

EUROPEAN UNION
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Development and Education



the material was prepared under the auspices of ESF II (No. CZ.02.2.69/0.0/0.0/18_056/0013239)

Electrostatics and stationary current fields

Modeling of multiphysics problems

Department of electrical engineering and computer modeling
Faculty of electrical engineering
University of West Bohemia in Pilsen

31. května 2021

Obsah

1 Maxwell's equations

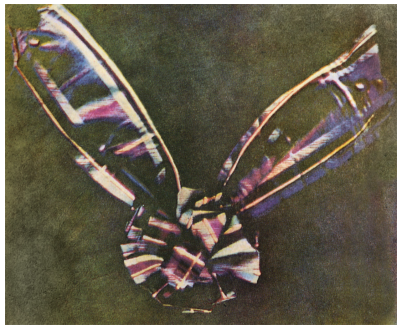
2 Electrostatic Field
- Basic Relationships
- Scalar potential
- Interface of two dielectrics
- Partial Differential Equations
- Energy in a Stationary Electric Field
- Forces in Electrostatic Field

3 Electric Current Field
- Basic Relationships
- Two Wire Interface
- Partial Differential Equations
- Energy and power in a stationary current field

Maxwell's equations

James Clerk Maxwell

- His work in the field of electromagnetism is based on Faraday's experiments.
- He postulated the basic relations between electricity and magnetism (20 equations).
- From the equations he predicted the existence of electromagnetic waves.
- He demonstrated the uniform nature of electromagnetism and light (electromagnetic waves).
- In 1864, he predicted the final speed of light propagation from electromagnetic experiments.
- Published the first color photograph as evidence of additive color mixing.



The first color photo



Scottish mathematician and physicist
(1831 - 1879)

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- The law of total current - Ampère's law

$$\oint_c \boldsymbol{H}\,\mathrm{d}\boldsymbol{l} = I + \frac{\partial \boldsymbol{\psi}}{\partial t}$$

- Faraday's law of induction

$$\oint_c \boldsymbol{E}\,\mathrm{d}\boldsymbol{l} = -\frac{\partial \boldsymbol{\phi}}{\partial t}$$

- Gauss's law of electrostatics

$$\oint_S \boldsymbol{D}\,\mathrm{d}\boldsymbol{S} = Q$$

- Law of continuity of induction flow

$$\oint_S \boldsymbol{B}\,\mathrm{d}\boldsymbol{S} = 0$$

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- Total current law - Ampere's law (shear currents are zero)

$$\oint_c \boldsymbol{H}\,\mathrm{d}\boldsymbol{l} = I$$

- Faraday's law of induction (induced voltage is zero)

$$\oint_c \boldsymbol{E}\,\mathrm{d}\boldsymbol{l} = 0$$

- Gauss's law of electrostatics

$$\oint_S \boldsymbol{D}\,\mathrm{d}\boldsymbol{S} = Q$$

- Law of continuity of induction flow

$$\oint_S \boldsymbol{B}\,\mathrm{d}\boldsymbol{S} = 0$$

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Maxwell's equations

Differential form - non-stationary field

- The law of total current - Ampere's law

$$\mathrm{curl}\boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

- Faraday's law of induction

$$\mathrm{curl}\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

- Gauss's law of electrostatics

$$\mathrm{div}\,\boldsymbol{D} = \rho$$

- Law of continuity of induction flow

$$\mathrm{div}\,\boldsymbol{B} = 0$$

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- Total current law - Ampere's law (shear currents are zero)

$$\mathrm{curl}\boldsymbol{H} = \boldsymbol{J}$$

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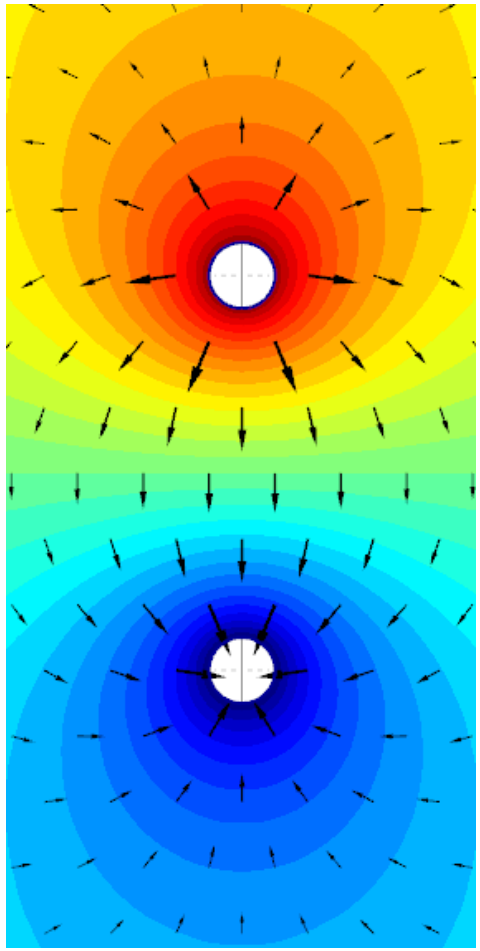
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Electrostatic Field

Electrostatic Field

- The source of the electric field are charges.
- Electric charge is a scalar quantity and can have positive or negative value.
- Quantum character (its magnitude is equal to an integral multiple of the elementary charge $e^- = -1{,}602 \times 10^{-19}\ \mathrm{C}$).
- Bulk charge density $\rho = \frac{\mathrm{d}Q}{\mathrm{d}V}$.
- Surface charge density $\sigma = \frac{\mathrm{d}Q}{\mathrm{d}S}$.
- Linear charge density $\tau = \frac{\mathrm{d}Q}{\mathrm{d}l}$.
- Free charges (electrons in metals, ions in electrolytes).
- Bound charges (dielectric polarization).



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Electrostatic Field

- Faraday's law of induction (induced voltage is zero)

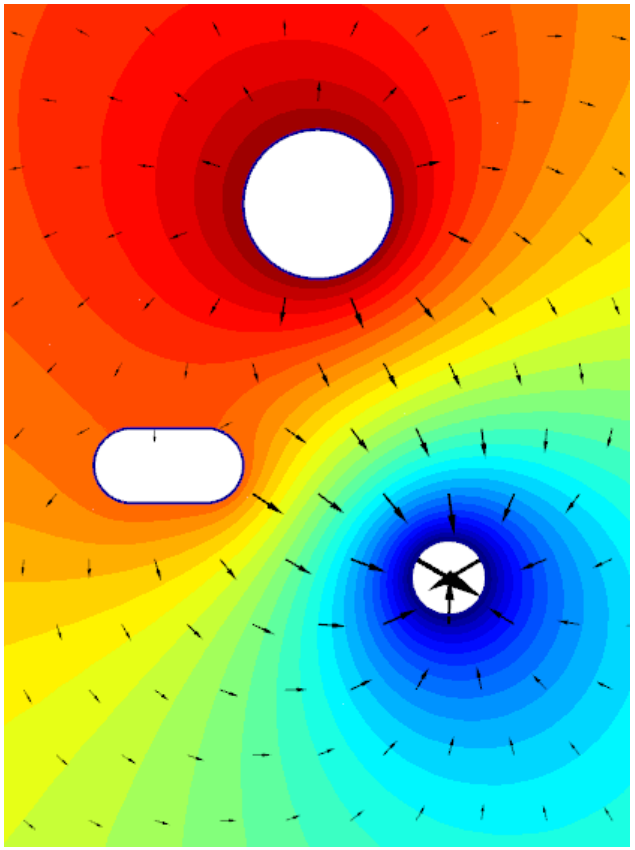
$$\operatorname{curl} \boldsymbol{E} = 0$$

  the electrostatic field is circulation-free

- Gauss's law of electrostatics

$$\operatorname{div} \boldsymbol{D} = \rho$$

  the source of the electrostatic field are electric charges (positive or negative)



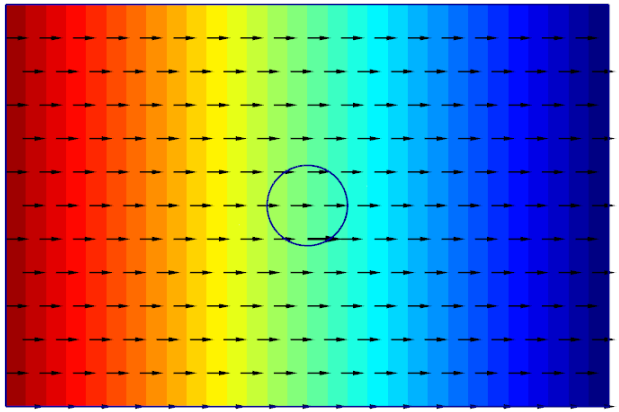
Positive source

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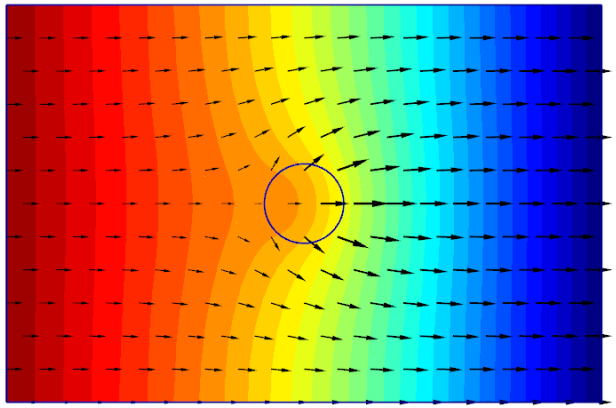
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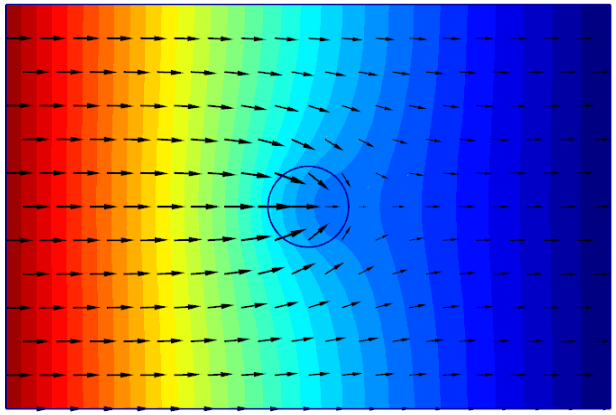
Electrostatic Field



Without source ($Q = 0\ \mathrm{C}$)



Spring ($Q > 0\ \mathrm{C}$)



Negative source ($Q < 0\ \mathrm{C}$)

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- The intensity of an electric field is defined as the force acting on a unit charge

$$\boldsymbol{F}_{\mathrm{L,e}} = Q\boldsymbol{E}$$

- Electric induction

$$\boldsymbol{D} = \varepsilon\boldsymbol{E}$$

- Voltage

$$U_{\mathrm{AB}} = \int_A^B \boldsymbol{E}\,\mathrm{d}\boldsymbol{l} = \varphi_{\mathrm{A}} - \varphi_{\mathrm{B}}$$

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Scalar potential ($\mathrm{curl}\boldsymbol{E} = 0$ and at the same time $\mathrm{curlgrad}\,\varphi = 0$)

$$\boldsymbol{E} = -\mathrm{grad}\,\varphi$$

(the minus sign respects the physical idea of the direction of the electric field intensity in the direction of the decreasing potential)

Properties of scalar potential:

- Suitable for indirect solutions (boundary value problems for potential)
- Ambiguous - may vary by constant

$$\boldsymbol{E} = -\mathrm{grad}\,\varphi = -\mathrm{grad}\,(\varphi + K)$$

- Continuity of electric potential - potential φ is a continuous function of position at all points.
- Physical interpretation: voltage is equal to the work done by transferring a unit charge between these points.

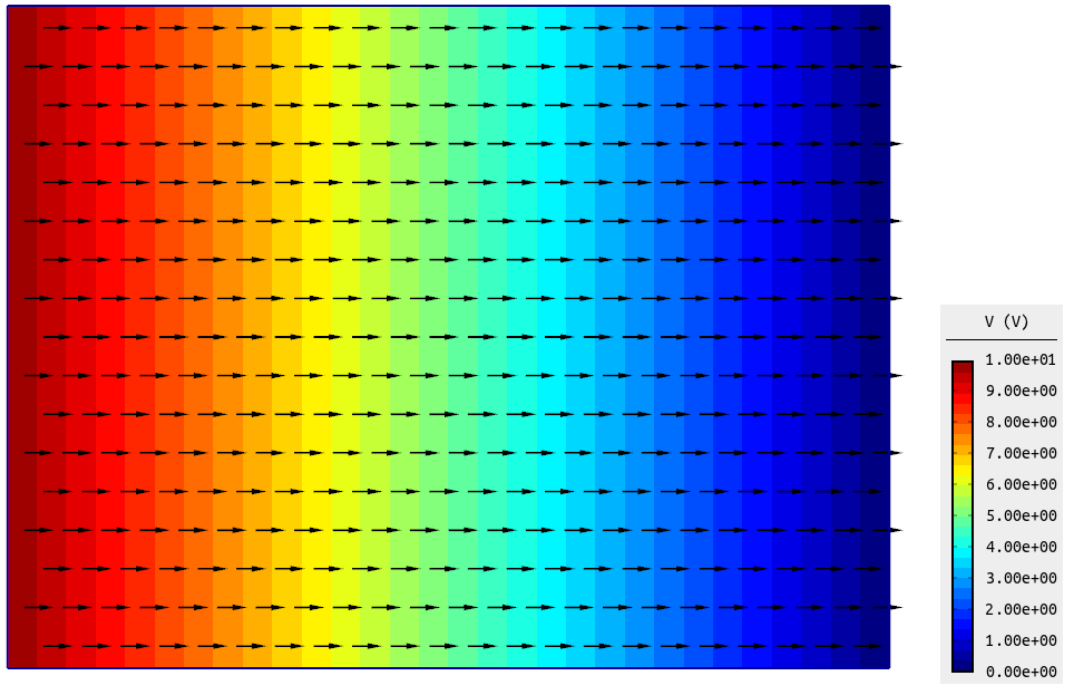
$$A = Q(\varphi_{\mathrm{A}} - \varphi_{\mathrm{B}}) = QU_{\mathrm{AB}}$$

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Consider the field in a plate capacitor. It has a value of $E = \frac{U}{d}$ at each point and is therefore homogeneous.



Electric field of plate capacitor ($E = \mathrm{konst.}$)
homogeneous field

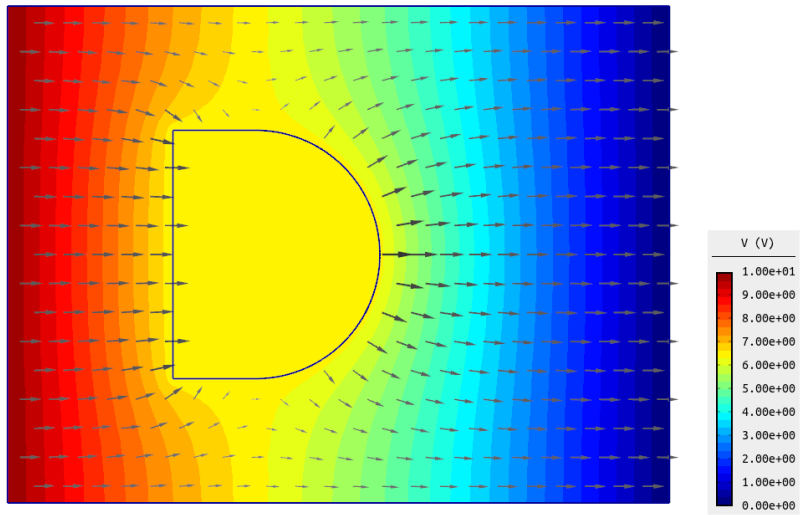
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If we apply a free electric charge to a conductor, it is distributed on its surface with a surface density σ. The inner part of the conductor is without macroscopic charge.

If we insert the conductor into the external field, surface charges are formed on its opposite edges $\pm\,\sigma$. The field inside the conductor is fully compensated (floating electrode $\varphi = \mathrm{konst.}$).



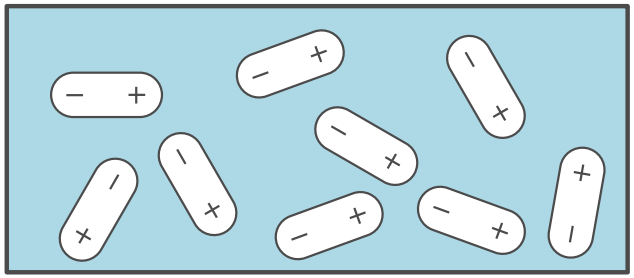
Conductor in electric field - electrostatic induction ($E_\mathrm{v} = 0$)

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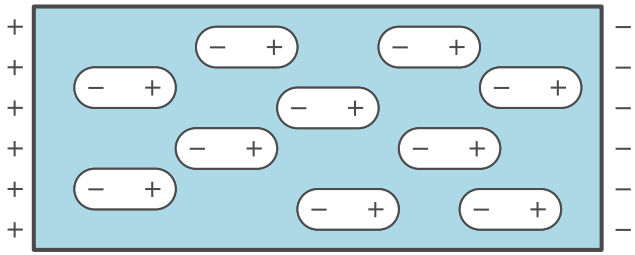
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- Dielectrics do not conduct a conductive current - they do not contain free charges
- A simple dielectric model consists of elementary dipoles that are randomly arranged.



- After applying an external electric field, the dipoles orient themselves in the direction of the electric field intensity.



- If we cancel the external electric field, the dipoles return to their original random order. However, there are materials we call *electrets* that remain polarized (similar to permanent magnets).

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- We define electrical induction

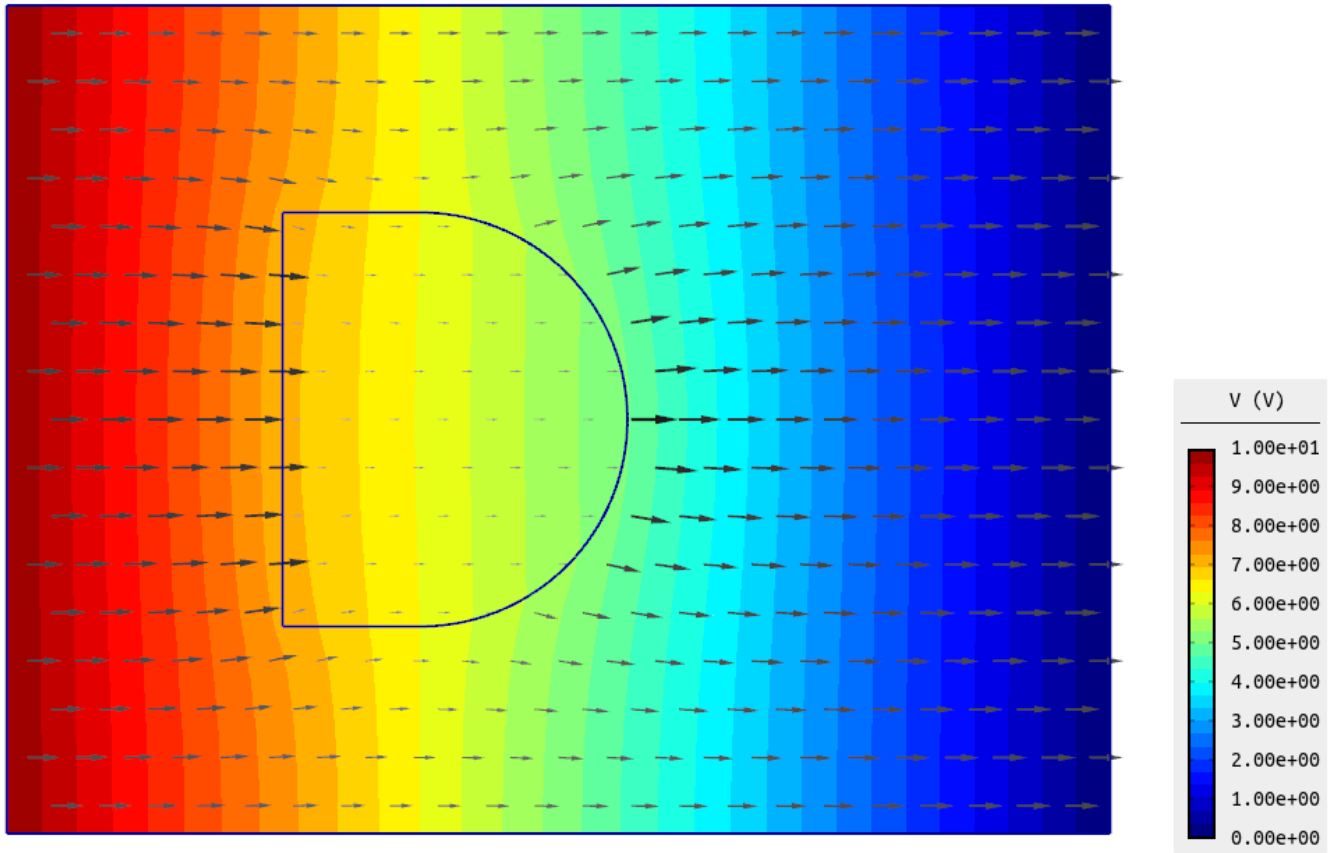
$$\boldsymbol{D} = \varepsilon_0 \varepsilon_\mathrm{r} \boldsymbol{E} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P},$$

  where the quantity $\boldsymbol{P}$ represents the polarization of the dielectric. From the above equation, the polarization vector in the form can be written

$$\boldsymbol{P} = \varepsilon_0 (\varepsilon_\mathrm{r} - 1) \boldsymbol{E} = \varepsilon_0 \varepsilon_\mathrm{r} \boldsymbol{E} - \varepsilon_0 \boldsymbol{E}.$$

  The quantity $\kappa_\mathrm{e} = \varepsilon_0 (\varepsilon_\mathrm{r} - 1)$ is called electrical susceptibility and represents an alternative description of the material properties of the dielectric.
- Polarization can be physically interpreted as an electrical induction acting against the effects of the field that caused it. The resulting field is therefore attenuated by the dielectric.
- No polarization occurs in air and metals ($\varepsilon_\mathrm{r} = 1 \Rightarrow \boldsymbol{P} = \boldsymbol{0}$).

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Dielectric in electric field ($E_{\mathrm{d}} \neq 0$)
electrostatic polarization

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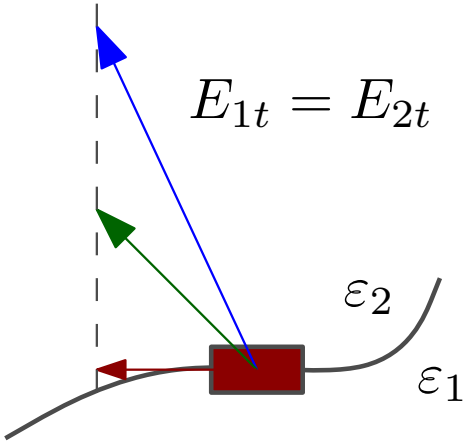
Maxwell's equations in differential form apply only at regular points in the environment. For example, regular points are not points that separate one environment from another and where there is a jump in permittivity ε. The state vectors of electric intensity $\boldsymbol{E}$ and induction $\boldsymbol{D}$ change discontinuously.

- Condition for $\boldsymbol{E}$

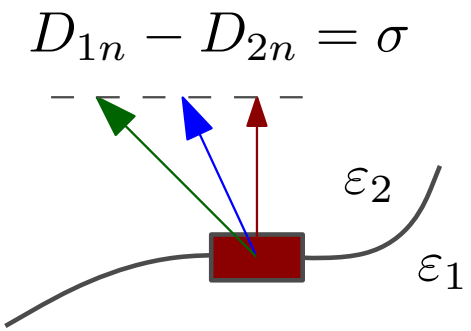
$$E_{1t} - E_{2t} = 0$$

- Condition for $\boldsymbol{D}$

$$D_{1n} - D_{2n} = \sigma$$



Electric field strength



Electrical induction

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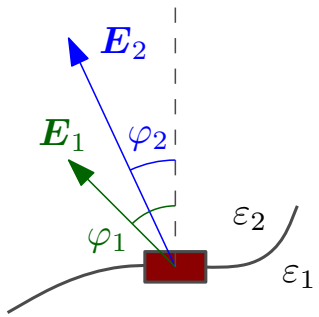
The law of refraction applies to electric lines of force. We start from the conditions at the interface (we neglect the surface charge density σ).

$$E_{1t} = E_{2t}, \quad D_{1n} = D_{2n}$$

After a simple adjustment we get

$$E_1 \sin \varphi_1 = E_2 \sin \varphi_2, \quad \varepsilon_1 E_1 \cos \varphi_1 = \varepsilon_2 E_2 \cos \varphi_2$$

Angles φ_1 and φ_2 are the angles between the field intensity vectors and the normal.



After further adjustment we obtain

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{\operatorname{tg} \varphi_1}{\operatorname{tg} \varphi_2}$$

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We start from Gauss's theorem in differential form

$$\operatorname{div} \boldsymbol{D} = \operatorname{div} \varepsilon \boldsymbol{E} = \rho$$

and a relationship introducing a scalar electric potential

$$\boldsymbol{E} = -\operatorname{grad} \varphi.$$

Substituting the relation for the potential into the Gaussian theorem, we obtain a partial differential equation in the form

$$\operatorname{div} \varepsilon \operatorname{grad} \varphi = -\rho.$$

In the linear case, ε can be placed before the divergence operator and we get the equation in the form

$$\operatorname{div} \operatorname{grad} \varphi = -\frac{\rho}{\varepsilon}.$$

It follows from the vector identity

$$\operatorname{div} \operatorname{grad} \varphi = \triangle \varphi.$$

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Derived equation

$$\triangle\varphi = -\frac{\rho}{\varepsilon}$$

we call the Poisson equation (it contains a nonzero right-hand side). In an environment without free charge ($\rho = 0$), the equation then goes to Laplace's equation in the form

$$\triangle\varphi = 0.$$

The scalar potential describing the electrostatic field in a linear environment must satisfy Laplace's or Poisson's equation.

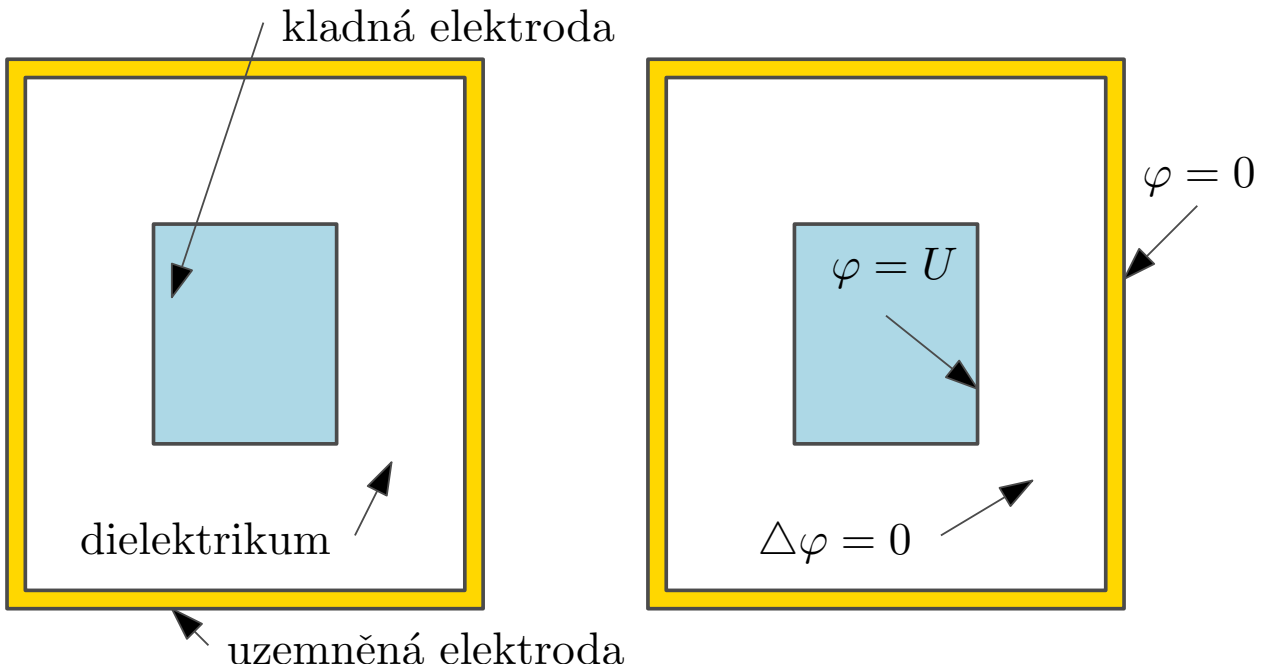
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The unambiguity of the solution must be ensured by suitable boundary conditions.

- Dirichlet boundary condition - knowledge of potential φ along the boundary of the region.
- Neumann boundary condition - knowledge of normal derivative of potential $\frac{\partial \varphi}{\partial n}$ along the boundary of the region.



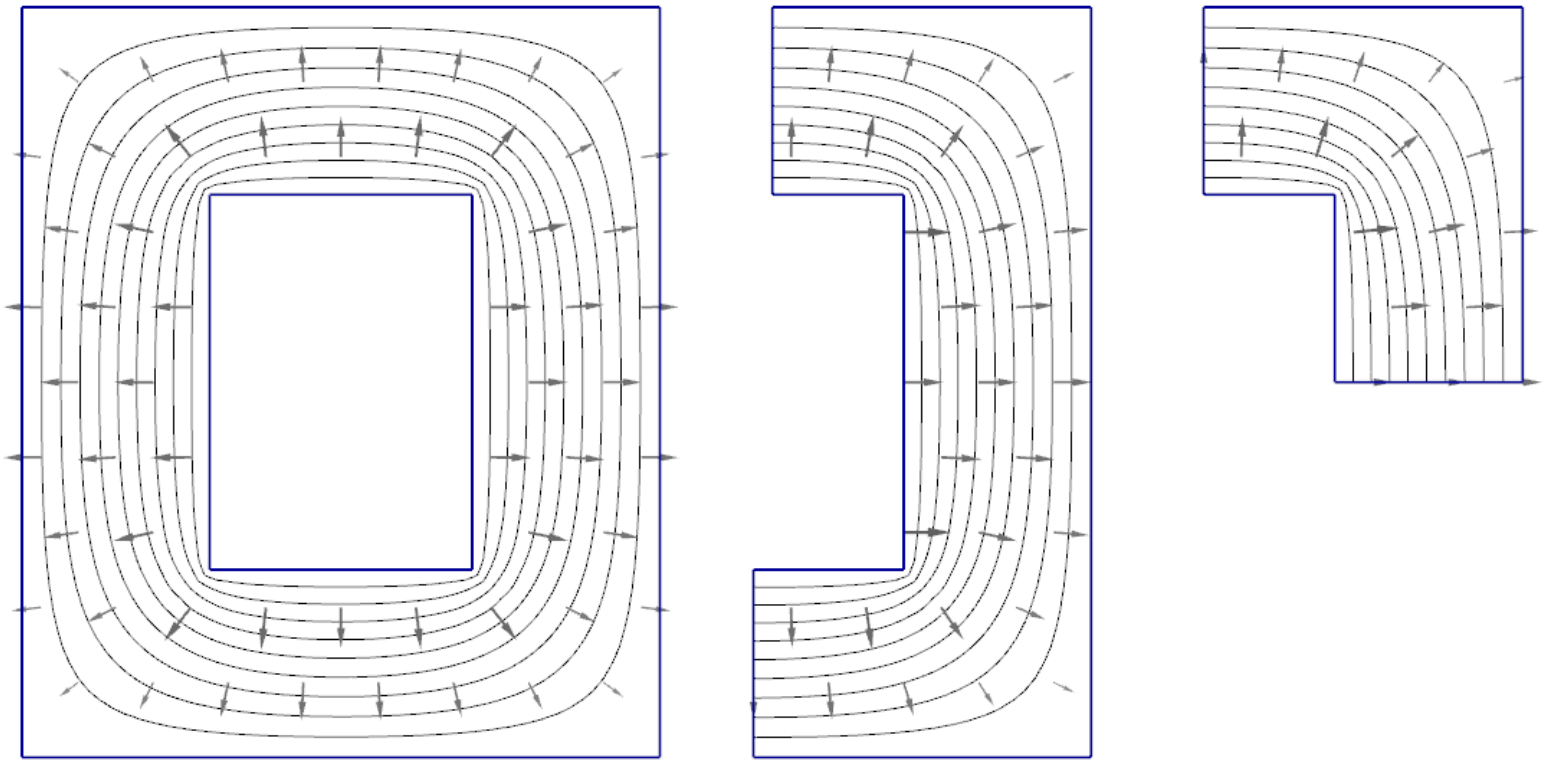
Capacitor in 2D

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- Dirichlet boundary condition - knowledge of the potential φ at the boundary of the region.
- Neumann's boundary condition - knowledge of the normal derivation of the potential $\frac{\partial \varphi}{\partial n}$ at the region boundary.



Capacitor in 2D - equipotentials and field vectors

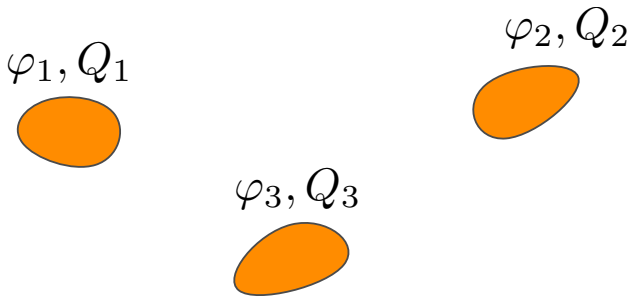
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The capacitor is an accumulation element. Thus, there is no dissipation of energy, but its storage. Energy can be stored in the ideal capacitor for any time.

Let us have a set of n conductors with charges $Q_1, \ldots, Q_n$ and with potentials $\varphi_1, \ldots, \varphi_n$ in an environment with constant permittivity ε.



The energy of the system is

$$W_{\mathrm{e}} = \frac{1}{2} \sum_{1}^{n} \varphi_i Q_i .$$

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In the case of a plate capacitor, it can be written

$$W_{\mathrm{e}} = \frac{1}{2}(\varphi_A(+Q) + \varphi_B(-Q)) = \frac{1}{2}Q(\varphi_A - \varphi_B) = \frac{1}{2}QU = \frac{1}{2}CU^2 = \frac{1}{2}\frac{Q^2}{C}$$

The energy definition of capacity follows from this relationship

$$C = \frac{2W_{\mathrm{e}}}{U^2}$$

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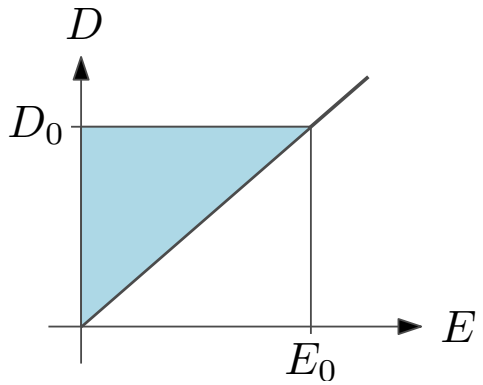
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Electric field energy can also be expressed directly using field vectors $\boldsymbol{E}$ and $\boldsymbol{D}$. The energy density can be written in the form

$$w_\mathrm{e} = \int_0^{\boldsymbol{D}} \boldsymbol{E}\,\mathrm{d}\boldsymbol{D}.$$

In the case of a linear isotropic medium (LIH), the permittivity is constant

$$w_\mathrm{e} = \int_0^D \frac{D}{\varepsilon}\,\mathrm{d}D = \frac{1}{2}\frac{D^2}{\varepsilon} = \frac{1}{2}ED.$$



The total energy of the system is then

$$W_\mathrm{e} = \int_V w_\mathrm{e}\,\mathrm{d}V$$

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In a plate capacitor, the energy density is constant.



Electric field strength



Energy density

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Lorentz force on a charge in an electric field

$$\boldsymbol{F}_{\mathrm{L,e}} = Q\boldsymbol{E}$$

Force derived from electric field energy

$$\boldsymbol{F}_{\mathrm{e}} = -\frac{\mathrm{d}W_{\mathrm{e}}}{\mathrm{d}\boldsymbol{s}} \Rightarrow \boldsymbol{F}_{\mathrm{e}} = -\mathrm{grad}\, W_{\mathrm{e}}$$

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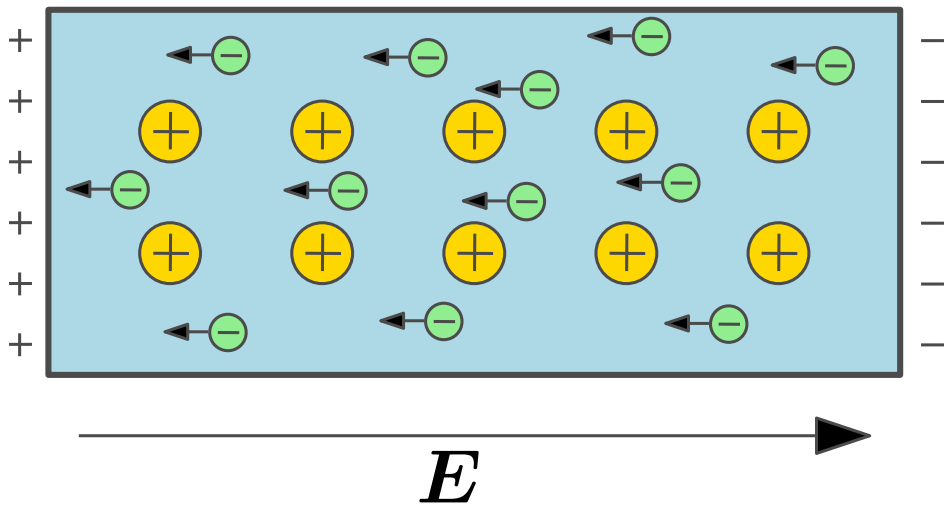
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Electric Current Field

Electric Current Field

There is no constant current in electrostatics (an electric field excited by stationary charges). Voltage sources have the property that there is a permanent excess of free charge carriers on their terminals. This allows us to permanently maintain their flow in the conductor. If these charges move, we speak of a stationary electric current field.



External electric field acting on electrons in a conductor ($\boldsymbol{F}_{\mathrm{L,e}} = Q\boldsymbol{E}$)

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- Maxwell's second equation

$$\mathrm{curl}\boldsymbol{E} = 0$$

the electric current field is circulation-free.

- The law of continuity

$$\mathrm{div}\,\boldsymbol{J} = 0$$

non - source electric current field.

- Scalar potential ($\mathrm{curl}\boldsymbol{E} = 0$ and valid at the same time $\mathrm{curlgrad}\,\varphi = 0$)

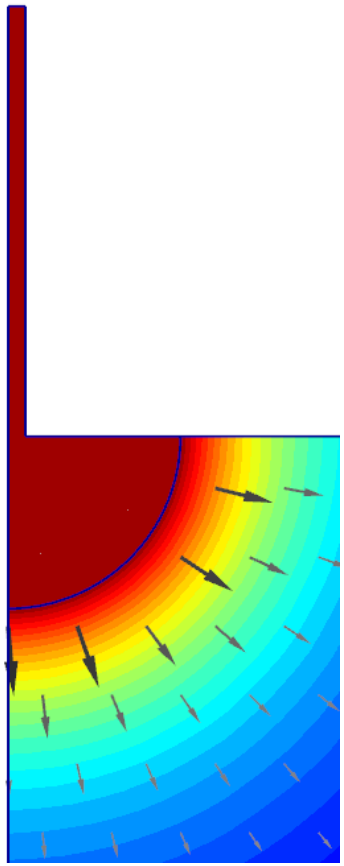
$$\boldsymbol{E} = -\mathrm{grad}\,\varphi.$$

- Current density

$$\boldsymbol{J} = \gamma\boldsymbol{E}.$$

- Voltage

$$U_{\mathrm{AB}} = \int_A^B \boldsymbol{E}\,\mathrm{d}\boldsymbol{l} = \varphi_{\mathrm{A}} - \varphi_{\mathrm{B}}.$$



Grounding electrode

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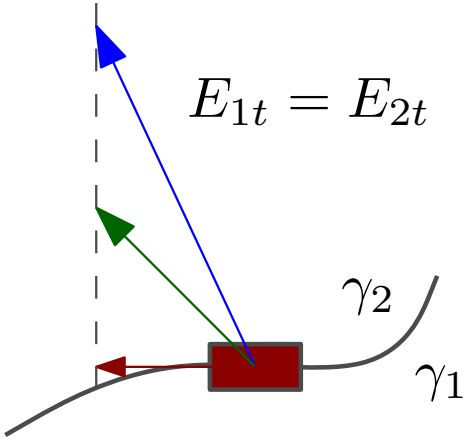
Maxwell's equations in differential form apply only at regular points in the environment. For example, regular points are not points that separate one environment from another and where there is a step change in conductivity γ. The state vectors of electric intensity $\boldsymbol{E}$ and current density $\boldsymbol{J}$ change discontinuously.

- Condition for $\boldsymbol{E}$

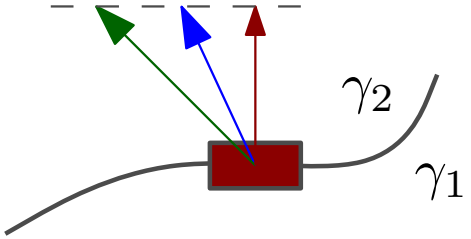
$$E_{1t} - E_{2t} = 0$$

- Condition for $\boldsymbol{J}$

$$J_{1n} - J_{2n} = 0$$



Electric field strength



Current density

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We start from the continuity equation in differential form

$$\operatorname{div} \boldsymbol{J} = \operatorname{div} \gamma \boldsymbol{E} = 0$$

and a relationship introducing a scalar electric potential

$$\boldsymbol{E} = -\operatorname{grad} \varphi.$$

By combining both relations we get a partial differential equation in the form

$$\operatorname{div} \gamma \operatorname{grad} \varphi = 0.$$

In the linear case, γ can be placed before the divergence operator and we get the equation in the form

$$\operatorname{div} \operatorname{grad} \varphi = 0.$$

The scalar potential describing a stationary current field in a linear medium must satisfy Laplace's equation

$$\triangle \varphi = 0.$$

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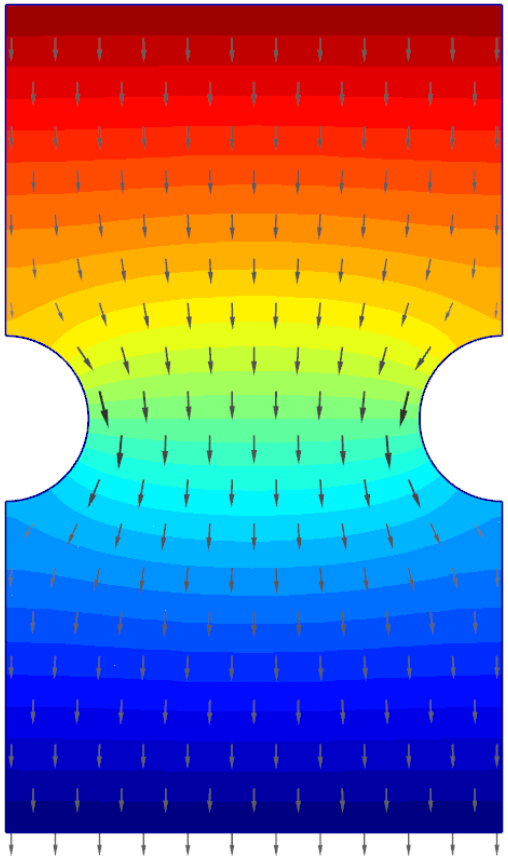
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Electric Current Field

Energy and power in a stationary current field

- The passage of electric current through a conductor of finite conductivity γ results in the dissipation of electric energy.
- It is an irreversible process in which electrical energy is converted into thermal energy (Joule heat).
- The conductor is heated by the passage of current and the heat is transferred to the surroundings by convection and radiation.
- Physically, this phenomenon can be explained by collisions of free charge carriers with a fixed lattice.



Potential

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The work proportional to the transfer of charge from point A to B is

$$A = W_{\mathrm{AB}} = \int_A^B \boldsymbol{F}\,\mathrm{d}\boldsymbol{l} = Q\int_A^B \boldsymbol{E}\,\mathrm{d}\boldsymbol{l} = QU.$$

This work turns into heat with power

$$P = \frac{\mathrm{d}W_{\mathrm{AB}}}{\mathrm{d}t} = U\frac{\mathrm{d}Q}{\mathrm{d}t} = UI.$$

Using Ohm's law, the relationship can be further transferred

$$P = UI = RI^2 = \frac{U^2}{R}.$$

We call the derived equation the integral form of Joule's law.

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However, sometimes it is more advantageous to work with the distribution of losses in volume, and therefore we introduce a differential form of Joule's law: the specific Joule's law. Let's get out from the relationships

$$\mathrm{d}Q = I\,\mathrm{d}t = \boldsymbol{J}\,\mathrm{d}\boldsymbol{S}\,\mathrm{d}t, \qquad \mathrm{d}U = \boldsymbol{E}\,\mathrm{d}l.$$

The increase in energy is then

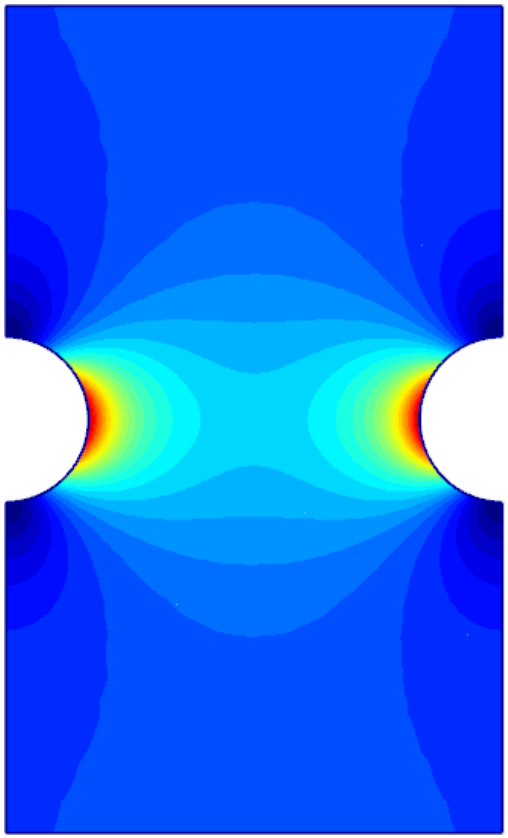
$$\mathrm{d}W = \mathrm{d}Q\,\mathrm{d}U = \boldsymbol{J}\boldsymbol{E}\,\mathrm{d}\boldsymbol{S}\,\mathrm{d}l\,\mathrm{d}t = \boldsymbol{J}\boldsymbol{E}\,\mathrm{d}V\,\mathrm{d}t.$$

The specific power is therefore equal to

$$p = \frac{\mathrm{d}W}{\mathrm{d}V\,\mathrm{d}t} = \boldsymbol{J}\boldsymbol{E} = \frac{J^2}{\gamma} = \gamma\boldsymbol{E}^2.$$

The total losses in the conductor are then

$$P = \int_V p\,\mathrm{d}V.$$



Specific losses

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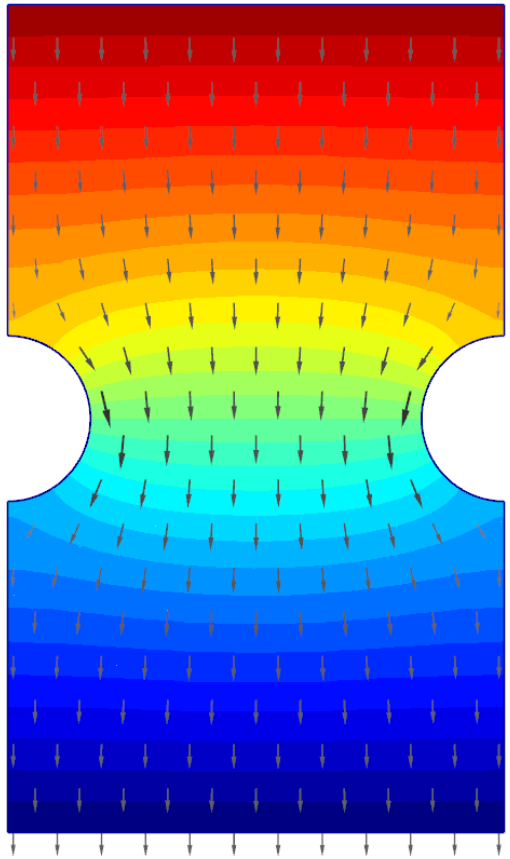
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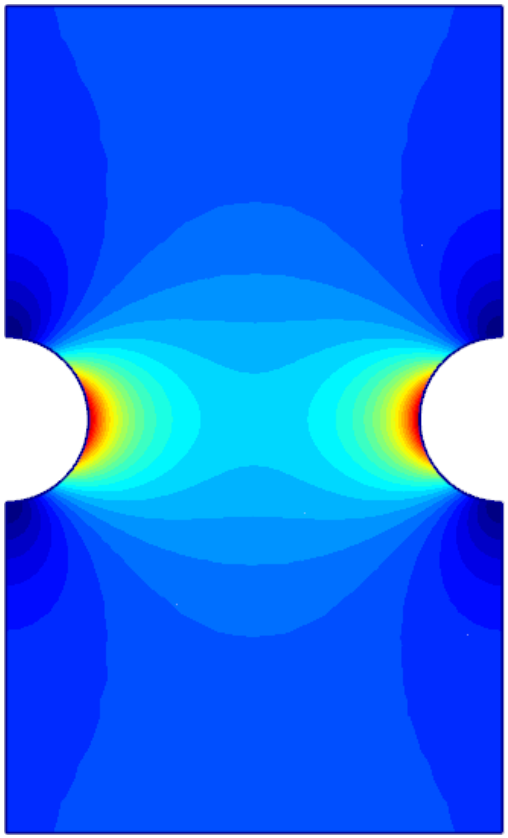
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There are then total losses in the conductor

$$P = \int_V p\,\mathrm{d}V.$$



Specific losses

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Děkuji za pozornost

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Faculty of electrical engineering
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