8 examples (tasks for practice to 4th lecture)

Comparative investment costs and interest rate

The calculation of comparative investment cost

- in economic calculations, which do not implement the investment in one year, the use so-called comparative capital costs N_{ip} [CZK], converted to 1 year putting the investment into operation.

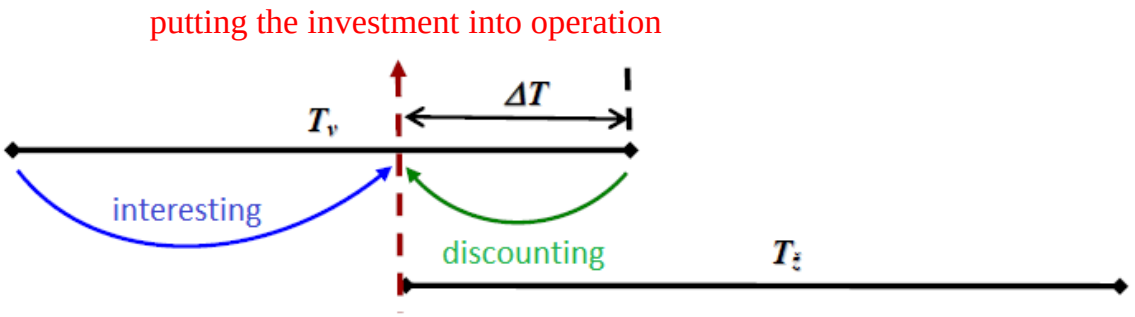


Fig. 1:

T_v - construction time [years], the time during which investment takes place construction of a facility (is bounded at the beginning of the beginning project work and at the end of running of the whole device to operation, can exceed launch investment)

$T_ž$ - lifetime (operation) [years]

ΔT - overlap construction period until the operation (lifetime) [years]

- updated number of investment costs incurred during each years of construction time for the beginning of the operation, we determine the relationship

$$N_{ip} = \sum_{t=1}^{T_v} N_{it} \cdot r^{T_v - \Delta T - t}$$

where:

N_{ip} - comparative investment costs [CZK]

N_{it} - investment costs issued in *t*-th year of construction [CZK]

r - interest

T_v - construction time investment [years]

ΔT - number of years for which construction investment exceeds its launch operation [years]

t - serial number for each year of construction

ESF II project of the University of West Bohemia in Pilsen, reg. number: CZ.02.2.69/0.0/0.0/18_056/0013239

Ex. 1:

Calculate the comparative investment costs for the construction of two variants energy source when the construction time T_v = ***5 years***, total investment costs N_i = ***100 mil. CZK*** and pumping investments in the construction of individual years N_{it} indicates tab. 1. The number of years for which exceeds the construction commencement of operations, ***ΔT = 1 year*** and the interest rate is ***10 %***.

Tab. 1:

Variant	***t***	[years]	1	2	3	4	5
A	N_{it}	[mil. CZK]	5	15	30	40	10
B	N_{it}	[mil. CZK]	10	40	30	15	5

Interesting

- determine what value will have economic variable for the period ***T*** (if applicable designation ***n***) at an interest rate of ***p***

Methods:

a) simple interesting
b) compound interesting

ad a) The amount of interest is calculated only on the original amount, not considering the interests of other period.

$$K_n = K_0 \cdot (1 + n \cdot p)$$

where:
- K_n - future value of economic variables [CZK]
- K_0 - current (original) value of economic variables [CZK]
- p - interest rate [%, -]
- n - period [years]

ad b) Interest shall be payable only from the original amount, but also the already implemented interest.

$$K_n = K_0 \cdot (1 + p)^n = K_0 \cdot r^n$$

where:
- K_n - future value of the variable (value for ***n*** years) [CZK]
- K_0 - current (original) value of economic variables [CZK] ***p***
- - interest rate [%, -]

  $r = 1+p$
- n - period [years]

This method of interest is used for the calculation of economic efficiency of investment.

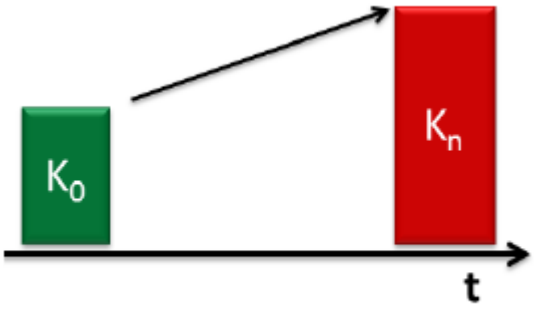


Fig. 2:

Interest in intervals of less than one year

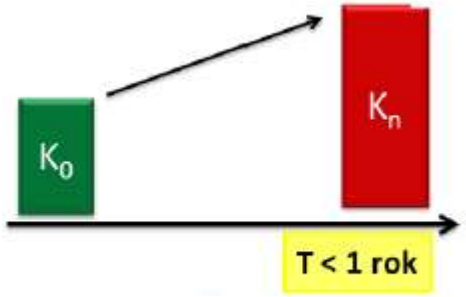


Fig. 3:

$$(1+\frac{p}{m})^{n \cdot m}$$

where: ***p*** - annual interest rate
n - number of years
m - number of interest period in 1 year

Discounting

- It is the inverse operation of the remuneration by which we determine the current (initial) the value of economic fundamentals, which we have available at the time period ***n*** (we know its value in ***n*** years)

$$K_0 = K_n \cdot (1+p)^{-n} = K_n \cdot r^{-n}$$

where:
- K_n - value economic quantities of n years [CZK]
- K_0 - current (original) value of economic variables [CZK]
- p - interest rate [%, -]

$$r^{-1} = \frac{1}{1+p}$$

- n - period [years]



Fig. 4:

Ex. 2:
At how high the interest rate is better to have today 100 000 CZK, than two years 200 000 CZK?

Ex. 3:
How much you need to save now to over 5 years at an interest rate of 0.04 was a sum of 20 000 CZK?

Ex. 4:
Mr. Usurer lends money at an annual interest rate of 36% or monthly interest rate of 3% or quarterly interest rate of 9%. Which interest rate is more favourable for Mr. Usurer and which is more favourable to his clients in simple and compound interest?

- simple interesting:

$1 + p_1 \cdot n_1 =$

$1 + p_2 \cdot n_2 =$

$1 + p_3 \cdot n_3 =$

- compound interesting:

$(1 + p_1)^{n_1} =$

$(1 + p_2)^{n_2} =$

$(1 + p_3)^{n_3} =$

Accumulation

- when economic calculations frequently used variables (costs, revenues, sales), which are regularly repeated during a certain period of time ***T***, while we need to determine their updated sum for ***T*** years (we add and bear interest at the same time)

$$K_n = \sum_{T=1}^{n} V_T \cdot r^{(n-T)} = V \cdot h_n = V \cdot \frac{r^n - 1}{r - 1}$$

where:

K_n	- future value (updated sum) magnitudes [CZK]
$V_T = V$	- periodic amount (constant) [CZK]
r	- interest
h_n	- saver [-]
n	- period [years]

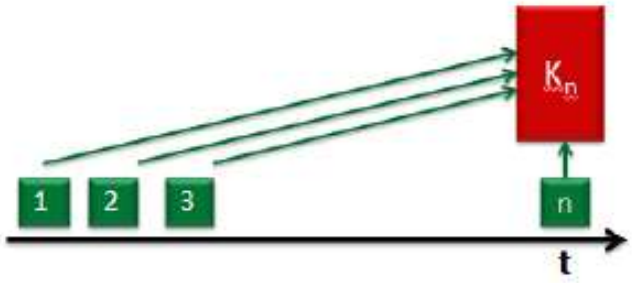


Fig. 5:

Ex. 5:

Compare the two options for obtaining 25 000 CZK in five years:

a) disposable insertion of 15 000 CZK at the beginning of the five-year period

b) periodically storing 4 000 CZK at the end of each year

The sum of economic variables updated at the beginning of the reporting period

- updates mean the transfer of economic variables spent in various periods at a selected point in time

- mostly in the calculations of economic efficiency of investment performs updates of the future at the beginning of the period (we use discounting)

- in this case it is an update that is performed when the sum annually recurring economic variables over a certain period of time (usually a period lifetime or for comparison)

$$K_0 = \sum_{T=1}^{n} V_T \cdot r^{-T} = V \cdot z_n = V \cdot \frac{r^n - 1}{r^n (r-1)}$$

where:
- K_0 - current (initial) value of quantity [CZK]
- V - regularly recurring amount [CZK]
- r - interest [-]
- z_n - suppliers' [-]
- n - period [years]

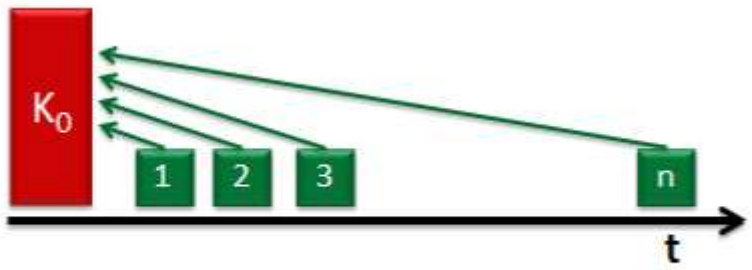


Fig. 6:

Ex. 6:
How big is the sum of sales that year after year the same amount of 200 000 CZK discounted to the beginning (for ex. commissioning of the facility) at an interest rate of 8% and a time span of 10 years?

Ex. 7:
How much you need to have a bank account, if we want the next 5 years annually collect 20 000 CZK with an annual interest rate of 4%? On account will be remitted no other deposits and the last choice we assume a zero balance.

Amortization (repayment)

- the process of repaying the amount borrowed, including the relevant interest is called amortization
- If we use a simple amortization paid each pay period constant amount, a sum which must include not only the proportion of the amount borrowed, but interest
- a constant amount that we pay so regularly is called **an annuity**
- when determining the size of the annuity is based on the condition that the sum of the annuity for the entire repayment period, discounted at the time of the loan is equal to the size of the amount borrowed

$$A = K_0 \cdot a_n = K_0 \cdot \frac{r^n (r-1)}{r^n - 1}$$

where:
- A - annuity, constant payment at the end of each year [CZK]
- K_0 - total amount borrowed [CZK]
- r - interest [-]
- a_n - pro rata annuity [-]
- n - period [years]

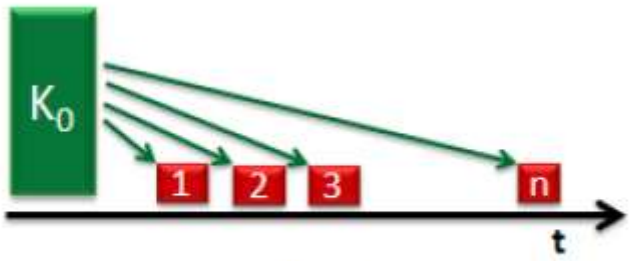


Fig. 7:

Ex. 8:
Mr. Rich borrowed 10 000 CZK Mrs. Clever, which would like return the amount, so that it was convenient for her. Eventually they agreed to return the money in two instalments as follows:

- after a year returned 8 000 CZK and two years remaining 4 000 CZK in interest rate of 10%.

a) It is advantageous for Mrs. Clever?
b) At what interest rate it will be advantageous for Mrs. Clever?
c) How big is the second instalment to be that it was for Mrs. Clever advantage when interest rate of 0.1?