4 examples (other tasks for practice to 3rd lecture)

Enterprise resources - CA optimization with using a dynamic deterministic model

We want to determine the optimal amount of CA that is to be repeatedly stocked in a given time period and the optimal time (time interval between two deliveries) when it is to be taken (i.e. when and how many stocks are to be added).

We will use a **dynamic deterministic model** that:

- does not contain random variables
- is time-dependent

The graphical representation of changes in inventories of the single unit model (with satisfied demand) is shown in Fig. 1

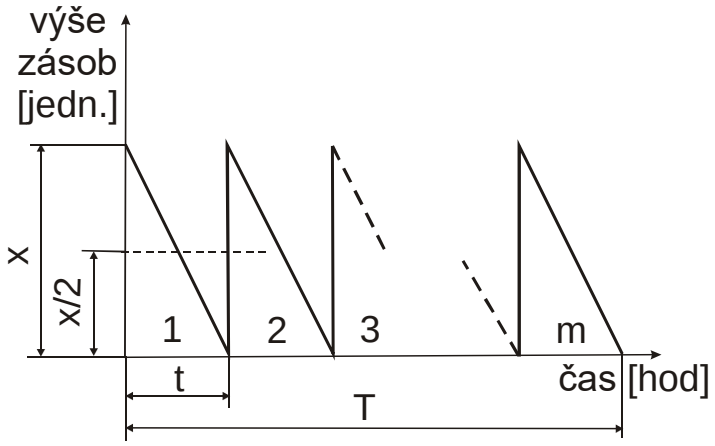


Fig. 1:

Model characteristics:

- we investigate the time period *T*, during which it is cyclically consumed and generating an CA of *x* units
- when all CA are exhausted, we will get a new one again at *x* units
- this cycle is repeated during the observed period m - times
- the need for CA in the amount of $Q = m.\ x$ units for the period *T* is uniform and continuous (*Q* - total stock consumption for *T*)
- we assume the linear consumption of CA, therefore the average stock available during *T* is *x/2*
- there is no time lag between exhaustion and CA acquisition

The time interval between two consecutive deliveries, i.e. the stock replenishment cycle, is denoted t.

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Our solutions are based on cost minimization, which can be influenced by stock replenishment.

The acquisition costs are:

$$N_p = n_p \cdot \frac{Q}{x} = n_p \cdot m \quad \text{[CZK]}$$

where:

n_p - the cost of purchasing 1 supply

Q - total unit demand for the T period

x - delivery size

m - number of supply cycles

The storage costs are:

$$N_s = n_s \cdot T \cdot \frac{x}{2} \quad \text{[CZK]}$$

where: n_s - unit cost of storage of 1 unit (pc) per unit of time

Total costs are:

$$N_x = N_p + N_s = n_p \cdot \frac{Q}{x} + n_s \cdot T \cdot \frac{x}{2} \quad \text{[CZK]} \qquad (*)$$

We look for the minimum cost function, ie its derivation by x is equal to 0. From the resulting equation we get the optimal size of one xo supply:

$$\frac{dN_x}{dx} = -\frac{n_p \cdot Q}{x^2} + \frac{n_s \cdot T}{2} = 0$$

$$x_o = \sqrt{\frac{2 \cdot n_p \cdot Q}{T \cdot n_s}} \quad \text{[units] or [pcs]}$$

After placing xo in the equation (*) and after adjusting, we get the minimum cost for adding N_{xo}:

$$N_{x_o} = \sqrt{2 \cdot n_p \cdot n_s \cdot T \cdot Q} \quad \text{[CZK]}$$

The optimal length of stock replenishment t_o:

$$t_o = \frac{x_o \cdot T}{Q} = \sqrt{\frac{2 \cdot n_p \cdot T}{Q \cdot n_s}} \quad \text{[hours, days, years]}$$

The optimal number of orders for the T period:

$$m_o = \frac{Q}{x_o} = \sqrt{\frac{T \cdot Q \cdot n_s}{2 \cdot n_p}} \text{ ...[-]}$$

Examples:

Ex. 1:

In the production plant, the final product is completed in a year by assembling 16,000 parts that the company receives from its subcontractor. The purchase of one parts delivery is CZK 200. Costs associated with storing one piece per unit per day is 0.05 CZK.

Determine the optimal delivery size, the optimum number and length of stock replenishment cycle. Note that the stock completion cycle length will be an integer (total number of days).

Ex. 2:

In the plant, it is necessary to assemble 200 pieces of the parts to complete a product that this company receives from its supplier within 1 month. Costs for the purchase of one delivery are CZK 5,000, storage costs per day are quoted to CZK 20 per delivery.

Specify the optimal delivery size, the optimal number and length of the stock replenishment cycle, provided the stock replenishment cycle length is an integer.

Solution Procedure:

We will specify:

1/ Optimum delivery size:

$x_o =$

2/ Optimal cost of stock acquisition

$$N_{x_o} = \sqrt{2 \cdot n_p \cdot n_s \cdot T \cdot Q} =$$

3/ Optimal number of stock replenishment cycles

$$m_o = \frac{Q}{x_o} =$$

4/ Optimum stock replenishment cycle length:

$$t_o = \frac{x_o \cdot T}{Q} =$$

We consider T = 260 days (number of working days per year) – Ex. 1, 22 days (number of working days per month) - Ex. 2

The condition specified in the assignment implies that we need to optimize the total cost for the $t_{o1} = t_o$ rounded down and for the $t_{o2} = t_o$ rounded up:

$t_{o1} =$

$t_{o2} =$

5/ Delivery size of individual types of parts for t_{o1}

$$x_{o1} = \frac{Q \cdot t_{o1}}{T} =$$

6/ Total cost of stock replenishment for t_{o1}

$$N_{x_{o1}} = n_p \cdot \frac{Q}{x_{o1}} + n_s \cdot T \cdot \frac{x_{o1}}{2} =$$

7/ Number of stock replenishment cycles for t_{o1}

$$m_{o1} = \frac{Q}{x_{o1}} =$$

8/ Delivery size of individual parts for t_{o2}

9/ Total cost of stock replenishment for t_{o2}

10/ Number of stock replenishment cycles for t_{o2}

Conclusion:

Minimizing the cost of stock replenishment implies that,

Dynamic deterministic model multipolar with satisfactory demand

This model optimizes the choice of orders for different stock (supply) items of varying demand.

The same principles apply as for a single-item model, i.e.:

- the stock is replenished at the time of exhaustion
- the acquisition time is negligible
- it is always supplemented by deliveries of same volume
- there is no shortage of supplies

In the solution, we want to find the length of cycle of ***t*** common to all items so that the costs associated with upkeep and stocking are minimal.

We express the cost function as:

$$N_x = N_p + N_s = \sum_{i=1}^{k} \left(n_p \cdot \frac{Q_i}{x_i} + n_{si} \cdot \frac{x_i}{2} . T \right)$$
$$= n_p \cdot \frac{T}{t} + \frac{1}{2} \sum_{i=1}^{k} Q_i \cdot n_{si} \cdot t = n_p \cdot m + \frac{1}{2} \sum_{i=1}^{k} Q_i \cdot n_{si} \cdot t \quad \text{...[CZK]}$$

where:
- Q_i - total demand for i-th item during period T [pcs]
- x_i - delivery size for i-th item [pcs]
- t - stock replenishment cycle length [hours, days, years]
- m - number of supply cycles [-]
- n_{si} - the cost of storing 1 piece of i-th item per unit of time
- n_p - the cost of acquiring all k-items in one delivery

For the optimal cycle length of stock refill t_o (common to all items):

$$t_o = \sqrt{\frac{2 \cdot n_p \cdot T}{\sum_{i=1}^{k} Q_i \cdot n_{si}}} \quad \text{[hours, days, years]}$$

The optimum delivery size of the i-th item can be calculated as:

$$x_{io} = \frac{Q_i \cdot t_o}{T} = Q_i \sqrt{\frac{2 \cdot n_p}{T \cdot \sum_{i=1}^{k} Q_i n_{si}}} \quad \text{[pcs]}$$

Total stock replenishment costs at the optimum cycle length are:

$$N_{x_{io}} = \sqrt{2 \cdot n_p \cdot T \sum_{i=1}^{k} Q_i \cdot n_{si}} \quad \text{[CZK]}$$

Examples:

Ex. 3:

Manufacturing needs 3 types of parts to complete its product from the supplier. To assemble the product you need 2 pieces A, 1 piece B and 3 pieces C. Each year 1,500 pieces of this product are produced.

The cost of storing 1 piece A is 10 CZK per year, 1 piece B CZK 8 and 1 piece C 5 CZK. Since all parts are acquired from one vendor, they are purchased at the same time. The cost of simultaneous purchase of all three parts for one delivery is CZK 200.

Determine the optimal cycle time for replenishment of the 3 supply items, the optimum delivery size for each of them, and the total cost of replenishing stock for the optimal cycle length. Note that the stock completion cycle length must be an integer.

Solution Procedure:

We will specify:

1/ Total annual need for parts

Q_A =

Q_B =

Q_C =

2/ Optimum stock replenishment cycle length:

t_o =

T is equal to 365 days.

The condition stated in the assignment implies that we have to optimize the total cost for the variant $t_{o1} = t_o$ that rounded down and for the $t_{o2} = t_o$ rounded up option.

$t_{o1} =$

$t_{o2} =$

3/ Delivery size of individual types of parts for t_{o1}

$$x_{A\,o1} = \frac{Q_A \cdot t_{o1}}{T} =$$

$x_{B\,o1} =$

$x_{C\,o1} =$

4/ Total cost of stock replenishment for t_{o1}

$$N_{x_{o1}} = n_p \cdot \frac{T}{t_{o1}} + \frac{1}{2} \sum_{i=A}^{C} Q_i \cdot n_{Si} \cdot t_{o1} =$$

5/ Delivery size of individual types of parts for t_{o2}

6/ Total cost of replenishing stock for t_{o2}

Conclusion:

Minimizing stock replenishment costs implies that the optimum cycle length of all parts at the same time is .

The optimal size of one supply of Part A is , Part B is and Part C .

Dynamic deterministic model with unsatisfied demand

The graphical representation of changes in stock state of the single item model (with unsatisfied demand) is shown in Figure 2

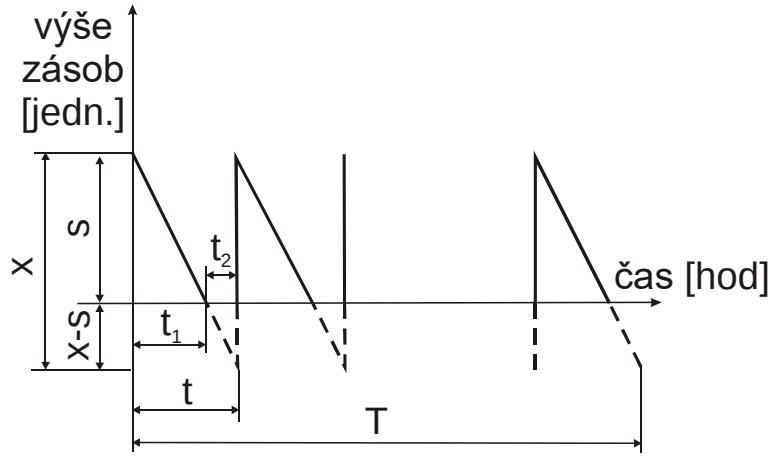


Fig. 2:

Compared to the previous model with satisfied demand, we are deliberately admitting the lack of stock, which also results in additional cost N_D [CZK / T]

We want to determine the amount of stock x [un.] And the size of sufficient stock ***s*** [single.]

We also define:

- $x - s$ - lack of stock [units]
- t_1 - the period for which the stock is available
- t_2 - the period for which the stock does not exist
- $t_1 + t_2$ - length of replenishment cycle t
- n_D - specific cost of lack of units of stock per unit of time

The total cost function has 3 components for this model:

1) the cost of acquisition of the CA for the period ***T***

$$N_p = n_p \cdot \frac{Q}{x} = n_p \cdot m \quad \text{[CZK]}$$

2) storage costs of the OP for the period ***T***

$$N_s = n_s \cdot \frac{s}{2} \cdot t_1 \cdot m \quad \text{[CZK]}$$

3) costs of lack of CA for the period T

$$N_D = n_D \cdot \frac{x-s}{2} \cdot t_2 \cdot m \quad \text{[CZK]}$$

where: $s/2$, $(x-s)/2$ - average volume of stock/deficiency

Then the total costs [CZK / T] can be expressed as

$$N_x = N_p + N_s + N_D \quad \text{[CZK]}$$

We are looking for the minimum of cost function

$$N_x = f(x,s) \overset{!}{=} min$$

From the derivations equalled to 0 we determine (see lecture):

4) optimum stock size

$$x_o = \sqrt{\frac{2n_p Q}{T n_s}} \sqrt{\frac{n_s + n_D}{n_D}} \text{ ... [units]}$$

5) optimal size of sufficient stock [units]

$$s_o = \sqrt{\frac{2n_p Q}{T n_s}} \sqrt{\frac{n_D}{n_s + n_D}} \text{ ... [units]}$$

6) optimal stock replenishment cycle length

$$t_o = \sqrt{\frac{2 \cdot T \cdot n_p}{Q \cdot n_s}} \sqrt{\frac{n_s + n_D}{n_D}} \text{ ... [hours, days, years]}$$

7) the cost of stock acquisition

$$N(x_o, s_o) = \sqrt{2 \cdot T \cdot n_s \cdot n_p \cdot Q} \sqrt{\frac{n_D}{n_s + n_D}} \text{ ... [CZK]}$$

Examples:

Ex. 4:

Optimize the need for measuring instruments installed in the distribution panel that is produced in the plant. There are two devices in each panel, with an annual production of 80,000 units of panels.

The cost of purchasing one delivery is 800 CZK, the cost of storing 1 piece of the measuring instrument is CZK 0.05 / day. In the absence of measuring instruments in production because costs of CZK 10 per 1 missing.

Note that the stock replenishment cycle length is an integer.

Solution Procedure:

We will specify:

1/ The total annual need for measuring instruments

$$Q =$$

1/ Optimum stock size:

$$x_o =$$

2/ Optimal amount of sufficient supplies:

$$s_o =$$

3/ Optimum stock replenishment cycle length:

$$t_o =$$

T is 260 days (number of working days per year).

The condition stated in the assignment implies that we have to optimize the total cost for the variant $t_{o1} = t_o$ rounded down and for the $t_{o2} = t_o$ rounded up option.

4/ Optimal cost of stock acquisition

$$N(x_o, s_o) =$$

5/ For t_{o1}

a/ Optimum stock size

$$x_{o1} = \frac{Q \cdot t_{o1}}{T} =$$

b/ A sufficient amount of stock s_o:

- we will use equation

$$\frac{x_o}{s_o} = \frac{\sqrt{\frac{n_s + n_D}{n_D}}}{\sqrt{\frac{n_D}{n_s + n_D}}} =$$

- then

$$s_{o1} = x_{o1} \frac{s_o}{x_o} =$$

c/ Total stock acquisition costs

$$N(x_{o1}, s_{o1}) = \frac{1}{2} \frac{{s_{o1}}^2}{x_{o1}} n_S T + \frac{1}{2} n_D T \frac{(x_{o1} - s_{o1})^2}{x_{o1}} + n_p \frac{Q}{x_{o1}} =$$

d/ Number of missing measuring devices

$$Q - m_{o1} \cdot s_{o1} = Q - \frac{T}{t_{o1}} \cdot s_{o1} =$$

6/ For t_{o2}

a/ Optimum stock size

x_{o2} =

b/ A sufficient amount of stock s_o:

- we will use equation

x_o / s_o =

- then

s_{o2} =

c/ Total stock acquisition costs

$N(x_{o2}, s_{o2})$ =

d/ Number of missing measuring devices

$Q - m_{o2} \cdot s_{o2}$ =

Conclusion: