ESF II project of the University of West Bohemia in Pilsen
reg. number: CZ.02.2.69/0.0/0.0/18_056/0013239

EUROPEAN UNION
European Structural and Investment Funds
Operational Programme Research,
Development and Education

THEORY OF TRANSMISSION AND DISTRIBUTION OF ELECTRIC ENERGY

7th lectures - example

Simple LV network solution by the method of gradual simplification – example 7-1 (1)

Task:

- The DC network according to the Fig. 6-11 is supplied at points **A, B** with voltage U_A = 220 V and U_B = 221 V. Determine I_A, I_B and current distribution in branches if the cross-section of aluminum (ρ = 0,0359 $\mu\Omega$m) conductors is S = 10 mm^2.
- Next, identify the points of greatest voltage drop and check that the voltage drop at these points does not exceed the permitted voltage drop of 5%.

Current values in [A] and length values in [m] are:

I_1	I_2	I_3	I_4	l_1	l_2	l_3	l_4	l_5	l_6	l_7
6	8	10	12	36	48	20	18	32	12	28

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Simple LV network solution by the method of gradual simplification – example 7-1 (2)

Calculation of branch currents:

A/ Assumption $U_A = U_B$

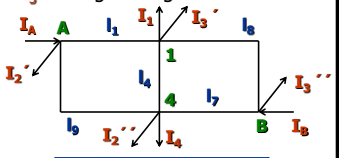
1) Transfer of I_2 and I_3 to neighboring nodes

$$I_2' = \frac{I_2 \cdot l_6}{l_9} = \frac{8 \cdot 12}{60} = 1{,}6 \text{ A}$$

$$I_2'' = \frac{I_2 \cdot l_2}{l_9} = \frac{8 \cdot 48}{60} = 6{,}4 \text{ A}$$

$$I_3' = \frac{I_3 \cdot l_5}{l_8} = \frac{10 \cdot 32}{52} = 6{,}15 \text{ A}$$

$$I_3'' = \frac{I_3 \cdot l_3}{l_8} = \frac{10 \cdot 20}{52} = 3{,}85 \text{ A}$$



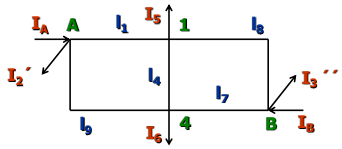
$l_8 = l_3 + l_5 = 20 + 32 = 52$ m
$l_9 = l_2 + l_6 = 48 + 12 = 60$ m

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Simple LV network solution by the method of gradual simplification – example 7-1 (3)

Calculation of branch currents:

2) Addition of currents in nodes 1, 4



$I_5 = I_1 + I_3' = 6 + 6{,}15 = 12{,}15$ A

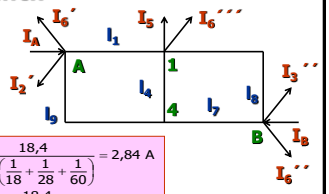
$I_6 = I_2'' + I_4 = 6{,}4 + 12 = 18{,}4$ A

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Simple LV network solution by the method of gradual simplification – example 7-1 (4)

Calculation of branch currents:

3) Transfer of I_6 from the center of the star



$$I_6' = \frac{I_6}{l_9 \cdot \left(\frac{1}{l_4} + \frac{1}{l_7} + \frac{1}{l_9}\right)} = \frac{18{,}4}{60 \cdot \left(\frac{1}{18} + \frac{1}{28} + \frac{1}{60}\right)} = 2{,}84 \text{ A}$$

$$I_6'' = \frac{I_6}{l_7 \cdot \left(\frac{1}{l_4} + \frac{1}{l_7} + \frac{1}{l_9}\right)} = \frac{18{,}4}{28 \cdot \left(\frac{1}{18} + \frac{1}{28} + \frac{1}{60}\right)} = 6{,}09 \text{ A}$$

$$I''_6 = \frac{I_6}{l_4 \cdot \left(\frac{1}{l_4} + \frac{1}{l_7} + \frac{1}{l_9}\right)} = \frac{18{,}4}{18 \cdot \left(\frac{1}{18} + \frac{1}{28} + \frac{1}{60}\right)} = 9{,}47 \text{ A}$$

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Simple LV network solution by the method of gradual simplification – example 7-1 (5)

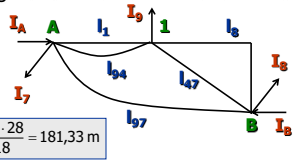
Calculation of branch currents:

4) Star-triangle transfiguration and addition of currents in A, B and 1

$$l_{94} = l_9 + l_4 + \frac{l_9 \cdot l_4}{l_7} = 60 + 18 + \frac{60 \cdot 18}{28} = 116{,}57 \text{ m}$$

$$l_{97} = l_9 + l_7 + \frac{l_9 \cdot l_7}{l_4} = 60 + 28 + \frac{60 \cdot 28}{18} = 181{,}33 \text{ m}$$

$$l_{47} = l_4 + l_7 + \frac{l_4 \cdot l_7}{l_9} = 18 + 28 + \frac{18 \cdot 28}{60} = 54{,}40 \text{ m}$$



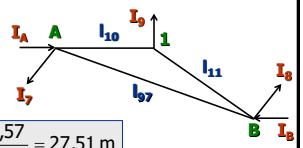
$I_7 = I_2' + I_6' = 1{,}60 + 2{,}84 = 4{,}44$ A
$I_8 = I_3'' + I_6'' = 3{,}85 + 6{,}09 = 9{,}94$ A
$I_9 = I_5 + I_6''' = 12{,}15 + 9{,}47 = 21{,}62$ A

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Simple network solution nn by the method of gradual simplification – example 7-1 (6)

Calculation of branch currents:

5) Addition of parallel branches l_1 and l_{94}
l_8 and l_{47}



$$l_{10} = \frac{l_1 \cdot l_{94}}{l_1 + l_{94}} = \frac{36 \cdot 116{,}57}{36 + 116{,}57} = 27{,}51 \text{ m}$$

$$l_{11} = \frac{l_8 \cdot l_{47}}{l_8 + l_{47}} = \frac{52 \cdot 54{,}4}{52 + 54{,}4} = 26{,}59 \text{ m}$$

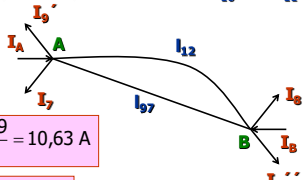
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Simple LV network solution by the method of gradual simplification – example 7-1 (7)

Calculation of branch currents:

6) Transfer of I_9 to the points A, B and addition l_{10} and l_{11}

$$l_{12} = l_{10} + l_{11} = 27{,}51 + 26{,}59 = 54{,}1 \text{ m}$$



$$I_9' = \frac{I_9 \cdot l_{11}}{l_{12}} = \frac{21{,}62 \cdot 26{,}59}{54{,}1} = 10{,}63 \text{ A}$$

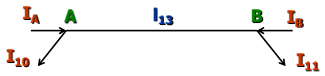
$$I_9'' = \frac{I_9 \cdot l_{10}}{l_{12}} = \frac{21{,}62 \cdot 27{,}51}{54{,}1} = 11 \text{ A}$$

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Simple LV network solution by the method of gradual simplification – example 7-1 (8)

Calculation of branch currents:

7) Adding parallel branches and consumption



$$l_{13} = \frac{l_{12} \cdot l_{97}}{l_{12} + l_{97}} = \frac{54{,}1 \cdot 181{,}33}{54{,}1 + 181{,}33} = 41{,}67 \text{ m}$$

$I_A = I_{10} = I_7 + I_9' = 4{,}44 + 10{,}63 = 15{,}07$ A
$I_B = I_{11} = I_8 + I_9'' = 9{,}94 + 11 = 20{,}94$ A

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Simple LV network solution by the method of gradual simplification – example 7-1 (9)

Calculation of branch currents:

B/ $U_A \neq U_B \rightarrow$ equalizing current

$$I_0 = \frac{U_B - U_A}{R_V} = \frac{U_B - U_A}{\frac{2 \cdot \rho \cdot l_{13}}{S}} = \frac{221 - 220}{\frac{2 \cdot 0{,}0359 \cdot 10^{-6} \cdot 41{,}67}{0{,}000010}} = 3{,}34 \text{ A}$$

8)



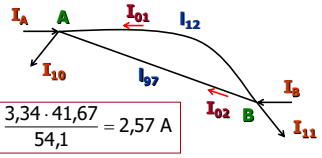
$I_A' = I_A - I_0 = 15{,}07 - 3{,}34 = 11{,}73$ A
$I_B' = I_B + I_0 = 20{,}94 + 3{,}34 = 24{,}28$ A

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Simple LV network solution by the method of gradual simplification – example 7-1 (10)

Calculation of branch currents:

9) Dividing of current I_0 into parallel branches



$$I_{01} = \frac{I_0 \cdot l_{13}}{l_{12}} = \frac{3{,}34 \cdot 41{,}67}{54{,}1} = 2{,}57 \text{ A}$$

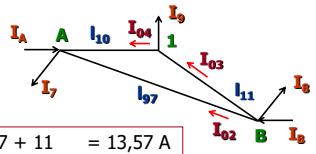
$$I_{02} = \frac{I_0 \cdot l_{13}}{l_{97}} = \frac{3{,}34 \cdot 41{,}67}{181{,}33} = 0{,}77 \text{ A}$$

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Simple LV network solution by the method of gradual simplification – example 7-1 (11)

Calculation of branch currents:

10) Return of current I_9 to point **1**



$I_{03} = I_{01} + I_9'' = 2,57 + 11 = 13,57$ A

$I_{04} = I_{01} - I_9' = 2,57 - 10,63 = -8,06$ A

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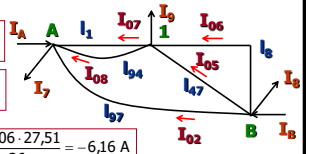
Simple LV network solution by the method of gradual simplification – example 7-1 (12)

Calculation of branch currents:

11) Dividing of I_{04} and I_{03} currents into parallel branches between points **A** and **1**, **B** and **1**

$$I_{05} = \frac{I_{03} \cdot l_{11}}{l_{47}} = \frac{13,57 \cdot 26,59}{54,4} = 6,63 \text{ A}$$

$$I_{06} = \frac{I_{03} \cdot l_{11}}{l_8} = \frac{13,57 \cdot 26,59}{52} = 6,94 \text{ A}$$



$$I_{07} = \frac{I_{04} \cdot l_{10}}{l_1} = \frac{-8,06 \cdot 27,51}{36} = -6,16 \text{ A}$$

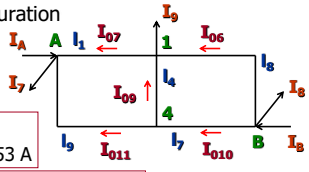
$$I_{08} = \frac{I_{04} \cdot l_{10}}{l_{94}} = \frac{-8,06 \cdot 27,51}{116,57} = -1,9 \text{ A}$$

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Simple LV network solution by the method of gradual simplification – example 7-1 (13)

Calculation of branch currents:

12) Reverse transfiguration star – triangle



$I_{09} = I_{05} - I_{08} = 6,63 - (-1,9) = 8,53$ A

$I_{010} = I_{02} + I_{05} = 0,77 + 6,63 = 7,40$ A

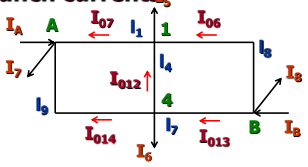
$I_{011} = I_{02} + I_{08} = 0,77 + (-1,9) = -1,13$ A

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Simple LV network solution by the method of gradual simplification – example 7-1 (14)

Calculation of branch currents:

13) Return of current I_6



$I_{012} = I_{09} - I_6'''' = 8,53 - 9,47 = -0,94$ A

$I_{013} = I_{010} + I_6'''' = 7,40 + 6,09 = 13,49$ A

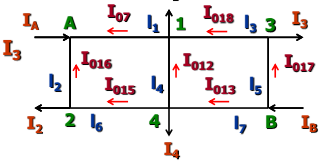
$I_{014} = I_{011} - I_6' = -1,13 - 2,84 = -3,97$ A

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Simple LV network solution by the method of gradual simplification – example 7-1 (15)

Calculation of branch currents:

14) Return of current I_2 and I_3



$I_{015} = I_{014} + I_2'''' = -3,79 + 6,4 = 2,43$ A

$I_{016} = I_{014} - I_2' = -3,97 - 1,6 = -5,57$ A

$I_{017} = I_{06} + I_3'''' = 6,94 + 3,85 = 10,79$ A

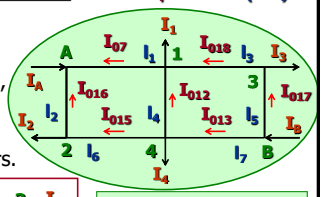
$I_{018} = I_{06} - I_3' = 6,94 - 6,15 = 0,79$ A

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Simple LV network solution by the method of gradual simplification – example 7-1 (16)

Calculation of voltage drop:

- Currents I_{04}, I_{07}, I_{08}, I_{011}, I_{012}, I_{014}, I_{016} negative, their direction will be revers.



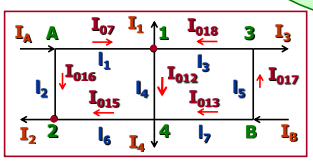


Figure with the currents from the calculation.

Figure after turning the currents with maximum ΔU points.

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Simple LV network solution by the method of gradual simplification – example 7-1 (17)

Calculation of voltage drop:

For the node 1:

$$\Delta u_A\,\% = \frac{200 \cdot 0{,}0359}{10 \cdot 220}(I_{07} \cdot l_1) = \frac{200 \cdot 0{,}0359}{10 \cdot 220}(6{,}16 \cdot 36) = 0{,}72\ \%$$

$$\Delta u_B\,\% = \frac{200 \cdot 0{,}0359}{10 \cdot 220}(I_{017} \cdot l_5 + I_{018} \cdot l_3) = \\ = \frac{200 \cdot 0{,}0359}{10 \cdot 220}(10{,}79 \cdot 32 + 0{,}79 \cdot 20) = 1{,}18\ \%$$

or

$$\Delta u_B\,\% = \frac{200 \cdot 0{,}0359}{10 \cdot 220}(I_{013} \cdot l_7 - I_{012} \cdot l_4) = \\ = \frac{200 \cdot 0{,}0359}{10 \cdot 220}(13{,}49 \cdot 28 - 0{,}94 \cdot 18) = 1{,}18\ \%$$

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Simple LV network solution by the method of gradual simplification – example 7-1 (18)

Calculation of voltage drop:

For the node 2:

$$\Delta u_A\,\% = \frac{200 \cdot 0{,}0359}{10 \cdot 220}(I_{016} \cdot l_2) = \frac{200 \cdot 0{,}0359}{10 \cdot 220}(5{,}57 \cdot 48) = 0{,}87\ \%$$

$$\Delta u_B\,\% = \frac{200 \cdot 0{,}0359}{10 \cdot 220}(l_7 \cdot (I_4 - I_{012}) + I_{015} \cdot (l_7 + l_6)) \\ = \frac{200 \cdot 0{,}0359}{10 \cdot 220}(28 \cdot (12 - 0{,}94) + 2{,}43 \cdot (28 + 12)) = 1{,}33\ \%$$

$$\Delta u_B\,\% = \frac{200 \cdot 0{,}0359}{10 \cdot 220}(I_{013} \cdot l_7 + I_{015} \cdot l_6) = \\ = \frac{200 \cdot 0{,}0359}{10 \cdot 220}(13{,}48 \cdot 28 + 2{,}43 \cdot 12) = 1{,}33\ \%$$

Two ways calculation drop from point B

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