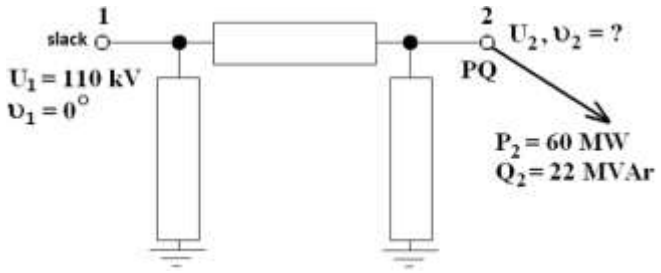
Example: Solve the load flow problem (see figure below), i.e. calculate the voltage phasor in bus no. 2 by using the G-S method (first 2 iterations) - version A, by using the N-R method (only 1 iteration) - version B.



Line parameters: R = 0.156 Ω/km, X = 0.4 Ω/km, B = $3 \cdot 10^{-6}$ S/km, l = 100 km.

Calculation:

$$\overline{Y}_{12} = \frac{1}{(R + jX)l} = 0.0084628 - j0.0217\,S$$

$$\overline{Y}_{120} = jB\frac{l}{2} = j0.00015\,S$$

$$\underline{\overline{A}} = \begin{bmatrix} \overline{Y}_{12} + \overline{Y}_{120} & -\overline{Y}_{12} \\ -\overline{Y}_{12} & \overline{Y}_{12} + \overline{Y}_{120} \end{bmatrix} = \begin{bmatrix} 0.0084628 - j0.02155 & -0.0084628 + j0.0217 \\ -0.0084628 + j0.0217 & 0.0084628 - j0.02155 \end{bmatrix} S$$

Version A - the G-S method:

$$\overline{U}_2^{(p+1)} = \frac{1}{\overline{A}_{22}}\left[\frac{P_2 - jQ_2}{\overline{U}_2^{(p)*}} - \overline{A}_{21}\overline{U}_1\right]$$

For first iteration:

$$\overline{U}_2^{(1)} = \frac{1}{0.0084628 - j0.02155}\left[\frac{(-60 + j22)\times 10^6}{110000} - (-0.0084628 + j0.0217)\times 110000\right]$$
$$= 95918.04754\angle -11°26'V$$

For second iteration:

$$\overline{U}_2^{(2)} = \frac{1}{0.0084628 - j0.02155}\left[\frac{(-60 + j22)\times 10^6}{95918.04754\angle 11°26'} - (-0.0084628 + j0.0217)\times 110000\right]$$
$$= 89417.65894\angle -11°20'V$$

Version B - the N-R method:

Initial guess: $U_2 = 110000$, $\theta_2 = 0°$

Mismatches for first iteration:

$$\Delta P_2 = P_2 - U_2\left[U_1\left(G_{21}\cos\theta_{21} + B_{21}\sin\theta_{21}\right) + U_2\left(G_{22}\cos\theta_{22} + B_{22}\sin\theta_{22}\right)\right] =$$
$$= P_2 - U_2\left[G_{21}U_1 + G_{22}U_2\right] = -60\times10^6$$
$$\Delta Q_2 = Q_2 - U_2\left[U_1\left(G_{21}\sin\theta_{21} - B_{21}\cos\theta_{21}\right) + U_2\left(G_{22}\sin\theta_{22} - B_{22}\cos\theta_{22}\right)\right] =$$
$$= Q_2 - U_2\left[-B_{21}U_1 - B_{22}U_2\right] = -20.185\times10^6$$

Jacobi matrix:

$$\underline{J} = \begin{bmatrix} H_{22} & N_{22} \\ J_{22} & L_{22} \end{bmatrix} = \begin{bmatrix} -Q_2 - B_{22}U_2^2 & P_2 + G_{22}U_2^2 \\ P_2 - G_{22}U_2^2 & Q_2 - B_{22}U_2^2 \end{bmatrix} = \begin{bmatrix} 282.755\times10^6 & 42.39988\times10^6 \\ -162.39988\times10^6 & 238.755\times10^6 \end{bmatrix}$$

Set of equations:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \underline{J}\begin{bmatrix} \Delta\theta \\ \Delta U/U \end{bmatrix}$$

After putting all the values inside:

$$\begin{bmatrix} -60\times10^6 \\ -20.185\times10^6 \end{bmatrix} = \begin{bmatrix} 282.755\times10^6 & 42.39988\times10^6 \\ -162.39988\times10^6 & 238.755\times10^6 \end{bmatrix}\begin{bmatrix} \Delta\theta \\ \Delta U/U \end{bmatrix}$$

Solution (e.g. by using Cramer's rule):

$$\Delta\theta = -0.18105\ rad,\ \Delta U/U = -0.20797 \rightarrow \theta_2 = -0.18105\ rad\ (-10°22'), U_2 = 87123.3V.$$