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THEORY OF TRANSMISSION AND DISTRIBUTION OF ELECTRIC ENERGY

10th lecture – theory

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- **Load flow analysis - continuation**
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Load flow analysis

6. Newton-Raphson method

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Newton-Raphson method

- Simple case (nonlinear equation with 1 unknown x)
  $f(x) = b$
- By Taylor's development we get:
  $$f\left(x^{(0)} + \Delta x\right) \approx f\left(x^{(0)}\right) + \Delta x \left.\frac{df}{dx}\right|_{x=x^{(0)}} + \frac{\Delta x^2}{2!}\left.\frac{d^2 f}{dx^2}\right|_{x=x^{(0)}} + \ldots + \frac{\Delta x^N}{N!}\left.\frac{d^N f}{dx^N}\right|_{x=x^{(0)}} = b$$
- Higher orders are neglected:
  $$f\left(x^{(0)} + \Delta x\right) \approx f\left(x^{(0)}\right) + \Delta x \left.\frac{df}{dx}\right|_{x=x^{(0)}} = b$$
- Multidimensional case:
  $$\begin{bmatrix} b_1 - f_1\left(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}\right) \\ b_2 - f_2\left(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}\right) \\ \vdots \\ b_n - f_n\left(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}\right) \end{bmatrix} = \begin{bmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|_{x_1=x_1^{(0)}} & \left.\frac{\partial f_1}{\partial x_2}\right|_{x_2=x_2^{(0)}} & \cdots & \left.\frac{\partial f_1}{\partial x_n}\right|_{x_n=x_n^{(0)}} \\ \left.\frac{\partial f_2}{\partial x_1}\right|_{x_1=x_1^{(0)}} & \left.\frac{\partial f_2}{\partial x_2}\right|_{x_2=x_2^{(0)}} & \cdots & \left.\frac{\partial f_2}{\partial x_n}\right|_{x_n=x_n^{(0)}} \\ \vdots & \vdots & \ddots & \vdots \\ \left.\frac{\partial f_n}{\partial x_1}\right|_{x_1=x_1^{(0)}} & \left.\frac{\partial f_n}{\partial x_2}\right|_{x_2=x_2^{(0)}} & \cdots & \left.\frac{\partial f_n}{\partial x_n}\right|_{x_n=x_n^{(0)}} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix}$$

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Newton-Raphson method

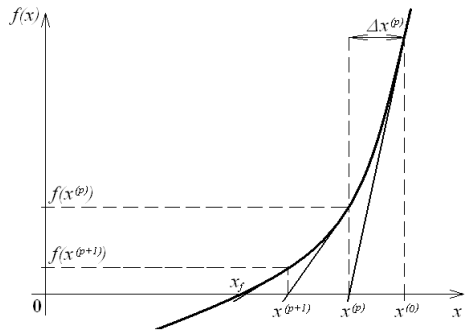
- Calculation of correction vector in new iteration:
  $$\begin{bmatrix} \Delta x_1^{(p+1)} \\ \Delta x_2^{(p+1)} \\ \vdots \\ \Delta x_n^{(p+1)} \end{bmatrix} = \begin{bmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|_{x_1=x_1^{(p)}} & \left.\frac{\partial f_1}{\partial x_2}\right|_{x_2=x_2^{(p)}} & \cdots & \left.\frac{\partial f_1}{\partial x_n}\right|_{x_n=x_n^{(p)}} \\ \left.\frac{\partial f_2}{\partial x_1}\right|_{x_1=x_1^{(p)}} & \left.\frac{\partial f_2}{\partial x_2}\right|_{x_2=x_2^{(p)}} & \cdots & \left.\frac{\partial f_2}{\partial x_n}\right|_{x_n=x_n^{(p)}} \\ \vdots & \vdots & \ddots & \vdots \\ \left.\frac{\partial f_n}{\partial x_1}\right|_{x_1=x_1^{(p)}} & \left.\frac{\partial f_n}{\partial x_2}\right|_{x_2=x_2^{(p)}} & \cdots & \left.\frac{\partial f_n}{\partial x_n}\right|_{x_n=x_n^{(p)}} \end{bmatrix}^{-1} \begin{bmatrix} b_1 - f_1\left(x_1^{(p)}, x_2^{(p)}, \ldots, x_n^{(p)}\right) \\ b_2 - f_2\left(x_1^{(p)}, x_2^{(p)}, \ldots, x_n^{(p)}\right) \\ \vdots \\ b_n - f_n\left(x_1^{(p)}, x_2^{(p)}, \ldots, x_n^{(p)}\right) \end{bmatrix}$$
- Updating process:
  $$\begin{bmatrix} x_1^{(p+1)} \\ x_2^{(p+1)} \\ \vdots \\ x_n^{(p+1)} \end{bmatrix} = \begin{bmatrix} x_1^{(p)} \\ x_2^{(p)} \\ \vdots \\ x_n^{(p)} \end{bmatrix} + \begin{bmatrix} \Delta x_1^{(p+1)} \\ \Delta x_2^{(p+1)} \\ \vdots \\ \Delta x_n^{(p+1)} \end{bmatrix}$$

Convergence criterion:

$$\max_i \begin{bmatrix} b_1 - f_1\left(x_1^{(p)}, x_2^{(p)}, \ldots, x_n^{(p)}\right) \\ b_2 - f_2\left(x_1^{(p)}, x_2^{(p)}, \ldots, x_n^{(p)}\right) \\ \vdots \\ b_n - f_n\left(x_1^{(p)}, x_2^{(p)}, \ldots, x_n^{(p)}\right) \end{bmatrix} \le \varepsilon$$

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Graphical interpretation of N-R method



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Modeling example

- Let's solve the set of 2 nonlinear equations:

$$x + 3\frac{y}{x} = 4$$

$$-\frac{5}{y} + 8xy = 11$$

for $p = 0$ ..... $x_0 = \{1;1\}$

ending condition: $\varepsilon_{lim} = 1 \cdot 10^{-4}$

Procedure: main numerical formula for N-R method

$$\begin{bmatrix}\Delta f_1 \\ \Delta f_2\end{bmatrix} = \begin{bmatrix}\frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y}\end{bmatrix}\begin{bmatrix}\Delta x \\ \Delta y\end{bmatrix}$$ + update: $x^{(p+1)} = x^{(p)} + \Delta x$, $y^{(p+1)} = y^{(p)} + \Delta y$

Jacobi matrix

Update is done at the end of each iteration!!

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Modeling example

- Numerical formula for this modeling example:

$$\begin{bmatrix}4 - x - 3\frac{y}{x} \\ 11 + \frac{5}{y} - 8xy\end{bmatrix} = \begin{bmatrix}1 - 3\frac{y}{x^2} & \frac{3}{x} \\ 8y & \frac{5}{y^2} + 8x\end{bmatrix}\begin{bmatrix}\Delta x \\ \Delta y\end{bmatrix}$$

- For p = 1 (where $x_0 = \{1;1\}$):

update

$$\begin{bmatrix}0 \\ 8\end{bmatrix} = \begin{bmatrix}-2 & 3 \\ 8 & 13\end{bmatrix}\begin{bmatrix}\Delta x \\ \Delta y\end{bmatrix} \rightarrow \Delta x = 0.48, \Delta y = 0.32 \rightarrow x^{(1)} = 1.48,\ y^{(1)} = 1.32$$

- Numerical calculation:

iteration	x	y	deviation
0	1	1	8
1	1.48	1.32	0.84092
2	1.4976	1.2502	0.020998
3	**1.5**	**1.25**	**7.1501e-006**

Found solution:

x = 1.5, y = 1.25

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N-R method for load flow analysis

- Basic iterative algorithm of N-R method:

$$\begin{bmatrix} \Delta P^{(p-1)} \\ \Delta Q^{(p-1)} \end{bmatrix} = \begin{bmatrix} \underline{H}^{(p-1)} & \underline{N}^{(p-1)} \\ \underline{J}^{(p-1)} & \underline{L}^{(p-1)} \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta^{(p)} \\ \dfrac{\Delta U^{(p)}}{U^{(p-1)}} \end{bmatrix}$$

where: power mismatch vector is

$$\Delta P_i = P_i - U_i \sum_{k=1}^{n} U_k \left(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}\right) \quad \text{or} \quad \Delta P_i = P_i - U_i \sum_{k=1}^{n} A_{ik} U_k \cos\left(\theta_{ik} - \varphi_{ik}\right)$$

$$\Delta Q_i = Q_i - U_i \sum_{k=1}^{n} U_k \left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right) \quad \text{or} \quad \Delta Q_i = Q_i - U_i \sum_{k=1}^{n} A_{ik} U_k \sin\left(\theta_{ik} - \varphi_{ik}\right)$$

Note.: $\theta_{ik} = \theta_i - \theta_k$

elements of Jacobi matrix are

$$H_{ik} = \frac{\partial P_i}{\partial \theta_k} \qquad N_{ik} = U_k \frac{\partial P_i}{\partial U_k} \qquad J_{ik} = \frac{\partial Q_i}{\partial \theta_k} \qquad L_{ik} = U_k \frac{\partial Q_i}{\partial U_k}$$

- Iterative algorithm contains: 2 equations for each PQ bus, 1 equation for PU bus, no equation for slack bus

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Jacobi matrix

- Properties:
  - symmetrical in structure, not in value (H_{ik} not in *ik* cell)
  - highly sparse (at minimum 2x sparser than ***A***)
  - strongly diagonally dominant = fast convergence
- Calculation of elements of Jacobi matrix:

$$H_{ii} = -Q_i - B_{ii}U_i^2 \quad N_{ii} = P_i + G_{ii}U_i^2 \quad J_{ii} = P_i - G_{ii}U_i^2 \quad L_{ii} = Q_i - B_{ii}U_i^2$$

$$H_{ik} = U_i U_k \left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right) \quad N_{ik} = U_i U_k \left(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}\right)$$

$$J_{ik} = -U_i U_k \left(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}\right) = -N_{ik}$$

$$L_{ik} = U_i U_k \left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right) = H_{ik}$$

- For PU buses, first Q_i needs to be calculated for H_{ii}:

$$Q_i^{(p-1)} = U_i^{(p-1)} \sum_{k=1}^{n} U_k^{(p-1)} \left(G_{ik}\sin\theta_{ik}^{(p-1)} - B_{ik}\cos\theta_{ik}^{(p-1)}\right) = Q_{Gi}^{(p-1)} + Q_{Li}$$

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Calculation of Jacobi matrix

- Elements of H submatrix:

$$H_{ii} = -\frac{\partial \Delta P_i}{\partial \theta_i} = \frac{\partial P_i}{\partial \theta_i} = -\left(Q_i + B_{ii}U_i^2\right)$$

$$H_{ik} = -\frac{\partial \Delta P_i}{\partial \theta_k} = \frac{\partial P_i}{\partial \theta_k} = U_i U_k \left(G_{ik}\,\mathbf{sin}\,\theta_{ik} - B_{ik}\,\mathbf{cos}\,\theta_{ik}\right)$$

- Elements of N submatrix:

$$N_{ii} = -U_i \frac{\partial \Delta P_i}{\partial U_i} = U_i \frac{\partial P_i}{\partial U_i} = P_i + G_{ii}U_i^2$$

$$N_{ik} = -U_k \frac{\partial \Delta P_i}{\partial U_k} = U_k \frac{\partial P_i}{\partial U_k} = U_i U_k \left(G_{ik}\,\mathbf{cos}\,\theta_{ik} + B_{ik}\,\mathbf{sin}\,\theta_{ik}\right)$$

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Calculation of Jacobi matrix

- Elements of J submatrix:

$$J_{ii} = -\frac{\partial \Delta Q_i}{\partial \theta_i} = \frac{\partial Q_i}{\partial \theta_i} = P_i - G_{ii}U_i^{\,2}$$

$$J_{ik} = -\frac{\partial \Delta Q_i}{\partial \theta_k} = \frac{\partial Q_i}{\partial \theta_k} = -U_i U_k \left(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}\right) = -N_{ik}$$

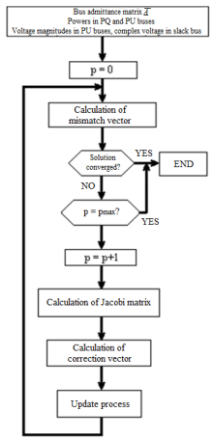
- Elements of L submatrix:

$$L_{ii} = -U_i \frac{\partial \Delta Q_i}{\partial U_i} = U_i \frac{\partial Q_i}{\partial U_i} = -B_{ii}U_i^{\,2} + Q_i$$

$$L_{ik} = -U_k \frac{\partial \Delta Q_i}{\partial U_k} = U_k \frac{\partial Q_i}{\partial U_k} = U_i U_k \left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right) = H_{ik}$$

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Calculation diagram of N-R method



Update process:

$$\theta_i^{(p)} = \theta_i^{(p-1)} + \Delta\theta_i^{(p)}$$

$$U_i^{(p)} = U_i^{(p-1)} + \frac{\Delta U_i^{(p)}}{U_i^{(p-1)}} U_i^{(p-1)}$$

Convergence criterion:

$$\max_i \left|\frac{\Delta P}{\Delta Q}\right| \le \varepsilon$$

Limit values:

$\varepsilon \le 10^{-5}$

$p_{max} \ge 20$ it.

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Example (a 3-bus system)

- Initial (starting) conditions:

$$U^{(0)} = \begin{bmatrix} U_2^{(0)} \\ U_3^{(0)} \end{bmatrix} = \begin{bmatrix} 110000 \\ 110000 \end{bmatrix}; \quad \theta^{(0)} = \begin{bmatrix} \theta_2^{(0)} \\ \theta_3^{(0)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- Basic iterative algorithm of N-R method for p = 1:

$$\begin{bmatrix} -50 \\ 100 \\ -8.185 \\ 10.9075 \end{bmatrix} = \begin{bmatrix} 310.685 & 0 & -50 & 0 \\ 0 & 594.0925 & 0 & 100 \\ -50 & 0 & 290.685 & 0 \\ 0 & 100 & 0 & 614.0925 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_2^{(p)} \\ \Delta\theta_3^{(p)} \\ \frac{\Delta U_2^{(p)}}{U_2^{(p-1)}} \\ \frac{\Delta U_3^{(p)}}{U_3^{(p-1)}} \end{bmatrix}$$

$$\rightarrow \Delta\theta_2 = -0.170177, \Delta\theta_3 = 0.169993, \frac{\Delta U_2}{U_2^{(0)}} = -0.057429, \frac{\Delta U_3}{U_3^{(0)}} = -0.009920$$

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Numerical calculation

- Update in iteration p = 1:

$$\theta_2^{(1)} = \theta_2^{(0)} + \Delta\theta_2 = -0.170177\,rad,\ \theta_3^{(1)} = \theta_3^{(0)} + \Delta\theta_3 = 0.169993\,rad$$

$$U_2^{(1)} = U_2^{(0)} + \frac{\Delta U_2}{U_2^{(0)}} \cdot U_2^{(0)} = 103682.81V$$

$$U_3^{(1)} = U_3^{(0)} + \frac{\Delta U_3}{U_3^{(0)}} \cdot U_3^{(0)} = 108908.80\,V$$

- Numerical calculation:

p	U_2	U_3
0	110000,00000	110000,00000
1	103682,77009	108908,78612
2	105170,26641	110477,69448
3	105171,47627	110476,12763
4	105171,47628	110476,12820
5	**105171,47628**	**110476,12820**

θ_2	θ_3
0	0
-9°45´1,54´´	9°44´23,73´´
-9°57´21,6´´	9°27´59,88´´
-9°57´18,67´´	9°28´21,56´´
-9°57´18,67´´	9°28´21,56´´
-9°57´18,67´´	**9°28´21,56´´**

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Modification of N-R method

- Inversion of Jacobi matrix – most time-consuming
  - calculation of new Jacobi matrix only in first 2 iterations
  - calculation of Jacobi matrix only 1x per 2-3 iterations (so-called Lazy N-R method)
  - use of sparsity techniques to speed-up inversion process
- For reactive power limits in PU buses – inclusion of Q_i row in Jacobi matrix: e.g. 4-bus network, bus 3 = PU bus

$$\begin{bmatrix}\Delta P_2\\ \Delta P_3\\ \Delta P_4\\ \Delta Q_2\\ \Delta Q_4\end{bmatrix} = \begin{bmatrix}H_{22} & H_{23} & H_{24} & N_{22} & N_{24}\\ H_{32} & H_{33} & H_{34} & N_{32} & N_{34}\\ H_{42} & H_{43} & H_{44} & N_{42} & N_{44}\\ J_{22} & J_{23} & J_{24} & L_{22} & L_{24}\\ J_{42} & J_{43} & J_{44} & L_{42} & L_{44}\end{bmatrix}\begin{bmatrix}\Delta\theta_2\\ \Delta\theta_3\\ \Delta\theta_4\\ \frac{\Delta V_2}{V_2}\\ \frac{\Delta V_4}{V_4}\end{bmatrix} \qquad \begin{bmatrix}\Delta P_2\\ \Delta P_3\\ \Delta P_4\\ \Delta Q_2\\ 0\\ \Delta Q_4\end{bmatrix} = \begin{bmatrix}H_{22} & H_{23} & H_{24} & N_{22} & 0 & N_{24}\\ H_{32} & H_{33} & H_{34} & N_{32} & 0 & N_{34}\\ H_{42} & H_{43} & H_{44} & N_{42} & 0 & N_{44}\\ J_{22} & J_{23} & J_{24} & L_{22} & 0 & L_{24}\\ 0 & 0 & 0 & 0 & 1 & 0\\ J_{42} & J_{43} & J_{44} & L_{42} & 0 & L_{44}\end{bmatrix}\begin{bmatrix}\Delta\theta_2\\ \Delta\theta_3\\ \Delta\theta_4\\ \frac{\Delta V_2}{V_2}\\ \frac{\Delta V_3}{V_3}\\ \frac{\Delta V_4}{V_4}\end{bmatrix}$$

Evaluation of N-R method

- Advantages:
  - fast (quadratic) convergence
  - no. of iterations - independent on network size (approx. 3-7 iterations are needed)
  - time requirements - low (even for larger networks), are increasing only linearly
- Disadvantages:
  - complex mathematical model (matrices, derivatives, etc.)
  - number of iterations can be increased when including reactive power limits in PU buses and OLTC transformers
  - strong dependence on initial (starting) conditions
  - strong uncertainty of N-R method during updating process
  - problems with sparsely connected and ill-conditioned networks

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Load flow analysis

7. Inclusion of reactive power limits in PU buses

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Inclusion of *Q* limits in PU buses

- Generated reactive power must be within its limits
  $$Q_{Gi\,\min} \le Q_{Gi} \le Q_{Gi\,\max}$$
- Limits can be positive and negative
- Voltage regulator maintains voltage magnitude in PU bus at preset value by moving reactive power within its limits
- In critical situations (loss of regulation) it switches the PU bus to PQ bus, where reactive power value is constant ($Q_{Gi\,\min}$ or $Q_{Gi\,\max}$) and new voltage magnitude must be found.
- In load flow analysis, it is the PU-PQ switching, but it is not clear if it is one-way only and definitive
- Different PU-PQ switching logics for G-S and N-R method

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Inclusion of *Q* limits in PU buses

- Bus-Type Switching Logics
- A) Standard PU-PQ logic

  - **iteratively** compares reactive power in all PU network buses

  - when exceeding the Q limit, it switches PU bus to PQ bus, sets reactive power on exceeded Q limit and releases the voltage magnitude for new calculation

$$\begin{aligned} &\textit{if}\ \ Q_{Gi} > Q_{Gi\,\max} \\ &\quad Q_{Gi} = Q_{Gi\,\max};\ PU \rightarrow PQ; \\ &\textit{else if}\ \ Q_{Gi} < Q_{Gi\,\min} \\ &\quad Q_{Gi} = Q_{Gi\,\min};\ PU \rightarrow PQ; \\ &\textit{end} \end{aligned}$$

Evaluation: Gives unreliable results in some LF calculations

Switches too many PU buses to PQ buses

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Inclusion of *Q* limits in PU buses

- B) Enhanced PU-PQ logic
  - it works only in convergence area, not iteratively
  - it gathers the distances of reactive powers from both limits
  - then it permanently switches the (one) PU bus which exceeds its limit at most

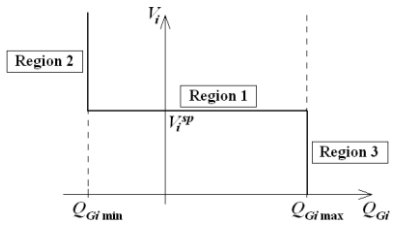
$$\begin{array}{l} \textit{if } Q_{Gi} > Q_{Gi\max} \\ \quad M_i = Q_{Gi} - Q_{Gi\max}; \\ \textit{else if } Q_{Gi} < Q_{Gi\min} \\ \quad M_i = Q_{Gi\min} - Q_{Gi}; \\ \textit{end} \end{array} \qquad \begin{array}{l} \textit{if } M_i == M_{\max} \\ \quad \textit{if } Q_{Gi} > Q_{Gi\max} \quad Q_{Gi} = Q_{Gi\max}; PU \to PQ; \\ \quad \textit{else if } Q_{Gi} < Q_{Gi\min} \quad Q_{Gi} = Q_{Gi\min}; PU \to PQ; \\ \quad \textit{end} \\ \textit{end} \end{array}$$

- Evaluation: Highly reliable and flexible for G and G-S methods, careful PU-PQ switching, risk of significant increase of number of iterations for networks with high number of PU buses.

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Inclusion of *Q* limits in PU buses

- C) Robust PU-PQ and PQ-PU logic
  - it works in each iteration, 2 steps: PU-PQ, PQ-PU
  - first step is standard PU-PQ logic (logic A)
  - second step – use of strong Q-U dependence



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Inclusion of *Q* limits in PU buses

- C) Robust PU-PQ and PQ-PU logic – cont.

$$\begin{array}{l} \textit{if } type1 == PQ \ \& \ type0 == PU \\ \quad \textit{if } \left(Q_{Gi} == Q_{Gi\max} \,\&\, U_i > U_i^{sp}\right) \text{or} \left(Q_{Gi} == Q_{Gi\min} \,\&\, U_i < U_i^{sp}\right) \\ \quad\quad U_i = U_i^{sp}; PQ \to PU; \\ \quad \textit{end} \\ \textit{end} \end{array}$$

Note: type1 – bus type in current iteration
type0 – bus type at the start of numerical calculation

Evaluation: flexible, reliable, fast for N-R method
only slight increase of iteration numbers needed
minimum impact on N-R method's performance
is applied in commercial software PowerWorld
danger in multiple oscillations between PU and PQ

Load flow analysis

8. Other calculations in electric power systems

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Bus outputs of LF analysis

- voltage magnitudes (detection of under-/over-voltages)
- voltage phase displacements (static stability)
- reactive power injections in PU buses (PU, PU --> PQ)
- power injections in the slack bus

$$\overline{S}_{ref} = \overline{U}_{ref} \sum_{i=1}^{n} \overline{A}^{*}_{ref\,i} \overline{U}^{*}_{i} \rightarrow P_{ref} = \mathrm{Re}\{\overline{S}_{ref}\}, Q_{ref} = \mathrm{Im}\{\overline{S}_{ref}\}$$

- power injections by shunt compensators / upstream network

$$P_{sh_i} = -G_{sh_i} U_i^2$$

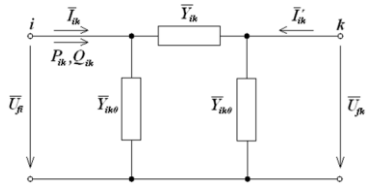
$$Q_{sh_i} = +B_{sh_i} U_i^2$$

- current injections $\overline{I}_i = \dfrac{P_i - jQ_i}{\sqrt{3}\overline{U}_i^{*}}$

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Line outputs of LF analysis

- active/reactive power flows (for power lines)



$$P_{ik} = (G_{ik} + G_{ik0}) U_i^2 - G_{ik} U_i U_k \cos(\theta_i - \theta_k) - B_{ik} U_i U_k \sin(\theta_i - \theta_k)$$

$$Q_{ik} = -(B_{ik} + B_{ik0}) U_i^2 + B_{ik} U_i U_k \cos(\theta_i - \theta_k) - G_{ik} U_i U_k \sin(\theta_i - \theta_k)$$

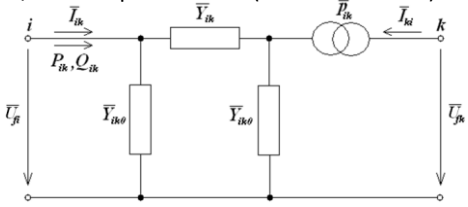
$$P_{ki} = (G_{ik} + G_{ik0}) U_k^2 - G_{ik} U_i U_k \cos(\theta_k - \theta_i) - B_{ik} U_i U_k \sin(\theta_k - \theta_i)$$

$$Q_{ki} = -(B_{ik} + B_{ik0}) U_k^2 + B_{ik} U_i U_k \cos(\theta_k - \theta_i) - G_{ik} U_i U_k \sin(\theta_k - \theta_i)$$

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Line outputs of LF analysis

- active/reactive power flows (for transformers)



$$P_{ik} = (G_{ik} + G_{ik0})U_i^2 - U_i U_k |\overline{p}_{ik}| [G_{ik}\cos(\theta_i - \theta_k - \alpha_{ik}) + B_{ik}\sin(\theta_i - \theta_k - \alpha_{ik})]$$

$$Q_{ik} = -(B_{ik} + B_{ik0})U_i^2 + U_i U_k |\overline{p}_{ik}| [B_{ik}\cos(\theta_i - \theta_k - \alpha_{ik}) - G_{ik}\sin(\theta_i - \theta_k - \alpha_{ik})]$$

$$P_{ki} = (G_{ik} + G_{ik0})U_k^2 |\overline{p}_{ik}|^2 - U_i U_k |\overline{p}_{ik}| [G_{ik}\cos(\theta_k + \alpha_{ik} - \theta_i) + B_{ik}\sin(\theta_k + \alpha_{ik} - \theta_i)]$$

$$Q_{ki} = -(B_{ik} + B_{ik0})U_k^2 |\overline{p}_{ik}|^2 + U_i U_k |\overline{p}_{ik}| [B_{ik}\cos(\theta_k + \alpha_{ik} - \theta_i) - G_{ik}\sin(\theta_k + \alpha_{ik} - \theta_i)]$$

Line outputs of LF analysis

- power losses (for power lines)

$$\Delta P_{ik} = P_{ik} + P_{ki} = (G_{ik} + G_{ik0})(U_i^2 + U_k^2) - 2G_{ik}U_i U_k \cos(\theta_i - \theta_k)$$

$$\Delta Q_{ik} = Q_{ik} + Q_{ki} = -(B_{ik} + B_{ik0})(U_i^2 + U_k^2) + 2B_{ik}U_i U_k \cos(\theta_i - \theta_k)$$

- no-load active power losses (for transformers)

$$\Delta P_{ik0} = G_{ik0}(U_i^2 + |p_{ik}U_k|^2)$$

- line currents, line/transformer loadings, power factors

$$I_{ik} = \frac{\sqrt{P_{ik}^2 + Q_{ik}^2}}{\sqrt{3}U_i} \quad I_{ki} = \frac{\sqrt{P_{ki}^2 + Q_{ki}^2}}{\sqrt{3}U_k} \quad \eta = \frac{\sqrt{P_{ik}^2 + Q_{ik}^2}}{S_{MVA}} \cdot 100 \quad \eta = \sqrt{\frac{\Delta P_{ik}}{\Delta P_{Cu}}} \cdot 100$$

$$\cos\varphi_{ik} = \cos\left(\arctan\left(\frac{Q_{ik}}{P_{ik}}\right)\right) \quad \cos\varphi_{ki} = \cos\left(\arctan\left(\frac{Q_{ki}}{P_{ki}}\right)\right)$$

- total active/reactive power losses

$$\Delta P = \sum_{i=1}^{n}(P_{Gi} + P_{Li} + P_{Ci}) \quad \Delta Q = \sum_{i=1}^{n}(Q_{Gi} + Q_{Li} + Q_{Ci}) \quad \Delta P = \sum_{i=1}^{n}\sum_{k=i+1}^{n}\Delta P_{ik} \quad \Delta Q = \sum_{i=1}^{n}\sum_{k=i+1}^{n}\Delta Q_{ik}$$

Next time

- **Failure states in electrical networks**
  - Decomposition of asymmetric system into symmetrical components (method of Fortescue)
- **Short-circuits**
  - Calculation of short-circuit currents (initial short-circuit current I_k'', 3ph, 2ph, 1ph short-circuit currents)

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