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Operational Programme Research,
Development and Education



ECONOMY AND MANAGEMENT IN POWER ENGINEERING

2nd lecture

Content

- Classification of costs for the electrical energy production, determination of the price of *1 produced kWh*
- Economic dividing of production and load of electric power system (EPS) among electric power plants (economic covering of load diagram)
  - A/ on the basis of minimization of total costs
  - B/ respecting only variable component of costs (using consumption characteristics)

Literature in CZ:

- Skriptum WBU - Elektrárny – příklady I, II

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Economy of load diagram covering by electric power sources (plants)

- Expectations:
  - Consumption load diagram (LD) is available
  - This load diagram can be covered by existing power plants
- Task:

  To cover the load diagram by electric power plants so that costs on electricity supply are minimal.

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Costs of electrical energy production (1)

- Decisive factor for the use of some power plant to covering of some load should be the resultant price of 1 produced kWh.
- Costs can be expressed:
  - In absolute values fo determinate time, usually 1 year (symbol ***N [CZK/year]***)
  - In specific values for example relative to production unit (symbol ***n [CZK/kWh]***), then:

$$n = \frac{N}{W_c}$$

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Costs of electrical energy production (2)

- Total production costs have two components:

$$N_c = N_{st} + N_{pr} = n_{st} \cdot P_i + n_{pr} . W_c \quad [CZK/year]$$

  - Costs dependent on purchase price of power plant (proportional to the installed capacity P_i *[MW]*) and independent on quantity of produced energy are so called **fixed costs** N_{st}
  - Costs dependent on quantity of energy W_c *[kWh]* produced in power plant for determinate time are **variable costs** N_{pr}

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Fixed costs

- **Fixed costs** consists of **depreciation** N_o and of **work costs** N_{pc} that are independent on quantity of produced electric energy:

$$N_{st} = N_o + N_{pc} = \frac{N_i}{T_ž} + N_{pc} = p_o \cdot N_i + n_{pc} \cdot P_i \quad [CZK/year]$$

$$n_{st} = \frac{N_{st}}{P_i} = \frac{N_o}{P_i} + \frac{N_{pc}}{P_i} = \frac{N_i}{T_ž \cdot P_i} + n_{pc} = \frac{n_i}{T_ž} + n_{pc} =$$

$$= p_o \cdot \frac{N_i}{P_i} + n_{pc} = p_o \cdot n_i + n_{pc} \quad [CZK/kW]$$

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where: n_{st} - *specific fixed costs [CZK/kW]*
N_i - *investment costs (purchase prize) [CZK]*
n_i - *specific investment costs (purchase prize of 1 install kilowatt) [CZK/kW]*
$T_ž$ - *life time [year]*
p_o - *relative depreciation [-]* ($p_o = 1/T_ž$)
n_{pc} - *specific fixed work costs [CZK/kW]*

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Variable costs

- **Variable costs** consists of **operation costs** N_{pz} and of **service and maintenance costs** N_{ou}:

$$N_{pr} = N_{pz} + N_{ou} \quad [CZK/year]$$

$$n_{pr} = \frac{N_{pr}}{W_c} = \frac{N_{pz}}{W_c} + \frac{N_{ou}}{W_c} = n_{pz} + n_{ou} \quad [CZK/kWh]$$

where: n_{pr} – *specific variable costs [CZK/kWh]*
n_{pz} - *specific costs of operating material [CZK/kWh]*
n_{ou} - *specific costs of service and maintenance [CZK/kWh]*

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Price of *1 produced kWh*

- Price of *1 kWh produced in electric power plant* can be determined from total costs:

$$N_c = N_{st} + N_{pr} = N_o + N_{pc} + N_{pz} + N_{ou}$$

$$N_c = \overbrace{n_{st} \cdot P_i}^{} + \overbrace{n_{pr} \cdot W_c}^{} =$$

$$= \overbrace{p_o \cdot n_i \cdot P_i + n_{pc} \cdot P_i}^{} + \overbrace{n_{pz} \cdot W_c + n_{ou} \cdot W_c}^{}$$

$$n_c = \frac{N_c}{W_c} = \frac{n_{st} \cdot P_i}{W_c} + n_{pr} = \frac{n_{st} \cdot \cancel{P_i}}{\cancel{P_{max}} \cdot T_m} + n_{pr}$$

$$[CZK/kWh] \qquad = \frac{n_{st}}{T_m} + n_{pr}$$

providing that $P_i = P_{max}$

T_m – time of maximum utilization

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A/ Dividing of electricity production and load of EPS among electric power plants on the basis of minimization of total costs

Example:

Divide electric power output and electricity production necessary to cover given load diagram between base-load and peak-load power plants so that total production costs will be minimal.

Supposed load diagram is known (by table, see below).

Cost equations of these two power plants are:

$$N_{c1} = 520.P_1 + 0{,}14.W_1 \; [CZK/year]$$

$$N_{c2} = 100.P_2 + 0{,}25.W_2 \; [CZK/year]$$

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Principle of the given example solving (1)

If we have the power output P_c and the elektricity production W_c of EPS to cover from two power plants with power outputs P_1 and P_2 and with productions W_1 and W_2, it must be valid:

$$P_c = P_1 + P_2$$

$$W_c = W_1 + W_2$$

At the same time, it must be ensured that total production costs will be minimal:

$$N_c = N_{c1} + N_{c2} = min \quad !$$

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Principle of the given example solving (2)

For resultant production costs we can written:

$$N_c = n_{st1}.P_1 + n_{pr1}.W_1 + n_{st2}.P_2 + n_{pr2}.W_2$$

Into cost equation we can make substitution

$$P_2 = P_c - P_1$$

$$W_2 = W_c - W_1 ,$$

whereby we get:

$$N_c = n_{st1}.P_1 + n_{pr1}.W_1 + n_{st2}.(P_c - P_1) + n_{pr2}.(W_c - W_1)$$

$$N_c = P_1.(n_{st1} - n_{st2}) + W_1.(n_{pr1} - n_{pr2}) + n_{st2}.P_c + n_{pr2}.W_c$$

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Principle of the given example solving (3)

- If we are looking for minimum of this function, we must to derive the equation and put the derivation equal to zero, providing that $W_1 = f(P_1)$:

$$\frac{dN_C}{dP_1} = 0 \quad n_{st1} - n_{st2} + \frac{dW_1}{dP_1} \cdot (n_{pr1} - n_{pr2}) = 0$$

whereof passes condition:

$$\frac{dW_1}{dP_1} = -\frac{n_{st1} - n_{st2}}{n_{pr1} - n_{pr2}} = \frac{dW_2}{dP_2} = k$$

- We will achieve the smallest production costs in both of power plants, if we will divide power output and elektricity production between both of them in the point of production curve, in which the angular coefficient (slope) of tangent have value k (see deduced formula).

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Procedure of graphic solution

a/ we construct load diagram

b/ we construct diagram of load duration (load duration curve)

c/ we make graphic integration of load duration curve, whereby get production curve

d/ we construct tangent to production curve and make dividing

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Construction of load diagram

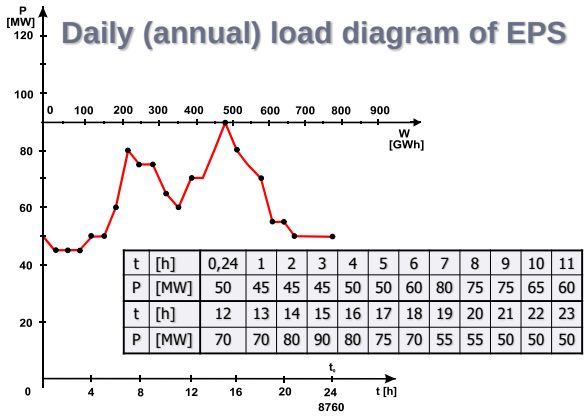
- ad a/

  Load diagram (red line) is determined by table, for example:

t	[h]	0,24	1	2	3	4	5	6	7	8	9	10	11
P	[MW]	50	45	45	45	50	50	60	80	75	75	65	60
t	[h]	12	13	14	15	16	17	18	19	20	21	22	23
P	[MW]	70	70	80	90	80	75	70	55	55	50	50	50

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Daily (annual) load diagram of EPS



t	[h]	0,24	1	2	3	4	5	6	7	8	9	10	11
P	[MW]	50	45	45	45	50	50	60	80	75	75	65	60
t	[h]	12	13	14	15	16	17	18	19	20	21	22	23
P	[MW]	70	70	80	90	80	75	70	55	55	50	50	50

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Construction of load duration curve

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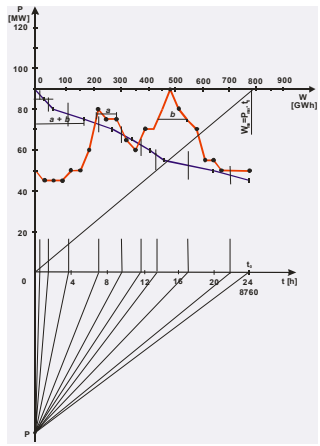
  We will get the diagram of load duration (= load duration curve – blue) by add for all time segments below the load diagram on given power level.

  As an example, it is presented construction on power level 75 MW.

  Note: It is necessary to know that daily load diagram is changing during the year. For simplicity we suppose that this diagram will be repeated each day in year so that we can mark time axis also in hours per year and get the year load diagram.

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Load duration diagram – decreasing from P_{max} to P_z



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Construction of production curve

- ad c/

  We can get the production curve by the graphical integration of duration load curve (green).

  It begins in origin of the coordinate system and ends on the level of the maximal power output, where the axis of produced energy is illustrated in a parallel way with time axis, on which straight line segment O-A give the total produced electrical energy W_c.

  Production curve has straight line running in interval from origin up to level of minimal power P_z , which is determined by load diagram. From this point it fluently diffracts up to work axis.

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Scale of produced energy axis

- We will get the direction of straight part of production curve (green) so that we will determine theoretical work, which would be done, if power plants have operated with maximal (available) power output for whole time t_0.

  $W_{max} = P_{max} \cdot t_0 = 90.8760 = 788{,}4$ GWh

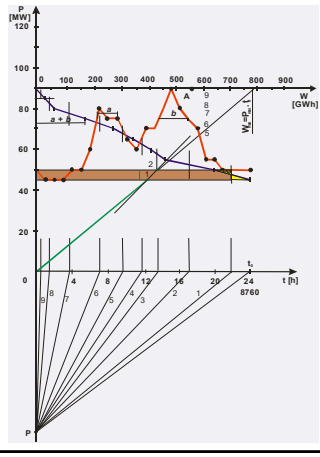
- Factual value W_{max} tells also the scale for work axis.

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Determination of integration pole

- We will connect the point W_{max} with coordinate system origin and through the end point of time axis (8760 hours) we will make parallel with this line so that cross the power output axis. In this intersection, the integration pole P will be.
- For obtaining of integration conditions, we can proceed also that we will choose the integration pole P, connect it with the end point of time axis and with this line make parallel through the coordinate system origin. In its point of intersection with produced energy axis we will get point $W_{max} = P_{max}.8760$, by which we will modify the scale of produced energy axis.

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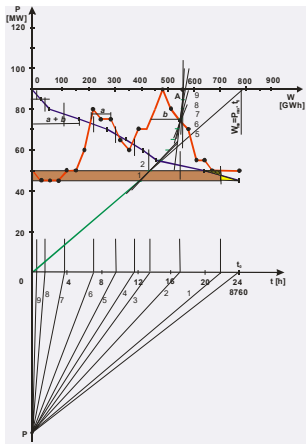
Integration of load duration curve between P_z a P_{max}

- We will find out the course of production curve within the range of P_z - P_{max} by graphical integration by verticl axis.
- We will replace the area below load duration curve by rectangles so that triangles below this curve and above will be equal.
- We will project the right edge of each rectangle to time axis and we will connect the point of intersection with the integration pole.
- Parallel with this line in the power output segment determined by chosen rectangle in this segment is then its integral and at the same time also part of production curve in this segment.

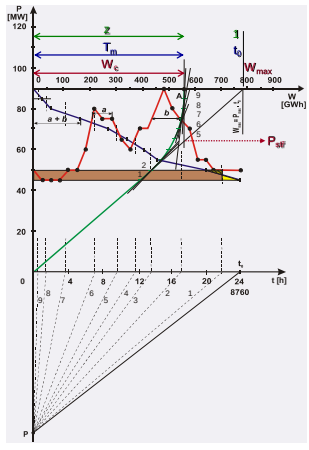
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The total produced energy we will get as:

$$W_c = \int_0^{P_{max}} t \, dP$$



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Graphical determination of the point of elektricity production and power output dividing between 2 power plants

- ad d/

We will achieve the smallest production costs, if we will divide power output and energy production between power plants in the point of production curve, in which the tangent slope have value ***k*** (see formula earlier).

$$\frac{dW_1}{dP_1} = -\frac{n_{st1} - n_{st2}}{n_{pr1} - n_{pr2}} = -\frac{520 - 100}{0{,}14 - 0{,}25} = \frac{420}{0{,}11} \; kWh/kW$$

$$\left[\frac{Kč/kW - Kč/kW}{Kč/kWh - Kč/kWh}\right] = [kWh/kW]$$

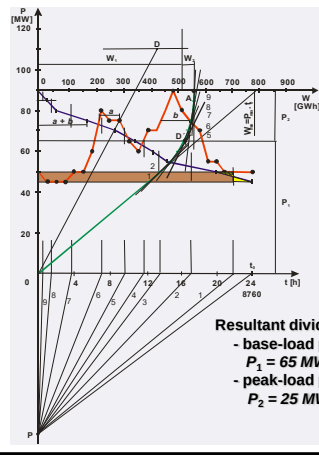
$$\frac{dW_1}{dP_1} = \frac{420}{110} \; GWh/MW$$

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- In the diagram we find the point *D*, which have coordinates quantified above.
- We will connect this point with the origin and make parallel so that it is tangent to production curve.
- In the gained point *D'*, there is the dividing place, in which next conditions are satisfied:

$$P_c = P_1 + P_2$$
$$W_c = W_1 + W_2$$
$$N_c = N_{c1} + N_{c2} \overset{!}{=} min$$

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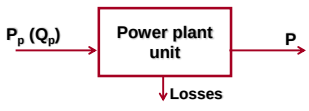


Resultant dividing is:
- **base-load power plant:**
  $P_1 = 65\ MW \quad W_1 = 510\ GWh$
- **peak-load power plant:**
  $P_2 = 25\ MW \quad W_2 = 45\ GWh$

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B/ Economic covering of load diagram respecting only variable components of costs (using consumption characteristic)

- We don't reflect fixed costs of production of electrical energy, but only variable that can be expressed by consumption characteristics of individual power units
- Consumption characteristic is the other expression of efficiency of transformation running during the electricity production in power plant unit



Direct method of efficiency determination:

$$\eta = \frac{P}{P_p}$$

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Consumption characteristic of electric power plant unit

- **Consumption characteristic** represented dependence of power input and output

  $P_p = f(P)$

or for thermal power plants (input = heat consumption)

  $Q_p = f(P)$

  - Power plant output can be theoretically changed from zero up to maximal power of plant, whereby the efficiency of plant is also changed.
  - In reality power plant unit works in the technical range from P_{min} to P_{max} .

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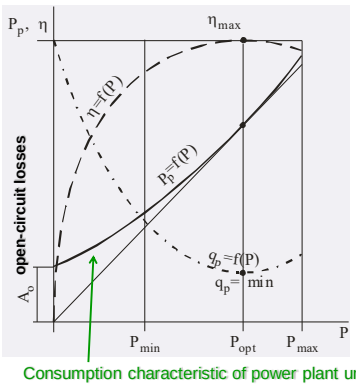
Graphical illustration of consumption characteristic

- Consumption characteristics can be constructed as graphs of quadratic function (polynom of 2nd degree), or can be linearized.
- In graph (see below) the consumption characteristic is substituted by the polynom of 2. degree with equation:

  $P_p = A_0 + B.P + C.P^2$

- Consumption (production) characteristic has the feature that tangent from origin touches it in the place of optimal power P_{opt}. At this power output the plant have maximal efficiency (also see graph below).

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Consumption characteristic of power plant unit $P_p = A_0 + B.P + C.P^2$

$q_p = f(P)$ running represented the dependence of specific heat consumption on power output.

$$q_p = \frac{P_p}{P} \quad [kJ/kWh]$$

$$\eta = \frac{3600}{q_p}$$

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Determination of optimal power output of electric power plant

- If we looking for the maximum of the function $\eta = \frac{P}{P_p}$, then

  $$\frac{d\eta}{dP} = 0 \Rightarrow \frac{P_p dP - P dP_p}{P_p^2} = 0$$

- From this results that $P_p = P\frac{dP_p}{dP}$,

  which is for concrete value $\frac{dP_p}{dP}$ (= direction) equation of bisector from the origin.

- For quadratic function $P_p = A_0 + B.P + C.P^2$

  we determine economic output $P_{opt} = \sqrt{\frac{A_0}{C}}$ (deduction in mimeographed)

  (determines point, in which function has $\eta = f(P)$ maximum)

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Determination of the slope of above mentioned tangent

- Calculation of the slope of straight line going from origin, which touches of consumption characteristic in P_{opt}:
  - Equation of this line (tangent), see above, is $P_p = P\frac{dP_p}{dP}$
  - For quadratic function $P_p = A_0 + B.P + C.P^2$ the slope is

$$\frac{dP_p}{dP} = B + 2CP = B + 2CP_{opt} =$$

$$= B + 2C\sqrt{\frac{A_0}{C}} = B + 2\sqrt{A_0 C}$$

- Tangent going from origin and touching of consumption characteristics (by polynom of 2nd degree) in point for P_{opt} has equation

$$P_p = (B + 2\sqrt{A_0 C})P$$

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Economic dividing of power output among more plants using relative increments

- During the calculation of optimal load of more plants is possible to go directly from **relative inkrement** (= slope of the tangent to the consumption characteristic going from the beginning)

  $$b = \frac{dP_p}{dP}$$,

  which is used in practice, above all in centrally controlled economic dividing of power output among individual power plant units at their parallel cooperation.

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Solving with the help of determination of relative increments

- At common thinking of quadratic consumption characteristics, at connecting of individual power plant units to EPS, we go from **equality** of relative increments, i.e.

  $b_1 = \ldots = b_i = \ldots = b_n$.

- At thinking of linear consumption characteristics we make dividing of load P_c by **size** of relative increments, i.e.

  $b_1 < \ldots < b_i < \ldots < b_n$,

  it means that at first units with small increments are connected and with the growing consumption of electric output units with higher increments, with smaller efficiency, are connected.

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Procedure of solving at use of consumption characteristics at parallel cooperation of power units

- Generally again, when power plant units are working together, the main task is to ensure the most economical operation, i.e. to generate electricity at the lowest possible cost.
- For simplification, at first, we will thinking only two power units.
- The whole problem can be mathematically written by the next equations:

$$P_c = P_1 + P_2 \quad !$$
$$N_c = N_1 + N_2 = \min,$$

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Determination of minimum of power input from collective consumption characteristic

- If we will thinking only variable costs, it is possible to represented them by consumption characteristics:

  $N_1 = f(Q_1),\ N_2 = f(Q_2)$

  Then, it is possible to formulate the task:

$$P_c = P_1 + P_2 = const \quad !$$
$$Q_c = Q_1 + Q_2 = \min$$

where: $Q_1 = f(P_1),\ Q_2 = f(P_2)$

- Function Q_c will have minimum in point: $\frac{dQ_c}{dP} = O$

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Principle of graphical solving illustrated by the example

Example:

Determinate the optimal parallel cooperation of two power plant units with nominal power outputs

$P_{1n} = 6\ MW$ a $P_{2n} = 9\ MW$

for achievement of total output $P_c = 9\ MW$.

Their consumption characteristics are determined by table expression from measured values.

P [MW]	0	1	2	3	4	5	6	7	8	9
Q_1 [MW]	5,0	5,4	5,9	6,5	7,5	8,7	10			
Q_2 [MW]	5,6	5,9	6,3	6,8	7,5	8,2	9,0	9,9	10,9	12

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a/ Construction of collective additive characteristic Q_c

$P_c = P_1 + P_2 = 9\ MW$

$Q_c = Q_1 + Q_2$

We will make combinations $P_1 + P_2$ with step $1\ MW$, it means for additive output $P_c = 9\ MW$ (combinations $0\text{-}9$, $1\text{-}8$, $2\text{-}7$, $3\text{-}6$, $4\text{-}5$, $5\text{-}4\ MW$) we will determinate summations of power inputs (see table below) and we will draw it to the graph.

The power output of the first block is plotted on the P axis From the minimum of the curve is then detected the optimal P_1, the output of the second unit we will calculate by take-off P_1 from P_c.

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Combinations of units outputs

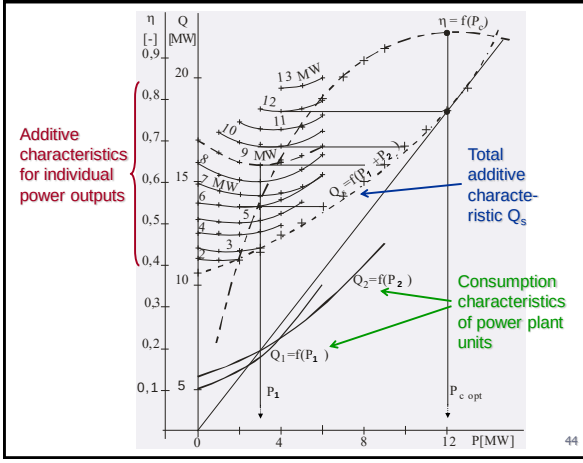
$P_c = const. = 9$ MW				
P_1 [MW]	P_2 [MW]	Q_1 [MW]	Q_2 [MW]	Q_c [MW]
0	9	5,0	12,0	17,0
1	8	5,4	10,9	16,3
2	7	5,9	9,9	15,8
3	6	6,5	9,0	15,5
4	5	7,5	8,2	15,7
5	4	8,7	7,5	16,2
6	3	10,0	6,8	16,8

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b/ Construction of total additive characteristic Q_s

- Parallel cooperation at the total output $P_c = 9\ MW$ does not have to be the optimal. At the other value P_c costs could be smaller. This we will find out by the construction of the total additive characteristic $Q_s = f(P_1 + P_2)$.
- Firstly, similarly as in the previous case, we will construct additive characteristics for values P_{ci}, where i mean the output of relevant parallel combination.
- We will draw the minimal values from additive curves to the graph, always to relevant value P_{ci}, and by the connection (interlay) of these points we will get characteristic $Q_s = f(P_1 + P_2)$.
- The optimal cooperation is determined by tangent to curve goes from the origin, it means that in the contact point we can find out that $P_{c\ opt} = 12\ MW$.

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Determination of power input minimum by Lagrange function

- At the bigger number of power plant units the previous graphical method of direct use of consumption characteristic is too complicated.
- Optimal parallel cooperation of n units can be determined from Lagrange function with bound extremes, where individual outputs of units P_i are used as these extremes:

$$\Phi = N_1 + N_2 + \ldots + N_i + \ldots + N_n + \\ + \lambda(P_c - P_1 - P_2 - \ldots - P_i - \ldots - P_n) = min \quad !$$

where: $N_i = f(P_i)$, $i = 1 \div n$

λ - Lagrange multiplicator

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Derivation of Lagrange function $\frac{\partial \Phi}{\partial N_i} = 0$

- Because we finding out the minimum of function with more variables, we will make partial derivation of it by individual costs. Costs are expressed by consumption characteristics of relevant units:

$$\frac{\partial \Phi}{\partial N_1} = \frac{\partial Q_1}{\partial P_1} - \lambda = 0 \Rightarrow \lambda = \frac{\partial Q_1}{\partial P_1}$$
$$\vdots$$
$$\frac{\partial \Phi}{\partial N_i} = \frac{\partial Q_i}{\partial P_i} - \lambda = 0 \Rightarrow \lambda = \frac{\partial Q_i}{\partial P_i}$$
$$\vdots$$
$$\frac{\partial \Phi}{\partial N_n} = \frac{\partial Q_n}{\partial P_n} - \lambda = 0 \Rightarrow \lambda = \frac{\partial Q_n}{\partial P_n}$$

- From this results:

$$\frac{\partial Q_1}{\partial P_1} = \ldots = \frac{\partial Q_i}{\partial P_i} \ldots = \frac{\partial Q_n}{\partial P_n}$$ - **equality of relative increments**

(see above).

Numerical solving of optimal parallel cooperation of power plant units – example

Task:

Power system is supplied by three power units.

Total power needed in the EPS is $P_c = 320\ MW$.

Find the optimal dividing of power outputs among these three units, if their consumption characteristics are available:

1. $P_{1n} = 200\ MW$ $\quad Q_1 = 23{,}6873 + 0{,}0993\ P_1 + 0{,}0007\ P_1^2$
2. $P_{2n} = 110\ MW$ $\quad Q_2 = 10{,}0120 + 0{,}2663\ P_2 + 0{,}0002\ P_2^2$
3. $P_{3n} = 110\ MW$ $\quad Q_3 = 9{,}3404 + 0{,}1501\ P_3 + 0{,}0011\ P_3^2$

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Optimal parallel cooperation of three power plant units - example

Solution:

Relative gains of units:

$$b_1 = \frac{\partial Q_1}{\partial P_1} = 0{,}0993 + 0{,}0014\ P_1$$
$$b_2 = \frac{\partial Q_2}{\partial P_2} = 0{,}2663 + 0{,}0004\ P_2$$
$$b_3 = \frac{\partial Q_3}{\partial P_3} = 0{,}1501 + 0{,}0022\ P_3$$

We will make dividing of load by equality of relative increments, it means: $b_1 = b_2 = b_3$

From this condition the system of equations results :

$$0{,}0993 + 0{,}0014\ P_1 = 0{,}2663 + 0{,}0004\ P_2$$
$$0{,}2663 + 0{,}0004\ P_2 = 0{,}1501 + 0{,}0022\ P_3$$
$$P_1 + P_2 + P_3 = 320\ MW$$

By solving of this system we will get:

$P_1 = 148{,}08\ MW$ $\quad P_2 = 100{,}78\ MW$ $\quad P_3 = 71{,}14\ MW$

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