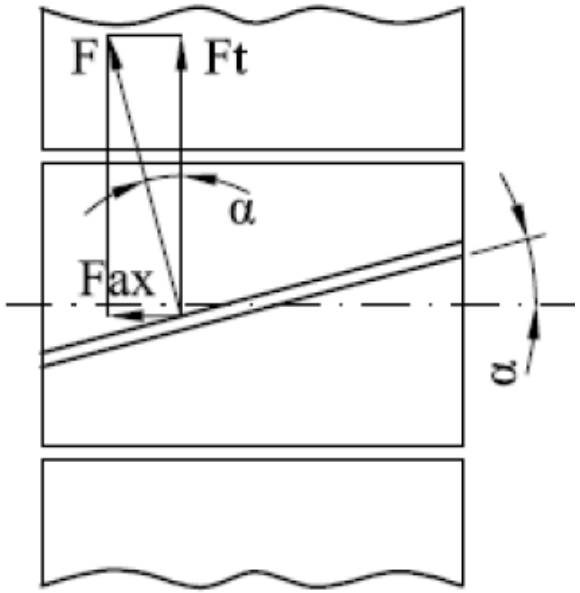
Magnetic forces in electrical machines

- **Axial magnetic forces – conical rotor and stator**
- Utilization in motors equiped with a mechanical brake
  - Conical rotor placed on the shaft, axial shift allowed – pulled to the break disc by a spring
  - Axial magnetic force larger than spring force => rotor shift and brake release
  - Axial force extinction after power cut => conical part pulled back to the break dics, rotor stops quickly
  - Difficult manufacture of rotor lamination + low magnetic utilization (saturation) of larger sheets



Magnetic forces in electrical machines

- **Axial magnetic forces – skewed slots**
- Armature slot skewing (IM often both rotor and stator)
  - Noise and vibration reduction
  - Surface power losses reduction
  - Slot harmonic elimination (one slot pitch skewing)



- Axial force derived from tangential force

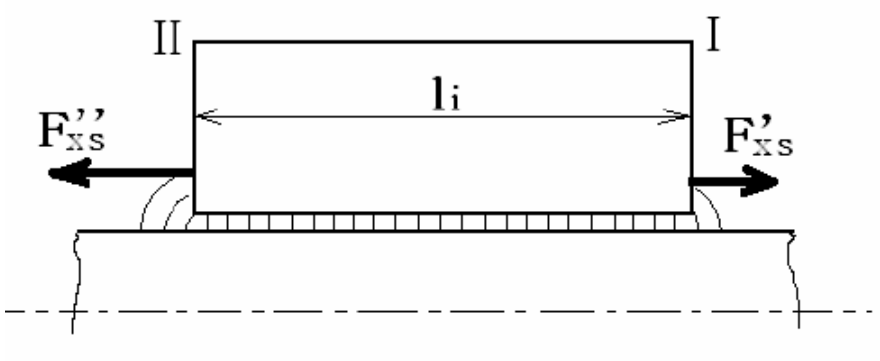
$$F_{ax} = F.\sin\alpha = \frac{F_t}{\cos\alpha}\sin\alpha = F_t.tg\alpha = \frac{2M}{D}tg\alpha$$

Magnetic forces in electrical machines

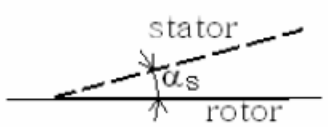
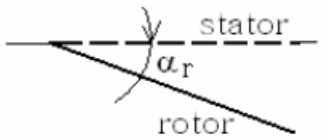
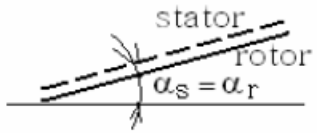
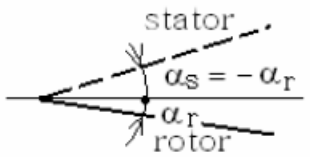
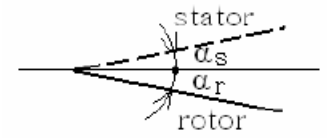
- **Axial magnetic forces – skewed slots**
- In a real machine axial forces act also due to leakage flux at the armature ends

- Forces at the both ends are equal, but have an opposite direction => in some cases resulting force can be zero

$$F''_{xs} - F'_{xs} = -F_{ax}$$



Magnetic forces in electrical machines

Configuration	α_s	α_r	$-(F'_x - F''_x)$	$F_{ax(\,stat\,)}$	$F_{ax(\,výsl\,)}$
	α	0	$-\frac{1}{2}F_t.tg\alpha$	$F_t.tg\alpha$	$\frac{1}{2}F_t.tg\alpha$
	0	–α	$-\frac{1}{2}F_t.tg\alpha$	0	$-\frac{1}{2}F_t.tg\alpha$
	α	α	0	$F_t.tg\alpha$	$F_t.tg\alpha$
	α_s	α_r	$-\frac{1}{2}F_t(tg\alpha_s - tg\alpha_r)$	$F_t.tg\alpha_s$	$\frac{1}{2}F_t(tg\alpha_s + tg\alpha_r)$
	α	–α	$-F_t.tg\alpha$	$F_t.tg\alpha$	0

Thank you

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