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THEORY OF TRANSMISSION AND DISTRIBUTION OF ELECTRIC ENERGY

6th lecture - theory

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Content

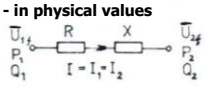
- **Solution of voltage, current and power output conditions in the electrical network**
  - Calculation of voltage drop on line (DC, AC-1ph and 3ph)
    - Procedure for line:
      - open (= powered from one side) with one consumption
      - open with multiple consumptions
      - powered from two sides
    - Examples (in a separate document)

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General calculation of ΔU on line, phasor diagrams

- **Voltage drop** $\Delta U = |\bar{U}_1| - |\bar{U}_2|$
  - When calculating ΔU in low voltage (**LV**) and middle (in CZ so called high) voltage **MV** distribution AC networks **we can neglect the transverse admittance**, i.e. we consider only R and L (X)
  - For DC networks, house distribution etc. - we neglect L
- **Substitute schema:**

- in physical values



- in relative values

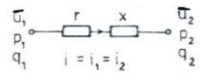
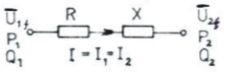


Fig. 6-1:

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General calculation of ΔU on the alternating line, phasor diagrams

- for inductive consumption (prevailing in the distribution system)



$$\Delta U_f = |\bar{U}_{1f}| - |\bar{U}_{2f}| = \underbrace{R \cdot I \cdot \cos\varphi}_{a} + \underbrace{X \cdot I \cdot \sin\varphi}_{b} + \underbrace{\frac{(X \cdot I \cdot \cos\varphi - R \cdot I \cdot \sin\varphi)^2}{2 \cdot U_f}}_{c}$$

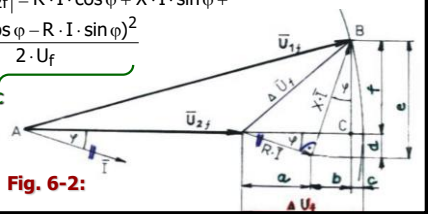
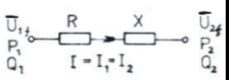


Fig. 6-2:

General calculation of ΔU on the alternating line, phasor diagrams

- for capacitive consumption



$$\Delta U_f = \underbrace{R \cdot I \cdot \cos\varphi}_{a} - \underbrace{X \cdot I \cdot \sin\varphi}_{b} + \underbrace{\frac{(X \cdot I \cdot \cos\varphi + R \cdot I \cdot \sin\varphi)^2}{2 \cdot U_f}}_{c}$$

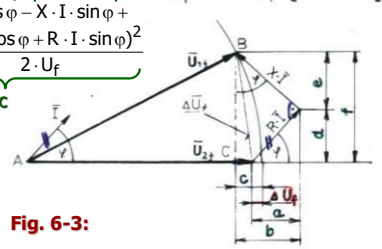


Fig. 6-3:

General calculation of ΔU on the alternating line, phasor diagrams

Note.: The relation for **c** is derived from **ΔABC**

BC ≙ f, AC, AB

$$(X \cdot I \cdot \cos\varphi \mp R \cdot I \cdot \sin\varphi)^2 + (U_{1f} - c)^2 = U_{1f}^2$$

$$(X \cdot I \cdot \cos\varphi \mp R \cdot I \cdot \sin\varphi)^2 = 2 \cdot U_{1f} \cdot c - c^2 = c \cdot (2 \cdot U_{1f} - c)$$

e, d

f = e - d

$c << U_{1f} \rightarrow$ we can neglect

$U_{1f} \sim U_f$

Fact: – for inductive character
+ for capacity character

- for inductive consumption

$$c = \frac{(X \cdot I \cdot \cos\varphi \mp R \cdot I \cdot \sin\varphi)^2}{2 \cdot U_f}$$

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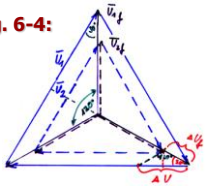
General calculation of ΔU on the alternating line, phasor diagrams

- **Associated value of voltage drop**

$$\Delta U = \sqrt{3} \cdot \Delta U_f$$

- representation of the relationship between the drop of the associated a phase voltage:

Fig. 6-4:



$$\frac{\frac{\Delta U}{2}}{\Delta U_f} = \cos 30° = \frac{\sqrt{3}}{2}$$

$$\frac{\Delta U}{2} = \frac{\sqrt{3}}{2} \Delta U_f$$

$$\Delta U = \sqrt{3} \cdot \Delta U_f$$

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Calculation of ΔU for open line with 1 consumption at the end (1)

(open line = powered from one side)

1) Line DC

- voltage drop is given by **Ohm's law**

$$\Delta U = R \cdot I = \rho \cdot \frac{2 \cdot l}{S} \cdot I \quad [V]$$

- where: l [m] … distance from supply point (2l - respects the return line)
  ρ [μΩm]↔[Ωmm²/m] … conductor resistivity
  S [mm²] … conductor cross section

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Calculation of ΔU for open line with 1 consumption at the end (2)

- percentage voltage drop (related to line voltage level = = voltage at the beginning)

$$\Delta u\% = \frac{\Delta U}{U} \cdot 100 = \frac{200 \cdot \rho \cdot l \cdot I}{S \cdot U}$$

- if the consumption is specified by the power outputof the appliance

$$P = U \cdot I \Rightarrow \Delta u\% = \frac{200 \cdot \rho \cdot l \cdot P}{S \cdot U^2}$$

- in calculations - can be used tab. 6-1 (from the Standard ČSN 341610)
  - gives the so-called equivalent resistance R_K for one km of conductor corresponding to different line cross-sections

$$\Delta u\% = \frac{200 \cdot R_K \cdot l \cdot P}{U^2}$$

where R_K [Ω/km] ⇒ l [km]

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Table for determining R_K and X_K of line

Cross-section mm²	Resistance at temperature 60 °C R_K [Ω/km]			Inductive reactance X_K [Ω/km]					
				Cable (Cu i Al)		Overhead line*)			
						Cu		AlFe 6	
	Cu	Al	AlFe 6	with metal sheath	without metal sheath	d = 40 cm	d = 50 cm	d = 40 cm	d = 50 cm
1,5	14,5	–	–	–	–	–	–	–	–
2,5	8,7	14,3	–	–	–	–	–	–	–
4	5,43	8,97	–	–	–	–	–	–	–
6	3,62	5,98	–	0,099 až 0,107	0,088 až 0,094	–	–	–	–
10	2,17	3,59	–	0,094 až 0,102	0,083 až 0,089	0,346	0,360	–	–
16	1,3	2,24	2,180	0,089 až 0,097	0,079 až 0,085	0,338	0,352	0,327	0,341
25	0,87	1,43	1,390	0,085 až 0,093	0,075 až 0,081	0,325	0,339	0,313	0,327
35	0,62	1,02	0,970	0,083 až 0,091	0,073 až 0,079	0,311	0,325	0,302	0,316
50	0,434	0,718	0,690	0,081 až 0,089	0,071 až 0,077	0,300	0,314	0,291	0,305
70	0,31	0,513	0,501	0,079 až 0,087	0,069 až 0,075	0,287	0,301	0,279	0,293
95	0,229	0,378	0,369	0,078 až 0,086	0,068 až 0,074	0,279	0,293	0,270	0,284
120	0,181	0,299	0,271	0,078 až 0,086	0,066 až 0,072	0,271	0,285	0,260	0,274
150	0,145	0,239	0,223	0,077 až 0,085	0,066 až 0,072	0,264	0,277	0,254	0,268
185	0,117	0,194	0,180	0,077 až 0,085	0,065 až 0,071	0,257	0,271	0,247	0,261
240	0,09	0,149	–	0,077 až 0,085	0,065 až 0,071	–	–	–	–

*) **d** is the distance between the conductors

Tab. 6.1:

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Calculation of ΔU for open line with 1 consumption at the end (3)

2) AC line, 1phase

- voltage drop $\Delta U = |\underline{U}_1| - |\underline{U}_2|$ (voltage difference at points 1, 2) → depends on the:
  - active resistance R
  - inductive reactance X
  - power factor cosφ
  - flowing current I

  } see general calculation

  - taking:
    - we consider only inductive load (it prevails; for capacitive → sign exchange)
    - at cosφ > 0,5 (i.e. φ < 60°) we can neglect the part marked c

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Calculation of ΔU for open line with 1 consumption at the end (4)

2) AC line, 1ph – continuation

$$\Delta U = R \cdot I \cdot \cos\varphi + X \cdot I \cdot \sin\varphi$$

- sub. for $R = 2 \cdot R_K \cdot l \quad X = 2 \cdot X_K \cdot l$ (see tab. 6.1)

$$I = \frac{P}{U \cdot \cos\varphi}$$

$$\Delta U = 2 \cdot R_K \cdot l \cdot \frac{P}{U \cdot \cos\varphi} \cdot \cos\varphi + 2 \cdot X_K \cdot l \cdot \frac{P}{U \cdot \cos\varphi} \cdot \sin\varphi$$

$$\Delta U = \frac{2 \cdot P \cdot l}{U} \cdot (R_K + X_K \cdot tg\varphi)$$

- where: R_K, X_K [Ω/km] ⇒ l [km]

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Calculation of ΔU for open line with 1 consumption at the end (5)

- percentage drop

$$\Delta u\% = \frac{\Delta U}{U} \cdot 100 = \frac{200 \cdot P \cdot l}{U^2} \cdot (R_K + X_K \cdot tg\varphi)$$

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Calculation of ΔU for open line with 1 consumption at the end (6)

3) AC line, 3 phase

- same relation for ΔU in phase of 3ph line as for 1ph line

$$\Delta U_f = R \cdot I \cdot \cos\varphi + X \cdot I \cdot \sin\varphi$$

- but sub.: $I = \frac{P}{3 \cdot U_f \cdot \cos\varphi} = \frac{P}{\sqrt{3} \cdot U \cdot \cos\varphi}$

$R = R_K \cdot l \qquad X = X_K \cdot l$

$$\Delta U_f = \frac{R_K \cdot l \cdot P}{\sqrt{3} \cdot U} + \frac{X_K \cdot l \cdot P}{\sqrt{3} \cdot U} \cdot tg\varphi$$

... values for simple length

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Calculation of ΔU for open line with 1 consumption at the end (7)

- associated (line) voltage drop (see Fig. 6-4)

$$\Delta U = \sqrt{3} \cdot \Delta U_f$$

$$\Delta U = \frac{R_K \cdot I \cdot P}{U} + \frac{X_K \cdot I \cdot P}{U} \cdot tg\varphi$$

- percentage drop

$$\Delta u\% = \frac{\Delta U}{U} \cdot 100 = \frac{100 \cdot P \cdot l}{U^2} \cdot (R_K + X_K \cdot tg\varphi)$$

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Line dimensioning according to permissible voltage drop (1)

- The conductors of the individual parts of the distribution system must be dimensioned in such a way that the load **does not cause unauthorized voltage drop** at the terminals of the appliances → it will adversely affect the functioning of the appliances

! Check of voltage drop is important mainly in low voltage wires !

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Line dimensioning according to permissible voltage drop (2)

- Specific requirements for the size of permissible voltage drops - in various technical Standards

⇓

- E.g. for distribution in industrial plants :

ČSN 34 16 10

⇓

- For most appliances (motor and resistive)
  - long-term voltage fluctuations ± 5 % (for lighting 3 %)
  - short term to 10 %

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Line dimensioning according to permissible voltage drop (3)

- for a given voltage drop ΔU or $\Delta u\%$ we calculate the required line cross-section
  - **DC:** $S = \frac{2 \cdot \rho \cdot l \cdot I}{\Delta U} = \frac{200 \cdot \rho \cdot l \cdot I}{\Delta u\% \cdot U} = \frac{200 \cdot \rho \cdot l \cdot P}{\Delta u\% \cdot U^2}$
  - **AC, 1ph:** $R_K + X_K \cdot tg\varphi = \frac{\Delta U \cdot U}{2 \cdot P \cdot l} = \frac{\Delta u\% \cdot U^2}{200 \cdot P \cdot l}$
  - **AC, 3ph:** $R_K + X_K \cdot tg\varphi = \frac{\Delta U \cdot U}{P \cdot l} = \frac{\Delta u\% \cdot U^2}{100 \cdot P \cdot l}$
    - Based on the estimated R_K, X_K **we determine S** and **check it**

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Line dimensioning according to permissible voltage drop (4)

Note:

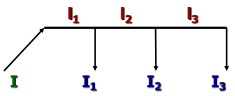
- According to the Standard for dimensioning the line according to voltage drop, it is necessary to count with :
  - **specific resistivity at 60°C** (ρ_{Cu} = 0,0217 $\mu\Omega$m, ρ_{Al} = 0,0359 $\mu\Omega$m)
  - **the smallest electrical cross-section**, which is 95 % of the nominal cross-section (e.g. for ropes AlFe = sum of S of all Al wires)

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Open line (= power supply from 1 side) with several elektricity consumption

- We use the same relationships as for open line with 1 consumption at the end, only instead of
  - **Pl =>∑Pl**
  - **Il =>∑Il**

Fig. 6-5:



- **∑Il =$I_1 l_1$+ I_2 (l_1+ l_2) + I_3 (l_1+ l_2+ l_3)**
- **∑Il =l_1 (I_1+ I_2+ I_3) + l_2 (I_2+ I_3) + l_3 I_3**

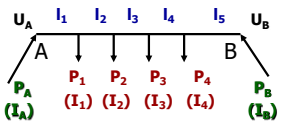
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Calculation of ΔU - line powered from 2 sides (1)

- Calculation procedure:
  - **We assume $U_A = U_B$**

1) We calculate the size of the supply power output (current) from balance condition of power output (current) moments to the opposite supply point.

Fig. 6-9:



2) We determine the **X** point of the maximum voltage drop, i.e. the point to which the sum of power output (current) moments is greatest. At this point the power outputs (currents) from both sides add up.

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Calculation of ΔU - line powered from 2 sides (2)

- Calculation procedure:
  - **$U_A = U_B$**

3) For calculating the ΔU or cross-section dimensioning we divide the line at the point of greatest voltage drop and then we count these parts as a line powered from one side. The results for the ΔU from both sides must be the same.

  - **If $U_A \neq U_B$, then**

4) We calculate the equalizing current

$$I_0 = \frac{U_A - U_B}{R_V}$$

R_V... resulting resistance

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Calculation of ΔU - line powered from 2 sides (3)

- Calculation procedure:
  - **$U_A \neq U_B$**

5) We determine new supply currents (outputs)

$$I_A' = I_A + I_0$$
$$I_B' = I_B - I_0$$

=> X point of the largest voltage drop shifts

Next ad 3), only the sums of current moments and ΔU from both sides will not be the same, but it is valid that:

$\Delta U_A - \Delta U_B = U_A - U_B$ (because there is only one voltage at X point)

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Next time

- **Solution of voltage, current and power output conditions in the electrical network**
  - Solution of simple network by the method of the schema gradual simplification

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