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EUROPEAN UNION
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Operational Programme Research,
Development and Education



ECONOMY AND MANAGEMENT IN POWER ENGINEERING

5th lecture

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Content

- Discounted sum of incomes (costs) at the beginning of the 1st year
  - in n years
  - for terms T_1 to T_2

Discounted sum of incomes (costs) in n years at the beginning of the 1st year

- In economic effectiveness calculations are economics quantity which are annual repeated in specifics time term (T_{LT}) and our task is performed its updating and also summarization
- **Presumption:** R (or C) are changing in time with some specifics dependence. We solve following options:
  - Incomes are **constant** in each year
  - Annual change by β%
  - Annual change by β^2%
  - Annual growth by D [KCZ]

Incomes are constant in each year (1)

- Discounted sum of incomes (revenues) in n years at the beginning of the 1st year

$$R_1 r^{-1} + R_2 r^{-2} + \ldots + R_n r^{-n} = \sum_{T=1}^{n} R_T r^{-T}$$

- Presump.: $R_T = R_1 = const.$

		discount
1st year	$R_1 = R_1$	$. r^{-1}$
2nd year	$R_2 = R_1$	$. r^{-2}$
3rd year	$R_3 = R_1$	$. r^{-3}$
⋮		
n-th year	$R_n = R_1$	$. r^{-n}$

$$R_1 r^{-1} + R_1 r^{-2} + \ldots + R_1 r^{-n} = R_1 \sum_{T=1}^{n} r^{-T}$$

Incomes are constant in each year (2)

- We perform a geometric progression sum in ()

$$R_1 (r^{-1} + r^{-2} + \ldots + r^{-n}) = R_1 \sum_{T=1}^{n} r^{-T} \quad ,$$

where $a_1 = 1/r$, $q = 1/r$

- generally

$$S_n = a_1 + a_2 + \ldots + a_n = a_1 + a_1 q + \ldots + a_1 q^{n-1} = a_1 \cdot \frac{1 - q^n}{1 - q}$$

- then

$$R_1 \sum_{T=1}^{n} r^{-T} = R_1 \cdot \frac{1}{r} \cdot \frac{1 - \left(\frac{1}{r}\right)^n}{1 - \frac{1}{r}} = R_1 \cdot \frac{1}{r} \cdot \frac{r^n - 1}{r^n} \cdot \frac{r}{r - 1} =$$

$$= R_1 \frac{r^n - 1}{r^n (r - 1)}$$

supplier of 0-th level for the period n years

$s_n^{(0)}$ (mark event. z_n)

Ex. – Incomes sum (constant)

- How big will be discounted sum of incomes, which has constant annual repetition, at the beginning of the 1st year of putting in operation the investment. Economic time term is $T_{LT} = 25$ years, incomes in the 1st year $R_1 = 16\,000$ CZK and time factor $q = 1{,}08$?

$$\sum_{T=1}^{T_{LT}} R_T q^{-T} = R_1 \frac{q^{T_{LT}} - 1}{q^{T_{LT}}(q-1)} = 16000 \frac{1{,}08^{25} - 1}{1{,}08^{25}(1{,}08 - 1)} =$$

$$= 16000 \cdot 10{,}6748 = \underline{170\,797 \text{ CZK}}$$

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Annual change by β% (B -times) (1)

- Discounted sum of incomes in n years at the beginning of the 1st year

$$R_1 r^{-1} + R_2 r^{-2} + \ldots + R_n r^{-n} = \sum_{T=1}^{n} R_T r^{-T}$$

- Annual growth B -times [-], where $B = 1 + \frac{\beta\%}{100}$

		discount
1st year	$R_1 = R_1$	$. r^{-1}$
2nd year	$R_2 = R_1 . B$	$. r^{-2}$
3rd year	$R_3 = R_2 . B = R_1 . B^2$	$. r^{-3}$
⋮		
n-th year	$R_n = R_1 . B^{n-1}$	$. r^{-n}$

$$R_1 r^{-1} + R_1 B r^{-2} + \ldots + R_1 B^{n-1} r^{-n} = \sum_{T=1}^{n} R_1 B^{T-1} r^{-T}$$

Annual change by β% (B -times) (2)

- We perform a geometric progression sum

$$R_1 r^{-1} + R_1 B r^{-2} + \ldots + R_1 B^{n-1} r^{-n} = \sum_{T=1}^{n} R_1 B^{T-1} r^{-T} =$$

where $a_1 = B/r$, $q = B/r$ $= R_1 B^{-1} \sum_{T=1}^{n} \left(\frac{B}{r}\right)^T =$,

- Then
  - if $B \neq r$, $= R_1 B^{-1} \left(\frac{B}{r}\right) \frac{1 - \left(\frac{B}{r}\right)^n}{1 - \frac{B}{r}} = R_1 \left(\frac{r^n - B^n}{r^n (r - B)}\right)$
  - if $B = r$, $= R_1 B^{-1} \sum_{T=1}^{n} 1^T = R_1 \left(\frac{n}{B}\right)$

composite supplier of 1st level for the period n years

$s_n^{(1)}$

Annual change by β^2 % (B^2 -times) (1)

- Discounted sum of incomes in n years at the beginning of the 1st year

$$R_1 r^{-1} + R_2 r^{-2} + \ldots + R_n r^{-n} = \sum_{T=1}^{n} R_T r^{-T}$$

- Annual growth B^2 -times [-] , where $B = 1 + \frac{\beta\%}{100}$

		discount
1st year	$R_1 = R_1$	$. r^{-1}$
2nd year	$R_2 = R_1 . B^2$	$. r^{-2}$
3rd year	$R_3 = R_2 . B^2 = R_1 . B^4$	$. r^{-3}$
⋮		
n-th year	$R_n = R_1 . B^{2(n-1)}$	$. r^{-n}$

$$R_1 r^{-1} + R_1 B^2 r^{-2} + \ldots + R_1 B^{2(n-1)} r^{-n} = \sum_{T=1}^{n} R_1 B^{2(T-1)} r^{-T} =$$

Annual change by β^2 % (B^2 -times) (2)

- We perform a geometric progression sum

$$R_1 r^{-1} + R_1 B^2 r^{-2} + \ldots + R_1 B^{2(n-1)} r^{-n} = \sum_{T=1}^{n} R_1 B^{2(T-1)} r^{-T} =$$

$$= R_1 B^{-2} \sum_{T=1}^{n} \left(\frac{B^2}{r}\right)^T = ,$$

where $a_1 = B^2/r$, $q = B^2/r$

- Then
  - if $B^2 \neq r$,

$$= R_1 B^{-2} \left(\frac{B^2}{r}\right) \frac{1 - \left(\frac{B^2}{r}\right)^n}{1 - \frac{B^2}{r}} = R_1 \frac{r^n - B^{2n}}{r^n (r - B^2)}$$

  - if $B^2 = r$,

$$= R_1 B^{-2} \sum_{T=1}^{n} 1^T = R_1 \frac{n}{B^2}$$

composite supplier of 2nd level for the period n years
$s_n^{(II)}$

Annual growth by D [KCZ] (1)

- Discounted sum of incomes in n years at the beginning of the 1st year

$$R_1 r^{-1} + R_2 r^{-2} + \ldots + R_n r^{-n} = \sum_{T=1}^{n} R_T r^{-T}$$

- Annual growth by D [KCZ]

		discount
1st year	$R_1 = R_1$	$. r^{-1}$
2nd year	$R_2 = R_1 + D$	$. r^{-2}$
3rd year	$R_3 = R_2 + D = R_1 + 2D$	$. r^{-3}$
⋮		
n-th year	$R_n = R_1 + (n-1)D$	$. r^{-n}$

$$R_1 r^{-1} + (R_1 + D) r^{-2} + (R_1 + 2D) r^{-3} + \ldots + [R_1 + (n-1)D] r^{-n} =$$

$$= \sum_{T=1}^{n} [R_1 + (T-1)D] r^{-T} = R_1 \sum_{T=1}^{n} r^{-T} + D \sum_{T=1}^{n} T \cdot r^{-T} + D \sum_{T=1}^{n} r^{-T}$$

Annual growth by *D* [CZK] (2)

$$R_1 \sum_{T=1}^{n} r^{-T} + D \sum_{T=1}^{n} T \cdot r^{-T} + D \sum_{T=1}^{n} r^{-T} = (R_1 + D) \sum_{T=1}^{n} r^{-T} + D \sum_{T=1}^{n} T \cdot r^{-T} =$$

$\sum_{T=1}^{n} T \cdot r^{-T}$ ... combination of geometric and arithmetic progression which sum we perform from relation (as in lecture *4*):

$$\sum_{T=1}^{n} T x^T = x + 2x^2 + 3x^3 + \ldots + nx^n = \frac{x}{1-x}\left(\frac{1-x^n}{1-x} - nx^n\right)$$, where $x = r^{-1} = 1/r$ is quotient of geom. progress.

$$= (R_1 + D)\frac{r^n - 1}{r^n(r-1)} + D\frac{\frac{1}{r}}{1-\frac{1}{r}}\left[\frac{1-\left(\frac{1}{r}\right)^n}{1-\frac{1}{r}} - n\left(\frac{1}{r}\right)^n\right] =$$

$$= (R_1 + D)\frac{r^n - 1}{r^n(r-1)} + D\frac{1}{r-1}\left[\frac{r(1-r^{-n})}{r-1} - nr^{-n}\right]$$

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Ex. – Incomes sum (change by *D*)

- Profit in 1st year of running the investment is $P_1 = 10$ **mil. CZK**. What is the profit in the lifetime $T_{LT} = 20$ **years** when $q = 1{,}08$, if its value growth each year by **0,2 mil. CZK**?

$$P_{T_{LT}} = \sum_{T=1}^{T_{LT}} P_T \cdot q^{-T} =$$

$$= (P_1 + D)\frac{q^{T_{LT}} - 1}{q^{T_{LT}}(q-1)} + D\frac{1}{q-1}\left[\frac{q(1-q^{-T_{LT}})}{q-1} - T_{\check{z}} q^{-T_{LT}}\right] =$$

$$= (10 + 0{,}2)\frac{1{,}08^{20} - 1}{1{,}08^{20}(1{,}08-1)} + 0{,}2\frac{1}{(1{,}08-1)} \cdot$$

$$\cdot\left[\frac{1{,}08(1-1{,}08^{-20})}{(1{,}08-1)} - 20 \cdot 1{,}08^{-20}\right] = \underline{115{,}89 \text{ mil. CZK}}$$

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Generalization of above-mentioned terms (1)

- Terms for calculation the incomes (costs) were derived providing that the change proceed for time $T = 1$ to n (for example lifetime T_{LT})
- In general the relation (incomes are constant, change by β %, by β^2 %, by *D* [money]) needn't to proceed in the same way for all time term but different in different time intervals -> change proceed in time term T_r ($T = T_1$ to T_2)
- Then

$$\sum_{T=T_1}^{T_2} r^{-T} = r^{-T_1} + r^{-(T_1+1)} + \ldots + r^{-T_2} =$$

rebuke r^{-T1}

(continue at next page)

Generalization of above-mentioned terms (2)

$= r^{-T_1}(1 + r^{-1} + r^{-2} + r^{-3} \ldots + r^{-T_2+T_1}) =$ /rebuke r

$= r \cdot r^{-T_1}(r^{-1} + r^{-2} + r^{-3} \ldots + r^{-T_2+T_1-1}) =$

$= r^{1-T_1}(r^{-1} + r^{-2} + r^{-3} \ldots + r^{-(T_2-T_1+1)}) = r^{1-T_1} \cdot \sum_{T=1}^{T_r} r^{-T}$

$T_r = T_2 - T_1 + 1$ ... years in monitored period where the change proceed

$= r^{1-T_1} \frac{r^{T_r} - 1}{r^{T_r}(r-1)}$ **supplier of 0th level for the period T_r** $s_{T_r}^{(0)}$

- Then we speculate about the change in 1st year of the term did not come !

Incomes are constant in each year – generally

- Discounted sum of incomes in T_r years at the beginning of the 1st year

		discount
1st year of term T_1 to T_2	R_{T1}	$. r^{-T1}$
2nd year of term T_1 to T_2	$R_{T1+1} = R_{T1}$	$. r^{-(T1+1)}$
3rd year of term T_1 to T_2	$R_{T1+2} = R_{T1}$	$. r^{-(T1+2)}$
⋮		
T_2 -th year	$R_{T2} = R_{T1}$	$. r^{-T2}$

$R_{T_1} r^{-T_1} + R_{T_1} r^{-(T_1+1)} + \ldots + R_{T_1} r^{-T_2} = R_{T_1} \sum_{T=T_1}^{T_2} r^{-T}$

$= R_{T_1} \cdot r^{1-T_1} \cdot \frac{r^{T_r} - 1}{r^{T_r}(r-1)}$ **supplier of 0th level for the period T_r years** $s_{T_r}^{(0)}$

Annual change by β% (B -times) – generally (1)

- Discounted sum of incomes in T_r years at the beginning of the 1st year
- Annual growth B -times [-] , where $B = 1 + \frac{\beta\%}{100}$

		discount
1st year of term T_1 to T_2	R_{T1}	$. r^{-T1}$
2nd year of term T_1 to T_2	$R_{T1+1} = R_{T1} B$	$. r^{-(T1+1)}$
3rd year of term T_1 to T_2	$R_{T1+2} = R_{T1} B^2$	$. r^{-(T1+2)}$.
⋮		
T_2-th year	$R_{T2} = R_{T1} B^{T2-T1}$	$. r^{-T2}$

$R_{T_1} r^{-T_1} + R_{T_1} B r^{-(T_1+1)} + \ldots + R_{T_1} B^{T_2-T_1} r^{-T_2} = \sum_{T=T_1}^{T_2} R_{T_1} B^{T-T_1} r^{-T}$

Annual change by β% (B -times) – generally (2)

- Discounted sum of incomes in T_r years at the beginning of the 1st year

$$\sum_{T=T_1}^{T_2} R_{T_1} B^{T-T_1} r^{-T} = R_{T_1} \cdot B^{-1} \cdot \sum_{T=T_1}^{T_2} (B^{T-T_1+1} \cdot r^{-T}) =$$

$$= R_{T_1} \cdot B^{-1} \cdot r^{1-T_1} \cdot \sum_{T=T_1}^{T_2} B^{T-T_1+1} \cdot \sum_{1}^{T_r} r^{-T}$$

- If $B \neq r$,

$$= R_{T_1} \left(\frac{r^{T_r} - B^{T_r}}{r^{T_r}(r - B)} \right) r^{1-T_1}$$

- If $B = r$

$$= R_{T_1} B^{-1} \sum_{T=T_1}^{T_2} 1^T = R_{T_1} \left(\frac{T_r}{B} \right) r^{1-T_1}$$

composite supplier of 1st level for the period T_r years $s_{Tr}^{(I)}$

Annual change by β^2% (B^2 -times) - generally (1)

- Discounted sum of incomes in T_r years at the beginning of the 1st year
- Annual growth B^2 –times [-], where $B = 1 + \frac{\beta\%}{100}$

		discount
1st year of term T_1 to T_2	R_{T1}	$. r^{-T1}$
2nd year of term T_1 to T_2	$R_{T1+1} = R_{T1} B^2$	$. r^{-(T1+1)}$
3rd year of term T_1 to T_2	$R_{T1+2} = R_{T1} B^4$	$. r^{-(T1+2)}$.
.		
.		
T_2-th year	$R_{T2} = R_{T1} B^{2(T2-T1)}$	$. r^{-T2}$

$$R_{T_1} r^{-T_1} + R_{T_1} B^2 r^{-(T_1+1)} + \ldots + R_{T_1} B^{2(T_2-T_1)} r^{-T_2} =$$

$$= \sum_{T=T_1}^{T_2} R_{T_1} B^{2(T-T_1)} r^{-T}$$

Annual change by β^2% (B^2 -times) – generally (2)

- Discounted sum of incomes in T_r years at the beginning of the 1st year

$$\sum_{T=T_1}^{T_2} R_{T_1} B^{2(T-T_1)} r^{-T} =$$

- If $B^2 \neq r$,

$$= R_{T_1} \left(\frac{r^{T_r} - B^{2T_r}}{r^{T_r}(r - B^2)} \right) r^{1-T_1}$$

- If $B^2 = r$,

$$= R_{T_1} B^{-2} \sum_{T=T_1}^{T_2} 1^T = V_{T_1} \left(\frac{T_r}{B^2} \right) r^{1-T_1}$$

composite supplier of 2nd level for the period T_r years $s_{Tr}^{(II)}$

Annual growth by *D* [KCZ] – generally (1)

- Discounted sum of incomes in T_r years at the beginning of the 1st year
- Annual growth by ***D*** [money]

		discount
1st year of term T_1 to T_2	R_{T1}	$. r^{-T1}$
2nd year of term T_1 to T_2	$R_{T1+1} = R_{T1} + D$	$. r^{-(T1+1)}$
3rd year of term T_1 to T_2	$R_{T1+2} = R_{T1} + 2D$	$. r^{-(T1+2)}$
⋮		
T_2-th year	$R_{T2} = R_{T1} (T_2-T_1)D$	$. r^{-T2}$

$$R_{T_1} r^{-T_1} + (R_{T_1} + D) r^{-(T_1+1)} + (R_{T_1} + 2D) r^{-(T_1+2)} + \dots$$
$$\dots + [R_{T_1} + (T_2 - T_1)D] r^{-T_2} = \sum_{T=T_1}^{T_2} [R_{T_1} + (T - T_1)D] r^{-T}$$

Annual growth by *D* [KCZ] – generally (2)

- Discounted sum of incomes in T_r years at the beginning of the 1st year

$$\left[\sum_{T=T_1}^{T_2} [R_{T_1} + (T - T_1)D] r^{-T} =\right]$$

$$= R_{T_1} \sum_{T=T_1}^{T_2} r^{-T} + D \sum_{T=T_1}^{T_2} (T - T_1) r^{-T} =$$

$$= R_{T_1} r^{1-T_1} \frac{r^{T_r} - 1}{r^{T_r}(r-1)} + D r^{1-T_1} \left[\frac{r^{T_r} - 1}{r^{T_r}(r-1)} \cdot \frac{1}{r-1} - \frac{T_r}{r^{T_r}(r-1)}\right] =$$

$$= R_{T_1} r^{1-T_1} \frac{r^{T_r} - 1}{r^{T_r}(r-1)} + D r^{1-T_1} \frac{r^{T_r} - 1}{r^{T_r}(r-1)} \cdot \left(\frac{1}{r-1} - \frac{T_r}{r^{T_r} - 1}\right) =$$

$$\left[= r^{1-T_1} \frac{r^{T_r} - 1}{r^{T_r}(r-1)} \left[R_{T_1} + D \cdot \left(\frac{1}{r-1} - \frac{T_r}{r^{T_r} - 1}\right)\right]\right]$$

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Ex. – discounted sum of costs (1)

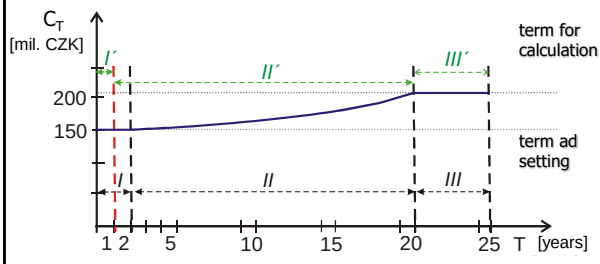
- We can use the above remarks relations only if the change in the 1st year of the term did not **come !!!**
- **Exercise:**
  - Count the amount of the costs in economic life time. Its time behaviour is visible in the fallowing table (on next page)

C_T [mil. CZK]	150,6	β = 2%	const.
T [year]	1 – 2	3 – 20	21 – 25

T_{LT} = 25 year q = 1,06

Ex. – discounted sum of costs (2)

- **Figure** shows the above remarks exercise and change of solved time terms



Ex. – discounted sum of costs (3)

- **Solving:**

I´. term – only one year

$$C_T^I = C_{T_1}\, q^{-1} = 150{,}6 \cdot 10^6 \cdot 1{,}06^{-1} = \underline{142{,}1 \cdot 10^6\ CZK}$$

II´. term: $B = 1 + \beta/100 = 1 + 0{,}02$

$T_1 = 2,\ T_2 = 20 \rightarrow T_r = T_2\text{-}T_1\text{+}1 = 20\text{-}2\text{+}1\text{=}19$

$C_{T1} = C_2 = C_1 = 150{,}6 \,.\, 10^6\ CZK$

$$\sum_2^{20} C_T^{II} = C_{T_1} \cdot \frac{q^{T_r} - B^{T_r}}{q^{T_r}(q - B)} \cdot q^{1-T_1} =$$

$$= 150{,}6 \cdot 10^6 \cdot \frac{1{,}06^{19} - 1{,}02^{19}}{1{,}06^{19}(1{,}06 - 1{,}02)} \cdot 1{,}06^{1-2} =$$

$$= \underline{1841{,}7 \cdot 10^6\ CZK}$$

Ex. – discounted sum of costs (4)

III´. term: $const. = C_{T1}$

$T_1 = 21,\ T_2 = 25 \rightarrow T_r = T_2\text{-}T_1\text{+}1 = 25\text{-}21\text{+}1\text{=}5$

$C_{T1} = C_{21} = C_2 \,.\, B^{(T2\text{-}T1)_{II}} = 150{,}6 \,.\, 10^6 \,.\, 1{,}02^{(20\text{-}2)} =$

$= 215{,}1 \,.\, 10^6\ CZK$

$$\sum_{21}^{25} C_T^{III} = C_{T_1} \cdot q^{1-T_1} \cdot \frac{q^{T_r} - 1}{q^{T_r}(q - 1)} = 215{,}1 \cdot 10^6 \cdot 1{,}06^{1-21} \cdot$$

$$\cdot \frac{1{,}06^5 - 1}{1{,}06^5(1{,}06 - 1)} = \underline{282{,}5 \cdot 10^6\ CZK}$$

Total costs:

$$C_{T_{Lr}} = C_T^I + \sum_2^{20} C_T^{II} + \sum_{21}^{25} C_T^{III} = (142{,}1 + 1841{,}7 +$$

$$+ 282{,}5) \cdot 10^6 = \underline{2266{,}3 \cdot 10^6\ CZK}$$