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Stationary and nonstationary magnetic fields

Modeling of multiphysics problems

Department of electrical engineering and computer modeling
Faculty of electrical engineering
University of West Bohemia in Pilsen

26. května 2021

FACULTY OF ELECTRICAL
ENGINEERING
UNIVERSITY
OF WEST BOHEMIA

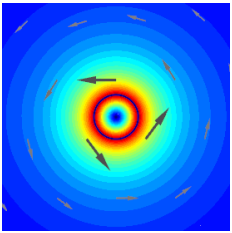
Magnetic field

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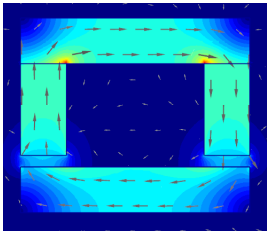
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Magnetic field

The magnetic field arises in the vicinity of moving charges, eg in the vicinity of current-carrying conductors. Permanent magnets are also a source of magnetic fields.



Magnetic field excited by a current-carrying conductor



Magnetic field excited by a permanent magnet

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the material was prepared under the auspices of ESF II (No. CZ.02.2.69/0.0/0.0/18_056/0013239)

- Magnetic induction is defined as the force acting on a unit charge (magnetic component of the Lorentz force)

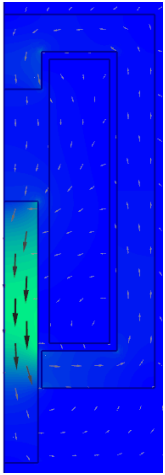
$$\boldsymbol{F}_{\mathrm{L,m}} = Q\,(\boldsymbol{v} \times \boldsymbol{B})$$

- Material equation

$$\boldsymbol{B} = \mu \boldsymbol{H}$$

- Magnetic voltage

$$U_{\mathrm{m,AB}} = \int_A^B \boldsymbol{H}\,\mathrm{d}\boldsymbol{l}$$



Actuator

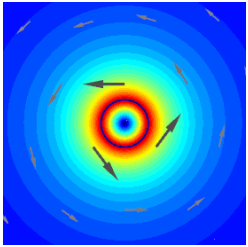
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- Maxwell's first equation

$$\mathrm{rot}\, \boldsymbol{H} = \boldsymbol{J}$$

the magnetic field is vortex



Wire (power lines are closed)

- Maxwell's fourth equation (law of continuity)

$$\mathrm{div}\, \boldsymbol{B} = 0$$

the magnetic field is non-spring

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The relationship between magnetic induction and field strength is given by permeability

$$\mu = \mu_0 \mu_r = \frac{B}{H},$$

where $\mu_0 = 4\pi \cdot 10^{-7}\,\mathrm{H/m}$ is the vacuum permeability.

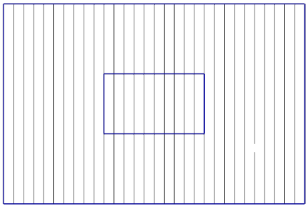
Magnetic materials can generally be divided into:

- Diamagnetics - their relative permeability is a little less than 1, in the magnetic field there is a slight weakening of the magnetic induction (copper)
- Paramagnetics - their relative permeability is slightly greater than 1, in the magnetic field there is a slight increase in magnetic induction (aluminum)
- Ferromagnets - their relative permeability is significantly greater than 1, in the magnetic field there is a strong amplification of magnetic induction (iron)

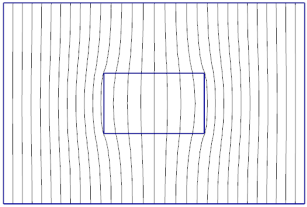
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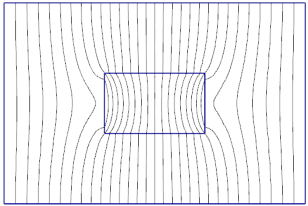
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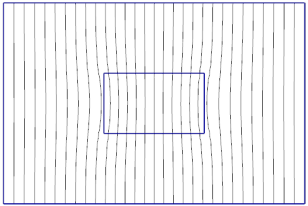
Vacuum ($\mu_\mathrm{r} = 1$)



Diamagnetic ($\mu_\mathrm{r} < 1$)



Ferromagnet ($\mu_\mathrm{r} \gg 1$)



Paramagnetic ($\mu_\mathrm{r} > 1$)

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Diamagnetismus

- Diamagnetics are composed of atoms whose resultant moment is zero
- In a diamagnetic substance, the arrangement in the external magnetic field changes electron shells of the atoms of matter, creating magnetic dipoles, whose magnetic field acts against an external magnetic field
- In diamagnetics, there is a slight weakening of the external magnetic field

Paramagnetismus

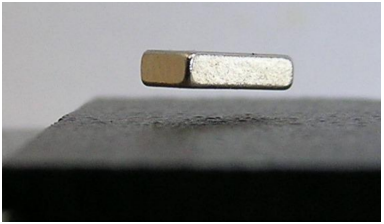
- Particles of paramagnetic matter have a permanent magnetic moment even in the absence of an external field (influence of unpaired electrons in orbitals)
- In paramagnetics, dipoles do not act on each other and in the absence of an external one the fields are randomly oriented (due to the thermal oscillations of the grid) and the total the magnetic moment is zero
- The dipoles of the paramagnetics in the external magnetic field rotate in the direction external field and a total magnetic moment is generated which is oriented in the direction of the external field
- In paramagnetics, the external magnetic field is slightly amplified

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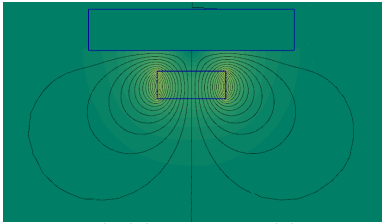
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- The ideal diamagnetic (superconductor) completely pushes the magnetic field out of its volume
- Superconductors are materials that, when cooled below a certain temperature, show immeasurable resistance to the conduction of electric current and very much they intensely displace the magnetic field from their volume
- Superconductors are not just metals, but modern superconductors are ceramic-based materials
- In terms of magnetic properties, they are superconductors so - called perfect diamagnets (they push the force lines of the magnetic field from their volume)



Superconductor - ideal diamagnetic



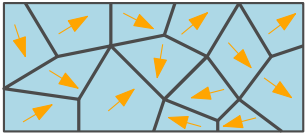
Ideal diamagnet model

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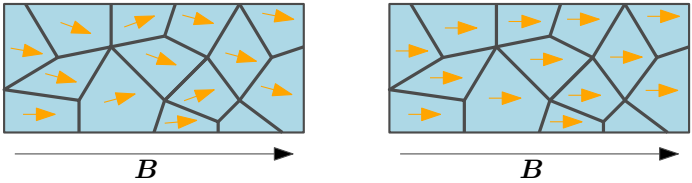
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- The magnetic substance consists of spontaneous magnetization domains (Weiss domains separated by Bloch walls) - small Areas of dimensions of the order of $10^{-3}\,\mathrm{mm}$
- Each domain is itself an elementary magnet
- The individual domains are oriented towards each other completely randomly



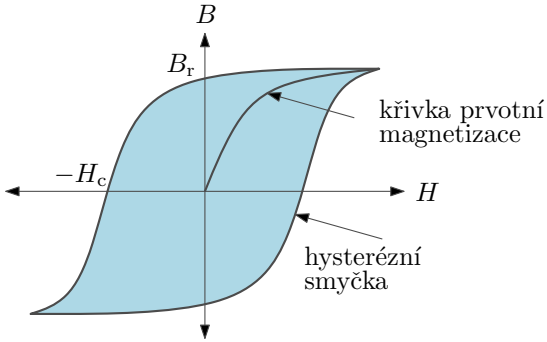
- By the action of an external magnetic field, domains can be arranged (magnetization)



Partially (left) and fully (right) magnetized ferromagnet

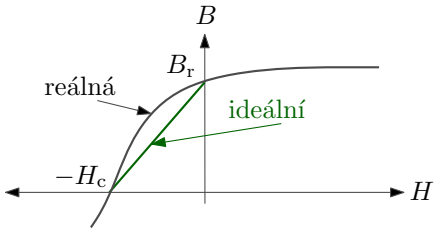
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- In cyclic magnetization of ferromagnets, the dependence of magnetic induction on field strength is given by a hysteresis loop
- Remanent induction B_r, coercive force H_c
- wide hysteresis curve (permanent magnets), narrow loop (magnetic materials for the manufacture of electrical machines)



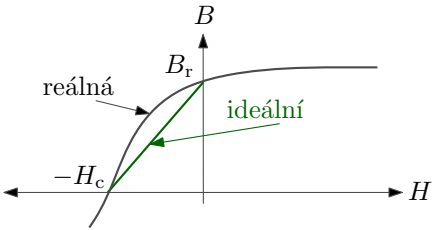
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- Magnetically hard ferromagnets have a curve with a small slope and a wide hysteresis curve. They usually have low permeability and a high value of coercive force H_c. Magnetically hard materials are therefore used as permanent magnets.
- Hard ferrites - 80 % iron oxide + 20 % barium (BaO) or strontium oxide (SrO) - cheap and reliable permanent magnets
- Rare earth magnets - neodymium-iron-boron (NdFeB) - this material is currently the strongest type of magnet with excellent magnetic properties and the highest internal energy. However, they can only be operated at a relatively low operating temperature (between 80 - 150 °C)



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- We can easily get from the picture

$$B = \frac{B_r}{H_c} \cdot H + B_r = \mu_0 \mu_r H + B_r$$

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We start from the continuity of magnetic flux ($\operatorname{div} \boldsymbol{B} = 0$ and at the same time $\operatorname{div} \operatorname{rot} \boldsymbol{A} = 0$)

$$\boldsymbol{B} = \operatorname{rot} \boldsymbol{A}\,.$$

Properties of vector potential:

- Continuity of vector potential - potential $\boldsymbol{A}$ is a continuous function of position at all points.
- Ambiguous - may differ by constant K and in addition by gradient of any scalar function ψ

$$\boldsymbol{B} = \operatorname{rot} \boldsymbol{A} = \operatorname{rot} (\boldsymbol{A} + \boldsymbol{K} + \operatorname{grad} \psi)\,.$$

  We impose an additional condition, the choice of which limits the class of functions ψ in the form

$$\operatorname{div} \boldsymbol{A} = 0.$$

  This condition is called Coulomb (sometimes also calibration).
- Suitable for indirect solutions (boundary value problems for potential).

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Otázka:
Why are we introducing another vector quantity? Both magnetic induction $\boldsymbol{B}$ and magnetic potential $\boldsymbol{A}$ are vector functions.

Odpověd:

- Continuity of potential.
- Special case of a two-dimensional problem

$$\boldsymbol{B}(x,y) = \operatorname{rot} \boldsymbol{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & A_z \end{vmatrix} = \mathbf{i}\frac{\partial A_z}{\partial y} - \mathbf{j}\frac{\partial A_z}{\partial x}.$$

For the magnetic field $\boldsymbol{B}(x,y)$, the vector magnetic potential $\boldsymbol{A}$ has only the component A_z in the direction of the axis z. It can therefore be considered as a scalar quantity.

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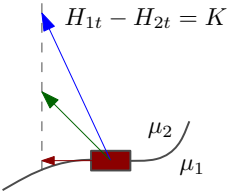
Maxwell's equations in differential form apply only at regular points in the environment. For example, regular points are not points that separate one environment from another and where there is a jump in permeability μ. The state vectors of magnetic induction $\boldsymbol{B}$ and magnetic intensity $\boldsymbol{H}$ change discontinuously.

- condition for $\boldsymbol{H}$

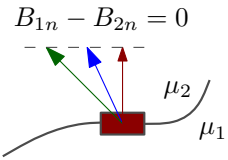
$$H_{1t} - H_{2t} = K$$

- condition for $\boldsymbol{B}$

$$B_{1n} - B_{2n} = 0$$



Magnetic field strength



Magnetic induction

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We start from Ampère's law in differential form

$$\operatorname{rot} \boldsymbol{H} = \boldsymbol{J}$$

and a relationship introducing a magnetic vector potential

$$\boldsymbol{B} = \operatorname{rot} \boldsymbol{A}.$$

Substituting into the previous equation we get a partial differential equation in the form

$$\operatorname{rot} \frac{1}{\mu} \operatorname{rot} \boldsymbol{A} = \boldsymbol{J}.$$

In the linear case, μ can be placed before the rotation operator and we get the equation in the form

$$\operatorname{rot} \operatorname{rot} \boldsymbol{A} = \mu \boldsymbol{J}.$$

It follows from the vector identity

$$\operatorname{rot} \operatorname{rot} \boldsymbol{A} = \operatorname{grad} \operatorname{div} \boldsymbol{A} - \operatorname{div} \operatorname{grad} \boldsymbol{A}\,.$$

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Using Coulomb calibration conditions

$$\operatorname{div} \boldsymbol{A} = 0$$

we get

$$\operatorname{div} \operatorname{grad} \boldsymbol{A} = -\mu \boldsymbol{J}.$$

It follows from the vector identity ($\operatorname{div} \operatorname{grad} \boldsymbol{A} = \triangle \boldsymbol{A}$)

$$\triangle \boldsymbol{A} = -\mu \boldsymbol{J}\,.$$

The derived equation is called the Poisson equation (it contains a nonzero right side). In an environment without an external current source ($\boldsymbol{J} = \boldsymbol{0}$), the equation then goes to Laplace's equation in the form

$$\triangle \boldsymbol{A} = \boldsymbol{0}.$$

The vector potential describing a stationary magnetic field in a linear medium must satisfy Laplace's or Poisson's equation. A clear solution must be provided by boundary conditions.

Magnetic circuits are often made of ferromagnetic materials, so it is usually necessary to solve the equation in its nonlinear form. Including the influence of permanent magnets, it then takes shape

$$\operatorname{rot} \frac{1}{\mu} \left(\operatorname{rot} \boldsymbol{A} - \boldsymbol{B}_{\mathbf{r}}\right) = \boldsymbol{J}.$$

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In areas without electricity, the right side of Ampère's law is zero. The field is not vortex and can be introduced analogously to the electric field magnetic scalar potential φ_m

$$\boldsymbol{H} = \operatorname{grad} \varphi_m .$$

The relationship between magnetic induction and magnetic field strength in the presence of a region with permanent magnets can be written

$$\boldsymbol{B} = \mu \boldsymbol{H} + \boldsymbol{B}_{\mathrm{r}},$$

where $\boldsymbol{B}_{\mathrm{r}}$ represents remanent magnetic induction. By analogy, a partial differential equation in the form can be derived

$$-\operatorname{div} \left(\mu \cdot \operatorname{grad} \ \varphi_m - \boldsymbol{B}_{\mathrm{r}}\right) = 0 .$$

Using this equation for scalar potential φ_m it is possible to solve relatively complex problems in 3D with a significant reduction in the number of variables (magnetic vector potential has 3 components and scalar only one)

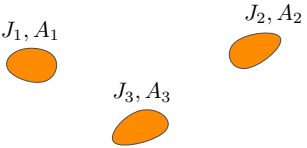
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The coil is an accumulation element. Thus, there is no dissipation of energy, but its storage.

Let us have a system of n conductors flowing through currents with density $\boldsymbol{J}_1, \ldots, \boldsymbol{J}_n$ with potentials $\boldsymbol{A}_1, \ldots, \boldsymbol{A}_n$ in an environment with constant permeability μ.



The energy of the system is

$$W_{\mathrm{m}} = \frac{1}{2} \sum_{1}^{n} \int_{V} \boldsymbol{J}_i \boldsymbol{A}_i \, \mathrm{d}V.$$

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In the case of a thin conductor, the relationship can be further shaped

$$W_{\mathrm{m}} = \frac{1}{2}\int_V \boldsymbol{J}\boldsymbol{A}\,\mathrm{d}V = \frac{1}{2}\oint_c \boldsymbol{A}\,\mathrm{d}\boldsymbol{l}\int_S \boldsymbol{J}\,\mathrm{d}\boldsymbol{S} = \frac{1}{2}\phi I = \frac{1}{2}LI^2.$$

Let's remember

$$\phi = \int_S \boldsymbol{B}\,\mathrm{d}\boldsymbol{S} = \int_S \mathrm{rot}\,\boldsymbol{A}\,\mathrm{d}\boldsymbol{S} = \oint_c \boldsymbol{A}\,\mathrm{d}\boldsymbol{l}, \quad L = \frac{\phi}{I}.$$

The energy definition of inductance follows from this relation

$$L = \frac{2W_{\mathrm{m}}}{I^2}\,.$$

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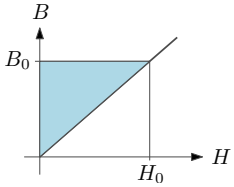
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The energy of the magnetic field can also be expressed directly using the field vectors $\boldsymbol{H}$ and $\boldsymbol{B}$. The energy density can be written in the form

$$w_{\mathrm{m}} = \int_0^{\boldsymbol{B}} \boldsymbol{H}\,\mathrm{d}\boldsymbol{B}.$$

In the case of a linear isotropic medium (LIH), the permeability is constant

$$w_{\mathrm{m}} = \int_0^B \frac{B}{\mu}\,\mathrm{d}B = \frac{1}{2}\frac{B^2}{\mu} = \frac{1}{2}HB.$$

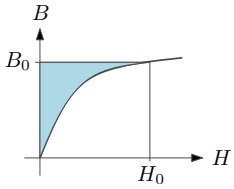


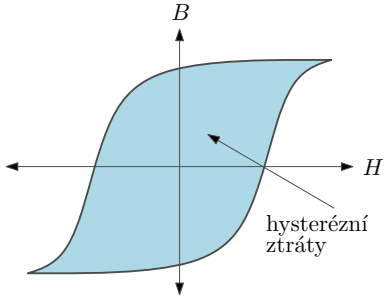
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Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ MLÁDEŽE A TĚLOVÝCHOVY

the material was prepared under the auspices of ESF II (No. CZ.02.2.69/0.0/0.0/18_056/0013239)

However, ferromagnetic materials are nonlinear. Permeability is a function of magnetic induction $\mu(\boldsymbol{B})$. The hysteresis losses associated with the periodic magnetization of the ferromagnet are associated with the area of the hysteresis curve. Electrical machines are therefore made of sheets with a narrow hysteresis curve.

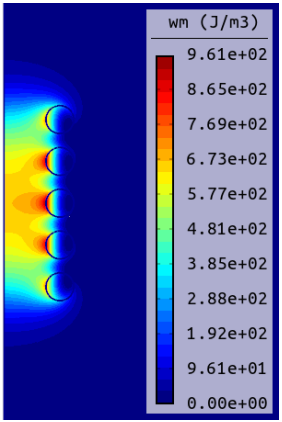




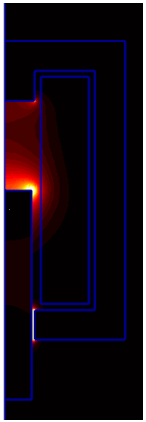
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Energy density is accumulated mainly in air gaps



Solenoid



Actuator (linear motor)

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The Lorentz force on a charge in an electromagnetic field can generally be written

$$\boldsymbol{F}_{\mathrm{L}} = Q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})\,.$$

The Lorentz force on the charge in the magnetic field is then

$$\boldsymbol{F}_{\mathrm{L,m}} = Q(\boldsymbol{v} \times \boldsymbol{B}).$$

The magnetic part can be further converted using

$$Q\boldsymbol{v} = \int_V \rho \boldsymbol{v}\,\mathrm{d}V = \int_V \boldsymbol{J}\,\mathrm{d}V$$

on face

$$\boldsymbol{F}_{\mathrm{L,m}} = \int_V (\boldsymbol{J} \times \boldsymbol{B})\,\mathrm{d}V\,.$$

In the case of a thin wire

$$\boldsymbol{F}_{\mathrm{L,m}} = I \int_c (\,\mathrm{d}\boldsymbol{l} \times \boldsymbol{B})\,.$$

Force derived from magnetic field energy

$$\boldsymbol{F}_{\mathrm{m}} = \frac{\mathrm{d}W_{\mathrm{m}}}{\mathrm{d}\boldsymbol{s}} \Rightarrow \boldsymbol{F}_{\mathrm{m}} = \mathrm{grad}\, W_{\mathrm{m}}\,.$$

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Non-Stationary Electromagnetic Field

- The law of total current - Ampere's law

$$\operatorname{rot} \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

- Faraday's law of induction

$$\operatorname{rot} \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

- Gauss's law of electrostatics

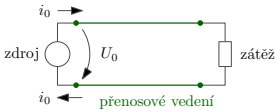
$$\operatorname{div} \boldsymbol{D} = \rho$$

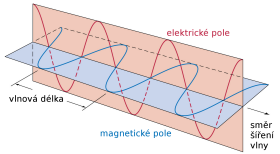
- Law of continuity of induction flow

$$\operatorname{div} \boldsymbol{B} = 0$$

- Law of continuity of electric current (does not belong to MR)

$$\operatorname{div} \boldsymbol{J} = -\frac{\partial \rho}{\partial t}, \quad \left(I = \oint_S \boldsymbol{J}\,\mathrm{d}\boldsymbol{S} = -\frac{\partial Q}{\partial t} = -\frac{\partial}{\partial t}\int_V \rho\,\mathrm{d}V \right)$$





Electromagnetic wave propagation

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If we scalar multiply the first Maxwell's equation in differential form

$$\operatorname{rot} \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

vector $\boldsymbol{E}$ and the second

$$\operatorname{rot} \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

vector $\boldsymbol{H}$, their difference has shape

$$\boldsymbol{E} \operatorname{rot} \boldsymbol{H} - \boldsymbol{H} \operatorname{rot} \boldsymbol{E} = \boldsymbol{J}\boldsymbol{E} + \boldsymbol{E}\frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{H}\frac{\partial \boldsymbol{B}}{\partial t}.$$

It follows from the vector identity that the expression on the left side of the equation can be written in the form

$$\boldsymbol{E} \operatorname{rot} \boldsymbol{H} - \boldsymbol{H} \operatorname{rot} \boldsymbol{E} = -\operatorname{div} (\boldsymbol{E} \times \boldsymbol{H})$$

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We will further integrate the obtained equation over the volume V

$$0 = \int_V \boldsymbol{J}\boldsymbol{E}\,\mathrm{d}V + \int_V \boldsymbol{E}\frac{\partial \boldsymbol{D}}{\partial t}\,\mathrm{d}V + \int_V \boldsymbol{H}\frac{\partial \boldsymbol{B}}{\partial t}\,\mathrm{d}V + \int_V \mathrm{div}\,(\boldsymbol{E}\times\boldsymbol{H})\,\mathrm{d}V.$$

We convert the last term to the area integral using the Gauss-Ostrogradsky theorem, substitute the intensity of the electric field including the imprinted forces for the product $\boldsymbol{J}\boldsymbol{E}$ and

$$\int_V \boldsymbol{J}\boldsymbol{E}_{\boldsymbol{v}}\,\mathrm{d}V = \int_V \frac{\boldsymbol{J}^{\boldsymbol{2}}}{\gamma}\,\mathrm{d}V + \int_V \boldsymbol{E}\frac{\partial \boldsymbol{D}}{\partial t}\,\mathrm{d}V + \int_V \boldsymbol{H}\frac{\partial \boldsymbol{B}}{\partial t}\,\mathrm{d}V + \oint_S (\boldsymbol{E}\times\boldsymbol{H})\,\mathrm{d}\boldsymbol{S},$$

which can be rewritten into a form

$$P_{\mathrm{v}} = P_{\mathrm{J}} + P_{\mathrm{e}} + P_{\mathrm{m}} + P_{\mathrm{N}},$$

where P_{v} is the power of the imprinted forces, P_{J} are the Joule losses, P_{e} is the power required to create an electric field, P_{m} is the power required to create a magnetic field and finally P_{N} can be interpreted as transmitted power.

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Product

$$\boldsymbol{N} = \boldsymbol{E} \times \boldsymbol{H} \tag{1}$$

we call it the Poynting vector and has the meaning of the radiated power density $(\mathrm{W}/\mathrm{m}^2)$. The total power transmitted by the Poynting vector is obtained by integrating this vector across the considered boundary

$$P_{\mathrm{N}} = \oint_S \boldsymbol{N}\,\mathrm{d}\boldsymbol{S}.$$

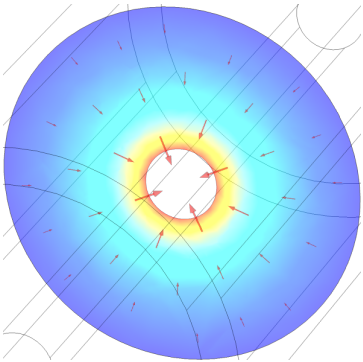
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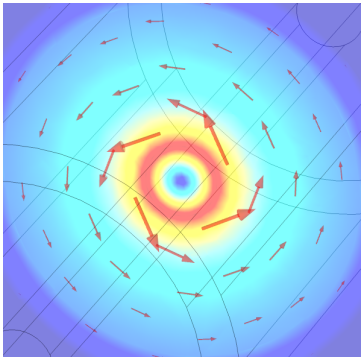
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Non-Stationary Electromagnetic Field

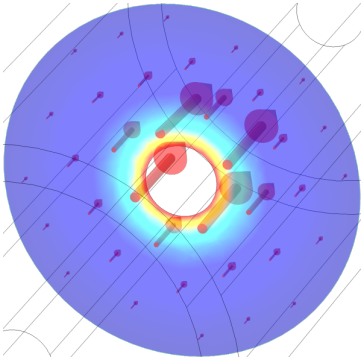
Energy propagation in coaxial cable



Electric field



Magnetic field



Poynting vector

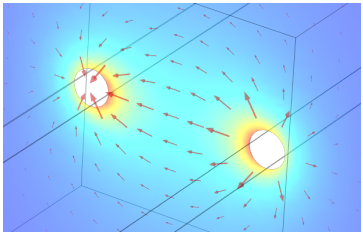
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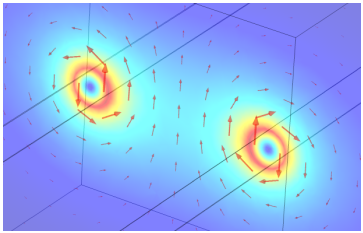
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Non-Stationary Electromagnetic Field

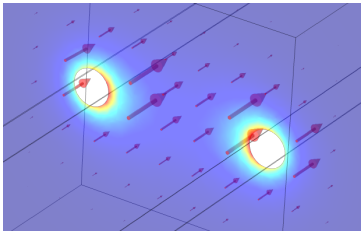
Energy propagation along a two-wire line



Electric field



Magnetic field



Poynting vector

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In the case of harmonic power supplies, SKM can be used to advantage

$$\frac{\mathrm{d}}{\mathrm{d}t} \longrightarrow \mathrm{j}\omega,$$

where $\omega = 2\pi f$ is the angular frequency. Maxwell's equations can then be written in the form

- The law of total current - Ampere's law

$$\mathrm{rot}\ \underline{\boldsymbol{H}} = \underline{\boldsymbol{J}} + \mathrm{j}\omega\underline{\boldsymbol{D}}$$

- Faraday's law of induction

$$\mathrm{rot}\ \underline{\boldsymbol{E}} = -\mathrm{j}\omega\underline{\boldsymbol{B}}$$

- Gauss's law of electrostatics

$$\mathrm{div}\ \underline{\boldsymbol{D}} = \rho$$

- Law of continuity of induction flow

$$\mathrm{div}\ \underline{\boldsymbol{B}} = 0$$

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According to Maxwell's second equation (Faraday's law of induction), a vortex electric field is created by a time change of a magnetic field

$$u_\mathrm{i} = \oint_c \boldsymbol{E}\,\mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}\phi}{\mathrm{d}t}.$$

The closed loop voltage c flowing through the time-varying inductive current ϕ is non-zero. We call it induced voltage u_i. In addition, if this loop is conductive, a current given by the relation flows through it (the inductance of the coil L is neglected)

$$i_\mathrm{c} = \frac{u_\mathrm{i}}{R}.$$

This current is called the eddy current. Note that the flux ϕ is the total magnetic flux through the loop. Derivation $\frac{\mathrm{d}}{\mathrm{d}t}$ is substantial (full) and involves a change in the flow of transformations and motion.

Depth of penetration (current density drops to 37 % of its value on the surface)

$$\delta = \sqrt{\frac{2}{\omega\gamma\mu}}$$

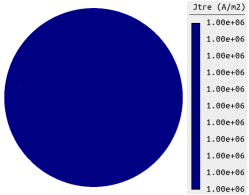
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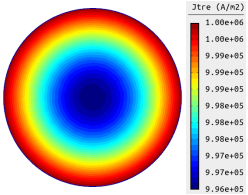
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Non-Stationary Electromagnetic Field

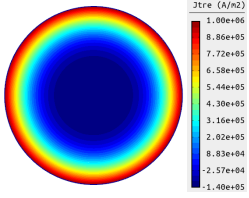
Induced Voltage



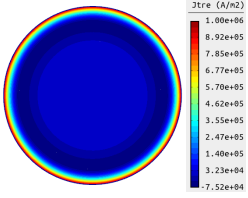
f=0 Hz



f=50 Hz



f=5000 Hz



f=50,000 Hz

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With slow time changes, the shear current given by the second member of the right side in I. MR can be neglected. The model is therefore based on the following equations

$$\operatorname{rot} \boldsymbol{H} = \boldsymbol{J},$$

rot $\boldsymbol{E}$=- d$\boldsymbol{B}$ $\overline{\mathrm{d}t}$ $and equations based on Lorentz force \boldsymbol{E}' = \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = \boldsymbol{E} + \boldsymbol{v} \times \operatorname{rot} \boldsymbol{A}, where \boldsymbol{E}$'$is a generalized electric fieldi$
$\times \boldsymbol{B}$.

Substituting the Lorentz force into the II. MR we obtain

$$\operatorname{rot} \left(\boldsymbol{E}' - \boldsymbol{v} \times \operatorname{rot} \boldsymbol{A} \right) = - \frac{\mathrm{d} \operatorname{rot} \boldsymbol{A}}{\mathrm{d}t}.$$

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By exchanging the time derivative and the spatial operator of rotation, we obtain

$$\mathrm{rot}\,(\boldsymbol{E}' - \boldsymbol{v} \times \mathrm{rot}\,\boldsymbol{A}) = -\mathrm{rot}\,\frac{\partial \boldsymbol{A}}{\partial t}$$

and by removing the rotation operator we finally get a solution in the form

$$\boldsymbol{E}' = \boldsymbol{v} \times \mathrm{rot}\,\boldsymbol{A} - \frac{\partial \boldsymbol{A}}{\partial t} - \mathrm{grad}\,\varphi + \boldsymbol{g}(t)$$

(applying the rotation operator to the second and third members of the right-hand side of the equation gives zero). We now multiply the equation by the electrical conductivity γ and obtain the total current density in the form

$$\boldsymbol{J} = \gamma \boldsymbol{E}' = \gamma \boldsymbol{v} \times \mathrm{rot}\,\boldsymbol{A} - \gamma \frac{\partial \boldsymbol{A}}{\partial t} - \gamma \mathrm{grad}\,\varphi + \gamma \boldsymbol{g}(t)$$

and we assess the physical significance of all members in the acquired relationship.

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The left side of the equation represents the total current density at a given point and the right side its component. The first of them, given by the motion in the magnetic field, expresses the motion component of the eddy currents

$$\boldsymbol{J}_{\mathrm{pohyb}} = \gamma \boldsymbol{v} \times \mathrm{rot}\, \boldsymbol{A}.$$

The second of them, given by the time derivative of the magnetic field, expresses the transformation component of the eddy currents. Their density is given by the relation

$$\boldsymbol{J}_{\mathrm{trans}} = -\gamma \frac{\partial \boldsymbol{A}}{\partial t}.$$

The third term represents a potential component of the current density (without rotation), which can be interpreted as the current density from an external field source.

$$\boldsymbol{J}_{\mathrm{pot}} = -\gamma \mathrm{grad}\, \varphi\,.$$

Finally, the fourth term (vector function dependent only on time) can represent a current density of another origin of magnitude

$$\boldsymbol{J}_{\mathrm{ext}} = -\gamma \boldsymbol{g}(t)\,.$$

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The motion component of eddy currents can be converted to an integral shape

$$u_{\mathrm{i,p}} = \oint_c \boldsymbol{E}_{\mathrm{pohyb}}\,\mathrm{d}\boldsymbol{l} = \oint_c (\boldsymbol{v} \times \mathrm{rot}\,\boldsymbol{A})\,\mathrm{d}\boldsymbol{l} = \oint_c (\boldsymbol{v} \times \boldsymbol{B})\,\mathrm{d}\boldsymbol{l}\,.$$

Note that in the case of perpendicular movement of conductors in a magnetic field ($(\boldsymbol{v} \perp \boldsymbol{B}) \| \mathrm{d}\boldsymbol{l}$), the relationship can be simplified to the form

$$u_{\mathrm{i,p}} = \int_A^B vB\,\mathrm{d}l\,.$$

The transformation component of eddy currents can be converted to an integral form

$$u_{\mathrm{i,t}} = \oint_c \boldsymbol{E}_{\mathrm{trans}}\,\mathrm{d}\boldsymbol{l} = -\oint_c \frac{\mathrm{d}\boldsymbol{A}}{\mathrm{d}t}\,\mathrm{d}\boldsymbol{l} = -\frac{\partial \boldsymbol{\phi}}{\partial t}$$

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Děkuji za pozornost

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Faculty of electrical engineering
University of West Bohemia in Pilsen