Environmental friendly technologies
KEE/ENT

ESF II project of the University of West Bohemia in Pilsen

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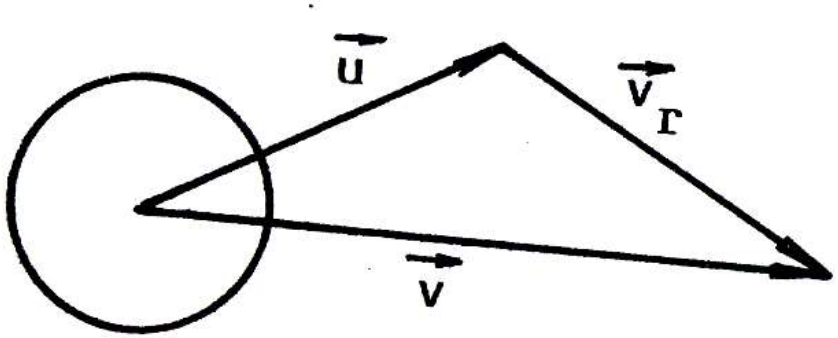
Lecture 2

Separation of solid particles – motion equation

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Motion equation



$$\vec{v}_r = \vec{v} - \vec{u}$$

$$\vec{F}_1 = M_č \frac{d\vec{u}}{dt} = \frac{\pi a^3}{6} \rho_č \frac{d\vec{u}}{dt}$$

$$M_p = V_p \rho = \frac{V_č}{2} \rho = \frac{\pi a^3}{12} \rho$$

$$\vec{F}_2 = M_p \frac{d(\vec{u} - \vec{v})}{dt} = \frac{\pi a^3}{12} \rho \left(\frac{d\vec{u}}{dt} - \frac{d\vec{v}}{dt} \right)$$

$$\vec{F}_1 + \vec{F}_2 = \vec{F}_\tau + \vec{F}_v + \vec{F}_e$$

$$\vec{F}_\tau = \xi \frac{\pi a^2}{4} \frac{|\vec{v} - \vec{u}|(\vec{v} - \vec{u})}{2} \rho$$

$$\vec{F}_v = -V_č \vec{\nabla} p$$

$$\frac{\pi a^3}{6} \rho_č \frac{d\vec{u}}{dt} + \frac{\pi a^3}{12} \rho \left(\frac{d\vec{u}}{dt} - \frac{d\vec{v}}{dt} \right) = \xi \frac{\pi a^2}{4} \frac{|\vec{v} - \vec{u}|(\vec{v} - \vec{u})}{2} \rho - \frac{\pi a^3}{6} \vec{\nabla} p + \vec{F}_e$$

Aerodynamical resistance of a particle

$$\xi = \frac{24}{\mathrm{Re}_r} \qquad \mathbf{Re_r} < 0{,}2$$

$$\mathrm{Re}_r = \frac{|\vec{v}_r| a\,\rho}{\eta}$$

$$\xi = \frac{24}{\mathrm{Re}_r}\left(1 + \frac{3}{16}\mathrm{Re}_r\right) \qquad \mathbf{Re_r} = 0{,}5$$

$$\xi = \frac{24}{\mathrm{Re}_r} + \frac{4}{\mathrm{Re}_r^{1/3}} = \frac{24}{\mathrm{Re}_r}\left(1 + \frac{1}{6}\mathrm{Re}_r^{2/3}\right) \qquad \mathbf{Re_r} = 0{,}2 \text{ až } 400$$

$$\vec{F}_\tau = \frac{24}{\mathrm{Re}_r}\frac{\pi a^2}{4}\frac{|\vec{v} - \vec{u}|(\vec{v} - \vec{u})}{2}\rho = 3\,\pi\,\eta a \vec{v}_r \qquad \psi = 1 \quad \mathbf{Re_r} < 0{,}2$$

$$\vec{F}_\tau = 3\,\pi\,\eta a_V\,\vec{v}_r\,\kappa_V \qquad \psi < 1 \quad \mathbf{Re_r} < 0{,}2$$

$$\kappa_V \doteq \frac{1}{\Psi^{1/2}}$$

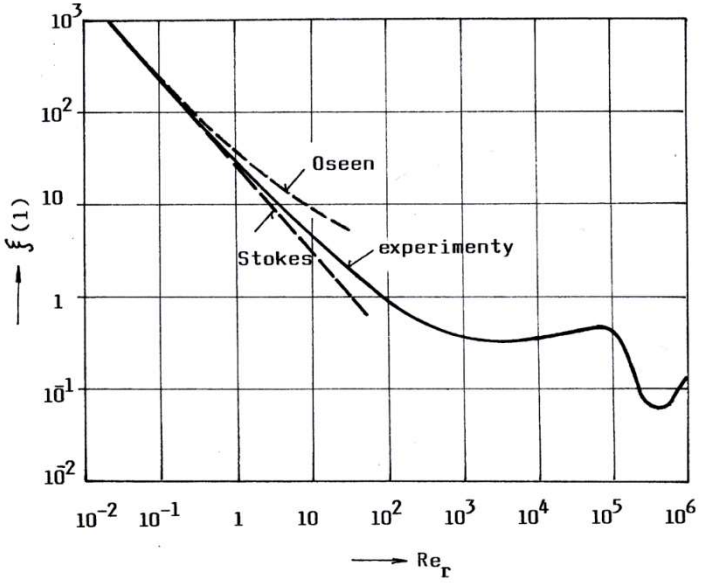
$$\vec{F}_\tau = \xi_n \frac{\pi a_V^2}{4}\frac{|\vec{v}_r|\vec{v}_r}{2}\rho \qquad \mathbf{Re_r} = 100.$$

$$\xi_n = \kappa_V \xi$$

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Aerodynamical resistance of a particle



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Size and shape of a particle

Microscopical size - Feret and Martin diameter

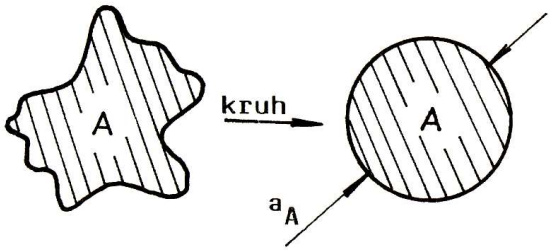


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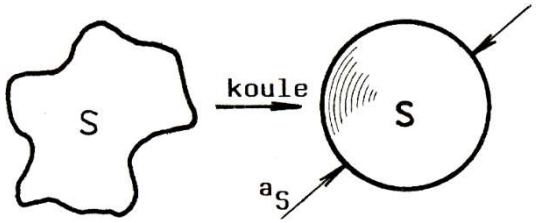
Size and shape of a particle

Equivalent size according to projection



$$a_A = \sqrt{\frac{4A}{\pi}}$$

Equivalent size according to surface

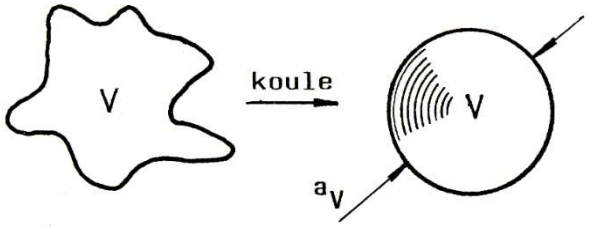


$$a_S = \sqrt{\frac{S}{\pi}}$$

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Equivalent size according to volume



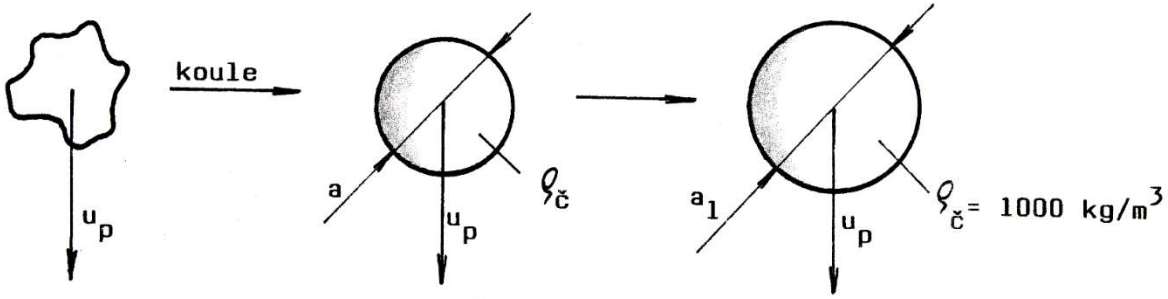
$$a_V = \sqrt[3]{\frac{6V}{\pi}}$$

Equivalent size according to surface and volume

$$\frac{V}{S} = \frac{a_V^3}{6a_S^2}$$

$$a_{VS} = \frac{a_V^3}{a_S^2}$$

Equivalent size according to sedimentation speed



$$a_1 = a\sqrt{\frac{\rho_č}{1000}}$$

Sieve size

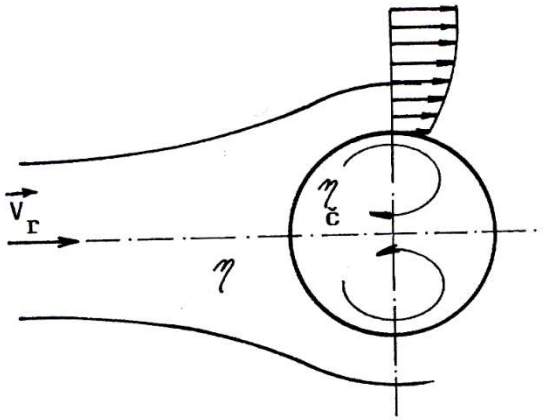
Sphericity

$$\psi = \frac{\pi a_V^2}{S} = \left(\frac{a_V}{a_s}\right)^2$$

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Aerodynamical resistance of a liquid particle



$$\vec{F}_\tau = \pi \eta a \vec{v}_r \frac{2\eta + 3\eta_č}{\eta + \eta_č}$$

$\eta_č \to \infty$ $\quad \vec{F}_\tau = \pi \eta a \vec{v}_r \cdot \lim_{\eta_č \to \infty} \frac{2\eta + 3\eta_č}{\eta + \eta_č} = 3\pi \eta a \vec{v}_r$

$\eta_č << \eta$ $\quad \vec{F}_\tau = \pi \eta a \vec{v}_r \cdot \lim_{\eta_č \to 0} \frac{2\eta + 3\eta_č}{\eta + \eta_č} = 2\pi \eta a \vec{v}_r$

$$\vec{F}_\tau = 3\pi \eta a \vec{v}_r \frac{1}{C}$$

$$C = 1 + 2\frac{l_m}{a}\left[1{,}257 + 0{,}400 \cdot \exp\left(-1{,}10\frac{a}{2\,l_m}\right)\right]$$

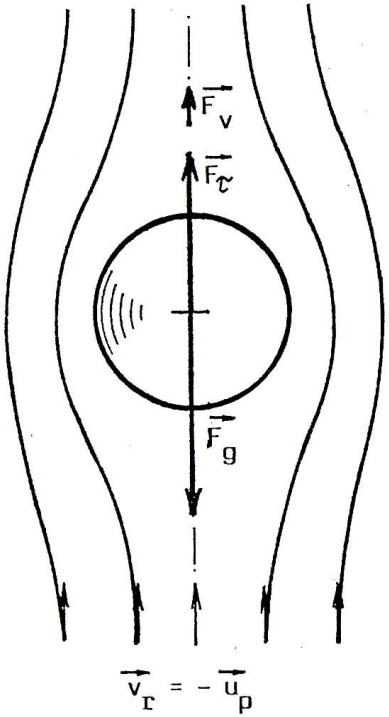
$\mathbf{l_m} = 0{,}065\ \mu m$

$$C \doteq 1 + K_C \frac{l_m}{a} \doteq 1 + 2{,}7\frac{l_m}{a}$$

$\mathbf{K_C} \doteq 2{,}7$

a (μm)	0,01	0,1	1	10
C (1) - vztah 1	22,119	2,857	1,163	1,016
C (1) - vztah 2	18,550	2,755	1,176	1,018

Straight constant movement



$$\vec{v}_r = -\vec{u}_p$$

$$0 = \vec{F}_\tau + \vec{F}_v + \vec{F}_g$$

$$\vec{F}_\tau = -\xi \frac{\pi a^2}{4} \frac{|\vec{u}_p|\vec{u}_p}{2} \rho$$

$$\vec{F}_v = -V_č \rho \vec{g}$$

$$\vec{F}_g = V_č \rho_č \vec{g}$$

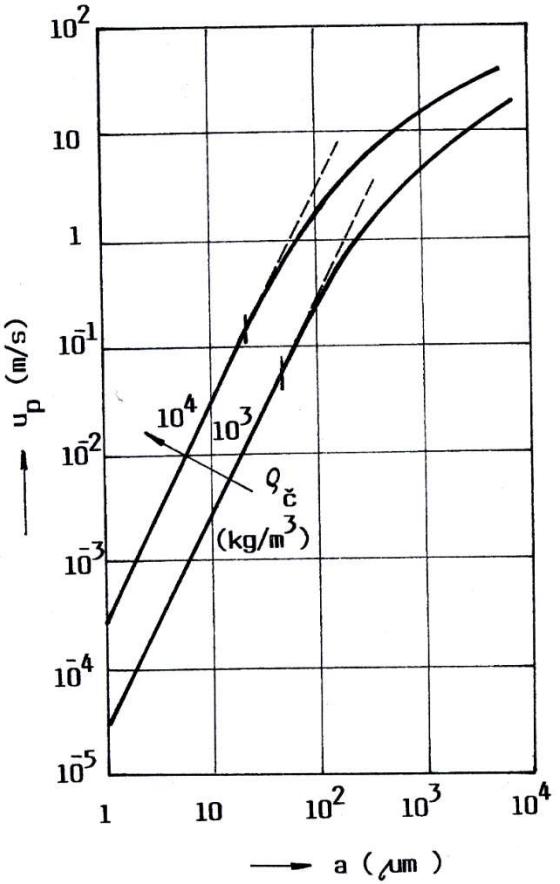
$$\xi \frac{\pi a^2}{4} \frac{u_p^2}{2} \rho = \frac{\pi a^3}{6} (\rho_č - \rho) g$$

$$u_p = \sqrt{\frac{4a(\rho_č - \rho)g}{3\xi\rho}}$$

$$\xi \mathrm{Re}_r^2 = \frac{4a^3(\rho_č - \rho)\rho g}{3\eta^2}$$

$$u_p = \frac{\mathrm{Re}_r \eta}{a\rho}$$

Straight constant movement



$\mathbf{Re_r} < 0{,}2$

$$\vec{F}_\tau = -3\pi\eta a\vec{u}_p$$

$\mathbf{Re_r} < 0{,}2$

$$3\pi\eta a u_p = \frac{\pi a^3}{6}(\rho_č - \rho)g$$

$$u_p = \frac{a^2(\rho_č - \rho)g}{18\eta}$$

$$\mathrm{Re}_r = \frac{u_p a\rho}{\eta}$$

$$\xi = \frac{4a(\rho_č - \rho)g}{3\rho u_p^2}$$

Definition of size from sedimentation speed

$$\frac{\mathrm{Re}_r}{\xi} = \frac{3u_p^3\rho^2}{4\,\eta(\rho_č - \rho)g}$$

$\mathbf{Re_r} < 0{,}2$

$$a = \sqrt{\frac{u_p 18\ \eta}{(\rho_č - \rho)g}}$$

$$\vec{F}_\tau = -3\,\pi\,\eta a\vec{u}_p$$

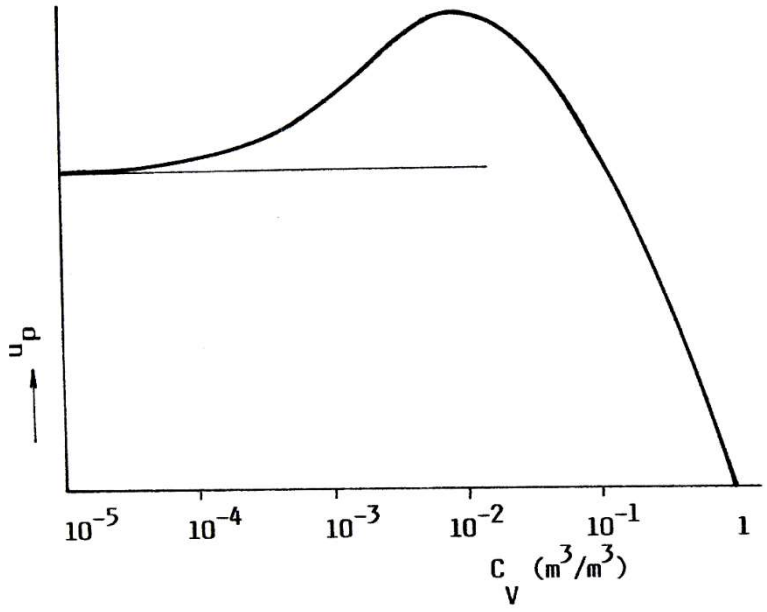
$$3\,\pi\,\eta a_V\,u_p\,\kappa_V = \frac{\pi a_V^3}{6}(\rho_č - \rho)g$$

$$u_p = \frac{a_V^2(\rho_č - \rho)g}{18\,\eta\kappa_V}$$

$$\kappa_V = \left(\frac{a_V}{a}\right)^2$$

$$\vec{F}_\tau \doteq -3\,\pi\,\eta a\vec{u}_p\left(1 + 0{,}28\frac{a}{s}\right)$$

Set of particles, nonhomogennous environment



s < a

$$\vec{F}_\tau \doteq -3\pi\eta a\vec{u}_p\left(1+0{,}28\frac{a}{s}\right)$$

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Thank you for your attention...

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