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Example 3:

Dividing of power output and electricity production among more power plants on the basis of total cost minimization

Task:

Production of electricity for part of the electric power system (EPS) is covered by the two electric power plants (basic and peak) having the following cost equation:

$$N_{c1} = 535P_1 + 0{,}16W_1 \quad \text{[CZK/year]}$$
$$N_{c2} = 105P_2 + 0{,}22W_2 \quad \text{[CZK/year]}$$

For DLD (daily load diagram) of the EPS part given in Table 3.1 divide the electric power output and the amount of produced electricity between the plants so that the total cost of electricity production in both plants es minimal.

Table 3.1: Loads of EPS part during the day

t [hour]	0	1	2	3	4	5	6	7	8
P [MW]	75	70	65	60	75	90	115	125	120

t [hour]	9	10	11	12	13	14	15	16	16
P [MW]	115	110	120	115	105	120	120	125	135

t [hour]	18	19	20	21	22	23	24
P [MW]	140	135	110	105	90	85	75

Assumption: The same load is repeated every day of the year, i.e. DLD = ALD (annual LD).

Note: The following division includes minimization of fixed and variable (operating and investment) costs. In actual dispatching control of EPS, the load is usually divided according to consumption characteristics (i.e. only operating costs are included in the calculation) - see Example 4.

Solution:

If we want to cover the power output P and the production W of the EPS part from two power plants with power outputs P_1 and P_2 and the electricity productions W_1 and W_2, the following must apply:

$$P = P_1 + P_2$$
$$W = W_1 + W_2$$

We also want to ensure that the total cost of electricity production is minimal:

$$N_c = N_{c1} + N_{c2} \overset{!}{=} min$$

We know (see example 1) that $N_c = n_{st} P + n_{pr} W$, i.e. for the two power plants we have:

$$N_c = n_{st1} P_1 + n_{pr1} W_1 + n_{st2} P_2 + n_{pr2} W_2$$

After substitution: $P_2 = P - P_1 \qquad W_2 = W - W_1$

$$N_c = n_{st1} P_1 + n_{pr1} W_1 + n_{st2} (P - P_1) + n_{pr2} (W - W_1)$$

Further after adjustment:

$$N_c = P_1 (n_{st1} - n_{st2}) + W_1 (n_{pr1} - n_{pr2}) + n_{st2} P + n_{pr2} W$$

We want to obtain minimum costs, therefore we are looking for the minimum cost function N_c, i.e. we perform its first derivation and put it equal to 0.

Assumption: $W_1 = f(P_1)$, $W_2 = f(P_2)$.

$$\frac{dN_c}{dP_1} = 0 = n_{st1} - n_{st2} + \frac{dW_1}{dP_1} (n_{pr1} - n_{pr2}) + 0 + 0$$

$$\frac{dW_1}{dP_1} = -\frac{n_{st1} - n_{st2}}{n_{pr1} - n_{pr2}} = k = \frac{dW_2}{dP_2} \quad [Wh/W] \ldots$$ to be converted according to the chart axes scale to $[GWh/MW]$

Note: In a similar way, it is possible to differentiate according to P_2 (instead of P_2 and W_2 we substitute for P_1 and W_1, the result will be the same).

The calculation shows that the lowest production costs is achieved by dividing electric power output and electricity production between the two power plants at the point of the production line where the slope of the tangent line to production line is k.

After finding k we proceed as follows:

- we plot DLD (= ALD), i.e. $P = f(t)$
- we construct a load duration diagram (the line of power output duration)
- we perform graphical integration of the load duration diagram
  $W = \int_0^{P_{max}} t\, dP$ (=> construction of the production line)
- we draw a point determining the slope k of the tangent, which we connect to the origin of the coordinate system
- we construct the tangent to the production line and perform dividing

Illustrative Fig. (with different values):

