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EUROPEAN UNION
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Operational Programme Research,
Development and Education

THEORY OF TRANSMISSION AND DISTRIBUTION OF ELECTRIC ENERGY

7th lecture - theory

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Content

- **Solution of voltage, current and power output conditions in the electrical network**
  - Solution of simple network by the method of the schema gradual simplification
    - Procedure
    - Example (in a separate document)

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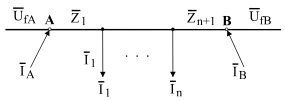
Simple network solution by the method of gradual simplification of the schema (1)

- **Calculation goal:**
  - to determine the currents (powers) flowing in individual branches of the network
  - to find places (points) suitable for network deviding to beam operation
- **Procedure:**
  - simplifying of the schema assuming the same voltages in the supply nodes (see circuit simplification techniques shown on the next page)
  - deviding of consumption currents (power outputs) into supply nodes
  - in case of voltage are not the same size in supply nodes, calculation of equalization current
  - determination of branch currents (power outputs)
  - finding of places with the greatest voltage drop

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Simple network solution by the method of gradual simplification of the schema (2)

- Techniques used for circuit simplification
  - Load transfer from branches to nodes and back



$$\sum_{k=1}^{n} \underline{I}_k = \underline{I}_A + \underline{I}_B$$

$$\underline{I}_A = \frac{\sum_{k=1}^{n}\left(\underline{I}_k \sum_{l=k+1}^{n} \underline{Z}_l\right)}{\sum_{k=1}^{n+1} \underline{Z}_k}$$

$$\underline{I}_B = \frac{\sum_{k=1}^{n}\left(\underline{I}_k \sum_{l=1}^{n} \underline{Z}_l\right)}{\sum_{k=1}^{n+1} \underline{Z}_k}$$

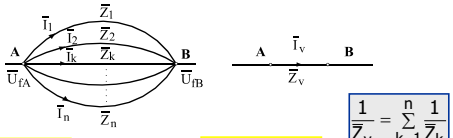
Equalizing current - when $\underline{U}_{fA} \neq \underline{U}_{fB}$

$$\underline{I}_o = \frac{\underline{U}_{fA} - \underline{U}_{fB}}{\sum_{k=1}^{n+1} \underline{Z}_k}$$

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Simple network solution by the method of gradual simplification of the schema (3)

- Techniques used for circuit simplification
  - Merging of parallel branches



$$\frac{1}{\underline{Z}_v} = \sum_{k=1}^{n} \frac{1}{\underline{Z}_k}$$

$$\underline{I}_k = \frac{\underline{U}_{fA} - \underline{U}_{fB}}{\underline{Z}_k} \qquad \underline{I}_v = \frac{\underline{U}_{fA} - \underline{U}_{fB}}{\underline{Z}_v}$$

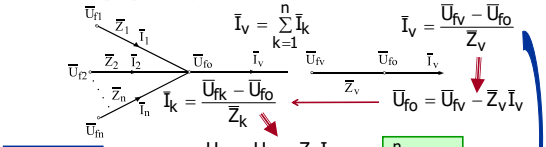
$$\underline{I}_k = \underline{I}_v \frac{\underline{Z}_v}{\underline{Z}_k}$$

$$\underline{I}_v = \sum_{k=1}^{n} \underline{I}_k$$

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Simple network solution by the method of gradual simplification of the schema (4)

- Techniques used for circuit simplification
  - Merging of supply branches



$$\bar{I}_v = \sum_{k=1}^{n} \bar{I}_k \qquad \bar{I}_v = \frac{\bar{U}_{fv} - \bar{U}_{fo}}{\bar{Z}_v}$$

$$\bar{I}_k = \frac{\bar{U}_{fk} - \bar{U}_{fo}}{\bar{Z}_k} \qquad \bar{U}_{fo} = \bar{U}_{fv} - \bar{Z}_v \bar{I}_v$$

$$\bar{I}_k = \frac{U_{fk}}{\bar{Z}_k} - \frac{U_{fv} - Z_v I_v}{\bar{Z}_k}$$

$$\frac{1}{\bar{Z}_v} = \sum_{k=1}^{n} \frac{1}{\bar{Z}_k} \qquad \sum_{k=1}^{n} \bar{I}_k = \bar{I}_v$$

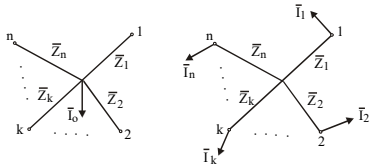
$$\bar{U}_{fv} = \frac{\sum_{k=1}^{n} \frac{\bar{U}_{fk}}{\bar{Z}_k}}{\sum_{k=1}^{n} \frac{1}{\bar{Z}_k}}$$

$$\sum_{k=1}^{n} \frac{\bar{U}_{fk}}{\bar{Z}_k} - \sum_{k=1}^{n} \frac{\bar{U}_{fo}}{\bar{Z}_k} = \frac{\bar{U}_{fv}}{\bar{Z}_v} - \frac{\bar{U}_{fo}}{\bar{Z}_v}$$

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Simple network solution by the method of gradual simplification of the schema (5)

- Techniques used for circuit simplification
  - Transfer the load from the star node to the its top

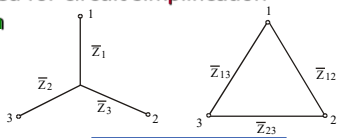


$$\bar{I}_k = \bar{I}_o \frac{1}{\bar{Z}_k \sum_{i=1}^{n} \frac{1}{\bar{Z}_i}}$$

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Simple network solution by the method of gradual simplification of the schema (6)

- Techniques used for circuit simplification
  - Transfiguration star - triangle



$$\bar{Z}_1 = \frac{\bar{Z}_{12}\bar{Z}_{13}}{\bar{Z}_{12} + \bar{Z}_{13} + \bar{Z}_{23}}$$

$$\bar{Z}_2 = \frac{\bar{Z}_{12}\bar{Z}_{23}}{\bar{Z}_{12} + \bar{Z}_{13} + \bar{Z}_{23}}$$

$$\bar{Z}_3 = \frac{\bar{Z}_{23}\bar{Z}_{13}}{\bar{Z}_{12} + \bar{Z}_{13} + \bar{Z}_{23}}$$

$$\bar{Z}_{12} = \frac{\bar{Z}_1\bar{Z}_2 + \bar{Z}_1\bar{Z}_3 + \bar{Z}_2\bar{Z}_3}{\bar{Z}_3}$$

$$\bar{Z}_{13} = \frac{\bar{Z}_1\bar{Z}_2 + \bar{Z}_1\bar{Z}_3 + \bar{Z}_2\bar{Z}_3}{\bar{Z}_2}$$

$$\bar{Z}_{23} = \frac{\bar{Z}_1\bar{Z}_2 + \bar{Z}_1\bar{Z}_3 + \bar{Z}_2\bar{Z}_3}{\bar{Z}_1}$$

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Solution of simple LV network by the method of gradual simplification – procedure (1)

Task network:

- The DC network as shown (see Fig. 6-11, next page) is powered in points **A** , **B** voltage $U_A \neq U_B$. Specify I_A , I_B and dividing of currents in branches, if **S** cross-section of aluminum conductors is given. Next, identify the points of greatest voltage drop and check that the voltage drop in these locations does not exceed 5 %.

Note:

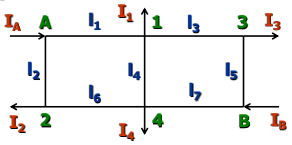
- We consider Al conductors of the same S, so instead of **Z** we can solve only with **l** ($l \approx Z$).

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Solution of simple LV network by the method of gradual simplification – procedure (2)

Task network:

- **Fig. 6-11:**



- Current values in [A] and length values in [m]:

I_1	I_2	I_3	I_4	l_1	l_2	l_3	l_4	l_5	l_6	l_7

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Solution of simple LV network by the method of gradual simplification – procedure (3)

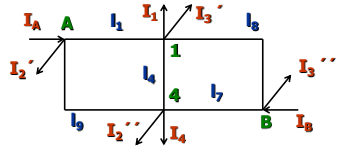
Calculation of branch currents:

A/ Assumption $U_A = U_B$

1) Transfer of I_2 and I_3 to neighboring nodes

$l_8 = l_3 + l_5$

$l_9 = l_2 + l_6$



$I_2' = \frac{I_2 \cdot l_6}{l_9}$ $I_2'' = \frac{I_2 \cdot l_2}{l_9}$ $I_3' = \frac{I_3 \cdot l_5}{l_8}$ $I_3'' = \frac{I_3 \cdot l_3}{l_8}$

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Solution of simple LV network by the method of gradual simplification – procedure (4)

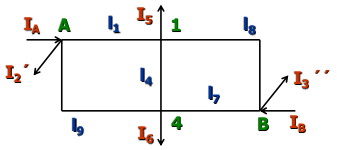
Calculation of branch currents:

A/ Assumption $U_A = U_B$

2) Addition of currents in nodes 1, 4

$I_5 = I_1 + I_3'$

$I_6 = I_2'' + I_4$



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Solution of simple LV network by the method of gradual simplification – procedure (5)

Calculation of branch currents:

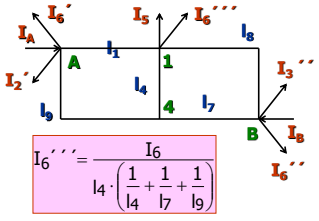
A/ Assumption $U_A = U_B$

3) I_6 transfer from the center of the star

$$I_6' = \frac{I_6}{l_9 \cdot \left(\frac{1}{l_4} + \frac{1}{l_7} + \frac{1}{l_9}\right)}$$

$$I_6'' = \frac{I_6}{l_7 \cdot \left(\frac{1}{l_4} + \frac{1}{l_7} + \frac{1}{l_9}\right)}$$

$$I_6''' = \frac{I_6}{l_4 \cdot \left(\frac{1}{l_4} + \frac{1}{l_7} + \frac{1}{l_9}\right)}$$



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Solution of simple LV network by the method of gradual simplification – procedure (6)

Calculation of branch currents:

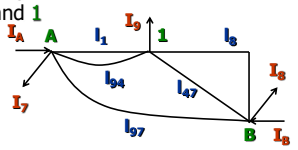
A/ Assumption $U_A = U_B$

4) Star-triangle transfiguration and addition of currents in A, B and 1

$$l_{94} = l_9 + l_4 + \frac{l_9 \cdot l_4}{l_7}$$

$$l_{97} = l_9 + l_7 + \frac{l_9 \cdot l_7}{l_4}$$

$$l_{47} = l_4 + l_7 + \frac{l_4 \cdot l_7}{l_9}$$



$I_7 = I_2' + I_6'$ $\quad I_8 = I_3'' + I_6''$

$I_9 = I_5 + I_6'''$

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Solution of simple LV network by the method of gradual simplification – procedure (7)

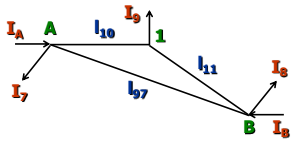
Calculation of branch currents:

A/ Assumption $U_A = U_B$

5) Addition of parallel branches I_1 and I_{94}, I_8 and I_{47}

$$I_{10} = \frac{I_1 \cdot I_{94}}{I_1 + I_{94}}$$

$$I_{11} = \frac{I_8 \cdot I_{47}}{I_8 + I_{47}}$$



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Solution of simple LV network by the method of gradual simplification – procedure (8)

Calculation of branch currents:

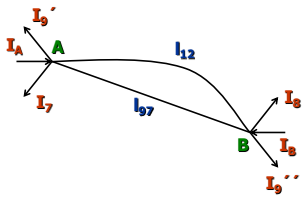
A/ Assumption $U_A = U_B$

6) Transfer of I_9 to points A, B and adding I_{10} and I_{11}

$$I_9' = \frac{I_9 \cdot I_{11}}{I_{12}}$$

$$I_9'' = \frac{I_9 \cdot I_{10}}{I_{12}}$$

$$I_{12} = I_{10} + I_{11}$$



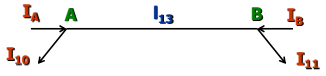
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Solution of simple LV network by the method of gradual simplification – procedure (9)

Calculation of branch currents:

A/ Assumption $U_A = U_B$

7) Addition of parallel branches and consumption



$$I_{13} = \frac{I_{12} \cdot I_{97}}{I_{12} + I_{97}}$$

$$I_{10} = I_7 + I_9'$$

$$I_{11} = I_8 + I_9''$$

Platí: $I_A = I_{10}$, $I_B = I_{11}$

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Solution of simple LV network by the method of gradual simplification – procedure (10)

Calculation of branch currents:

B/ $U_A \neq U_B \rightarrow$ equalizing current

$$I_0 = \frac{U_B - U_A}{R_V} = \frac{U_B - U_A}{\frac{2 \cdot \rho \cdot l_{13}}{S}}$$

8)



$I_A' = I_A - I_0$ $\quad$ $I_B' = I_B + I_0$

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Solution of simple LV network by the method of gradual simplification – procedure (11)

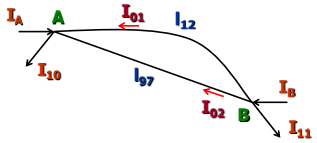
Calculation of branch currents:

B/ $U_A \neq U_B$

9) Dividing of current I_0 into parallel branches

$$I_{01} = \frac{I_0 \cdot l_{13}}{l_{12}}$$

$$I_{02} = \frac{I_0 \cdot l_{13}}{l_{97}}$$



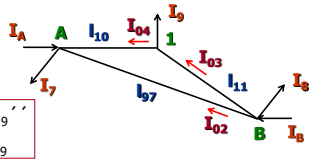
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Solution of simple LV network by the method of gradual simplification – procedure (12)

Calculation of branch currents:

B/ $U_A \neq U_B$

10) Return of current I_9 to point 1



$I_{03} = I_{01} + I_9''$

$I_{04} = I_{03} - I_9$

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Solution of simple LV network by the method of gradual simplification – procedure (13)

Calculation of branch currents:

B/ $U_A \neq U_B$

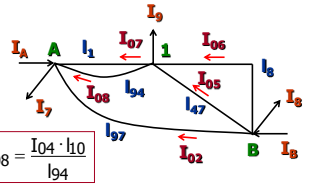
11) Dividing of currents I_{04} and I_{03} into parallel branches between points **A** and **1**, **B** and **1**

$$I_{05} = \frac{I_{03} \cdot l_{11}}{l_{47}}$$

$$I_{06} = \frac{I_{03} \cdot l_{11}}{l_8}$$

$$I_{07} = \frac{I_{04} \cdot l_{10}}{l_1}$$

$$I_{08} = \frac{I_{04} \cdot l_{10}}{l_{94}}$$



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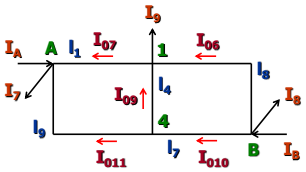
Solution of simple LV network by the method of gradual simplification – procedure (14)

Calculation of branch currents:

B/ $U_A \neq U_B$

12) Star-triangle back-configuration

$$I_{09} = I_{05} - I_{08}$$
$$I_{010} = I_{02} + I_{05}$$
$$I_{011} = I_{02} + I_{08}$$



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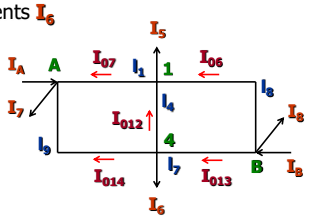
Solution of simple LV network by the method of gradual simplification – procedure (15)

Calculation of branch currents:

B/ $U_A \neq U_B$

13) Return of currents I_6

$$I_{012} = I_{09} - I_6'''$$
$$I_{013} = I_{010} + I_6''$$
$$I_{014} = I_{011} - I_6'$$



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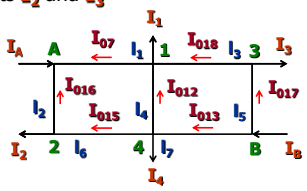
Solution of simple LV network by the method of gradual simplification – procedure (16)

Calculation of branch currents:

B/ $U_A \neq U_B$

14) Return of currents I_2 and I_3

$$I_{015} = I_{014} + I_2''$$
$$I_{016} = I_{014} - I_2'$$
$$I_{017} = I_{06} + I_3''$$
$$I_{018} = I_{06} - I_3'$$

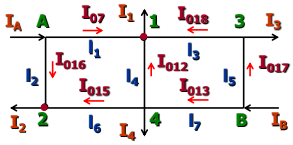


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Solution of simple LV network by the method of gradual simplification – procedure (17)

Calculation of voltage drop:

- We determine the points with the greatest voltage drop (here red dots)
- We calculate the size of the voltage drop by relation:



$$\Delta u\% = \frac{200 \cdot \rho}{S \cdot U_n} \sum_k I_k \cdot l_k$$

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Solution of simple LV network by the method of gradual simplification – procedure (18)

Calculation of voltage drop:

Ex. for the node 1:

$$\Delta u_A\% = \frac{200 \cdot 0{,}0359}{10 \cdot 220}(I_{07} \cdot l_1)$$

$$\Delta u_B\% = \frac{200 \cdot 0{,}0359}{10 \cdot 220}(I_{017} \cdot l_5 + I_{018} \cdot l_3)$$

or

$$\Delta u_B\% = \frac{200 \cdot 0{,}0359}{10 \cdot 220}(I_{013} \cdot l_7 + I_{012} \cdot l_4)$$

Ex. for the node 2:

$$\Delta u_A\% = \frac{200 \cdot 0{,}0359}{10 \cdot 220}(I_{016} \cdot l_2)$$

$$\Delta u_B\% = \frac{200 \cdot 0{,}0359}{10 \cdot 220}(I_{013} \cdot l_7 + I_{015} \cdot l_6)$$

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Next time

- **Load flow analysis**
  - Introduction (scope, assumptions, input data, outputs, bus types)
  - Bus admittance matrix - inclusion of power lines and compensators
  - Bus admittance matrix - inclusion of 2- and 3-winding transformers

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