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THEORY OF TRANSMISSION AND DISTRIBUTION OF ELECTRIC ENERGY

5th lecture
(Theory and examples)

Topic - Active Parameters of Overhead Lines

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Content

- **Solutions of voltage, current and power output conditions in the electric network**
  - Voltage and current conditions on power line
    - Possibility of some passive parameter neglecting
    - Solution with using replacement two-ports (Π, T, Γ– cells)
    - Example
  - Determination power output flows in branch, branch losses
    - Derivation of relation
    - Example

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Solutions of voltage, current and power conditions in the electrical network

- **Calculation goal** = determination of active parametrs, i.e. $\bar{U}$, $\bar{I}$ or P, Q in nodes and branches
- **Premise for calculation** – knowledge of substitude schema of elctrical network
- We choose the procedure for solving according to the complexity of the network:
  - simple networks (neglected transverse paramters) → method of gradual simplification of schema (without using PCs, usually for LV or HV networks)
  - complex networks → method Kirhoff´s (node) voltages

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Voltage and current conditions on power line (1)

- **Assumption:**
  - the line is symmetrical or transposed
  - the passive parameters are constant
  - voltages and currents are sinusoidal

- Current and voltage expansion along lines can be described by so-called **telegraph equations:**

$$-\frac{\partial u}{\partial x} = Ri + L\frac{\partial i}{\partial t}$$
$$-\frac{\partial i}{\partial x} = Gu + C\frac{\partial u}{\partial t}$$

  - where: $u(x, t) = U(x)\, e^{j\omega t}$, $i(x, t) = I(x)\, e^{j\omega t}$

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Voltage and current conditions on power line (2)

- The solution of telegraph equations are **wave equations in the form:**

$$\bar{U}_{f1} = \bar{U}_{f2} \cdot \cosh \bar{\gamma} l + \bar{Z}_v \bar{I}_2 \cdot \sinh \bar{\gamma} l$$
$$\bar{I}_1 = \frac{\bar{U}_{f2}}{\bar{Z}_v} \cdot \sinh \bar{\gamma} l + \bar{I}_2 \cdot \cosh \bar{\gamma} l$$

where:

$\bar{U}_{f1}, \bar{I}_1$ ... phasors at the beginning of the line

$\bar{U}_{f2}, \bar{I}_2$ ... phasors at the end of the line

$\bar{Z}_v$ ... wave impedance $\quad \bar{Z}_v = \sqrt{\frac{R + jX}{G + jB}} = \sqrt{\frac{\bar{Z}}{\bar{Y}}}$

$\bar{\gamma}$ ... spreading factor

$$\bar{\gamma} = \alpha + j\beta = \sqrt{(R + jX) \cdot (G + jB)} = \sqrt{\bar{Z} \cdot \bar{Y}}$$

α ... damping factor

β ... phase rotation factor

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Voltage and current conditions on power line (3)

- Wave equations can be further solved in a simplified way:

a/ using the first two members of the **development of hyperbolic functions in a row**

$$\bar{U}_{f1} = \bar{U}_{f2} \cdot (1 + \frac{\bar{Z}\bar{Y}l^2}{2}) + \bar{I}_2 \cdot (1 + \frac{\bar{Z}\bar{Y}l^2}{6}) \cdot \bar{Z}l$$

$$\bar{I}_1 = \bar{U}_{f2} \cdot (1 + \frac{\bar{Z}\bar{Y}l^2}{6}) \cdot \bar{Y}l + \bar{I}_2 \cdot (1 + \frac{\bar{Z}\bar{Y}l^2}{2})$$

- Note:

$$\cosh\alpha = 1 + \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} + \cdots$$

$$\sinh\alpha = \frac{\alpha}{1!} + \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} + \cdots$$

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Voltage and current conditions on power line (4)

- Wave equations can be further solved in a simplified way:

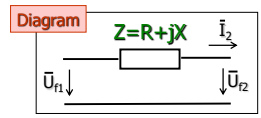
b/ by some **parametr neglecting**

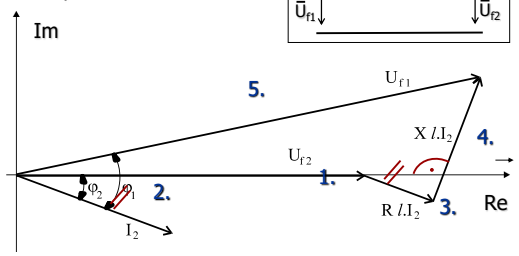
Voltage level	Can be omission	Cogitate
400, 220 kV	R, G	X, B
110 kV	G	R, X, B
vn	G, B ($\bar{Y}$)	R, X ($\bar{Z}$)
nn	G, B, X	R

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Voltage and current conditions on power line – transmission HV line (5)

- Phasor diagram, if **G = 0, B = 0**





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Voltage and current conditions on power line (6)

- Wave equations can be further solved in a simplified way:

c/ by using **replacement two-ports** (Π, T, Γ), to which these **matrix equations** apply:

$$\begin{bmatrix} \bar{U}_{f1} \\ \bar{I}_1 \end{bmatrix} = \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix} \cdot \begin{bmatrix} \bar{U}_{f2} \\ \bar{I}_2 \end{bmatrix},$$

where $\bar{A}, \bar{B}, \bar{C}, \bar{D}$ are tzv. **Blondel´s konstants**,

$$\bar{A}\bar{D} - \bar{B}\bar{C} = 1 \qquad \bar{A} = \bar{D}$$

Written:

$$\bar{U}_{f1} = \bar{A} \cdot \bar{U}_{f2} + \bar{B} \cdot \bar{I}_2$$

$$\bar{I}_1 = \bar{C} \cdot \bar{U}_{f2} + \bar{D} \cdot \bar{I}_2$$

Only for symmetric cells

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Voltage and current conditions on power line - replacement two-ports (7)

- Π cell

schema

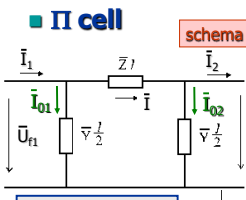
$\bar{A}$ [-] $\quad \bar{B}$ [Ω]

$$\bar{U}_{f1} = \bar{U}_{f2}\left(1 + \frac{\bar{Z}\bar{Y}l^2}{2}\right) + \bar{Z}l \cdot \bar{I}_2$$

$\bar{D}$ [-]

$$\bar{I}_1 = \bar{U}_{f2}\left(1 + \frac{\bar{Z}\bar{Y}l^2}{4}\right) \cdot \bar{Y}l + \bar{I}_2\left(1 + \frac{\bar{Z}\bar{Y}l^2}{2}\right)$$

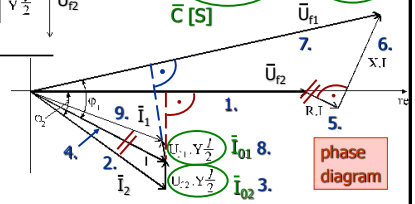
$\bar{C}$ [S]



$$\bar{U}_{f2} = \frac{U_2}{\sqrt{3}} \quad (\upsilon_2 = 0)$$

$$\bar{I}_{02} = \bar{U}_{f2} \cdot \frac{\bar{Y}l}{2} \perp \bar{U}_{f2}$$

$$\bar{I}_{01} = \bar{U}_{f1} \cdot \frac{\bar{Y}l}{2} \perp \bar{U}_{f1}$$



phase diagram

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Voltage and current conditions on power line - replacement two-ports (8)

- T cell

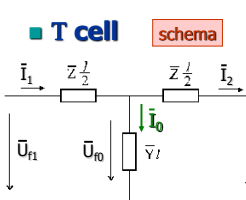
schema

$\bar{A}$ [-] $\quad \bar{B}$ [Ω]

$$\bar{U}_{f1} = \bar{U}_{f2}\left(1 + \frac{\bar{Z}\bar{Y}l^2}{2}\right) + \bar{I}_2\left(1 + \frac{\bar{Z}\bar{Y}l^2}{4}\right) \cdot \bar{Z}l$$

$$\bar{I}_1 = \bar{U}_{f2} \cdot \bar{Y}l + \bar{I}_2\left(1 + \frac{\bar{Z}\bar{Y}l^2}{2}\right)$$

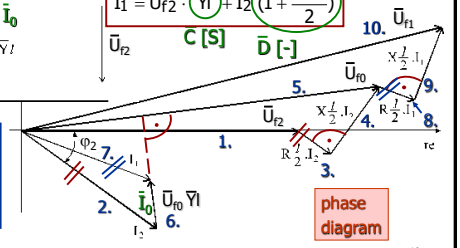
$\bar{C}$ [S] $\quad \bar{D}$ [-]



$$\bar{U}_{f2} = \frac{U_2}{\sqrt{3}}$$

$$(\upsilon_2 = 0)$$

$$\bar{I}_0 = \bar{U}_{f0} \cdot \bar{Y}l \perp \bar{U}_{f0}$$



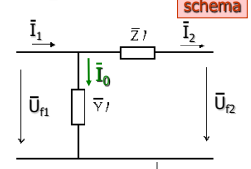
phase diagram

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Voltage and current conditions on power line - replacement two-ports (9)

- Γ cell

schema



$\bar{A} = 1$ [-] $\quad \bar{B}$ [Ω] $\quad \bar{D}$ [-] $\quad \bar{C}$ [S]

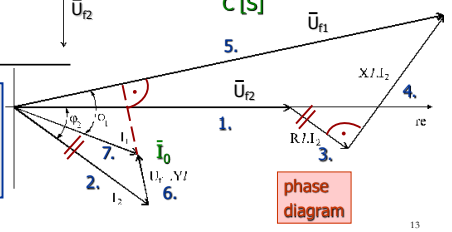
$\bar{U}_{f1} = \bar{U}_{f2} + \bar{I}_2 \cdot \bar{Z}l$

$\bar{I}_1 = \bar{U}_{f2} \cdot \bar{Y}l + \bar{I}_2 (1 + \bar{Z}\bar{Y}l^2)$

$\bar{U}_{f2} = \frac{U_2}{\sqrt{3}}$

$(\upsilon_2 = 0)$

$\bar{I}_0 = \bar{U}_{f1} \cdot \bar{Y}l \perp \bar{U}_{f1}$

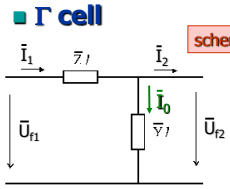


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Voltage and current conditions on power line - replacement two-ports (10)

- Γ cell

schema



$\bar{A}$ [-] $\quad \bar{B}$ [Ω] $\quad \bar{D} = 1$ [-] $\quad \bar{C}$ [S]

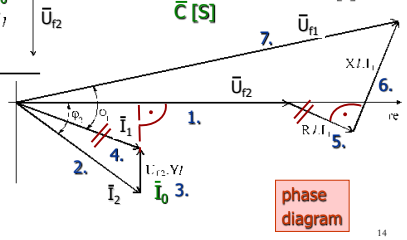
$\bar{U}_{f1} = \bar{U}_{f2}(1 + \bar{Z}\bar{Y}l^2) + \bar{I}_2 \cdot \bar{Z}l$

$\bar{I}_1 = \bar{U}_{f2} \cdot \bar{Y}l + \bar{I}_2$

$\bar{U}_{f2} = \frac{U_2}{\sqrt{3}}$

$(\upsilon_2 = 0)$

$\bar{I}_0 = \bar{U}_{f2} \cdot \bar{Y}l \perp \bar{U}_{f2}$



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Voltage and current conditions on power line - replacement two-ports (11)

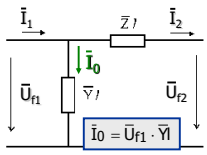
- Γ cell

$\bar{U}_{f1} = \bar{U}_{f2} + \bar{Z}l \cdot \bar{I}_2$

$\bar{I}_1 = \bar{I}_0 + \bar{I}_2 = \bar{Y}l \cdot \bar{U}_{f1} + \bar{I}_2$

$= (\bar{U}_{f2} + \bar{Z}l \cdot \bar{I}_2)\bar{Y}l + \bar{I}_2) =$

$= \bar{Y}l \cdot \bar{U}_{f2} + \bar{I}_2(1 + \bar{Z}\bar{Y}l^2)$



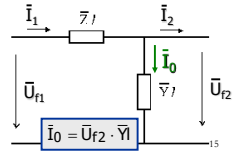
$\bar{I}_0 = \bar{U}_{f1} \cdot \bar{Y}l$

$\bar{U}_{f1} = \bar{U}_{f2} + \bar{Z}l \cdot \bar{I}_1$

$\bar{I}_1 = \bar{I}_0 + \bar{I}_2 = \bar{Y}l \cdot \bar{U}_{f2} + \bar{I}_2$

$\bar{U}_{f1} = \bar{U}_{f2} + \bar{Z}l \cdot (\bar{Y}l \cdot \bar{U}_{f2} + \bar{I}_2) =$

$= \bar{U}_{f2}(1 + \bar{Z}\bar{Y}l^2) + \bar{Z}l \cdot \bar{I}_2$



$\bar{I}_0 = \bar{U}_{f2} \cdot \bar{Y}l$

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Voltage and currect conditions on line – example 5-2 (1)

- **Task:**
  - Calculate volatage and current at the beginning of transmisson line and draw phase diagram (for check your results), if you know conditions at the end of line.

Transmission line 110 kV:
R = 0,096 Ω/km, X = 0,39 Ω/km, B = 2,92 . 10^{-6} S/km
l = 100 km, P_2 = 50 MW, Q_2 = 10 MVAr

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Voltage and currect conditions on line – example 5-2 (2)

- **Solution Π-cell**
  - It is necessary to calculate in phase values
  - We put phasor U_2 into real axis

$$U_{2f} = \frac{U_2}{\sqrt{3}} = \frac{110}{\sqrt{3}} = 63{,}5\ kV$$

$$\bar{Z} = 0{,}096 + j0{,}39\ \Omega/km \qquad \bar{Y} = j2{,}92 \cdot 10^{-6}\ S/km$$

$$\bar{I}_2 = \frac{P_2 - jQ_2}{3 \cdot U_{2f}} = \frac{(50 - j10) \cdot 10^3}{\sqrt{3} \cdot 110} = 262{,}42 - j52{,}49 =$$
$$= 267{,}63\angle -11{,}3°\ A$$

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Voltage and currect conditions on line – example 5-2 (3)

Blondel's constants:

$$\bar{D} = \bar{A} = 1 + \bar{Y} \cdot \bar{Z} \cdot \frac{l^2}{2} = 1 + j2{,}92 \cdot 10^{-6} \cdot (0{,}096 + j0{,}39) \cdot \frac{100^2}{2} = 1 +$$
$$+ (j0{,}28032 - 1{,}1388) \cdot \frac{10^{-2}}{2} = 1 + j0{,}14016 \cdot 10^{-2} - 0{,}5694 \cdot 10^{-2} =$$
$$0{,}994306 + j0{,}0014 \doteq 0{,}994306\ [-]$$

$$\bar{B} = \bar{Z} \cdot l = 9{,}6 + j39 = 40{,}16\angle 76{,}17°\Omega$$

$$\bar{C} = \bar{Y} \cdot l \cdot \left(1 + \bar{Y} \cdot \bar{Z} \cdot \frac{l^2}{4}\right) =$$

$\bar{Z} = 0{,}096 + j0{,}39\ \Omega/km$
$\bar{Y} = j2{,}92 \cdot 10^{-6}\ S/km$
$l = 100\ km$

$$= j2{,}92 \cdot 10^{-6} \cdot 100 \cdot \left(1 + j2{,}92 \cdot 10^{-6} \cdot (0{,}096 + j0{,}39) \cdot \frac{100^2}{4}\right) =$$
$$= j2{,}92 \cdot 10^{-4} \cdot \left(1 + j0{,}73 \cdot 10^{-2} \cdot (0{,}096 + j0{,}39)\right) =$$
$$= j2{,}92 \cdot 10^{-4} \cdot (1 + j0{,}07008 \cdot 10^{-2} - 0{,}2847 \cdot 10^{-2}) =$$
$$= j2{,}92 \cdot 10^{-4} - j0{,}831324 \cdot 10^{-6} - 0{,}2046336 \cdot 10^{-6} =$$
$$= -0{,}2046336 \cdot 10^{-6} + j291{,}17 \cdot 10^{-6} = 291\angle 90° \cdot 10^{-6}\ S$$

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Voltage and current conditions on line – example 5-2 (4)

Input voltage and current: $U_{2f} = \frac{110}{\sqrt{3}} = 63{,}5\ kV$

$\bar{U}_{1f} = \bar{A} \cdot \bar{U}_{2f} + \bar{B} \cdot \bar{I}_2$

$= 0{,}9943 \cdot 63{,}5 \cdot 10^3 + 40{,}16\angle 76 \cdot 267{,}693\angle -11{,}3 =$

$= 63146{,}53 + 10748{,}02\angle 65 =$

$= 63146{,}53 + 10748{,}02 \cdot (\cos 65 + j \sin 65) =$

$= 63146{,}53 + 4542{,}3 + j9741{,}01 = 67689 + j9741 = 68386\angle 8{,}1°\ V$

$\bar{U}_1 = 118448\ V$

$\bar{I}_2 = \frac{(50 - j10) \cdot 10^3}{\sqrt{3} \cdot 110} = 267{,}63\angle -11{,}3°\ A$

$\bar{I}_1 = \bar{C} \cdot \bar{U}_{2f} + \bar{A} \cdot \bar{I}_2$

$= 291 \cdot 10^{-6}\angle 90 \cdot 63{,}5 \cdot 10^3 + 0{,}9943 \cdot 267{,}63\angle -11{,}3 =$

$= j18{,}49 + 266{,}1 \cdot (\cos(-11{,}3)) + j(\sin(-11{,}3)) =$

$= j18{,}49 + 260{,}94 - j52{,}14 = 260{,}94 - j33{,}66 = 263\angle -7{,}35°\ A$

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Voltage and current conditions on line – example 5-2 (5)

For phasor diagram:

$\bar{I}_{02} = \bar{Y} \cdot \frac{l}{2} \cdot \bar{U}_{2f} = j2{,}92 \cdot 10^{-6} \cdot \frac{100}{2} \cdot 63{,}5 \cdot 10^3 = j9{,}272\ A$

$\bar{I}_{01} = \bar{Y} \cdot \frac{l}{2} \cdot \bar{U}_{1f} = j2{,}92 \cdot 10^{-6} \cdot \frac{100}{2} \cdot 68386\angle 8{,}1 = 9{,}984\angle 98{,}1°\ A$

$\bar{I} = \bar{I}_2 + \bar{I}_{02} = 262{,}43 - j52{,}49 + j9{,}272 = 262{,}43 - j43{,}22 =$

$= 265{,}97\angle -9{,}35°\ A$

$\bar{I}_1 = \bar{I} + \bar{I}_{01} = 262{,}43 - j43{,}22 + 9{,}984 \cdot (\cos 98{,}1 + j \sin 98{,}1) =$

$= 262{,}43 - j43{,}22 - 1{,}55 + j9{,}87 = 260{,}88 - j33{,}35 =$

$= 263\angle -7{,}28°\ A$

$\bar{Y} = j2{,}92 \cdot 10^{-6}\ S/km$,

$l = 100\ km$

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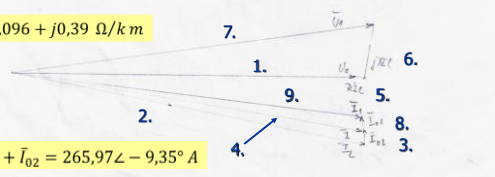
Voltage and current conditions on line – example 5-2 (6)

$R \cdot l \cdot \bar{I} = 0{,}096 \cdot 100 \cdot 265{,}97\angle -9{,}35 = 2{,}553\angle -9{,}35°\ kV$

$jX \cdot l \cdot \bar{I} = j0{,}39 \cdot 100 \cdot 265{,}97\angle -9{,}35 = 10{,}373\angle -80{,}65°\ kV$

Volba měřítka: $U_1: 1cm = 5\ kV$ $I: 1cm = 20\ A$

$\bar{Z} = 0{,}096 + j0{,}39\ \Omega/km$



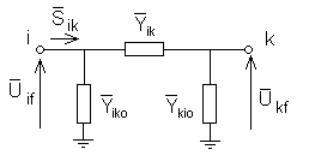
$\bar{I} = \bar{I}_2 + \bar{I}_{02} = 265{,}97\angle -9{,}35°\ A$

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Determination of power output flow in branch (1)

Derivation of relation for calculation of branch power outputs

- Substitution branch by **Π-cell**, which parametrs are:
  - R_{ik} , L_{ik} -> X_{ik}
  - G_{ik0} … we neglect
  - C_{ik} -> B_{ik0}



$$\bar{Y}_{ik} = G_{ik} + jB_{ik} = \frac{1}{Z_{ik}} = \frac{1}{R_{ik} + jX_{ik}} = \frac{R_{ik}}{R_{ik}^2 + X_{ik}^2} - j\frac{X_{ik}}{R_{ik}^2 + X_{ik}^2}$$

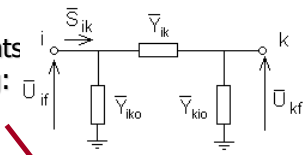
G_{ik} B_{ik}

$$\bar{Y}_{ik0} = G_{ik0} + jB_{ik0} = 0 + jB_{ik0} = \frac{1}{2}j\omega C_{ik}$$

Determination of power output flow in branch (2)

Derivation of relation for calculation of branch power outputs

- The same like for currents **- 2 parts**, tj. S flowing:
  - into longitudinal element
  - into transverse element



$$\bar{S}_{ik} = P_{ik} + jQ_{ik} = \sqrt{3}\cdot\bar{U}_i\cdot\bar{I}_{ik}^{*} =$$

$$= \sqrt{3}\cdot\bar{U}_i\cdot[\bar{Y}_{ik0}^{*}\cdot\bar{U}_{if}^{*} + \bar{Y}_{ik}^{*}\cdot(\bar{U}_{if}^{*} - \bar{U}_{kf}^{*})] =$$

$$= \bar{U}_i[\bar{Y}_{ik0}^{*}\bar{U}_i^{*} + \bar{Y}_{ik}^{*}(\bar{U}_i^{*} - \bar{U}_k^{*})] = U_i^2(\bar{Y}_{ik0}^{*} + \bar{Y}_{ik}^{*}) - \bar{U}_i\bar{U}_k^{*}\bar{Y}_{ik}^{*}$$

$$\bar{I} = \frac{\bar{U}_f}{\bar{Z}} = \bar{Y}\cdot\bar{U}_f = \bar{Y}\cdot\frac{\bar{U}}{\sqrt{3}}$$

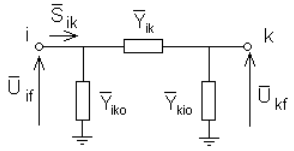
converted to line voltage

!size (U_i $\bar{U}_i^{*}$)

multiply and reproached

Determination of power output flow in branch (3)

Derivation of relation for calculation of branch power outputs



$$\bar{S}_{ik} = P_{ik} + jQ_{ik}$$

$$\bar{S}_{ik} = U_i^2(\bar{Y}_{ik0}^{*} + \bar{Y}_{ik}^{*}) - \bar{U}_i\bar{U}_k^{*}\bar{Y}_{ik}^{*}$$

$$\bar{S}_{ik} = \bar{Y}_{ik}^{*}(U_i^2 - \bar{U}_i\bar{U}_k^{*}) + U_i^2\bar{Y}_{ik0}^{*} =$$

$$= (G_{ik} - jB_{ik})\cdot(U_i^2 - U_i e^{j\vartheta_i}\cdot U_k e^{-j\vartheta_k}) - jU_i^2 B_{ik0} =$$

$$= (G_{ik} - jB_{ik})\cdot(U_i^2 - U_i U_k e^{j(\vartheta_i - \vartheta_k)}) - jU_i^2 B_{ik0} =$$

$$= (G_{ik} - jB_{ik})\cdot(U_i^2 - U_i U_k e^{j\vartheta_{ik}}) - jU_i^2 B_{ik0} =$$

$$= (G_{ik} - jB_{ik})\cdot[U_i^2 - U_i U_k(\cos\vartheta_{ik} + j\sin\vartheta_{ik})] - jU_i^2 B_{ik0}$$

$\bar{Y}_{ik} = G_{ik} + jB_{ik}$

$\bar{Y}_{ik0} = jB_{ik0}$

$\vartheta_{ik} = \vartheta_i - \vartheta_k$

Determination of power output flow in branch (4)

Derivation of relation for calculation of branch power outputs

- Dividing $\bar{S}_{ik} = P_{ik} + jQ_{ik}$ into active and reactive components

$$\bar{S}_{ik} = (G_{ik} - jB_{ik}) \cdot [U_i^2 - U_iU_k(\cos\vartheta_{ik} + j\sin\vartheta_{ik})] - jU_i^2B_{ik0}$$

- We split previus link for **Re** and **Im** part:

$$P_{ik} = G_{ik}U_i^2 - G_{ik}U_iU_k\cos\vartheta_{ik} - B_{ik}U_iU_k\sin\vartheta_{ik}$$

$$Q_{ik} = -U_i^2(B_{ik} + B_{ik0}) + B_{ik}U_iU_k\cos\vartheta_{ik} - G_{ik}U_iU_k\sin\vartheta_{ik}$$

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Determination of power output flow in branch (5)

Simplification for **VHV**:

- Assumption $R_{ik} << X_{ik} \Rightarrow G_{ik} << B_{ik}$, $B_{ik} = -\frac{1}{X_{ik}}$,
- then

$$P_{ik} = -B_{ik}U_iU_k\sin\vartheta_{ik} = \frac{1}{X_{ik}}U_iU_k\sin\vartheta_{ik}$$

$$\vartheta_{ik} \to 0 \Rightarrow \sin\vartheta_{ik} = \vartheta_{ik}\,[\text{rad}]$$

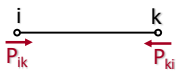
$$P_{ik} = \frac{U_iU_k}{X_{ik}}\vartheta_{ik}$$

- Same for Q_{ik}

$$Q_{ik} = \frac{1}{X_{ik}}(U_i^2 - U_iU_k\cos\vartheta_{ik}) - U_i^2B_{ik0}$$

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Calculation of active losses in the branch



$P_{ki} = -P_{ik}$

$\Delta P_{ik} = 0 \cdot\cdot$ lossless line

$\Delta P_{ik} = P_{ik} + P_{ki}$ - with the same electrical power orientation from node to branch as the index orientation

$$P_{ik} = G_{ik}(U_i^2 - U_iU_k\cos\vartheta_{ik}) - B_{ik}U_iU_k\sin\vartheta_{ik}$$

$$P_{ki} = G_{ki}(U_k^2 - U_kU_i\cos\vartheta_{ki}) - B_{ki}U_kU_i\sin\vartheta_{ki} =$$

$$= G_{ik}(U_k^2 - U_iU_k\cos\vartheta_{ik}) + B_{ik}U_iU_k\sin\vartheta_{ik}$$

- Reached: $G_{ki} = G_{ik}$, $B_{ki} = B_{ik}$, $\vartheta_{ki} = -\vartheta_{ik}$ } $\cos\vartheta_{ik} = \cos\vartheta_{ki}$, $\sin\vartheta_{ik} = -\sin\vartheta_{ki}$

$$\Delta P_{ik} = G_{ik}(U_i^2 + U_k^2 - 2 \cdot U_iU_k\cos\vartheta_{ik})$$

- When we calculate active power losses, we can never neglect the G_{ik} member.

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Calculation of active power losses in the branch – example 5-3 (1)

- **Task:**

  To compare the effect of differential voltage nodes and angles between voltages solve an example for the following entries:

a) $U_i = U_k = 400$ kV, $\upsilon_{ik} = 10^o$
b) $U_i = 405$ kV, ostatní jako v a)
c) $\upsilon_{ik} = 5^o$, ostatní jako v a)
d) $U_i = 405$ kV, $U_k = 400$ kV, $\upsilon_{ik} = 0^o$

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Calculation of active power losses in the branch – example 5-3 (2)

- **Calculation of passive paramters**

  $G_{ik}, B_{ik} (R_{ik}, X_{ik})$ a B_{ik0} :

$$R_{ik} = R \cdot l = 0{,}028 \cdot 100 = 2{,}8\ \Omega$$

$$X_{ik} = X \cdot l = 0{,}35 \cdot 100 = 35\ \Omega$$

$$G_{ik} = \frac{R_{ik}}{R_{ik}^2 + X_{ik}^2} = \frac{2{,}8}{2{,}8^2 + 35^2} = 2{,}27.10^{-3}\ \text{S}$$

$$B_{ik} = -\frac{X_{ik}}{R_{ik}^2 + X_{ik}^2} = -\frac{35}{2{,}8^2 + 35^2} = -0{,}0284\ \text{S}$$

$$B_{ik0} = \frac{B}{2} \cdot l = 3 \cdot 10^{-6} \cdot 100 = 0{,}00015\ \text{S}$$

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Calculation of active power losses in the branch – example 5-3 (3)

- **Solving of the variant a):**

  $U_i = U_k = 400$ kV, $\upsilon_{ik} = 10^o = \frac{10}{180} \cdot \pi = 0{,}1745$ rad

$$P_{ik} = G_{ik}(U_i^2 - U_i U_k \cos\vartheta_{ik}) - B_{ik} U_i U_k \sin\vartheta_{ik}$$

$$P_{ik} = 2{,}27 \cdot 10^{-3}(400^2 - 400 \cdot 400 \cdot 0{,}9848) + 0{,}0284 \cdot 400 \cdot 400 \cdot 0{,}1736 =$$
$$= 5{,}52 + 788{,}84 = 794{,}36\ \text{MW}$$

$$Q_{ik} = -U_i^2(B_{ik} + B_{ik0}) - G_{ik} U_i U_k \sin\vartheta_{ik} + B_{ik} U_i U_k \cos\vartheta_{ik}$$

$$Q_{ik} = -400^2(-0{,}0284 + 0{,}00015) - 2{,}27 \cdot 10^{-3} \cdot 400 \cdot 400 \cdot 0{,}1736 -$$
$$- 0{,}0284 \cdot 400 \cdot 400 \cdot 0{,}9848 = 4520 - 63{,}05 - 4474{,}9 = -17{,}95\ \text{MVAr}$$

$$\Delta P_{ik} = G_{ik}(U_i^2 + U_k^2 - 2 \cdot U_i U_k \cos\vartheta_{ik})$$

$$\Delta P_{ik} = 2{,}27 \cdot 10^{-3}(400^2 + 400^2 - 2 \cdot 400 \cdot 400 \cdot 0{,}9848) = 11{,}04\ \text{MW}$$

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Calculation of active power losses in the branch – example 5-3 (4)

- **Solving of the variant a):**

$U_i = U_k = 400$ kV, $\upsilon_{ik} = 10^o = \frac{10}{180} \cdot \pi = 0{,}1745$ rad

Calculation in a simplified form of neglecting members Gik:

$$P_{ik} = \frac{U_i U_k}{X_{ik}} \vartheta_{ik} = \frac{400^2}{35} \cdot 0{,}1745 = 797{,}7 \text{ MW}$$

$$Q_{ik} = \frac{1}{X_{ik}}(U_i^2 - U_i U_k \cos\vartheta_{ik}) - U_i^2 B_{ik0} =$$

$$= \frac{1}{35} 400^2 (1 - 0{,}9848) - 400^2 \cdot 0{,}00015 = 45{,}49 \text{ MVAr}$$

For comparison :

$P_{ik} = 794{,}36$ MW
$Q_{ik} = -17{,}95$ MVAr

The comparison of the results shows that a simplifying assumption can be used to calculate active power. We cannot afford simplification when calculating reactive power.

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Calculation of active losses in the branch – example 5-3 (5)

- **Solving of the variant b):**

$U_i = 405$ kV, $U_k = 400$ kV, $\upsilon_{ik} = 10^o$

$$P_{ik} = G_{ik}(U_i^2 - U_i U_k \cos\vartheta_{ik}) - B_{ik} U_i U_k \sin\vartheta_{ik}$$

$$P_{ik} = 2{,}27 \cdot 10^{-3}(405^2 - 405 \cdot 400 \cdot 0{,}9848) + 0{,}0284 \cdot 405 \cdot 400 \cdot 0{,}1736 =$$
$$= 10{,}1864 + 798{,}699 = 808{,}89 \text{ MW}$$

$$Q_{ik} = -U_i^2(B_{ik} + B_{ik0}) - G_{ik} U_i U_k \sin\vartheta_{ik} + B_{ik} U_i U_k \cos\vartheta_{ik}$$

$$Q_{ik} = -405^2(-0{,}0284 + 0{,}00015) - 2{,}27 \cdot 10^{-3} \cdot 405 \cdot 400 \cdot 0{,}1736 -$$
$$- 0{,}0284 \cdot 405 \cdot 400 \cdot 0{,}9848 = 4633{,}7 - 63{,}84 - 4530 = 39{,}86 \text{ MVAr}$$

$$\Delta P_{ik} = G_{ik}(U_i^2 + U_k^2 - 2 \cdot U_i U_k \cos\vartheta_{ik})$$

$$\Delta P_{ik} = 2{,}27 \cdot 10^{-3}(405^2 + 400^2 - 2 \cdot 405 \cdot 400 \cdot 0{,}9848) = 11{,}23 \text{ MW}$$

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Calculation of active losses in the branch – example 5-3 (6)

- **Solving of the variant c):**

$U_i = U_k = 400$ kV, $\upsilon_{ik} = 5^o = \frac{10}{180} \cdot \pi = 0{,}08725$ rad

$$P_{ik} = G_{ik}(U_i^2 - U_i U_k \cos\vartheta_{ik}) - B_{ik} U_i U_k \sin\vartheta_{ik}$$

$$P_{ik} = 2{,}27 \cdot 10^{-3}(400^2 - 400 \cdot 400 \cdot 0{,}9962) + 0{,}0284 \cdot 400 \cdot 400 \cdot 0{,}0872 =$$
$$= 1{,}382 + 396{,}237 = 397{,}62 \text{ MW}$$

$$Q_{ik} = -U_i^2(B_{ik} + B_{ik0}) - G_{ik} U_i U_k \sin\vartheta_{ik} + B_{ik} U_i U_k \cos\vartheta_{ik}$$

$$Q_{ik} = -400^2(-0{,}0284 + 0{,}00015) - 2{,}27 \cdot 10^{-3} \cdot 400 \cdot 400 \cdot 0{,}0872 -$$
$$- 0{,}0284 \cdot 400 \cdot 400 \cdot 0{,}9962 = 4520 - 38{,}37 - 4526{,}7 = -38{,}37 \text{ MVAr}$$

$$\Delta P_{ik} = G_{ik}(U_i^2 + U_k^2 - 2 \cdot U_i U_k \cos\vartheta_{ik}) - B_{ik} U_i U_k \sin\vartheta_{ik}$$

$$\Delta P_{ik} = 2{,}27 \cdot 10^{-3}(400^2 + 400^2 - 2 \cdot 400 \cdot 400 \cdot 0{,}9962) = 2{,}76 \text{ MW}$$

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Calculation of active power losses in the branch – example 5-3 (7)

- **Solving of the variant d):**

U_i = 405 kV, U_k = 400 kV, υ_{ik}= 0° (sin 0 = 0, cos 0 = 1)

$$P_{ik} = G_{ik}(U_i^2 - U_iU_k)$$

$$P_{ik} = 2{,}27 \cdot 10^{-3}(405^2 - 405 \cdot 400) = 4{,}6 \text{ MW}$$

$$Q_{ik} = -U_i^2(B_{ik} + B_{ik0}) + B_{ik}U_iU_k$$

$$Q_{ik} = -405^2(-0{,}0284 + 0{,}00015) + 0{,}0284 \cdot 405 \cdot 400 = -32{,}9 \text{ MVAr}$$

$$\Delta P_{ik} = G_{ik}(U_i^2 + U_k^2 - 2 \cdot U_iU_k)$$

$$\Delta P_{ik} = 2{,}27 \cdot 10^{-3}(405^2 + 400^2 - 2 \cdot 405 \cdot 400) = 0{,}057 \text{ MW}$$

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Calculation of active power losses in the branch – example 5-3 (8)

- **Summary of results:**

	a/	b/ change U	c/ change υ	d/ υ_{ik}= 0°
P_{ik} [MW]	794,30	808,60	397,28	4,60
P_{ik} [MW] (after simplification)	794,71	807,69	399,09	0
Q_{ik} [MVAr]	-18,01	38,95	-38,41	32,89
Q_{ik} [MVAr] after simplification)	45,49	103,61	-6,60	33,26
ΔP_{ik} [MW]	11,04	11,23	2,76	0,057

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Calculation of active power losses in the branch – example 5-3 (9)

Evaluation:

We can do these ressults from example 5-3

- Resistance can be neglected in the approximate calculation of active power output
- The size of the angle between the voltages in the nodes mainly determines the size of the active power output
- The difference in the size of voltage in the nodes mainly determines the size of reactive power output
- Active power losses depend only on equation members containing Gik

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Next time

- **Solution of voltage, current and power output conditions in the electrical network**
  - Calculation of voltage drop on line (DC, AC-1ph and 3ph)

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