ESF II project of the University of West Bohemia in Pilsen

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# THEORY OF TRANSMISSION AND DISTRIBUTION OF ELECTRIC ENERGY

## 9th lecture (Theory and example)

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## Content

- **Load flow analysis - continuation**
  4. Principles of numerical calculation (iteration, convergence, divergence)
  5. Gauss and Gauss-Seidel method
     - Characteristics
     - Calculation diagram
     - Example

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# Load flow analysis

4. Principles of numerical calculation (iteration, convegence, divergence)

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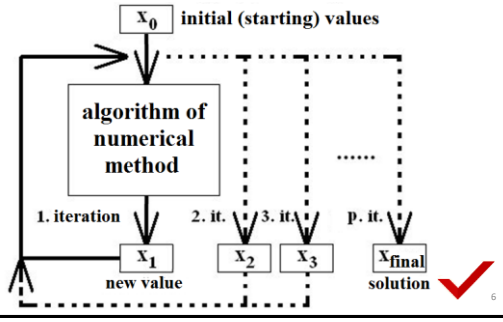
## Use of numerical approach

- First numerical methods:
  - **Gauss-Seidel method (G-S)**
  - **Newton-Raphson method (N-R)**
- Alternative methods (simplification of N-R method):
  - Decoupled method (DLF)
  - Fast-Decoupled method (F-D)
  - DC load flow
- by solving the system of nonlinear equations we get unknown variables $U_i, \theta_i$, which are needed for other calculations in the examined network

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## Principles of numerical calculation

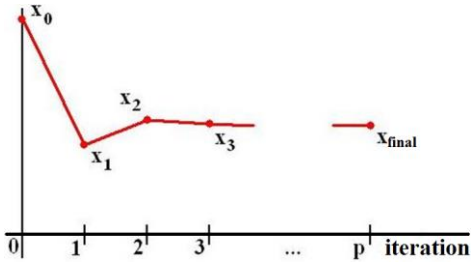
- Term „iteration “?




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## Convergence vs. divergence

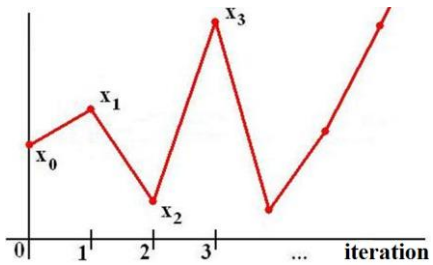
- Convergence of numerical calculation




solution is found with given accuracy after *p* iterations

## Convergence vs. divergence

- Divergence of numerical calculation




impossibility of finding the solution due to iterative oscillations

## Termination of numerical calculation

- Conditions:
  1) if the deviation between two adjacent iterations is smaller than the preset tolerance

$$\max\left|\frac{x_p - x_{p-1}}{x_{p-1}}\right| \le \varepsilon_{lim} \qquad \varepsilon_{lim} = 0.01, 0.0001, \text{ etc.}$$

  - activated at certain degree of convergence

  2) if the maximum number of iterations (preset for numerical calculation) is reached

$$p = p_{\max}$$

  - activated usually with divergence of numerical calculation

# Load flow analysis

## 5. Gauss and Gauss-Seidel method

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## Gauss-Seidel method

- for load flow analysis first applied in 1950s
- computationally simple, suitable for first computers
- only for solving small networks (max. 50 buses)
- more time-consuming for modeling larger networks (no requirement to obtain the solution very fast)
- today used for high calculation reliability (numerical stability)
- easily explainable, suitable for educational purposes

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## Modeling example

- Let's solve the set of 2 nonlinear equations:

$$x + 3\frac{y}{x} = 4$$
$$-\frac{5}{y} + 8xy = 11$$

for $p$ = 0 ..... $x_0$ = {1;1}

Step 1: create an iterative algorithm for each unknown variable

from 1st eqn.: $y^{(p+1)} = \frac{(4 - x^{(p)})x^{(p)}}{3}$ from 2nd eqn.: $x^{(p+1)} = \frac{\left(11 + \frac{5}{y^{(p+1)}}\right)}{8y^{(p+1)}}$

! Newly computed $y^{(p+1)}$ immediately replaces its old value from previous iteration $y^{(p)}$

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## Modeling example

- Numerical calculation

| iteration | x | y | deviation |
|---|---|---|---|
| 0 | 1 | 1 | --- |
| 1 | 2 | 1 | 1 |
| 2 | 1.3828 | 1.3333 | 0.61719 |
| 3 | 1.5693 | 1.2064 | 0.18644 |
| 4 | 1.468 | 1.2715 | 0.10125 |
| 5 | 1.5169 | 1.239 | 0.048902 |
| ... | ... | ... | ... |
| 13 | 1.5001 | 1.25 | 0.00021508 |
| 14 | 1.5 | 1.25 | 0.00010898 |
| 15 | 1.5 | 1.25 | 5.5215e-005 |

➡ solution: x = 1.5, y = 1.25 (for $\varepsilon_{lim} = 1 \cdot 10^{-4}$ )

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## Gauss vs. Gauss-Seidel method

- Gauss method
  - updates all unknowns to new values at the end of the iteration ➡ slower convergence than Gauss-Seidel method (more iterations needed)
- Gauss-Seidel method
  - the value of an unknown variable is updated immediately and used for other calculations within the same iteration ➡ faster convergence
  - **linear convergence** (deviation decreases linearly)

## G-S method for load flow analysis

- G-S iterative algorithm (for PQ buses)

$$\overline{U}_i^{(p+1)} = \frac{1}{\overline{A}_{ii}}\left[\frac{P_i - jQ_i}{\overline{U}_i^{(p)*}} - \sum_{k=1}^{i-1}\overline{A}_{ik}\overline{U}_k^{(p+1)} - \sum_{k=i+1}^{n}\overline{A}_{ik}\overline{U}_k^{(p)}\right]$$

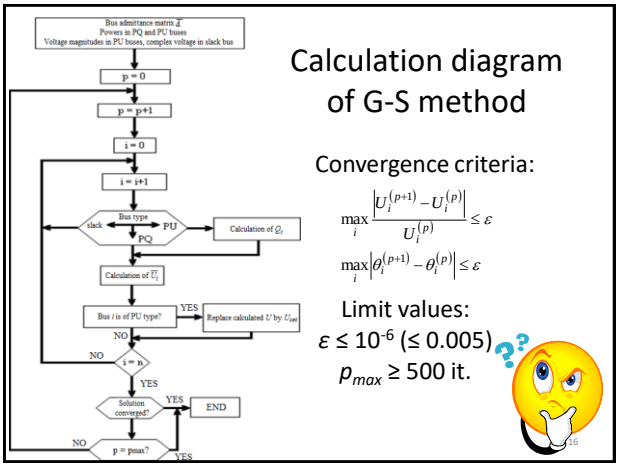
- for PU buses – compute $Q_i$ first, then use formula above and finally replace calculated voltage magnitude by preset value – see the formula below.

$$Q_i^{(p+1)} = \mathbf{Im}\left\{\overline{U}_i^{(p)}\left[\sum_{k=1}^{i-1}\overline{A}_{ik}^{*}\overline{U}_k^{(p+1)*} + \sum_{k=i}^{n}\overline{A}_{ik}^{*}\overline{U}_k^{(p)*}\right]\right\} = Q_{Gi}^{(p+1)} + Q_{Li}$$

$$\overline{U}_i^{(p+1)} = \frac{\overline{U}_i^{(p+1)}}{|\overline{U}_i^{(p+1)}|}\cdot U_{i_set}$$

- for slack bus – no calculation needed

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## Calculation diagram of G-S method

Convergence criteria:

$$\max_i \frac{\left|U_i^{(p+1)} - U_i^{(p)}\right|}{U_i^{(p)}} \le \varepsilon$$

$$\max_i \left|\theta_i^{(p+1)} - \theta_i^{(p)}\right| \le \varepsilon$$

Limit values:

$\varepsilon \le 10^{-6}$ ($\le 0.005$)

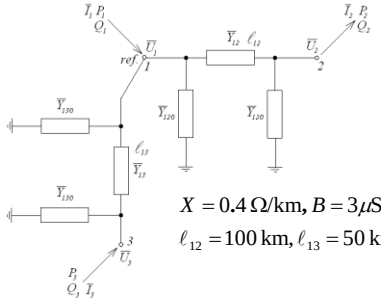
$p_{max} \ge 500$ it.





## Example (a 3-bus system)

- Solve the load flow of a 3-bus system




$$\overline{U}_1 = 110000 \cdot e^{j0^\circ}\ \text{V}$$
$$P_2 = 50\ \text{MW}$$
$$Q_2 = 10\ \text{MVAr}$$
$$P_3 = 100\ \text{MW}$$
$$Q_3 = 10\ \text{MVAr}$$

$X = 0.4\,\Omega/\text{km}$, $B = 3\mu S/\text{km}$

$\ell_{12} = 100\ \text{km}$, $\ell_{13} = 50\ \text{km}$

$U_2, \theta_2, U_3, \theta_3 = ?$

## Example (a 3-bus system)

- Iterative algorithm:

$$\overline{U}_2^{(p)} = \frac{1}{\overline{A}_{22}}\left[\frac{P_2 - jQ_2}{\overline{U}_2^{*(p-1)}} - \overline{A}_{21}\overline{U}_1 - \sum_3 \overline{A}_{ik}\overline{U}_k^{(p-1)}\right] = \frac{1}{\overline{A}_{22}}\left[\frac{P_2 - jQ_2}{\overline{U}_2^{*(p-1)}} - \overline{A}_{21}\overline{U}_1 - \overline{A}_{23}\overline{U}_3^{(p-1)}\right]$$

$$\overline{U}_3^{(p)} = \frac{1}{\overline{A}_{33}}\left[\frac{P_3 - jQ_3}{\overline{U}_3^{*(p-1)}} - \overline{A}_{31}\overline{U}_1 - \sum_{k=2} \overline{A}_{ik}\overline{U}_k^{(p)}\right] = \frac{1}{\overline{A}_{33}}\left[\frac{P_3 - jQ_3}{\overline{U}_3^{*(p-1)}} - \overline{A}_{31}\overline{U}_1 - \overline{A}_{32}\overline{U}_2^{(p)}\right]$$

- Admittance matrix:

$$\underline{\overline{A}} = j\begin{bmatrix} -0.074775 & 0.025 & 0.05 \\ 0.025 & -0.02485 & 0 \\ 0.05 & 0 & -0.049925 \end{bmatrix}$$

- Starting values:

$$\underline{\overline{x}}_0 = \begin{bmatrix} 110000\ e^{j0} \\ 110000\ e^{j0} \\ 110000\ e^{j0} \end{bmatrix} [\text{V}]$$

- Maximum no. of iterations: $p_{\max} = 500$
- Permitted tolerance: $\varepsilon_{lim} = 1 \cdot 10^{-4}$

## Numerical calculation

| $p$ | $U_2$ | $U_3$ |
|---|---|---|
| 0 | 110000,00000 | 110000,00000 |
| 1 | 108557,79539 | 113456,92025 |
| 2 | 105374,88998 | 110502,59407 |
| 3 | 105284,10663 | 110555,03645 |
| 4 | 105178,50904 | 110476,87553 |
| 5 | 105175,34781 | 110478,27315 |
| 6 | 105171,71834 | 110476,14855 |
| 7 | 105171,60950 | 110476,18655 |
| 8 | 105171,48461 | 110476,12876 |
| 9 | 105171,48086 | 110476,12979 |
| 10 | 105171,47656 | 110476,12822 |
| 11 | 105171,47643 | 110476,12824 |
| **12** | **105171,47629** | **110476,12820** |

| $p$ | $\theta_2$ | $\theta_3$ |
|---|---|---|
| 0 | 0 | 0 |
| 1 | -9°42´1,41´´ | 9°14´8,01´´ |
| 2 | -9°38´22,42´´ | 9°13´18,51´´ |
| 3 | -9°56´46,96´´ | 9°27´57,71´´ |
| 4 | -9°56´39,67´´ | 9°27´57,02´´ |
| 5 | -9°57´17,58´´ | 9°28´20,93´´ |
| 6 | -9°57´17,33´´ | 9°28´20,90´´ |
| 7 | -9°57´18,63´´ | 9°28´21,55´´ |
| 8 | -9°57´18,62´´ | 9°28´21,54´´ |
| 9 | -9°57´18,67´´ | 9°28´21,56´´ |
| 10 | -9°57´18,67´´ | 9°28´21,56´´ |
| 11 | -9°57´18,67´´ | 9°28´21,56´´ |
| **12** | **-9°57´18,67´´** | **9°28´21,56´´** |

$\Rightarrow \overline{U}_2 = 105171.47629\, e^{-j9°57'18.67"} V \quad \overline{U}_3 = 110476.12820\, e^{j9°28'21.56"} V$

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## Evaluation of G-S method

- Advantages:
  - simple mathematical model (no matrices, derivatives, etc.)
  - slow iterative approach to the solution (small updates between the iterations) ⇒ relatively high reliability for reaching the solution
  - low computational/time requirements per iteration
- Disadvantages:
  - working with complex numbers
  - strong dependence of no. of iterations on network size
  - only linear convergence ⇒ high no. of iterations
  - longer computing time (especially for larger networks)

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## Next time

- **Load flow analysis - continuation**
  - Newton-Raphson method
  - Inclusion of reactive power limits in PU buses
  - Other calculations in electric power systems

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