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Operational Programme Research,
Development and Education



THEORY OF TRANSMISSION AND DISTRIBUTION OF ELECTRIC ENERGY

8th lecture – theory

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2

Content

- **Load flow analysis**
  1. Introduction (scope, assumptions, input data, outputs, bus types)
  2. Bus admittance matrix - inclusion of power lines and compensators
  3. Bus admittance matrix - inclusion of 2- and 3-winding transformers

3

Load flow analysis

1. Introduction (scope, assumptions, input data, outputs, bus types)

4

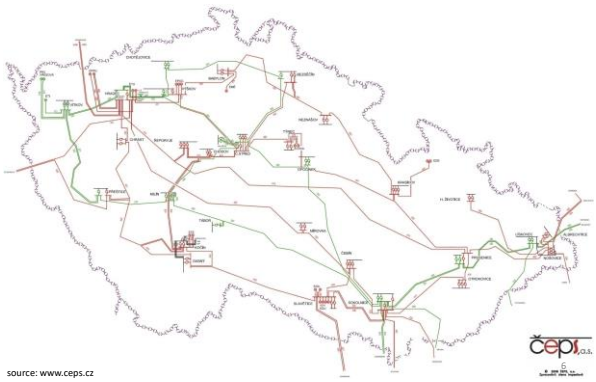
Czech electric power system

- consists of:
  - transmission system (400 kV, 220 kV, and part of 110 kV)
    - connection of major power stations
    - interconnection with foreign power systems
  - distribution systems (110, 35, 22, 10, 6, 0.4 kV)
    - connection of smaller power stations (in factories)
    - connection of consumers / consumption centres
- function:

  to ensure the power supply in required amount, time and quality; to required locations; with required reliability (i.e. with minimum number of outages) and with maximum cost efficiency

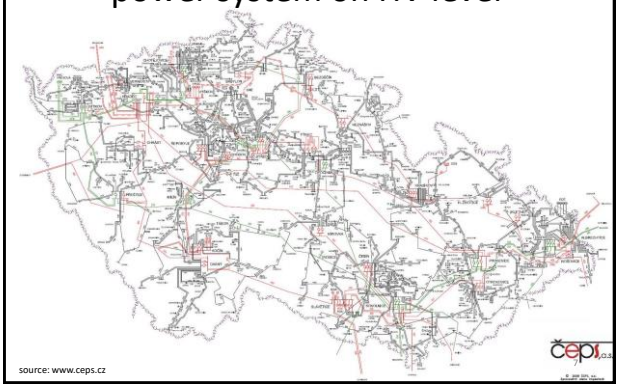
5

Czech transmission power system



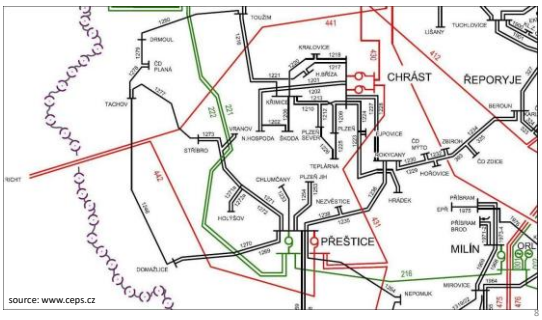
source: www.ceps.cz

Czech transmission + distribution power system on HV level



Sample of Czech power system

- Sample of Czech power system – Pilsen region



Modeling of electric power systems

- The goal is:
  - calculation of voltage and power conditions
  - calculation of total active and reactive power losses
  - optimum tap settings of OLTC transformers
  - optimum setting of generated active power in power stations to minimize generation costs
  - future design planning of electric power system
  - short-circuit analysis (1-/2-/3-phase short-circuit occurring in electric power system)
  - contingency analysis (N-1 or N-2 criterion) and more...

Load flow analysis

(british: „load flow“, american: „power flow“)

Conventional load flow analysis

- Goal:
  - analysis of voltage conditions in all network buses
  - analysis of power flows in all network lines
  - calculation of total active and reactive power losses
  - calculation of reactive power flows generated or consumed by generators, synchronous condensers and shunt compensators

10

Conventional load flow analysis

- Assumptions:
  - analysis of steady-state conditions only
  - 3-phase networks are symmetrical, passive parameters are constant in time and identical in all phases ⇒ calculation only for 1 phase, for remaining phases – rotation by ± 120 Degrees
  - sinusoidal waveform of voltages and currents ⇒ use of symbolic-complex method – use of phasors for active (U, I, S) and passive (Y, Z) variables

11

Loop current method, nodal voltage method

- Advantages of using the nodal voltage method
  - simple preparation of input data and of nodal equations (lower number of network buses, less parameters and equations)
  - shunt elements do not increase the number of unknown variables and of equations describing current flows
  - simple tap setting changes of OLTC transformers
  - comfortable use of computers
  - first, the nodal voltages are solved, then current/power flows can be simply calculated

12

Load flow calculation

- Input data
  - network topology with impedances / admittances of all network components (power lines, transformers, shunt compensators), limits for their maximum loading, fixed tap settings of transformers
  - active power load in PQ and PU buses, reactive power load in PQ buses, active power generation and voltage magnitude in PU buses, complex voltage in slack bus
  - lower/upper limits of generated reactive power for each PU bus

Definitions of PQ, PU and slack bus will be explained later.

Load flow calculation

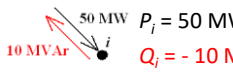
- Outputs
  - voltage magnitudes and voltage phase angles in all buses
  - injected active and reactive powers to each bus
  - power flows in all network lines
  - current flows, power factors, line losses and loadings
  - total active and reactive power losses

Note: All generated reactive powers, voltage magnitudes, line loadings and tap settings of OLTC transformers must stay in pre-defined limits.

14

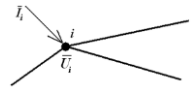
Modeling of network buses

- Network bus
  - represents consumption, substation, part of upstream network, power station, etc.
  - is uniquely defined by using active variables (either by $\bar{U}_i, \bar{I}_i$ or by U_i, θ_i, P_i, Q_i, where $\bar{U}_i = U_i e^{j\theta_i}$)
  - sign agreement:

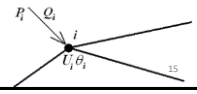


P_i = 50 MW

Q_i = - 10 MVAr



  - main formula: $\bar{I}_i = \frac{P_i - jQ_i}{\sqrt{3}\bar{U}_i^*}$



15

Modeling of network buses

- Bus type:
  - PQ bus (or load bus) – is the bus with connected load
    - P, Q are given; U, θ will be solved
  - PU bus (or generator bus) – is the bus with connected power station, generator or synchronous condenser
    - P, U are given; Q, θ will be solved
  - slack bus – is the bus with the largest connected power station or generator or the bus connecting the system to upstream or foreign network
    - **at least 1** to be defined for unique solution
    - U, θ are given; P, Q will be solved
    - θ is often defined equal to zero

!! 2 active variables are known, other 2 will be solved!!

16

Load flow analysis

2. Bus admittance matrix - inclusion of power lines and compensators

17

Bus admittance matrix ***A***

- Definition of bus admittance / impedance matrix

$$\sqrt{3}\underline{\overline{I}} = \underline{\overline{A}} \cdot \underline{\overline{U}} \quad \text{or} \quad \underline{\overline{U}} = \sqrt{3}\underline{\overline{Z}} \cdot \underline{\overline{I}}$$

where: $\underline{\overline{I}}$ - [n,1] vector of injected currents to network buses

$\underline{\overline{U}}$ - [n,1] vector of bus line-to-line voltages in network buses

$\underline{\overline{A}}$ - [n,n] bus admittance matrix

$\underline{\overline{Z}}$ - [n,n] bus impedance matrix

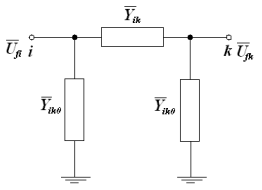
Note.: n – number of network buses

- Unique solution: 2n equations for 2n unknown variables

18

Derivation of bus admittance matrix

- Network built by power lines (as Π-elements)



$$\overline{Z}_{ik} = R_{ik} + jX_{ik} = \\ = R_{ik}[\Omega/km]\ell_{ik}[km] + \\ + jX_{ik}[\Omega/km]\ell_{ik}[km]\ [\Omega]$$

$$\overline{Y}_{ik0} = G_{ik0} + jB_{ik0} = \\ = G_{ik0}[S/km]\frac{\ell_{ik}[km]}{2} + \\ + jB_{ik0}[S/km]\frac{\ell_{ik}[km]}{2}\ [S]$$

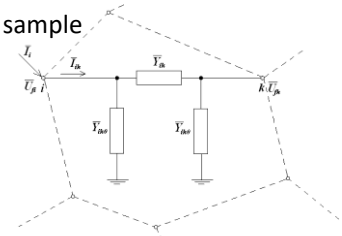
Admittances are more suitable for use (see later):

$$\overline{Y}_{ik} = \frac{1}{\overline{Z}_{ik}} = \frac{1}{R_{ik} + jX_{ik}} = \frac{R_{ik}}{R_{ik}^{\ 2} + X_{ik}^{\ 2}} - j\frac{X_{ik}}{R_{ik}^{\ 2} + X_{ik}^{\ 2}} = G_{ik} + jB_{ik}\ [S]$$

19

Derivation of bus admittance matrix

- Network sample



$$\overline{I}_{ik} = \overline{U}_{fi}\overline{Y}_{ik0} + (\overline{U}_{fi} - \overline{U}_{fk})\overline{Y}_{ik} = \overline{U}_{fi}(\overline{Y}_{ik0} + \overline{Y}_{ik}) - \overline{U}_{fk}\overline{Y}_{ik}$$

$$\rightarrow \overline{I}_i = \sum_{\substack{k=1 \\ k\neq i}}^{n} \overline{I}_{ik} = \overline{U}_{fi}\sum_{\substack{k=1 \\ k\neq i}}^{n}(\overline{Y}_{ik0} + \overline{Y}_{ik}) - \sum_{\substack{k=1 \\ k\neq i}}^{n}\overline{U}_{fk}\overline{Y}_{ik}$$

20

Derivation of bus admittance matrix

- By comparing with the definition of bus admittance matrix:

$$\overline{I}_i = \sum_{\substack{k=1 \\ k\neq i}}^{n} \overline{I}_{ik} = \overline{U}_{fi}\sum_{\substack{k=1 \\ k\neq i}}^{n}(\overline{Y}_{ik0} + \overline{Y}_{ik}) - \sum_{\substack{k=1 \\ k\neq i}}^{n}\overline{U}_{fk}\overline{Y}_{ik}$$

$$\overline{I}_i = \overline{A}_{i1}\overline{U}_{f1} + \overline{A}_{i2}\overline{U}_{f2} + \ldots + \overline{A}_{ii}\overline{U}_{fi} + \ldots + \overline{A}_{ik}\overline{U}_{fk} + \ldots + \overline{A}_{in}\overline{U}_{fn}$$

$$! \rightarrow \begin{matrix} \overline{A}_{ii} = \sum_{\substack{k=1 \\ k\neq i}}^{n}(\overline{Y}_{ik0} + \overline{Y}_{ik}) \\ \overline{A}_{ik} = -\overline{Y}_{ik} \end{matrix} !$$

- For bus impedance matrix applies: $\underline{\overline{Z}} = \underline{\overline{A}}^{-1}$

21

Admittance or impedance matrix?

- Bus admittance matrix
  - ✚ simple update after network topology changes

    switching-off the line *ik*:

    $\overline{A}_{ik-new} = 0$
    $\overline{A}_{ii-new} = \overline{A}_{ii-old} - \overline{Y}_{ik0} - \overline{Y}_{ik}$
    $\overline{A}_{kk-new} = \overline{A}_{kk-old} - \overline{Y}_{ik0} - \overline{Y}_{ik}$

    switching-on the line *ik*:

    $\overline{A}_{ik-new} = -\overline{Y}_{ik}$
    $\overline{A}_{ii-new} = \overline{A}_{ii-old} + \overline{Y}_{ik0} + \overline{Y}_{ik}$
    $\overline{A}_{kk-new} = \overline{A}_{kk-old} + \overline{Y}_{ik0} + \overline{Y}_{ik}$

  - ✚ very sparse – zero non-diagonal matrix elements for not interconnected network buses
  - ✚ symmetrical for networks containing **only** power lines
  - ✚ diagonally dominant

22

Admittance or impedance matrix?

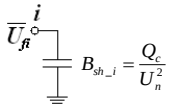
- Bus impedance matrix
  - ▬ does not contain any zero elements
  - ▬ impedance of 1 power line affects all matrix elements
  - ▬ can be obtained only by inversion of admittance matrix
  - ▬ problems during topology changes – conversion to admittance matrix, topology changes, conversion back to impedance matrix
  - ▬ not diagonally dominant

  ➡ exclusive use of admittance matrix ***A*** (operational states), impedance matrix ***Z*** for short-circuit calculation

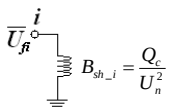
23

Inclusion of compensators into ***A***

- Shunt capacitors and inductors



$B_{sh_i} = \frac{Q_c}{U_n^2} > 0$ for LV, MV power factor correction



$B_{sh_i} = \frac{Q_c}{U_n^2} < 0$ for HV compensation of Q_{cap}

Update of matrix ***A***: $A_{ii} = A_{ii} + jB_{sh_i}$

24

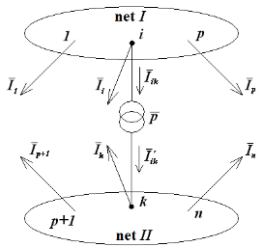
Load flow analysis

3. Bus admittance matrix - inclusion of 2- and 3- winding transformers

25

Inclusion of transformers into ***A***

- Network sample with one 2-winding transformer



$$\begin{bmatrix} \bar{I}_1 \\ \vdots \\ \bar{I}_i - \bar{I}_{ik} \\ \vdots \\ \bar{I}_p \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \cdots & \bar{A}_{1i} & \cdots & \bar{A}_{1p} \\ \vdots & & \vdots & & \vdots \\ \bar{A}_{i1} & \cdots & \bar{A}_{ii} & \cdots & \bar{A}_{ip} \\ \vdots & & \vdots & & \vdots \\ \bar{A}_{p1} & \cdots & \bar{A}_{pi} & \cdots & \bar{A}_{pp} \end{bmatrix} \cdot \begin{bmatrix} \bar{U}_{f1} \\ \vdots \\ \bar{U}_{fi} \\ \vdots \\ \bar{U}_{fp} \end{bmatrix}$$

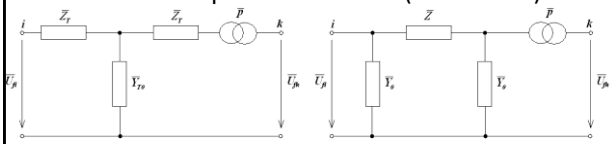
$$\begin{bmatrix} \bar{I}_{p+1} \\ \vdots \\ \bar{I}_k + \bar{I}'_{ik} \\ \vdots \\ \bar{I}_n \end{bmatrix} = \begin{bmatrix} \bar{A}_{p+1,p+1} & \cdots & \bar{A}_{p+1,k} & \cdots & \bar{A}_{p+1,n} \\ \vdots & & \vdots & & \vdots \\ \bar{A}_{k,p+1} & \cdots & \bar{A}_{kk} & \cdots & \bar{A}_{kn} \\ \vdots & & \vdots & & \vdots \\ \bar{A}_{n,p+1} & \cdots & \bar{A}_{nk} & \cdots & \bar{A}_{nn} \end{bmatrix} \cdot \begin{bmatrix} \bar{U}_{fp+1} \\ \vdots \\ \bar{U}_{fk} \\ \vdots \\ \bar{U}_{fn} \end{bmatrix}$$

- How to derive currents $\bar{I}_{ik}$, $\bar{I}'_{ik}$?

26

Inclusion of transformers into ***A***

- Transformer's equivalent scheme (Π-element)



$$\bar{Z}_T = R_1 + jX_{\sigma 1} = R_2' + jX_{\sigma 2}'$$

$$\bar{Y}_{T0} = G_{Fe} - jB_m$$

$$\bar{Z} = 2\bar{Z}_T + \bar{Y}_{T0}\bar{Z}_T^2 \cong 2\bar{Z}_T = R_k + jX_k$$

$$\rightarrow \bar{Y} = \frac{1}{\bar{Z}} = \frac{R_k}{R_k^2 + X_k^2} - j\frac{X_k}{R_k^2 + X_k^2}$$

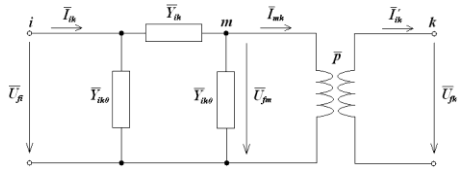
Wye – Delta conversion !!!

$$\bar{Y}_0 = \frac{\bar{Y}_{T0}\bar{Z}_T}{2\bar{Z}_T + \bar{Y}_{T0}\bar{Z}_T^2} \cong \frac{\bar{Y}_{T0}}{2} = \frac{1}{2}(G_{Fe} - jB_m)$$

27

Inclusion of transformers into ***A***

- Detailed scheme of a 2-winding transformer



Final formula for $\bar{I}_{ik}$ and $\bar{I}'_{ik}$ can be derived as:

$$\begin{bmatrix} \bar{I}_{ik} \\ \bar{I}'_{ik} \end{bmatrix} = \begin{bmatrix} \bar{Y}_{ik} + \bar{Y}_{ik0} & -p\bar{Y}_{ik} \\ p^{*}\bar{Y}_{ik} & -|p|^{2}\left(\bar{Y}_{ik} + \bar{Y}_{ik0}\right) \end{bmatrix} \begin{bmatrix} \bar{U}_{fi} \\ \bar{U}_{fk} \end{bmatrix}$$

28

Final form of ***A*** with one transformer

$$\begin{bmatrix} \bar{I}_1 \\ \vdots \\ \bar{I}_i \\ \vdots \\ \bar{I}_p \\ \bar{I}_{p+1} \\ \vdots \\ \bar{I}_k \\ \vdots \\ \bar{I}_n \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \cdots & \bar{A}_{1i} & \cdots & \bar{A}_{1p} & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \vdots & & \vdots & & \vdots \\ \bar{A}_{i1} & \cdots & \bar{A}_{ii} + \bar{Y}_{ik} + \bar{Y}_{ik0} & \cdots & \bar{A}_{ip} & 0 & \cdots & -p\bar{Y}_{ik} & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \vdots & & \vdots & & \vdots \\ \bar{A}_{p1} & \cdots & \bar{A}_{pi} & \cdots & \bar{A}_{pp} & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 & \bar{A}_{p+1,p+1} & \cdots & \bar{A}_{p+1,k} & \cdots & \bar{A}_{p+1,n} \\ \vdots & & \vdots & & \vdots & \vdots & & \vdots & & \vdots \\ 0 & \cdots & -p^{*}\bar{Y}_{ik} & \cdots & 0 & \bar{A}_{k,p+1} & \cdots & \bar{A}_{kk} + |p|^{2}\left(\bar{Y}_{ik} + \bar{Y}_{ik0}\right) & \cdots & \bar{A}_{kn} \\ \vdots & & \vdots & & \vdots & \vdots & & \vdots & & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \bar{A}_{n,p+1} & \cdots & \bar{A}_{nk} & \cdots & \bar{A}_{nn} \end{bmatrix} \cdot \begin{bmatrix} \bar{U}_{f1} \\ \vdots \\ \bar{U}_{fi} \\ \vdots \\ \bar{U}_{fp} \\ \bar{U}_{fp+1} \\ \vdots \\ \bar{U}_{fk} \\ \vdots \\ \bar{U}_{fn} \end{bmatrix}$$

Summary: each transformer is defined in ***A*** by 4 elements, passive parameters of transformer are calculated on primary (Π-element) side, power lines at their U_n

⇒ simple update of ***A*** in case of any tap change

29

Case of a 3-winding transformer?

- Network sample with one 3-winding transformer



30

Final system of load flow equations

$$\begin{bmatrix} \frac{P_1 - jQ_1}{\overline{U}_1^*} \\ \vdots \\ \frac{P_i - jQ_i}{\overline{U}_i^*} \\ \vdots \\ \frac{P_n - jQ_n}{\overline{U}_n^*} \end{bmatrix} = \begin{bmatrix} \overline{A}_{11} & \cdots & \overline{A}_{1i} & \cdots & \overline{A}_{1n} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ \overline{A}_{i1} & \cdots & \overline{A}_{ii} & \cdots & \overline{A}_{in} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ \overline{A}_{n1} & \cdots & \overline{A}_{ni} & \cdots & \overline{A}_{nn} \end{bmatrix} \begin{bmatrix} \overline{U}_1 \\ \vdots \\ \overline{U}_i \\ \vdots \\ \overline{U}_n \end{bmatrix}$$

$$\sqrt{3}\underline{\overline{I}} = \underline{\overline{A}} \times \underline{\overline{U}}$$

Conclusion:

- system of **n** non-linear equations with complex coefficients
- unknowns – voltage magnitudes and phase angles
- analytical approach possible only for very small networks

Next time

- **Load flow analysis**
  - Principles of numerical calculation (iteration, convergence, divergence)
  - Gauss and Gauss-Seidel method

32