ESF II project of the University of West Bohemia in Pilsen

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# THEORY OF TRANSMISSION AND DISTRIBUTION OF ELECTRIC ENERGY

## 3rd lecture
## (Theory and example)

## Topic - Transformers

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## Content

- **Equivalent circuit of two-winding transformer**
  - Short – circuit state of transformer
  - Open – circuit state of transformer
  - Recomputations real and relative Values
  - Calculation of transformer pasive parameters (series and shunt parameters)
  - Example

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## Equivalent Circuit of Two-winding Transformer (1)

- We can substitute two-winding transformer by **two elements in series** (see Fig. 3-5):
  - **Ideal transformer** without active and reactive losses with **transformer conversion (ratio) p** given to no-load ratio (i.e. nominal transformer conversion)
  - **Equivalent two-port network** (quadrupole), that contains **passive parameters** of real transformer (usually it is considered in form **T cell**)

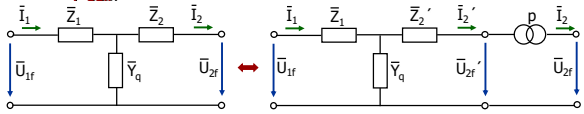



Fig. 3-5:

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## Equivalent Circuit of Two-winding Transformer (2)

- If we have on secondary side linked ideal transformer, **we must recount all secondary values** in T- cell **on level of primary voltage** (**in Fig. 3-5, 3-6** are marked by **comma**)
- On this assumption:

$$\bar{Z}_1 = \bar{Z}_2' \Rightarrow \textit{series impedance} \quad \bar{Z}_k = \bar{Z}_1 + \bar{Z}_2'$$

- We determine the parameters of equivalent T- cell from **no-load and short–circuit measurement**
- We suppose **uniform loss repartition** on both winding

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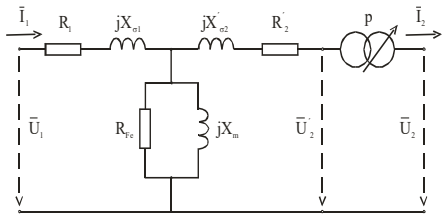
## Equivalent Circuit of Two-winding Transformer (3)

- In two-port network in form **T-cell**:
  - series elements (**= series impedance**) mean **resistance (effective resistance) of winding** and **leakage reactance of winding**
    At determination series elements we can go out from **short – circuit state**, where it is possible to neglect shunt parameter.
  - shunt elements (**= shunt admittance**) mean **conductivity (resistance) respecting no-load losses** and **exciting susceptance (reactance)**
    At determination shunt elements **open – circuit state** is important, where it is possible to neglect series parameter.

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## Equivalent Circuit of Two-winding Transformer (4)

Fig. 3-6:




- where:
  - $R_1$, $R_2$ resistance of primary (input), secondary (output) winding
  - $X_{\sigma 1}$, $X_{\sigma 2}$ leakage reactance of primary, secondary winding
  - $R_{Fe}$ resistance respecting no-load losses
  - $X_m$ reactance respecting exciting current

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## Transformer - Short – circuit State (Fig. 3-7)

- secondary side is short connected, on primary side we step up voltage until nominal current will flow in winding

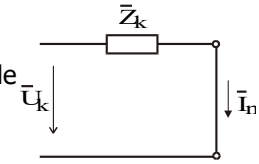



**Fig. 3-7:** Equivalent circuit of short connected two-winding transformer

- on primary side is short – circuit voltage
  - real value (in named units) $\bar{U}_k = \bar{Z}_k \cdot \bar{I}_n$
  - in relative units (relatid to nominal voltage)

$$\bar{u}_k[\text{p.j.}] = \bar{u}_k[\%]\,\frac{1}{100} = \frac{\bar{U}_k}{\bar{U}_n} = \frac{\bar{Z}_k \bar{I}_n}{\bar{Z}_n \bar{I}_n} = \bar{z}_k$$

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## Transformer - Open – circuit State (Fig. 3-8)

- transformer is fed in primary winding by nominal voltage and the same time the secondary side is disconnec-ted, i.e. is off load (open – circuit)

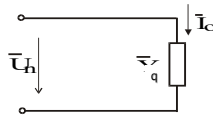



**Fig. 3-8:** Equivalent circuit of no-load two-winding transformer

- no-load current (eff. current of primary winding)
  - real values (in named units) $\bar{I}_0 = \bar{Y}_q \cdot \bar{U}_n$
  - relative value (related to nominal current) $\bar{i}_0 = \frac{\bar{I}_0}{\bar{I}_n} = \frac{\bar{Y}_q \cdot \bar{U}_n}{\bar{Y}_n \cdot \bar{U}_n} = \bar{y}_q$
  - it is consumed by no-load losses, which we can count from it (see further)

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## Recomputations Real and Relative Values

- In recomputation between **real values** and **values in relative units** we use:

  $S = \sqrt{3} U_s \cdot I$ , where from Ohm's law $I = \frac{U_f}{Z} = \frac{U_s}{\sqrt{3} \cdot Z}$

- after substitution

  $S = \frac{\sqrt{3} \cdot U_s \cdot U_s}{\sqrt{3} \cdot Z} = \frac{U_s^2}{Z} \Rightarrow Z = \frac{U_s^2}{S}$ , i.e. $\boxed{Z_n = \frac{U_n^2}{S_n}}$

- we determine impedance in r.u. as

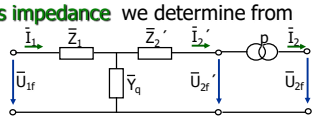
  $z = \frac{Z}{Z_n} = \frac{Z}{\frac{U_n^2}{S_n}} = Z \cdot \frac{S_n}{U_n^2} \Rightarrow \boxed{Z = z \cdot Z_n = z \cdot \frac{U_n^2}{S_n}}$

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## Derivation of Series Parameters (1)

- By determination of **series elements** we can go out from **short-circuit voltage** and **short-circuit losses**. In short-circuit state is possible to neglect shunt parameter.
  - Value of **relative series impedance** we determine from short-circuit voltage

    $\boxed{z_k = \frac{u_{k[\%]}}{100}}$

  - Value of **series impedance in Ohm** we put at recomputation from relative series impedance

    $\boxed{Z_k = z_k \cdot \frac{U_n^2}{S_n} = \frac{u_{k[\%]}}{100} \cdot \frac{U_n^2}{S_n}}$

    $\boxed{\bar{Z}_1 = \bar{Z}_2' \Rightarrow \bar{Z}_k = \bar{Z}_1 + \bar{Z}_2'}$

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## Derivation of Series Parameters (2)

- **Effective resistance of windings** we can determine from nominal short-circuit losses

  $\boxed{\Delta P_k = 3 R_k I_n^2}$

  $\boxed{R_k = \frac{\Delta P_k}{3 I_n^2} = \Delta P_k \left( \frac{U_n}{S_n} \right)^2 \ [\Omega]}$

  $\boxed{\begin{aligned} S_n &= \sqrt{3} U_n I_n \\ I_n &= \frac{S_n}{\sqrt{3} \cdot U_n} \\ \sqrt{3} \cdot I_n &= \frac{S_n}{U_n} \\ 3 I_n^2 &= \left( \frac{S_n}{U_n} \right)^2 \end{aligned}}$

- In **relative units** then

  $\boxed{r_k = R_k \cdot \frac{S_n}{U_n^2} = \Delta P_k \left( \frac{U_n}{S_n} \right)^2 \cdot \frac{S_n}{U_n^2} = \frac{\Delta P_k}{S_n}}$

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## Derivation of Series Parameters (3)

- Leakage reactance of the windings (short-circuit) v Ohm or relative leakage reactance of the windings we determine from known values of impedance and resistance

$$x_{\sigma k} = \sqrt{z_k^2 - r_k^2} = \sqrt{u_k^2 - \left(\frac{\Delta P_k}{S_n}\right)^2}$$

$$X_{\sigma k} = \sqrt{Z_k^2 - R_k^2} = x_k \cdot \frac{U_n^2}{S_n} = \sqrt{u_k^2 - \left(\frac{\Delta P_k}{S_n}\right)^2} \cdot \frac{U_n^2}{S_n} \quad [\Omega]$$

- **Conclusion** – total series impedance

$$\bar{Z}_k = R_k + jX_{\sigma k} \qquad \bar{z}_k = r_k + jx_{\sigma k}$$

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## Derivation of Series Parameters (4)

- Recomputation of parameters can be made to input voltage $U_1$ (node **1**) or eventually to input voltage $U_2$ (node **2**).
- Losses and leakage reactances at thinking recomputation on one voltage level divide by fifty-fifty on both winding.
  - For ohmic resistance of primary side and recomputed resistance of secondary side, as well as for leakage reactance then is valid:

$$R_1 \doteq R_2' \doteq \frac{R_k}{2} \qquad X_{\sigma 1} \doteq X_{\sigma 2}' \doteq \frac{X_{\sigma k}}{2}$$

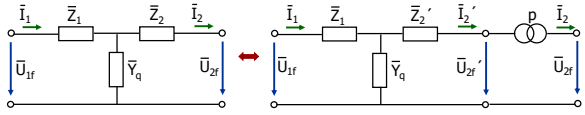
where: $R_2', X_{\sigma 2}'$ are values recomputed on level of voltage $U_n = U_1$ .

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## Derivation of Series Parameters (5)

- If recomputed parameters are executed to input voltage $U_1$ , we obtain real series parameters on secondary side by backward recomputation over reverse transformation ratio on second:

$$R_2 = R_2' \cdot \left(\frac{U_2}{U_1}\right)^2 \qquad X_{\sigma 2} = X_{\sigma 2}' \cdot \left(\frac{U_2}{U_1}\right)^2$$




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## Derivation of Shunt Parameters (1)

- By determination of **shunt parameters** is important open-circuit state, i.e. **no-load current** and **no-load losses**, when is possible to neglect series parameter.
  - Value of **relative shunt admittance** determinate from no-load current $$y_q = \frac{i_{0[\%]}}{100}$$
  - Value of **shunt admittance in [S]** we put at recomputation from relative shunt admittance $$Y_q = y_q \cdot \frac{S_n}{U_n^2} = \frac{i_{0[\%]}}{100} \cdot \frac{S_n}{U_n^2}$$

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## Derivation of Shunt Parameters (2)

- Value of **conductivity $G_{Fe}$ in [S]** can determine from the no-load (iron) losses:

$$\Delta P_0 = 3R_{Fe} \cdot I_0^2 = 3R_{Fe}\left(\frac{U_n}{\sqrt{3}R_{Fe}}\right)^2 = \frac{U_n^2}{R_{Fe}} = G_{Fe} \cdot U_n^2$$

$$G_{Fe} = \frac{1}{R_{Fe}} \doteq \frac{\Delta P_0}{U_n^2} \quad [S]$$

- In **relative units** then:

$$g_{Fe} = G_{Fe} \cdot \frac{U_n^2}{S_n} = \frac{\Delta P_0}{U_n^2} \cdot \frac{U_n^2}{S_n} = \frac{\Delta P_0}{S_n}$$

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## Derivation of Shunt Parameters (3)

- **Susceptance respecting magnetic flow v [S]** or **[r.u.]** we determine from known values of admittance and conductivity:

$$b_m = \sqrt{y_q^2 - g_{Fe}^2}$$

$$B_m = \sqrt{Y_q^2 - G_{Fe}^2} = b_m \cdot \frac{S_n}{U_n^2} \quad [S]$$

- **Conclusion** – total shunt admittance:

$$\bar{Y}_q = G_{Fe} + jB_m \qquad \bar{y}_q = g_{Fe} + jb_m$$

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## Equivalent Circuit of Two-winding Transformer - Example 3-1 (1)

- **Task:**
  - Create an equivalent circuit of the transformer in the form of T-cell type two-port and determine the specific values of the passive parameters, i.e. series impedance of the primary and secondary windings and shunt admittance:
    a) in the named units, i.e. in [Ω] and the [S]
    b) in relative units [r.u.]
  - Values on the label of solved transformer are:

| | |
|---|---|
| $S_n$=63 kVA | p=$U_1/U_2$=6/0,4 kV |
| $\Delta P_0$=0,34 kW | $\Delta P_k$=1,25 kW |
| $u_k$=4,5 % | $i_0$=3 % |

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## Equivalent Circuit of Two-winding Transformer - Example 3-1 (2)

- **Ad a/ Solution in [Ω] or [S]**
  - **Series impedance**

$$Z_k = \frac{u_{k[\%]}}{100} \cdot \frac{U_{n1}^2}{S_n} = 0{,}045 \cdot \frac{\left(6 \cdot 10^3\right)^2}{63 \cdot 10^3} = 25{,}71\,\Omega$$

  - **Effective resistance of windings**

$$R_k = \Delta P_k \cdot \left(\frac{U_{n1}}{S_n}\right)^2 = 1{,}25 \cdot 10^3 \left(\frac{6}{63}\right)^2 = 11{,}34\ \Omega$$

$$R_1 = R_2' = \frac{1}{2} R_k = 5{,}67\ \Omega$$

$$R_2 = R_2' \cdot \left(\frac{U_2}{U_1}\right)^2 = 5{,}67 \cdot \left(\frac{0{,}4}{6}\right)^2 = 2{,}52 \cdot 10^{-2}\ \Omega$$

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## Equivalent Circuit of Two-winding Transformer - Example 3-1 (3)

- **Ad a/ Solution in [Ω] or [S]**
  - **Series impedance**
    - **Leakage reactance of the windings**

$$X_k = \sqrt{Z_k^2 - R_k^2} = \sqrt{660{,}49 - 128{,}60} = 23{,}06\ \Omega$$

$$X_{\sigma 1} = X_{\sigma 2}' = \frac{1}{2} X_k = 11{,}53\ \Omega$$

$$X_{\sigma 2} = X_{\sigma 2}' \cdot \left(\frac{U_2}{U_1}\right)^2 = 11{,}53 \cdot \left(\frac{0{,}4}{6}\right)^2 = 5{,}12 \cdot 10^{-2}\ \Omega$$

$$\bar{Z}_k = R_k + jX_k = 11{,}34 + j23{,}06 = 25{,}71 \angle 63^\circ\ \Omega$$

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## Equivalent Circuit of Two-winding Transformer - Example 3-1 (4)

- **Ad a/ Solution in [Ω] or [S]**
  - **Shunt admittance**

$$Y_q = \frac{i_{0[\%]}}{100} \cdot \frac{S_n}{U_{n1}^2} = 0{,}03 \cdot \frac{63}{6^2.10^3} = 52{,}50 \cdot 10^{-6}\ \text{S}$$

  - **Conductivity respecting the no-load (iron) losses**

$$G_{Fe} = \frac{\Delta P_0}{U_{n1}^2} = \frac{0{,}34}{6^2 \cdot 10^3} = 9{,}44 \cdot 10^{-6}\ \text{S}$$

  - **(Effective resistance respecting no-load losses)**

$$R_{Fe} = \frac{1}{G_{Fe}} = 1{,}06 \cdot 10^5\ \Omega$$

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## Equivalent Circuit of Two-winding Transformer - Example 3-1 (5)

- **Ad a/ Solution in [Ω] or [S]**
  - **Shunt admittance**
    - **Susceptance respecting magnetic flow**

$$B_m = \sqrt{Y_q^2 - G_{Fe}^2} = 51{,}64 \cdot 10^{-6}\ \text{S}$$

    - **Reactance respecting magnetic flow**

$$X_m = \frac{1}{B_m} = \frac{1}{\sqrt{Y_q{}^2 - G_{Fe}{}^2}} = 1{,}94 \cdot 10^4\ \Omega$$

$$\bar{Y}_q = G_{Fe} + \frac{1}{jX_m} = G_{Fe} - jB_m = (9{,}44 - 51{,}64) \cdot 10^{-6} = 5{,}25 \cdot 10^{-5} \angle 280^\circ\ \text{S}$$

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## Equivalent Circuit of Two-winding Transformer - Example 3-1 (6)

- **Ad b/ Solution in r. u.**
  - **Series impedance**

$$z_k = \frac{u_{k[\%]}}{100} = 0{,}045$$

    - **Effective resistance of windings**

$$r_k = \frac{\Delta P_k}{S_n} = \frac{1{,}25 \cdot 10^3}{63 \cdot 10^3} = 0{,}0198$$

$$r_1 = r_2' = \frac{1}{2} r_k = 0{,}0099$$

$$r_2 = r_2' \cdot \left(\frac{U_2}{U_1}\right)^2 = 0{,}0099 \cdot \left(\frac{0{,}4}{6}\right)^2 = 7{,}33 \cdot 10^{-6}$$

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## Equivalent Circuit of Two-winding Transformer - Example 3-1 (7)

- **Ad b/ Solution in r. u.**
  - **Series impedance**
    - **Leakage reactance of the windings**

$$x_k = \sqrt{z_k^2 - r_k^2} = \sqrt{20{,}25 \cdot 10^{-4} - 3{,}92 \cdot 10^{-4}} = 0{,}0404$$

$$x_{\sigma 1} = x'_{\sigma 2} = \frac{1}{2} x_k = 0{,}0202$$

$$x_{\sigma 2} = x'_{\sigma 2} \cdot \left(\frac{U_2}{U_1}\right)^2 = 0{,}0202 \cdot \left(\frac{0{,}4}{6}\right)^2 = 89{,}78 \cdot 10^{-6}$$

$$\bar{z}_k = r_k + jx_k = 0{,}0198 + j0{,}0404 = 0{,}045 \angle 63° \ \Omega$$

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## Equivalent Circuit of Two-winding Transformer - Example 3-1 (8)

- **Ad b/ Solution in r. u.**
  - **Shunt admittance**

$$y_q = \frac{i_{0[\%]}}{100} = 0{,}03$$

    - **Conductivity respecting the no-load (iron) losses**

$$g_{Fe} = \frac{\Delta P_0}{S_n} = \frac{0{,}34}{63} = 5{,}397 \cdot 10^{-3}$$

    - **(Effective resistance respecting no-load losses)**

$$r_{Fe} = \frac{1}{g_{Fe}} = 185{,}29$$

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## Equivalent Circuit of Two-winding Transformer - Example 3-1 (9)

- **Ad b/ Solution in r. u.**
  - **Shunt admittance**
    - **Susceptance respecting magnetic flow**

$$b_m = \sqrt{y_q^2 - g_{Fe}^2} = \sqrt{(900 - 29{,}126) \cdot 10^{-6}} = 29{,}511 \cdot 10^{-3}$$

    - **Reactance respecting magnetic flow**

$$x_m = \frac{1}{b_m} = \frac{1}{\sqrt{y_q^2 - g_{Fe}^2}} = 33{,}89$$

$$\bar{y}_q = g_{Fe} + \frac{1}{jx_m} = g_{Fe} - jb_m = (5{,}397 - j29{,}511) \cdot 10^{-3} = 0{,}03 \angle 280°$$

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## Next time

- Creation of substitution schema of the EPS part

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