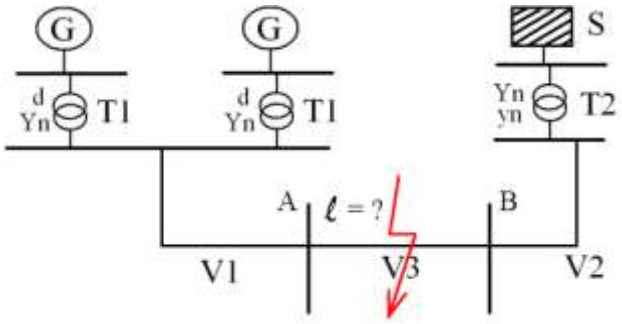
Example: Determine, how far from bus A on power line V3 is the place, where the 3-phase short-circuit current has the smallest value. Calculate with accuracy on 5 decimal places, in physical units, round the final distance on tens of meters. Network parameters are as follows:

G: $x_d''^{(1)} = 20\%$, $U_n = 15$ kV, $S_n = 100$ MVA;
T1: $u_k = 10\%$, $U_n = 15$ kV, $S_n = 200$ MVA, p = 15/110 kV; V1: X = 0.4 Ω/km, $\ell = 60$ km;
V2: X = 0.38 Ω/km, $\ell = 60$ km; T2: $u_k = 12\%$, $U_n = 220$ kV, $S_n = 250$ MVA, p = 220/110 kV;
V3: X = 0.35 Ω/km, $\ell = 80$ km; S: c = 1.1, $U_n = 220$ kV, $I_k'' = 30$ kA.

Hint: Express the total short-circuit impedance of the network as the function of parameter *k*, which states the distance of short-circuit location from bus A (e.g. k = 0 ... short circuit is in bus A, k = 1 ... short circuit is in bus B). By using the differential analysis, determine the maximum of this function and its value. For this maximum short-circuit impedance, there will be minimum 3-phase short-circuit current value.



Calculation:

$$X_G = \frac{x_d^{(1)''}}{100}\frac{U_n^2}{S_n}\frac{1}{\left(\frac{15}{110}\right)^2} = 24.2\,\Omega \quad X_{T1} = \frac{u_k}{100}\frac{U_n^2}{S_n}\frac{1}{\left(\frac{15}{110}\right)^2} = 6.05\,\Omega$$

$$X_{V1} = X\ell = 24\,\Omega \quad X_{V2} = X\ell = 22.8\,\Omega \quad X_{V3} = X\ell = 28\,\Omega$$

$$X_{T2} = \frac{u_k}{100}\frac{U_n^2}{S_n}\frac{1}{\left(\frac{220}{110}\right)^2} = 5.808\,\Omega \quad X_S = \frac{cU_n}{\sqrt{3}I_K''}\frac{1}{\left(\frac{220}{110}\right)^2} = 1.16432\,\Omega$$

Total left short-circuit impedance and total right short-circuit impedance (incl. coefficient *k*) is:

$$X_{CL} = \frac{X_G + X_{T1}}{2} + X_{V1} + kX_{V3} \quad X_{CP} = X_S + X_{T2} + X_{V2} + (1-k)X_{V3}$$

Final functions $Z_{k3f}'' = f(k)$ is as follows:

$$Z''_{k3f} = \frac{X_{CL}X_{CP}}{X_{CL}+X_{CP}} = \frac{\left(\frac{X_G+X_{T1}}{2}+X_{V1}+kX_{V3}\right)\left(X_S+X_{T2}+X_{V2}+(1-k)X_{V3}\right)}{\frac{X_G+X_{T1}}{2}+X_{V1}+X_{V3}+X_S+X_{T2}+X_{V2}}$$

By derivation of this function according to variable k and setting it to zero:

$$\frac{dZ''_{k3f}}{dk} = \frac{X_{V3}\left(X_S+X_{T2}+X_{V2}+(1-k)X_{V3}\right)-\left(\frac{X_G+X_{T1}}{2}+X_{V1}+kX_{V3}\right)X_{V3}}{\frac{X_G+X_{T1}}{2}+X_{V1}+X_{V3}+X_S+X_{T2}+X_{V2}} = 0$$

After a few steps we get:

$$X_S+X_{T2}+X_{V2}+(1-k)X_{V3} = \frac{X_G+X_{T1}}{2}+X_{V1}+kX_{V3}$$

Now we enter all the values we have already calculated and we get the value of k:

$$k = \frac{X_S+X_{T2}+X_{V2}+X_{V3}-\frac{X_G+X_{T1}}{2}-X_{V1}}{2X_{V3}} = 0{,}33299 \quad (\approx 26{,}64\,\text{km})$$

By entering values 0.33 and (1-0.33), and 0.34 and (1-0.34) we can verify that this value of k corresponds with the maximum short-circuit impedance value.

$$Z''_{k3fmax} = \frac{\left(\frac{X_G+X_{T1}}{2}+X_{V1}+0{,}33299X_{V3}\right)(X_S+X_{T2}+X_{V2}+(1-0{,}33299)X_{V3})}{\frac{X_G+X_{T1}}{2}+X_{V1}+X_{V3}+X_S+X_{T2}+X_{V2}}$$

$$= \underline{24{,}22433\ \Omega}$$

$$\rightarrow I''_{k3fmin} = \frac{1{,}1\times 110}{\sqrt{3}Z''_{k3fmax}} = \underline{2{,}884\ \text{kA}}$$

Minimum 3-phase short-circuit current is 2.884 kA and can occur some 26.64 km from bus A towards bus B.