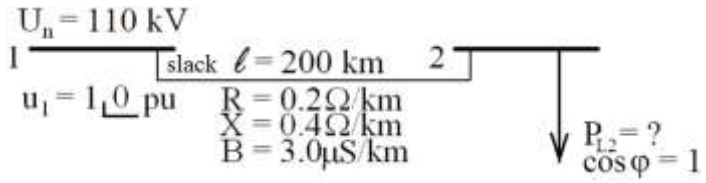
Example: As known, Feranti's phenomenon is the situation when $U_1 < U_2$. It occurs in three cases: a) power line is loaded by capacitive load, b) power line is not loaded, c) power line is loaded by inductive load close to no-load state (our case). For simple 2-bus network below determine the limiting active power load of the consumer in bus no. 2 with power factor = 1, from which the Feranti's phenomenon starts to occur. Do the calculations analytically, in per units (S_V = 100 MVA), with the accuracy on 5 decimal places.



Calculation:

Passive parameters of the power line - preparation of admittances in per units:

$$r = 0.2 \cdot 200 \cdot \frac{100}{110^2} = 0.33058 \text{ pu} \quad x = 0.4 \cdot 200 \cdot \frac{100}{110^2} = 0.66116 \text{ pu} \quad b = 3.0 \cdot 10^{-6} \cdot 200 \cdot \frac{110^2}{100} = 0.0726 \text{ pu}$$

$$y_{12} = \frac{1}{r + jx} = 0.605 - j1.21 \text{ pu} \quad y_{120} = j\frac{b}{2} = j0.0363 \text{ pu}$$

Load flow equations in the matrix form:

$$\begin{bmatrix} \dfrac{p_{ref} - jq_{ref}}{\bar{u}_1^*} \\ \dfrac{-p_{L2}}{\bar{u}_2^*} \end{bmatrix} = \begin{bmatrix} 0.605 - j1.1737 & -0.605 + j1.21 \\ -0.605 + j1.21 & 0.605 - j1.1737 \end{bmatrix} \begin{bmatrix} \bar{u}_1 \\ \bar{u}_2 \end{bmatrix}$$

Second equation can be re-written:

$$\frac{-p_{L2}}{\bar{u}_2^*} = (-0.605 + j1.21)\bar{u}_1 + (0.605 - j1.1737)\bar{u}_2$$

$$-p_{L2} = (-0.605 + j1.21)\bar{u}_1\bar{u}_2^* + (0.605 - j1.1737)u_2^2$$

For Feranti's phenomenon applies $u_1 = u_2$, therefore:

$$-p_{L2} = (-0.605 + j1.21)(\cos\theta_2 - j\sin\theta_2) + 0.605 - j1.1737$$

Separation on real and imaginary parts:

$$\begin{aligned} -p_{L2} &= -0.605\cos\theta_2 + 1.21\sin\theta_2 + 0.605 \\ 0 &= 1.21\cos\theta_2 + 0.605\sin\theta_2 - 1.1737 \end{aligned} \qquad (1)$$

Second equation can be analyzed for unknown θ_2:

$$0.605\sin\theta_2 = 1.1737\text{-}1.21\cos\theta_2 \quad /^2$$

$$0.366025\text{-}0.366025\cos^2\theta_2 = 1.37757169\text{-}2.840354\cos\theta_2 + 1.4641\cos^2\theta_2$$

$$1.830125\cos^2\theta_2\text{-}2.840354\cos\theta_2 + 1.01154669 = 0.$$

$$\rightarrow \cos\theta_{2_1} = 0.99838705 \quad \rightarrow \quad \sin\theta_2 = \frac{1.1737\text{-}1.21\cos\theta_2}{0.605} = -0.0567741$$

$\cos\theta_{2_2} = 0.55361295$ nonsense!!! it must be close to 0!

By entering the values for $\cos\theta_2$ and $\sin\theta_2$ into the first equation of (1), we get the limiting active power load:

$$p_{\text{L2lim}} = 0.605\cos\theta_2 - 1.21\sin\theta_2 - 0.605 = 0.067720826 \text{ pu} \approx 6.77 \text{ MW}.$$