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EUROPEAN UNION
European Structural and Investment Funds
Operational Programme Research,
Development and Education



ECONOMY ANBD MANAGEMENT IN POWER ENGINEERING

3rd lecture

Content

- Economic resources of the company (assets of the enterprise)
  - fixed (basic, long-term) assets – FA
  - circulating (current, short-term) assets – CA
- CA optimization (optimization methods, stock keeping theory)
  - static stochastic model
  - dynamic deterministic model (single-, multi-item)
    - with satisfied demand
    - with unsatisfied demand

3

Structure and purpose of the company

- The enterprise consists of the elements, by which realizes the „production" process.
- They have a certain structure. It means that these elements are in a certain interrelationship, and carry out a specific task.
  - The task of the power production company (electric power plant, heating plant) is primarily to produce electricity or heat and supply it to the PES or HS (heat system) networks.
  - The task of energy (distribution) companies is to ensure the distribution of electricity and heat and their supply to customers.

4

Economic resources of company

- = the means, by which the company is equipped, to fulfil its economic function.
- It's a type of ownership, which could be realized by manufacture or business activity of the company.



- They are divided according to participate in the production process:
  - **capital assets (fixed)** (real property – building, cars, machines) - FA
  - **floating (circulating) assets** (chattel – money, material, products) - CA

5

Circulating assets - CA

- Those assets still circulate. They merge in parts from the state of the circulation to the production phase and back (money on bank accounts - the purchase of stock material - work in progress - finished goods in stock - sale of goods, customer invoicing, claim, after the payment the money again)
- They are divided according to the relationship with the manufacturing process:
  - **CA in the sphere of production**
  - **CA in the sphere of circulation**

6

CA in the sphere of production

- **Manufacturing supplies - MS** (see next page)
- **Uncompleted manufacturing** = issued material where the processing has been started and costs that were expended on processing (products in various stages of completion, approximately 20% of the stock in the manufacturing companies)
- **Deferred expenses** – e.g. for the future innovations

7

Production supplies (1)

- **Raw materials** (ore, coal, grain) a **basic material** (steel, coal gas, starch) - passes into the product there are its essence
- **Auxiliary material** - either passes into the product, do not form the essence, gives him certain properties (corrosion resistant paint), or the product does not pass, but it is necessary to operate the production equipment (grease)
- **Fuel** – this is separate component of PS only if is used for energy purposes. (It represents not only the largest part of production supplies in energetic companies, but also in whole floating assets.) For other companies the fuel is an auxiliary material.

8

Production supplies (2)

- **Packages** - are separate parts only when they constitute a large proportion of the total cost. For the other companies this is supplies
- **Spare Parts** - the same like for packages
- **Subjects of progressive consumption** - they are repeatedly involved in the production process, but they are not consumed immediately. => They would have belong to the FA, but for simple planning and funding are allocated to the CA. For example:
  - Items that have a lower price than the limit is - regardless of their life (special devices).
  - Items that have a shorter life than one year - regardless of their price (boarding).

9

CA in the sphere of circulation

- **Products**, which have left the sphere of production, but haven't been delivered to customers yet. (in stock)
- **Customers claims** for delivered goods.
- **Financial resources** in the form of cash and cheques, and deposits in bank accounts.

10

CA Optimization

- We need to determine the extent of the CA, with which should company dispose during its business activities.
  - In case that company have more CA (mainly stock) than needs =>unnecessary costs (stockholding cost)
  - In case that company have lack of CA => losses due to dissatisfaction of company by the appropriate production
- =>Optimal management of supply processes is necessary
- **Task**– We need to determine the optimal size of reserves and the way to optimize their movement control.
- **Warehouse (stock) keeping theory (ST)** – use mathematical models of supply processes.

11

Mathematical models used in ST

- Partition according to the inclusion of random variable
  - stochastic (contains random variables)
  - deterministic (doesn't contain random variables)
- Partition according to the inclusion of time
  - static (variables aren't functions of time)
  - dynamic (variables are functions of time)

12

Static stochastic model

- It contains random variables and it isn't time-dependent
- It is characterized by a single order to replenish the stocks (an additional order is not possible)
- Unused stocks can't be used after the end of reference period (it becomes worthless)

  => Task – **determination the optimal amount of CA**, that is purchased only once as a stock for a given period of time (e.g. spare parts for some machinery for a period of its working life)

13

Model definition

- We expect that future demand for the product (CA), whose stock is purchased, is discrete random variable Y with known „probability distribution law" of its occurrence

$$P(Y=y) = p(y)$$

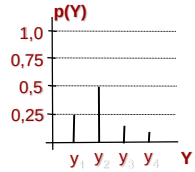
  $p(y)$ ... probability of supply needs, in amount y
- We want to find out with which probability we will need a specific amount of stocks CA (e.g. 1 spare part, 2, 3, ...)
- We are finding minimum of cost equation during the investigated period

14

Probability distribution law of discrete random variable

- Discrete random variable takes only isolated values often from the field of integral or natural numbers (e.g. number of defective products in manufacturing series, the number of equipment failures in a certain time interval etc., i. e. $Y = 0, 1, 2$ ...).
- Function which assigned appropriate corresponding probability of occurrence to each random variable is called a probability distribution (probability distribution law).

  For example.: $P(Y=y_1) = p(y_1) = 0{,}25$,
  This means: the probability that the phenomenon occurs y_1 is 25 %



15

Poisson distribution of random variable

- The probability that some random event occurs (hardly probably) is possible to describe by Poisson distribution of random variables (distributions rare phenomena)

$$p(m) = \frac{(\lambda T)^m}{m!} e^{-\lambda T}$$

$\lambda = 1/n$	- failure rate [failure / year]
n	- middle time before failure [years]
m	- number of failures
T	- investigation period

Generally: $p(y) = \frac{\mu^y}{y!} e^{-\mu}$

μ - median of the discrete random quantity y

16

Costs associated with supply acquisition

- Total cost C_x [money] depends on the size of stocks - x. There are represented by sum of overall costs in this model:
  - to the acquisition and maintenance (stock-keeping) C_{SK} [money]
  - and originating from deficiency C_D [money]

$$C_{SK} = C_{sk} \cdot x \qquad C_D = C_D \cdot \sum_{y=x+1}^{\infty} p(y)$$

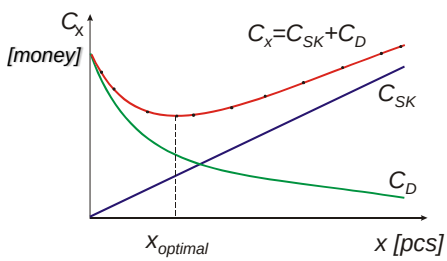
$$C_x = C_{SK} + C_D = c_{SK} \cdot x + c_D \cdot \sum_{y=x+1}^{\infty} p(y)$$

- where:
  - c_{SK} [money/pc] - specific costs of acquiring and maintaining unit supply
  - c_D [money/pc] - specific costs arising from a lack of unit supply

17

Cost characteristics which depends on the size of a one-shot supply.

- Behavior of cost function $C_x = f(x)$



$x_{optimal}$ – optimal range of single-shot supply

18

Model solution (1)

- Assump.: Known terms c_{SK} *[money/pc]*, c_D *[money/pc]*
- Target: We need to determine the size of the stock x to ensure that future demand and costs associated with the minimal size of stocks.
- We can order the supply of size x, and suppose that demand is y => true

$$C_x = c_{SK} \cdot x + c_D \cdot \sum_{y=x+1}^{\infty} p(y) \quad \textit{[money]}$$

$$C_x = c_{SK} \cdot x + c_D \cdot [p(x+1) + p(x+2) + \ldots + p(x+\infty)]$$

$p(y)$ ... probability of supplies requirement in amount y pieces

19

Model solution (2)

- We solve an example using the so-called cumulative probability, which we mark as P (= sum of probabilities in a specific situation, e.g. x)

$$P(x) = \sum_{y=0}^{x} p(y)$$

$$\underbrace{p(0) + p(1) + \ldots + p(x)}_{P(x)} + \underbrace{p(x+1) + \ldots + p(\infty)}_{\sum_{y=x+1}^{\infty} p(y)} = 1$$

$$\Rightarrow \sum_{y=x+1}^{\infty} p(y) = 1 - P(x)$$

- $P(x)$ – the probability that we will need a supply of CA in an amount x or less than x pieces

20

Model solution (3)

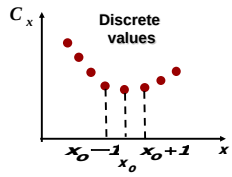
- Based on the determination of the cumulative probability, we can rewrite the cost equation in the form:

$$C_x = c_{SK} \cdot x + c_D \cdot [1 - P(x)]$$

- We are seeking a minimum of this function (optimal solution) with inequalities (when you buy a 1 piece more or less than optimal, C_x will be higher)

$$C_{x_o+1} \geq C_{x_o} \leq C_{x_o-1}$$

Assumption: C_x has only one local minimum



21

Model solution (4)

- We express the size of the cost C_{x+1}

$$C_{x_o+1} = c_{SK} \cdot (x_o + 1) + c_D \cdot [1 - P(x_o + 1)]$$

- After multiplication

$$C_{x_o+1} = c_{SK} \cdot x_o + c_{SK} + c_D - c_D \cdot P(x_o + 1)$$

- Substituting for the cumulative probability

$$P(x_0 + 1) = P(x_0) + p(x_0 + 1)$$

$$C_{x_o+1} = c_{SK} \cdot x_o + c_{SK} + c_D - c_D \cdot P(x_o) - c_D \cdot p(x_o + 1)$$

$$C_{x_o+1} = c_{SK} \cdot x_o + c_D [1 - P(x_o)] + c_{SK} - c_D \cdot p(x_o + 1)$$

$$\boxed{C_{x_o+1} = C_{x_o} + c_{SK} - c_D \cdot p(x_o + 1)}$$

22

Model solution (5)

- We express the size of the cost C_{x-1}

$$C_{x_o-1} = c_{Sk} \cdot (x_o - 1) + c_D \cdot [1 - P(x_o - 1)]$$

- After multiplication

$$C_{x_o-1} = c_{SK} \cdot x_o - c_{Sk} + c_D - c_D \cdot P(x_o - 1)$$

- Substitution for the cumulative probability

$$P(x_0 - 1) = P(x_0) - p(x_0)$$

$$C_{x_o-1} = c_{SK} \cdot x_o - c_{SK} + c_D - c_D \cdot P(x_o) + c_{SK} \cdot p(x_o)$$

$$C_{x_o-1} = c_{SK} \cdot x_o + c_{SK} [1 - P(x_o)] - c_{SK} + c_D \cdot p(x_o)$$

$$\boxed{C_{x_o-1} = C_{x_o} - c_{SK} + c_D \cdot p(x_o)}$$

23

Model solution (6)

$$C_{x_o+1} \geq C_{x_o} \leq C_{x_o-1}$$

$$C_{x_o} + c_{SK} - c_D \cdot p(x_o + 1) \geq C_{x_o} \leq C_{x_o} - c_{SK} + c_D \cdot p(x_o) \quad /\text{-}C_{xo}$$

$$c_{SK} - c_D \cdot p(x_o + 1) \geq 0 \leq -c_{SK} + c_D \cdot p(x_o) \quad /.(-1),$$

only left equation

$$-c_{SK} + c_D \cdot p(x_o + 1) \leq 0 \leq -c_{SK} + c_D \cdot p(x_o) \quad /+c_{SK}$$

$$/:c_D$$

$$\boxed{p(x_o + 1) \leq \frac{c_{SK}}{c_D} \leq p(x_o)}$$

→ To find the optimal size of reserves is sufficient to calculate c_{SK} / c_D and find out for which $x = x_o$ is the equation valid, ☐ i.e. for which x is $p(x_o)$ immediately above.

24

Example (1)

- **Task:** There should be installed a special machine in a company. Is necessary to know, how many pieces of specific spare part has to be one-shot ordered, together with this machine.
  - Exchange probability of components during operation time of this machine shows next table :

y	0	1	2	3	4	5	6
p(y)	0,90	0,05	0,02	0,01	0,01	0,01	0,00
P(y)	0,90	0,95	0,97	0,98	0,99	1,00	1,00

  - Stock-keeping costs are negligible. One piece price is independent on the amount of purchased components, namely c_{SK} ***= 4 900 CZK***. When the spare-part isn't available in the case of failure, the loss ***100 000 CZK*** arise (= the cost of lack c_D).

25

Example (2)

- **Solution:**
  - Calculate the ratio c_Z/c_D ***= 4 900/100 000 = 0,049***
  - The next higher ***p(y) = 0,05***, tj. x_0 ***= 1***

$$(0,02 \leq 0,049 \leq 0,05)$$

  - Next, we verify the accuracy of calculation by the computatation of the costs C_x for:

x = 0 $C_x = c_D (1 - 0,90) = 10\,000$ ***CZK***

x = 1 $C_x = c_{SK} + c_D (1 - 0,95) = 9\,900$ ***CZK***

x = 2 $C_x = 2n_{SK} + c_D (1 - 0,97) = 12\,800$ ***CZK***

Requirement of one spare part as the optimal stocks of the CA was confirmed by calculation of the cost.

26

Dynamic deterministic model

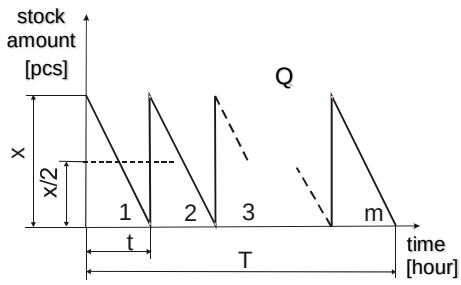
- Doesn't contain a random variables and is time-dependent
- Stock replenishment is performed repeatedly during a monitoring period
- Stocks are cyclically absorbed and bought

=> **Task**: We need to **determine the optimal amount** of CA, which should be repeatedly purchased as a stock in a given period of time, and **the optimal time** (the time interval between two deliveries), when should be acquired (when and how much stock should be resupplied)

27

Dynamic deterministic model with satisfied demand

- Graphic display of changes in stocks of one-item model



28

Model definition (1)

- We investigate the time period ***T*** during which the CA in the height of ***x*** units is cyclically consumed and bought
- At the moment, when all CA are exhausted, we buy new ***x*** units again
- This cycle is repeated during the monitoring period ***m*** - times
- The need ***Q*** = ***m.x*** units of CA during the period ***T*** is uniform and continuous (***Q*** - total consumption of stock in ***T***)

29

Model definition (2)

- We assume a linear consumption of CA => average amount of supply which is available during ***T***, is ***x/2***
- There is no time lag between the exhaustion and the acquisition of CA
- Time interval between two successive supplies, i.e. replenishment cycle, will be marked ***t***
- The solution is based on minimizing costs again, which should be affected by the way of replenishment

30

Replenishment (resupply) costs

- The purchase costs of the CA (in the order, acceptance, transportation, disposal invoices, etc.) for the period ***T*** [money/T]

$$C_p = c_p \cdot \frac{Q}{x} = c_p \cdot m$$

where: c_p – specific costs to purchase a supply of CA (c_p/x – cost of 1 piece)
Q – total number of units during the period ***T***
x – size of deliveries
m – number of supply cycles

- FA storage costs (interest on loans, handling costs, losses during storage, building operating costs and depreciation) during the period ***T*** [money/T]

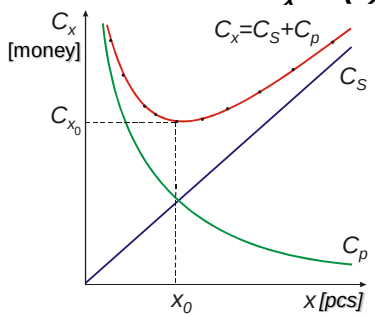
$$C_s = c_s \cdot T \cdot \frac{x}{2}$$

where: c_s – specific storage costs of 1 pc per time unit

31

Resupply cost characteristics

- Cost function behavior $C_x = f(x)$



x_o – optimal amount of one-shot supply

32

Total costs of resupply

- Total costs ***[money/T]***

$$C_x = C_p + C_s = c_p \cdot \frac{Q}{x} + c_s \cdot T \cdot \frac{x}{2} \qquad (*)$$

- We are looking for a cost function minimum, i.e. we set its derivative according to x equal to 0

$$\frac{dC_x}{dx} = -\frac{c_p \cdot Q}{x^2} + \frac{c_s \cdot T}{2}$$

$$\frac{dC_x}{dx} = 0 \Rightarrow -\frac{c_p \cdot Q}{x^2} + \frac{c_s \cdot T}{2} = 0$$

33

Model solution (1)

- We express ***x,*** so we get the optimal size of a delivery [simp.] (the so-called Wilson's formula)

$$x_o = \sqrt{\frac{2 \cdot c_p \cdot Q}{T \cdot c_s}} \;;\; n_s > 0$$

- We substitute x_o into the relation (*) and after adjustment, we can determine the minimum cost of replenishment C_{xo} ***[money/T]***

$$C_{x_o} = \sqrt{2 \cdot c_p \cdot c_s \cdot T \cdot Q}$$

34

Model solution (2)

- Optimal resupply cycle length t_o ***[days]*** is calculated from the correlation

$$(m_0 =) \frac{T}{t_0} = \frac{Q}{x_0} \quad \Rightarrow \quad t_o = \frac{x_o \cdot T}{Q} = \sqrt{\frac{2 \cdot c_p \cdot T}{Q \cdot c_s}}$$

- The optimal size of orders for the period T is then:

$$m_o = \frac{Q}{x_o} = \sqrt{\frac{T \cdot Q \cdot c_s}{2 \cdot c_p}}$$

35

Example (1)

- **Task:** Organization have to ensure consumption of a product in quantity ***90 000 pcs*** per year (***T***). The cost to the acquire of one's delivery are c_p ***= 500 CZK*** and storage costs of 1 pc are c_s ***= 2 CZK***.

  Determine the optimal size of delivery to replenish the stocks, the count of deliveries, the costs of replenishment and replenishment cycle length.

- **Solution:**
  - Cost function

$$C_x = c_p \cdot \frac{Q}{x} + c_s \cdot T \cdot \frac{x}{2} = 500 \cdot \frac{90000}{x} + 2 \cdot 1 \cdot \frac{x}{2}$$

36

Example (2)

- The optimal size of supply

$$x_o = \sqrt{\frac{2 \cdot c_p \cdot Q}{T \cdot c_s}} = \sqrt{\frac{2 \cdot 500 \cdot 90\,000}{1 \cdot 2}} = 6708{,}2 \ pcs$$

- The number of supplies corresponding to x_o

$$m_o = \frac{Q}{x_o} = \frac{90\,000}{6708{,}2} = 13{,}42$$

  - m_o round-off to an integer (see chart below) and repair the x_o, which is also round-off to integral number (up – this is inversely proportional to the m_o)

$$m_{o\,opr} = 13 \Rightarrow x_{o\,opr} = \frac{90\,000}{13} \doteq 6923 \ pcs$$

37

Example (3)

- The cost of replenishment of the supply size $x_{o\,opr}$

$$C_{x_{o\,opr}} = c_p \cdot \frac{Q}{x_{o\,opr}} + c_s \cdot T \cdot \frac{x_{o\,opr}}{2} =$$

$$= 500 \cdot \frac{90\,000}{6923} + 2 \cdot 1 \cdot \frac{6923}{2} = 13\,423 \ CZK/year$$

$$\left(C_{x_{o\,opr}} = \sqrt{2 \cdot c_p \cdot c_s \cdot T \cdot Q} = \sqrt{2 \cdot 500 \cdot 2 \cdot 1 \cdot 90\,000} \right.$$
$$\left. = \sqrt{180\,000\,000} = 13\,416 \ CZK/year \right)$$

- Replenishment cycle length

$$t_o = \frac{x_{o\,opr} \cdot T}{Q} = \frac{6923 \cdot 1}{90\,OOO} = 0{,}O77 \ year = 28 \ days$$

$$\left(t_o = \sqrt{\frac{2 \cdot c_p \cdot T}{Q \cdot c_s}} = \sqrt{\frac{2 \cdot 500 \cdot 1}{90\,000 \cdot 2}} = 0{,}075 \ year = 27 \ days \right)$$

38

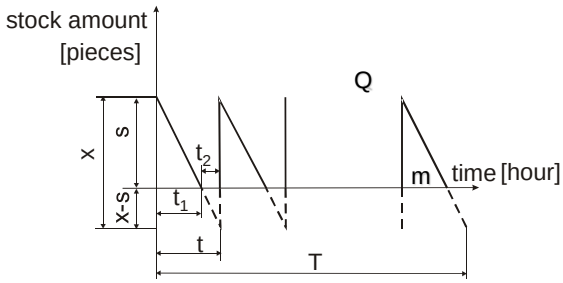
Dynamic deterministic model with satisfied demand (more items)

- Extends the previous model, i.e. optimize the selection of orders for k different stock items (supply items) with different sizes of demand
- Apply the same principles as for the single-item model
  - replenishment of stocks is carried out at the time of exhaustion
  - acquisition time is negligible
  - is always resupplied by equally large supply
  - lack of inventory is inadmissible
- **Target:** We want to find the cycle length, which is common for all items so that the costs of maintaining and purchasing will be minimal

39

Dynamic deterministic model with unsatisfied demand (one-item)

- Graphic display of changes in stocks of one-item model



40

Complete determination of the model

- In comparison with the previous model with satisfied demand the lack of stock is purposely admitted here. This results in extra costs from the deficiency ***C_D [money/T]***
- **Target** – We need to determine the amount of stock ***x [pcs]***, the amount of sufficient stock ***s [pcs]***
- Next, def.:
  - ***x-s*** - lack of stock [pcs]
  - ***t_1*** - period, during which that the stock exists
  - ***t_2*** - period, during which the stock doesn't exist
  - ***t_1+t_2*** - replenishment cycle length ***t***
  - ***c_D*** - specific costs of unit stock lack

41

Cost of resupply

- The total cost function has 3 components :
  - The acquisition cost of the CA during the period ***T*** *[money/T]*

$$C_p = c_p \cdot \frac{Q}{x} = c_p \cdot m$$

  - Storage cost CA during the period ***T*** *[money/T]*

$$C_s = c_s \cdot \frac{s}{2} \cdot t_1 \cdot m$$

  - The costs of deficiency CA during the period ***T*** *[money/T]*

$$C_D = c_D \cdot \frac{x-s}{2} \cdot t_2 \cdot m$$

where: ***s/2***, ***(x-s)/2*** - average height of the stock or the lack

42

Model solution (1)

- Then the **total cost** ***[money/T]*** can be expressed as $C_x = C_p + C_s + C_D$
- We are looking for a **cost function minimum**

$$C_x = f(x,s) \overset{!}{=} min$$

- We need to determine

$x_0 = ?$ - the optimal amount of stock ***[pcs]***
$s_0 = ?$ - the optimum amount of sufficient stock ***[pcs]***

43

Model solution (2)

- Assumption: $t_1 = t - t_2 \quad m = \frac{T}{t} \quad \frac{t_1}{t} = \frac{s}{x} \quad \frac{t_2}{t} = \frac{x-s}{x}$
- Than

$$C(x,s) = \frac{1}{2}\frac{s^2}{x}c_s T + \frac{1}{2}c_D T\frac{(x-s)^2}{x} + c_p\frac{Q}{x}$$

$$\frac{\partial C(x,s)}{\partial x} = -\frac{1}{2}\left(\frac{s}{x}\right)^2 c_s T + \frac{1}{2}c_D T - \frac{1}{2}\left(\frac{s}{x}\right)^2 c_D T - c_p\frac{Q}{x^2} = 0$$

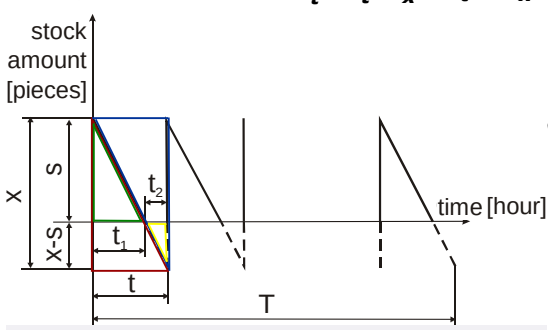
$$\frac{d}{dx}\left[\frac{(x-s)^2}{x}\right] = \frac{2(x-s)x-(x-s)^2}{x^2} =$$

$$= \frac{2x^2-2xs-x^2+2xs-s^2}{x^2} = \frac{x^2-s^2}{x^2} = 1-\left(\frac{s}{x}\right)^2$$

44

Model solution (3)

- Assumption: $t_1 = t - t_2 \quad m = \frac{T}{t} \quad \frac{t_1}{t} = \frac{s}{x} \quad \frac{t_2}{t} = \frac{x-s}{x}$
-



Model solution (4)

$$C(x,s)=\frac{1}{2}\frac{s^2}{x}c_sT+\frac{1}{2}c_DT\frac{(x-s)^2}{x}+c_p\frac{Q}{x}$$

$$\frac{\partial C(x,s)}{\partial s}=\frac{s}{x}c_sT-c_DT+\frac{s}{x}c_DT=0$$

$$\frac{d}{ds}\left[\frac{(x-s)^2}{x}\right]=\frac{2(x-s)(-1)}{x}=\frac{-2x+2s}{x}=2\left(-1+\frac{s}{x}\right)$$

$$\rightarrow \frac{s}{x}(c_sT+c_DT)=c_DT \rightarrow \frac{s}{x}=\frac{c_D}{c_s+c_D}$$

46

Model solution (5)

- After substitution s/x into the $\frac{\partial C(x,s)}{\partial x}$ we get:

$$-\frac{1}{2}T\left(\frac{c_D}{c_s+c_D}\right)^2(c_s+c_D)+\frac{1}{2}c_DT=c_p\frac{Q}{x^2} \quad /\cdot\frac{2}{T}$$

$$-\left(\frac{c_D}{c_s+c_D}\right)^2(c_s+c_D)+c_D=\frac{2c_pQ}{Tx^2}$$

$$\frac{-c_D^2+c_Dc_s+c_D^2}{c_s+c_D}=\frac{2c_pQ}{Tx^2}$$

→ optimal amount of stock *[pcs]*

$$x_o=\sqrt{\frac{2c_pQ}{Tc_s}}\sqrt{\frac{c_s+c_D}{c_D}}$$

47

Model solution (6)

- From x_o and from the derived relationship $\frac{s}{x}=\frac{c_D}{c_s+c_D}$ we determine the optimum amount of sufficient stock *[pcs]*

$$s_o=x_o\frac{c_D}{c_s+c_D}$$

$$s_o=\sqrt{\frac{2c_pQ}{Tn_s}}\sqrt{\frac{c_s+c_D}{c_D}\cdot\frac{c_D^2}{(c_s+c_D)^2}}$$

$$s_o=\sqrt{\frac{2c_pQ}{Tc_s}}\sqrt{\frac{c_D}{c_s+c_D}}$$

48

Model solution (7)

- Next, we derive optimal resupply cycle length , *[hours, days, years]*

$$t_o = \sqrt{\frac{2 \cdot T \cdot c_p}{Q \cdot c_s}} \sqrt{\frac{c_s + c_D}{c_D}}$$

- Than for optimal material acquisition costs:

$$C(x_o, s_o) = \sqrt{2 \cdot T \cdot c_s \cdot c_p \cdot Q} \sqrt{\frac{c_D}{c_s + c_D}}$$

49

Example (1)

- **Task:** LV switchgear manufacturer needs for its annual production of ***35 000*** pieces of circuit breakers. Optimize the size of the stocks of these circuit breakers during the year (we consider ***T = 260 days***, i.e. the number of working days per year), if the cost of one delivery c_p ***= 200 CZK***. Storage costs per circuit breaker per day is c_s ***= 0,2 CZK*** . In the case of absence of stock, daily cost rise to the c_D ***= 2 CZK*** per missing circuit breaker.

50

Example (2) – Solution

- The optimum amount of sufficient stock

$$s_o = \sqrt{\frac{2c_p Q}{Tc_s}} \sqrt{\frac{c_D}{c_s + c_D}} =$$

$$= \sqrt{\frac{2 \cdot 200 \cdot 35\,000}{260 \cdot 0{,}02}} \sqrt{\frac{2}{0{,}02 + 2}} = 1633 \text{ pcs}$$

- The optimal length of replenishment cycle

$$t_o = \sqrt{\frac{2Tc_p}{Qc_s}} \sqrt{\frac{c_s + c_D}{c_D}} =$$

$$= \sqrt{\frac{2 \cdot 260 \cdot 200}{35\,000 \cdot 0{,}02}} \sqrt{\frac{0{,}02 + 2}{2}} = 12{,}25 \text{ days}$$

51

Example (3) – Next decision procedure

- We respect the condition that the length of replenishment cycle will be an integer, so we consider $t_{o1} = 12$ **days** and $t_{o2} = 13$ **days**
- We will calculate the amount of stock and the amount of sufficient stocks for both values and next the replenishment cost. From calculated cost we determine the optimal replenishment cycle length
- We will use the formula
- Maybe also from $\frac{s}{x} = \frac{n_D}{n_s + n_D}$

$$\frac{x_o}{s_o} = \frac{\sqrt{\frac{c_s + c_D}{c_D}}}{\sqrt{\frac{c_D}{c_s + c_D}}} = \frac{\sqrt{\frac{0{,}02+2}{2}}}{\sqrt{\frac{2}{0{,}02+2}}} = 1{,}01$$

52

Ex. (4) – Total and sufficient amount of supply (stock)

- For $t_{o1} = 12$ **days**

$$x_{o1} = \frac{t_{o1} Q}{T} = \frac{12 \cdot 35\,000}{260} = 1616 \text{ pcs}$$

$$s_{o1} = \frac{x_{o1} s_o}{x_o} = \frac{1616}{1{,}01} = 1600 \text{ pcs}$$

- For $t_{o2} = 13$ **days**

$$x_{o2} = \frac{t_{o2} Q}{T} = \frac{13 \cdot 35\,000}{260} = 1750 \text{ pcs}$$

$$s_{o2} = \frac{x_{o2} s_o}{x_o} = \frac{1750}{1{,}01} = 1733 \text{ pcs}$$

53

Ex.(5) – Cost of replenishment

$$C(x,s) = \frac{1}{2}\frac{s^2}{x} c_s T + \frac{1}{2} c_D T \frac{(x-s)^2}{x} + c_p \frac{Q}{x}$$

- For $t_{o1} = 12$ **days**

$$C(x_{o1}, s_{o1}) = \frac{1}{2} \cdot \frac{1600^2}{1616} 0{,}02 \cdot 260 +$$

$$+ \frac{1}{2} \cdot 2 \cdot 260 \frac{(1600-1616)^2}{1616} + 200 \frac{35\,000}{1616} = 8491{,}68 \text{ CZK}$$

- For $t_{o2} = 13$ **days**

$$C(x_{o1}, s_{o1}) = \frac{1}{2} \cdot \frac{1733^2}{1750} 0{,}02 \cdot 260 +$$

$$+ \frac{1}{2} \cdot 2 \cdot 260 \frac{(1733-1750)^2}{1750} + 200 \frac{35\,000}{1750} = 8504{,}97 \text{ CZK}$$

54

Ex.(6) – Number of missing circuit breakers

- For $t_{o1} = 12$ days

$$Q - m \cdot s_{o1} = Q - \frac{T}{t_{o1}} \cdot s_{o1} =$$

$$= 35\,000 - \frac{260}{12} \cdot 1600 = 333 \text{ pcs}$$

- For $t_{o2} = 13$ days

$$Q - m \cdot s_{o2} = Q - \frac{T}{t_{o2}} \cdot s_{o2} =$$

$$= 35\,000 - \frac{260}{13} \cdot 1733 = 340 \text{ pcs}$$

55

Example (7) – Conclusion

- When we respect the conditions of the integer length of replenishment cycle than minimization of the cost of replenishment will be after 12 days.
- If we could keep the optimal replenishment cycle length $t_o = 12,25$ days, we reached next minimum costs:

$$C(x_o, s_o) = \sqrt{2 \cdot T \cdot c_s \cdot c_p \cdot Q} \sqrt{\frac{c_D}{c_s + c_D}} =$$

$$= \sqrt{2 \cdot 260 \cdot 0{,}02 \cdot 200 \cdot 35\,000} \sqrt{\frac{2}{0{,}02 + 2}} = 8490 \text{ CZK}$$

56