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THEORY OF TRANSMISSION AND DISTRIBUTION OF ELECTRIC ENERGY

6th lectures - examples

Table for determining R_K and X_K of line

Cross-section mm²	Resistance at temperature 60 °C R_K [Ω/km]			Inductive reactance X_K [Ω/km]					
				Cable (Cu i Al)		Overhead line*)			
						Cu		AlFe 6	
	Cu	Al	AlFe 6	with metal sheath	without metal sheath	d = 40 cm	d = 50 cm	d = 40 cm	d = 50 cm
1,5	14,5	–	–	–	–	–	–	–	–
2,5	8,7	14,3	–	–	–	–	–	–	–
4	5,43	8,97	–	–	–	–	–	–	–
6	3,62	5,98	–	0,099 až 0,107	0,088 až 0,094	–	–	–	–
10	2,17	3,59	–	0,094 až 0,102	0,083 až 0,089	0,346	0,360	–	–
16	1,3	2,24	2,180	0,089 až 0,097	0,079 až 0,085	0,338	0,352	0,327	0,341
25	0,87	1,43	1,390	0,085 až 0,093	0,075 až 0,081	0,325	0,339	0,313	0,327
35	0,62	1,02	0,970	0,083 až 0,091	0,073 až 0,079	0,311	0,325	0,302	0,316
50	0,434	0,718	0,690	0,081 až 0,089	0,071 až 0,077	0,300	0,314	0,291	0,305
70	0,31	0,513	0,501	0,079 až 0,087	0,069 až 0,075	0,287	0,301	0,279	0,293
95	0,229	0,378	0,369	0,078 až 0,086	0,068 až 0,074	0,279	0,293	0,270	0,284
120	0,181	0,299	0,271	0,078 až 0,086	0,066 až 0,072	0,271	0,285	0,260	0,274
150	0,145	0,239	0,223	0,077 až 0,085	0,066 až 0,072	0,264	0,277	0,254	0,268
185	0,117	0,194	0,180	0,077 až 0,085	0,065 až 0,071	0,257	0,271	0,247	0,261
240	0,09	0,149	–	0,077 až 0,085	0,065 až 0,071	–	–	–	–

*) **d** is the distance between the conductors

Tab. 6.1:

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Calculation of ΔU for open line with 1 consumption at the end – example 6-1

ad 1) (= DC) What voltage drop will be in the aluminum line of cross-section 25 mm², length 70 m, if a current of 50 A is drawn at 220 V at the end of the line?

$$\Delta U = \frac{2 \cdot \rho \cdot l}{S} \cdot I = \frac{2 \cdot 0{,}0359 \cdot 70}{25} \cdot 50 = 10{,}05 \text{ V}$$

$$\Delta u\% = \frac{\Delta U \cdot 100}{U} = \frac{10{,}05 \cdot 100}{220} = 4{,}57 \%$$

- using tab. 6-1: for 25 mm² and Al line →
  → R_K = 1,43 Ω/km
  $\Delta U = 2 \cdot R_K \cdot l \cdot I = 2 \cdot 1{,}43 \cdot 0{,}070 \cdot 50 = 10{,}01 \text{ V}$

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Dimensioning according to ΔU (open line with 1 consumption at the end) – example 6-2 (1)

ad 1) (= DC) Determine the cross-section of the aluminum cable leading to the appliance at 220 V with a power input of 12 kW, located at a distance of 25 m. The voltage drop must not exceed 4%.

$$S = \frac{2 \cdot \rho \cdot l \cdot 100 \cdot P}{\Delta u\% \cdot U^2} = \frac{2 \cdot 0{,}0359 \cdot 25 \cdot 12 \cdot 10^5}{4 \cdot 220^2} = 11{,}1 \text{ mm}^2$$

⇓

- We choose the next higher normalized cross-section, i.e. S = 16 mm²

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Dimensioning according to ΔU (open line with 1 consumption at the end) – example 6-2 (2)

- using tab. 6.1 → we need R_K and we determine S

$$R_K = \frac{\Delta u\% \cdot U^2}{200 \cdot l \cdot P} = \frac{4 \cdot 220^2}{200 \cdot 0{,}025 \cdot 12000} = 3{,}23 \text{ Ω/km}$$

⇓

- The nearest lower R_K = 2,24 Ω/km → S = 16 mm²

(S we are looking for a smaller R_K, because they are inversely proportional to each other)

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Dimensioning according to ΔU (open line with 1 consumption at the end) – example 6-3 (1)

ad 2) (= AC., 1ph) What cross-section would have 1ph AlFe 6 line, so that voltage drop at the 2 kW transmission and cosφ = 0,9 over a distance of 600 m does not exceed 5%. The voltage at the beginning of the line is 230 V and the distance between the wires is 50 cm.

- at cosφ = 0,9 is **tgφ = 0,48**

$$\Delta u\% = \frac{200 \cdot P \cdot l}{U^2} \cdot (R_K + X_K \cdot tg\varphi)$$

$$R_K + X_K \cdot tg\varphi = \frac{\Delta u\% \cdot U^2}{200 \cdot P \cdot l} = \frac{5 \cdot 230^2}{200 \cdot 2000 \cdot 0{,}6} = 1{,}10\ \Omega/km$$

⇓

%

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Dimensioning according to ΔU (open line with 1 consumption at the end) – example 6-3 (2)

- Select from **tab. 6.1**: **R_K = 0,97 Ω/km ; X_K = 0,316 Ω/km**
  - **check:** 0,97 + 0,316·0,48 = 1,12 Ω/km > 1,10 Ω/km

⇓

S = 35 mm² ← **NOT COMPLIANT**

- Take the nearest higher, i.e. S = 50 mm², **R_K = 0,69 Ω/km ; X_K = 0,305 Ω/km**
  - **check:** 0,69 + 0,305·0,48 = 0,836 Ω/km < 1,10 Ω/km

⇓

S = 50 mm² ← **COMPLIANT**

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Calculation of ΔU for open line with 1 consumption at the end – example 6-4 (1)

ad 3) (= AC, 3f) The pumping station is 800 m away from the substation. It is connected via a 3f line and has a power input of 15 kW at cosφ = 0.82 and a nominal voltage of 400 V. AlFe6 conductors with a cross-section of 50 mm2 and a distance of 40 cm are used. Determine the voltage drop.

- from **tab. 6.1** pro AlFe6, S = 50 mm² a d = 40 cm:

R_K = 0,69 Ω/km, X_K = 0,291 Ω/km

- for cosφ = 0,82 is **tgφ = 0,696**

⇓

%

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Calculation of ΔU for open line with 1 consumption at the end – example 6-4 (2)

- Voltage drop in **actual units:**

$$\Delta U = \frac{I \cdot P}{U} \cdot (R_K + X_K \cdot tg\varphi) =$$
$$= \frac{0{,}8 \cdot 15000}{400} \cdot (0{,}69 + 0{,}291 \cdot 0{,}696) = 26{,}78 \text{ V}$$

- Voltage drop **in %**

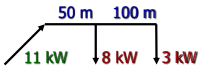
$$\Delta u\% = \frac{\Delta U}{U} \cdot 100 = \frac{26{,}78 \cdot 100}{400} = 6{,}69\ \%$$

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Calculation of ΔU for open lines with multiple consumption – example 6-5

- Calculate the voltage drop in the **DC line** with the voltage 220 V (see **Fig. 6-6**). The wires are aluminum (ρ_{Al} = 0,0359 μΩm) with cross-section 25 mm^2.

Fig. 6-6:



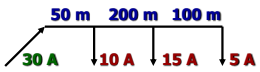
$$\Delta U = \frac{2 \cdot \rho \cdot \Sigma P \cdot l}{S \cdot U} = \frac{2 \cdot 0{,}0359 \cdot (8 \cdot 50 + 3 \cdot 150) \cdot 10^3}{25 \cdot 220} = 11{,}1 \text{ V}$$

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Dimensioning according to ΔU (open line with multiple consumption) – example 6-6

- Determine the cross-section of the **DC** copper line with the consumption according to **Fig. 6-7** if the voltage is 220 V and the allowed voltage drop is 5%. (ρCu = 0.0217μΩm)

Fig. 6-7:



$$\Sigma Il = 10 \cdot 50 + 15 \cdot 250 + 5 \cdot 350 = 6000 \text{ Am}$$

$$\Delta u\% = \frac{200 \cdot \rho \cdot \Sigma I \cdot l}{S \cdot U} \rightarrow S = \frac{200 \cdot \rho \Sigma Il}{\Delta u\% \cdot U} = \frac{200 \cdot 0{,}0217 \cdot 6000}{5 \cdot 220} = 23 \text{ mm}^2$$

- we select the cross-section **25 mm²**

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Dimensioning according to ΔU (open line with multiple consumption) – example 6-7 (1)

- Design the cross-section of the AlFe 6 line according to **fig. 6-8** so that at a voltage of **3** x 400 V and cosφ = 0.78 the voltage drop does not exceed 5%. The wires are arranged in an equilateral triangle of 500 mm side.

Fig. 6-8:



$$\Delta u\% = \frac{100 \cdot \Sigma Pl}{U^2}(R_K + X_K tg\varphi)$$

$$(R_K + X_K tg\varphi) = \frac{\Delta u\% \cdot U^2}{100 \cdot \Sigma Pl}$$

$$tg\varphi = 0{,}8 \ (\leftarrow \cos\varphi = 0{,}78)$$

$$\Sigma Pl = 20 \cdot 150 + 15 \cdot 350 = 8250 \text{ kWm} = 8250 \text{ Wkm}$$

$$R_K + X_K \cdot 0{,}8 = \frac{5 \cdot 400^2}{100 \cdot 8250} = 0{,}97 \ \Omega/\text{m}$$

%

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Dimensioning according to ΔU (open line with multiple consumption) – example 6-7 (2)

- We solve experimentally:
  - for S = 50 mm²: R_K = 0,69 Ω/km, X_K = 0,305 Ω/km
    0,69 + 0,305.0,8 = 0,934 < 0,97
    ↓
    compliant
  - for S=35 mm²: R_K = 0,97 Ω/km, X_K = 0,316 Ω/km
    0,97 + 0,316.0,8 = 1,223 > 0,97
    ↓
    not compliant

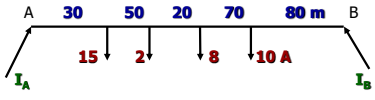
=> we select the cross-section 50 mm²

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Calculation of ΔU for power supply from 2 sides – examples 6-8 (1)

- Calculate the largest voltage drop in the aluminum (ρ_{Al} = 0,0359 μΩm) **DC** line (see **Fig. 6-10**) of cross-section 10 mm², if:
  a/ the same voltage **220 V** is at the supply points
  b/ $U_A \neq U_B$: **U_A=231 V, U_B=220 V**

Fig. 6-10:



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Calculation of ΔU for power supply from 2 sides – examples 6-8 (2)

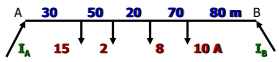
ad a) $U_A = U_B = 220$ V

1) **Supply currents** $\sum I_{(consumption)} = 35$ A

- from moment to point A

$$I_B \cdot 250 = 10 \cdot 170 + 8 \cdot 100 + 2 \cdot 80 + 15 \cdot 30 = 3110 \text{ Am}$$

$$I_B = \frac{3110}{250} = 12{,}44 \text{ A}$$



- from moment to point B

$$I_A \cdot 250 = 15 \cdot 220 + 2 \cdot 170 + 8 \cdot 150 + 10 \cdot 80 = 5640 \text{ Am}$$

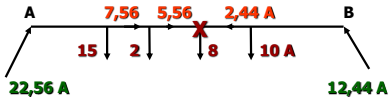
$$I_A = \frac{5640}{250} = 22{,}56 \text{ A}$$

- check: $I_B = \sum I - I_A = 35 - 22{,}56 = 12{,}44$ A

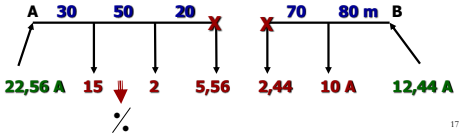
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Calculation of ΔU for power supply from 2 sides – examples 6-8 (3)

2) **Determination of X point with maximum voltage drop**



3) **Calculation of ΔU from divided line**



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Calculation of ΔU for power supply from 2 sides – examples 6-8 (4)

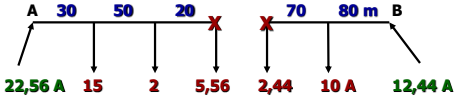
3) **Calculation of ΔU from divided line – contin.**

$$\sum_A^X Il = 15 \cdot 30 + 2 \cdot 80 + 5{,}56 \cdot 100 = 1166 \text{ Am}$$

$$\sum_B^X Il = 2{,}44 \cdot 150 + 10 \cdot 80 = 1166 \text{ Am}$$

$$\Delta U = \frac{2\rho \sum Il}{S} = \frac{2 \cdot 0{,}0359 \cdot 1166}{10} = 8{,}37 \text{ V}$$

$$\Delta u\% = \frac{100 \cdot \Delta U}{U} = \frac{837}{220} = 3{,}8\ \%$$



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Calculation of ΔU for power supply from 2 sides – examples 6-8 (5)

ad b) $U_A \neq U_B$: U_A=231 V, U_B=220 V

4) The equalizing current

$$I_0 = \frac{U_A - U_B}{R_V} = \frac{231 - 220}{1{,}8} = 6{,}1 \text{ A}$$

$$R_V = \frac{2 \cdot \rho \cdot l}{S} = \frac{2 \cdot 0{,}0359 \cdot 250}{10} = 1{,}8\ \Omega$$

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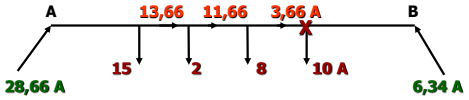
Calculation of ΔU for power supply from 2 sides – examples 6-8 (6)

5)

- New supply currents

$$I_A' = I_A + I_0 = 22{,}56 + 6{,}1 = 28{,}66 \text{ A}$$
$$I_B' = I_B - I_0 = 12{,}44 - 6{,}1 = 6{,}34 \text{ A}$$

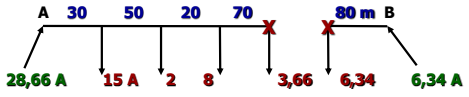
- Shift of the X point of greatest voltage drop



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Calculation of ΔU for power supply from 2 sides – examples 6-8 (7)

- Calculation of ΔU from devided line



$$\sum_{A}^{X} Il = 3{,}66 \cdot 170 + 8 \cdot 100 + 2 \cdot 80 + 15 \cdot 30 = 2032{,}2 \text{ Am}$$

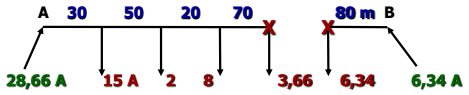
$$\Delta U_{AX} = \frac{2 \cdot 0{,}0359 \cdot 2032{,}2}{10} = 14{,}59 \text{ V} \ (\approx 14{,}6 \text{ V})$$

$$\Delta u\% = \frac{100 \cdot \Delta U_{AX}}{U} = \frac{1459}{220} = 6{,}63\%$$

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Calculation of ΔU for power supply from 2 sides – examples 6-8 (8)

- **Calculation of ΔU from devided line**



$$\sum_{B}^{X} Il = 6{,}34 \cdot 80 = 507{,}2 \text{ Am}$$

$$\Delta U_{BX} = \frac{2 \cdot 0{,}0359 \cdot 507{,}2}{10} = 3{,}64 \text{ V} \; (\approx 3{,}6 \text{ V})$$

$$\Delta u\% = \frac{100 \cdot \Delta U_{BX}}{U} = \frac{364}{220} = 1{,}65\%$$

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Calculation of ΔU for power supply from 2 sides – examples 6-8 (9)

- **Note.:**

$$\Delta U_{AX} - \Delta U_{BX} = 11 \text{ V}$$
$$U_A - U_B = 11 \text{ V}$$

- can be directly determined

$$\Delta U_{BX} = \Delta U_{AX} - (U_A - U_B) = 14{,}59 - 11 = 3{,}59 \text{ V} \; (\approx 3{,}6 \text{ V})$$

- at **X** point is the same voltage
  - $U_A - \Delta U_{AX}$ = 231 - 14,6 = 216,4 V
  - $U_B - \Delta U_{BX}$ = 220 - 3,6 = 216,4 V

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