ESF II project of the University of West Bohemia in Pilsen

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Regenerative Rankine-Clausius cycle (Recuperative heating of feed water)

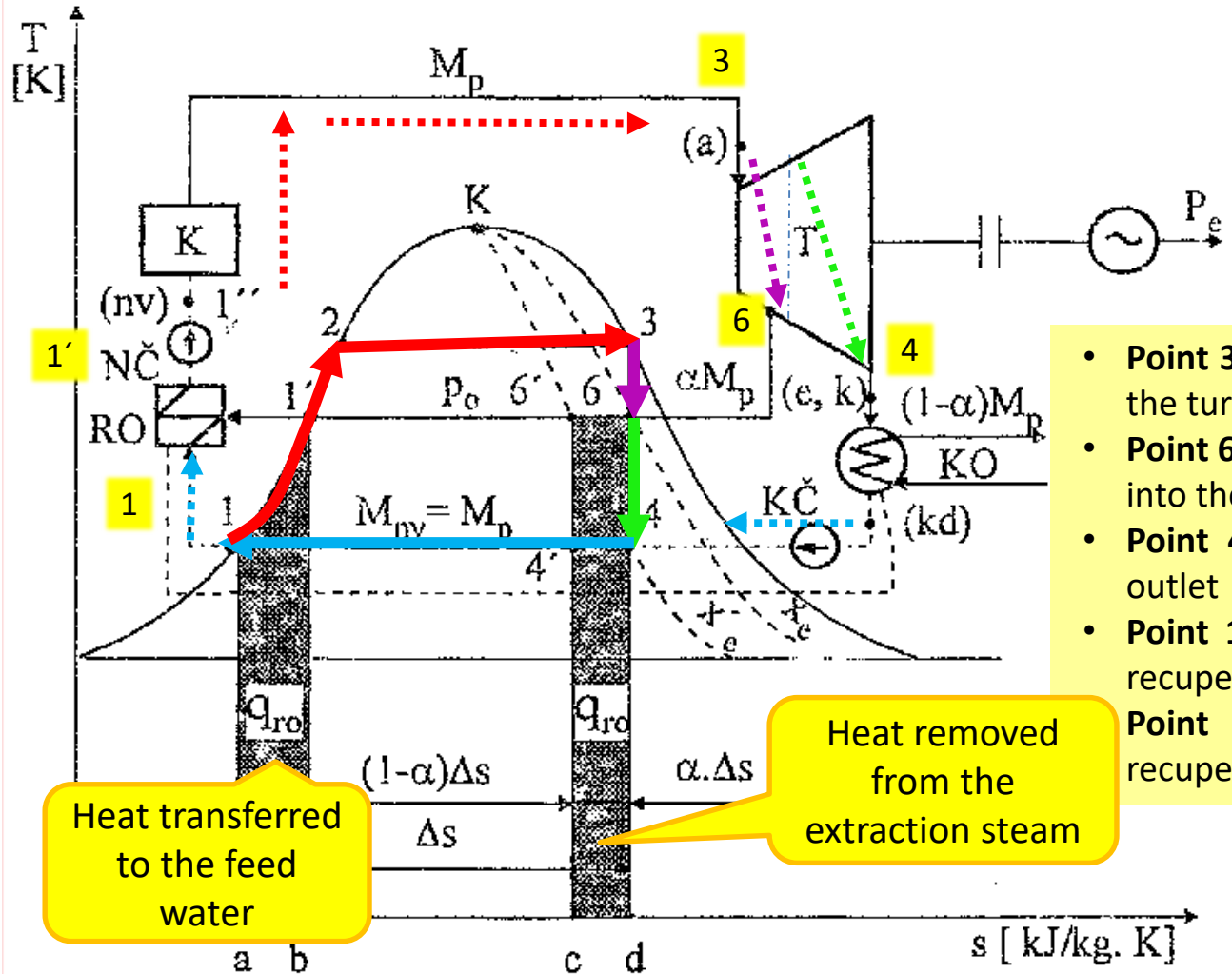
Exercises 5 (KEE/TTE)

EUROPEAN UNION
European Structural and Investment Funds
Operational Programme Research,
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MINISTRY OF EDUCATION, YOUTH AND SPORTS

Regenerative Rankine-Clausius cycle

Regeneration (Recuperation) = heating of feed water in the heat exchanger (regenerative heater or recuperative heater) by steam taken in the turbine (extraction steam)



- **Point 3** - input of the admission steam into the turbine
- **Point 6** - extraction of the part of the steam into the recuperation heater
- **Point 4** - emission steam at the turbine outlet
- **Point 1** - condensate before entering the recuperation heater

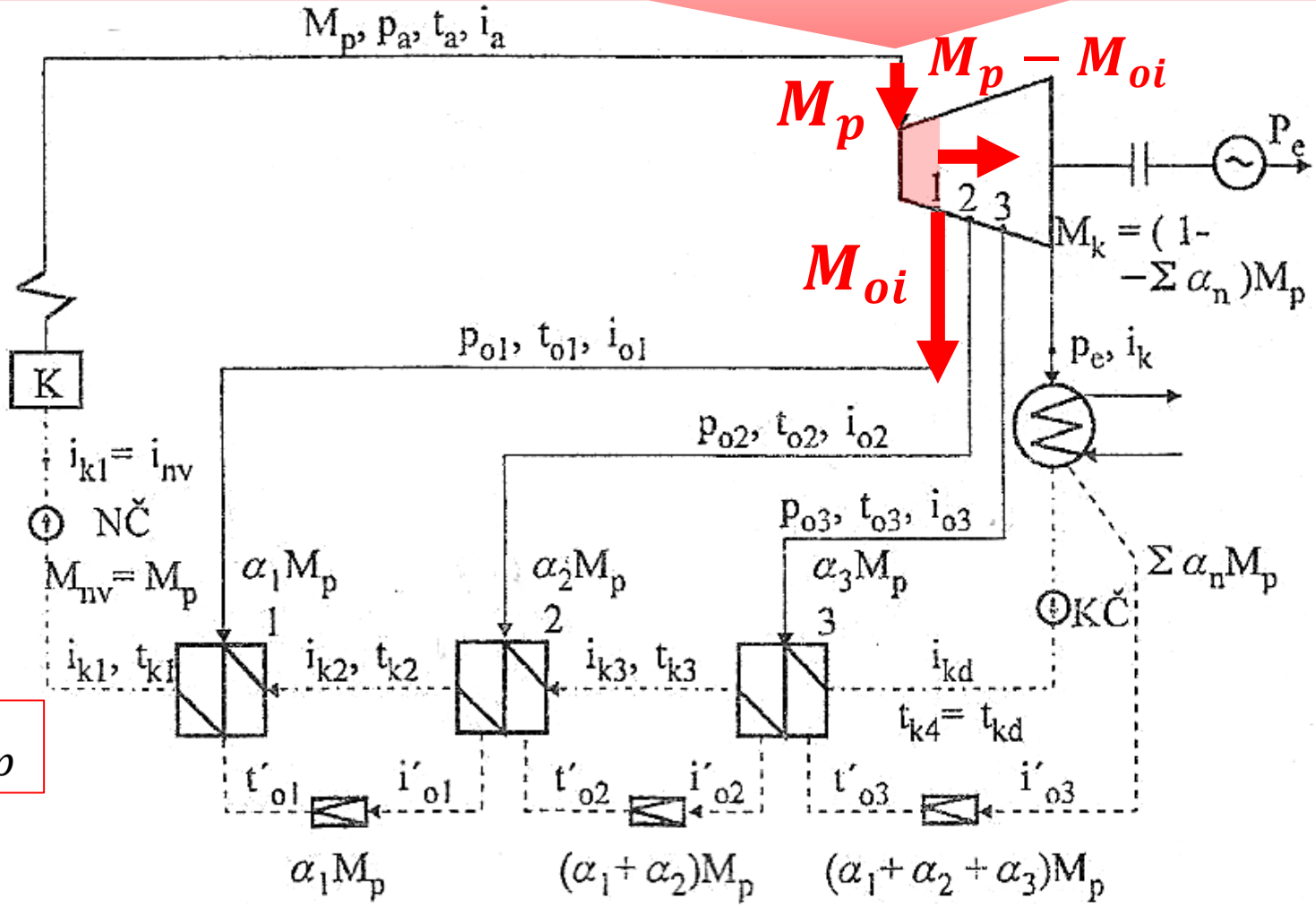
Point 1´ - condensate behind the recuperation heater

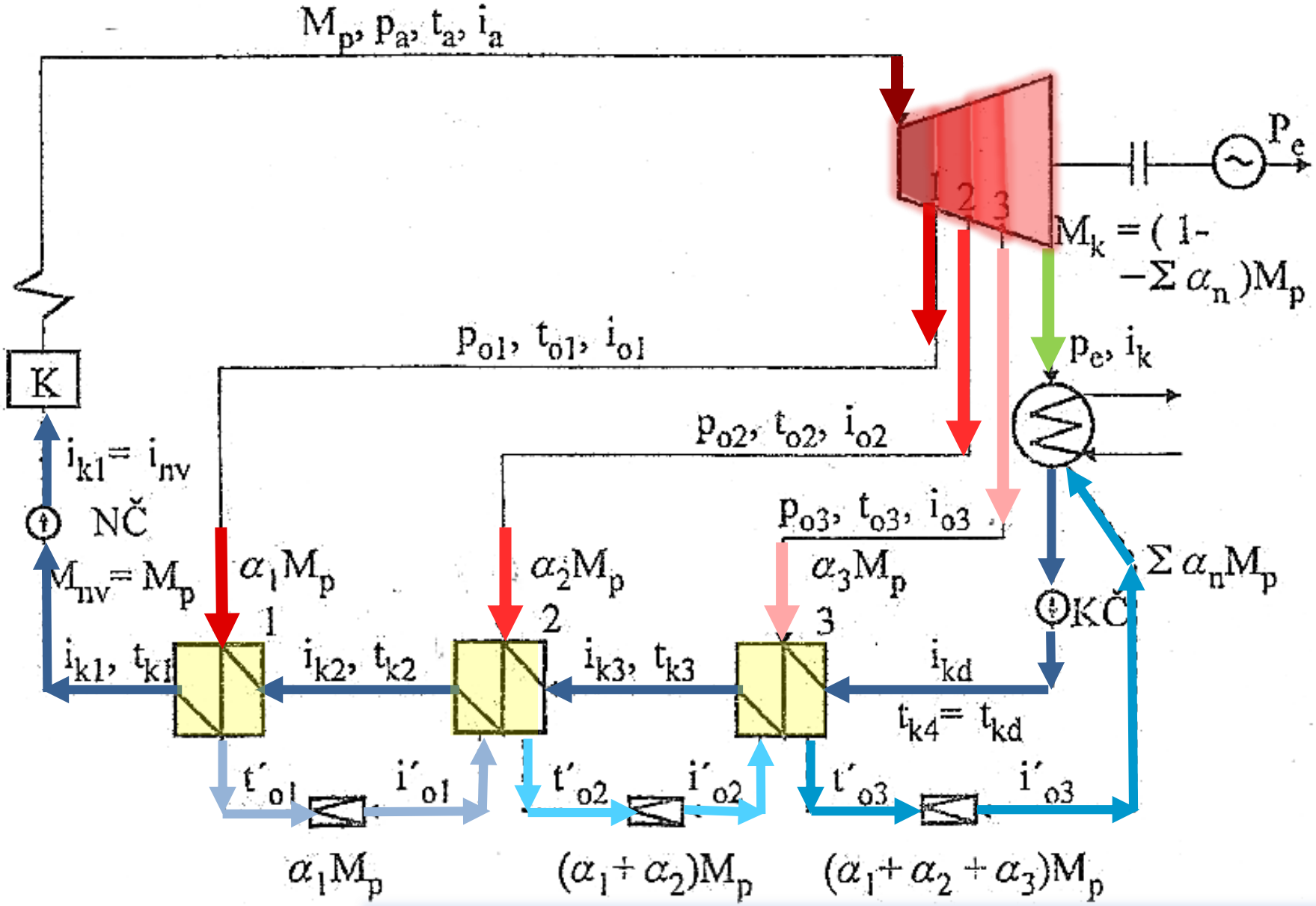
Regenerative Rankine-Clausius cycle

The amount of steam M_p expands in the turbine to the point where the steam is taken. From this point of the turbine, part of the steam (M_o) is fed to the recuperation heater. After the recuperative extraction of steam in the turbine, the remaining amount of steam is expanded until the next extraction, etc.

Relative amount of steam taken (extraction steam) in relation to the amount of steam at the turbine input

$$M_{oi} = \propto_i \cdot M_p$$

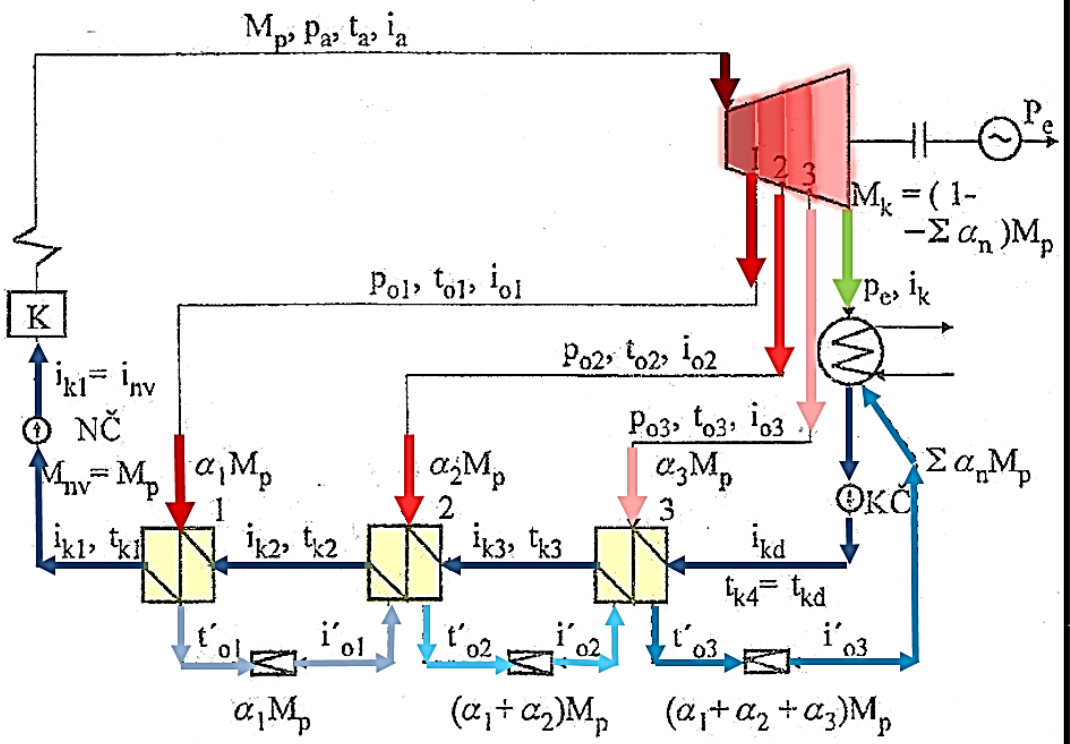


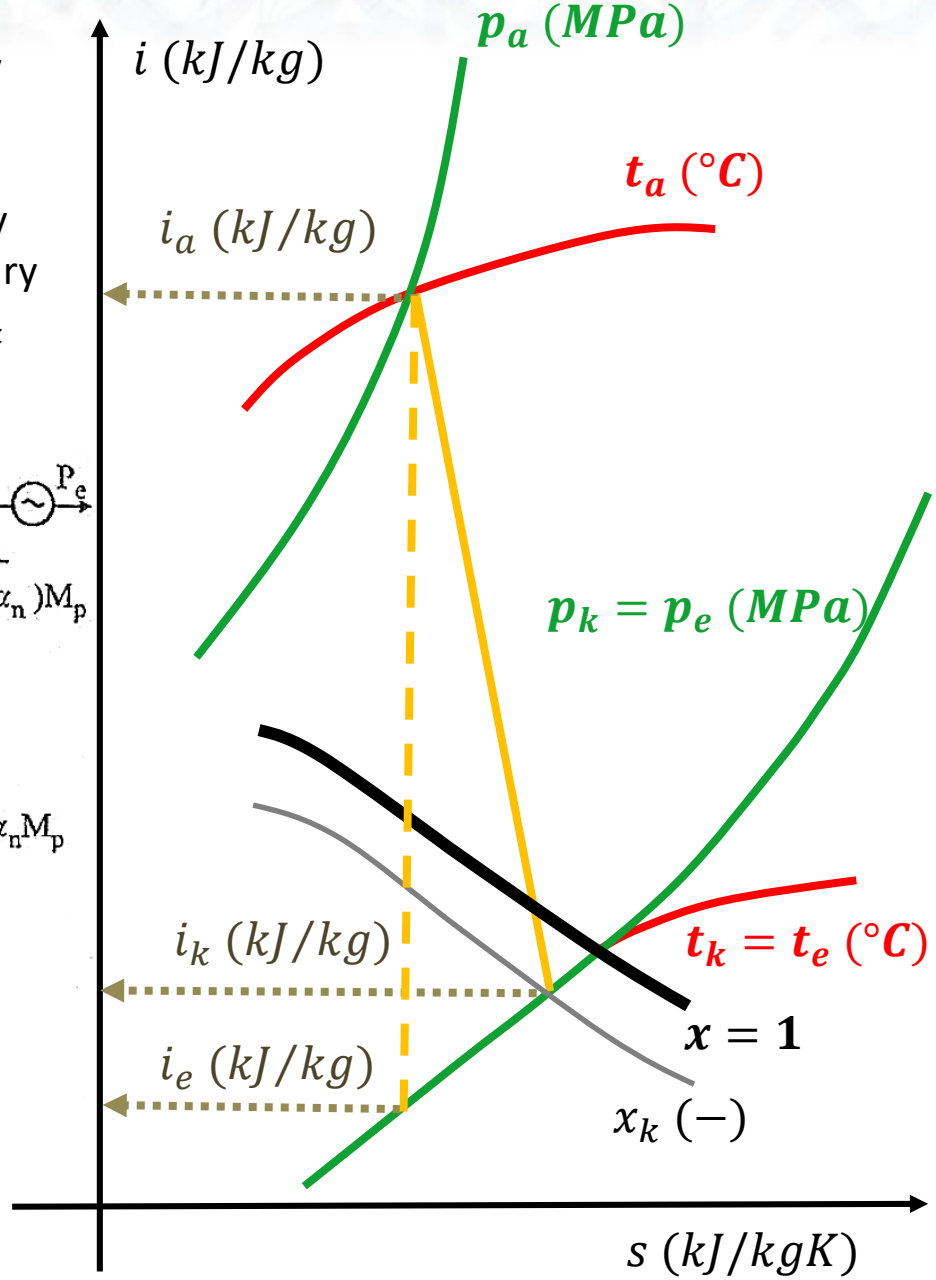


In the recuperative heater, the heating steam transfers its heat (superheat heat, condensing heat) to the feed water and its condensate is introduced into the condenser; so that the amount of feed water of value $M_p = M_{nv.}$ passes again through the recuperation heaters.

Procedure for plotting the thermal cycle in the i-s (h-s) diagram

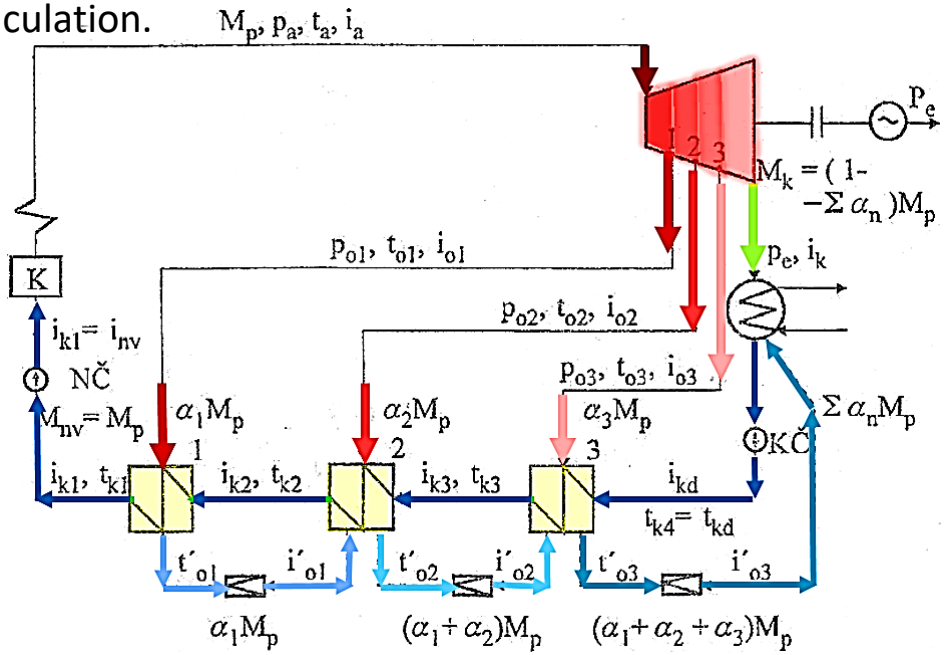
1. Izobar p_a ∩ isotherm t_a → $\boldsymbol{i_a}$
2. Perpendicular from p_a ∩ t_a towards the entropy axis, up to the isobar p_e (= isotherm t_e) → **ideal expansion curve** → $\boldsymbol{i_e}$
3. Calculation i_k from the thermodynamic efficiency or determination from the dryness value of the dry steam x_k ∩ t_k (p_k) → **real expansion curve** → $\boldsymbol{i_k}$

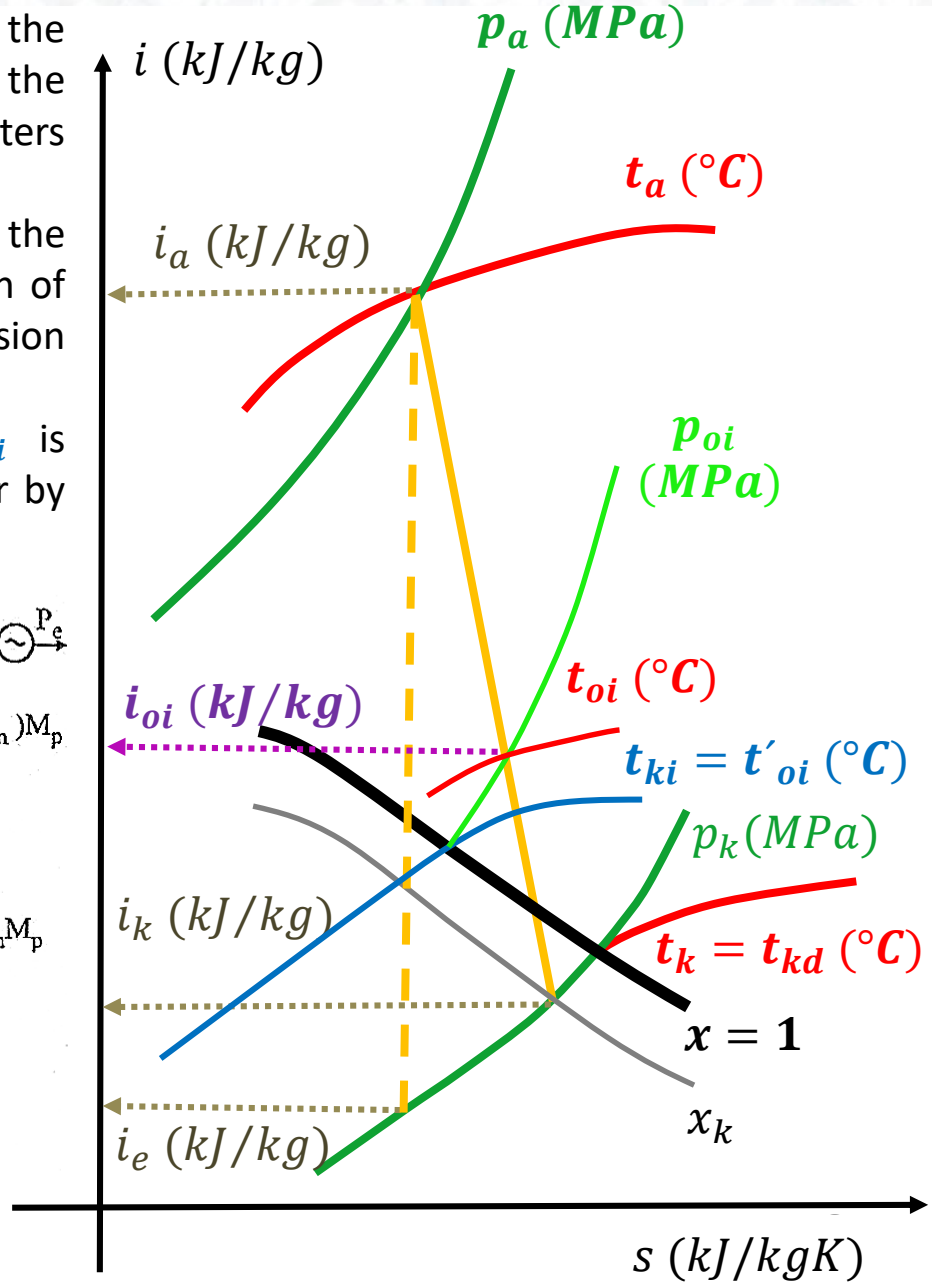




Procedure for plotting the thermal cycle in the i-s (h-s) diagram

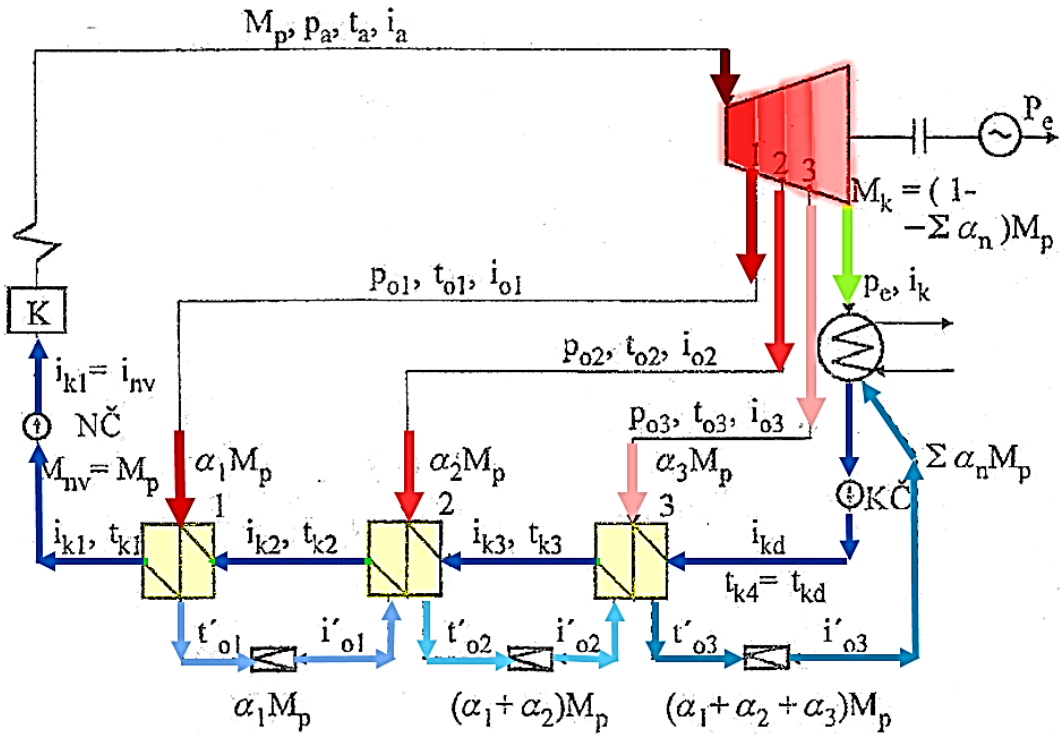
4. The pressure of the consumed heating steam (extraction steam) p_{oi} is always given by the isobar based on the intersections of the isotherms t_{ki} given by the temperature behind the individual recuperation heaters with the right limit curve, i.e. $x = 1 \cap t_{ki} \rightarrow (p_{oi})$.
5. The enthalpies of the extraction steam i_{oi} and the temperatures t_{oi} are obtained from the intersection of the isobar extraction steam p_{oi} with the real expansion curve.
6. The enthalpy of heating condensate (water) t'_{oi} is determined from steam tables (according to p_{oi}) or by calculation.

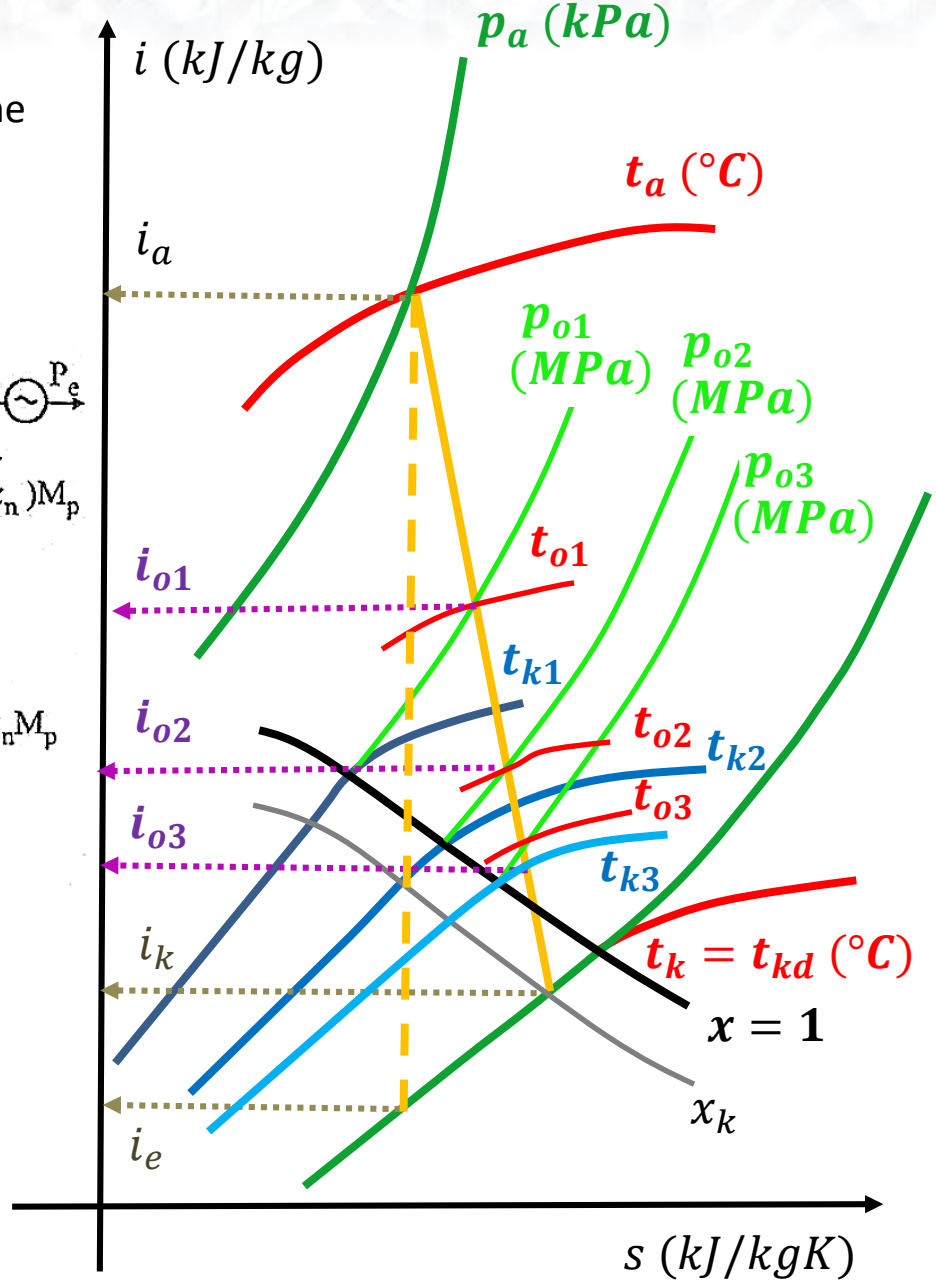




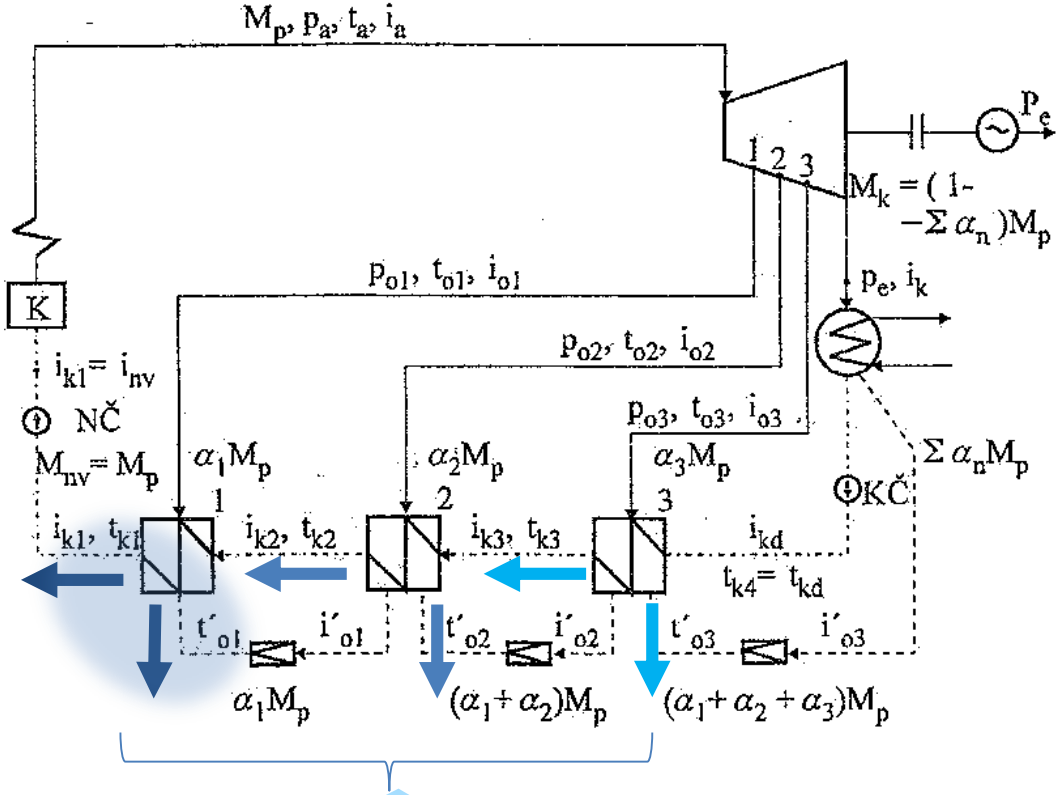
Procedure for plotting the thermal cycle in the i-s (h-s) diagram

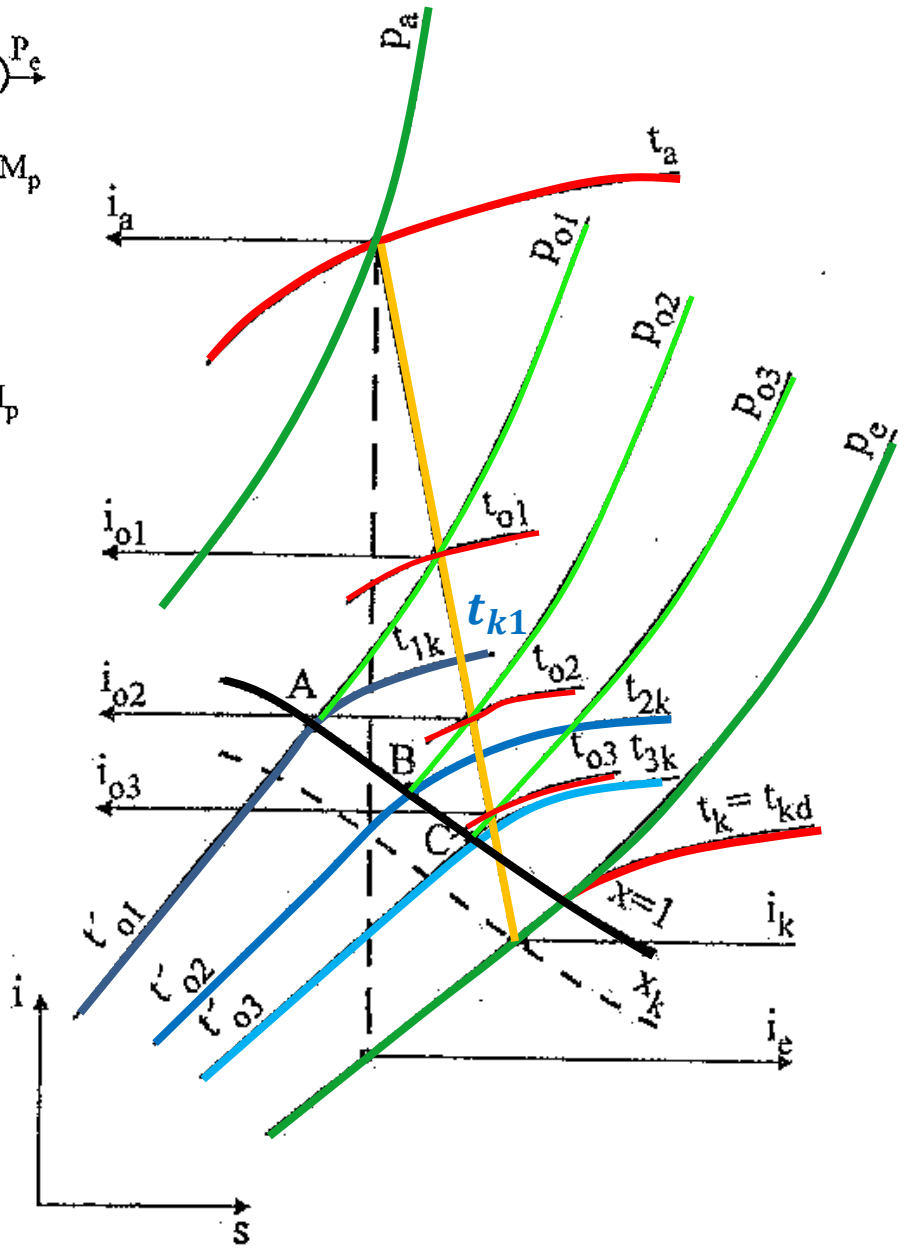
7. Display and reading of all steam extracts from the turbine in the i-s diagram





Regenerative Rankine-Clausius cycle





With ideal heat transfer (lossless), the temperature of the feed water and the temperature of the heating steam condensate at the output of the recuperation heater are the same.

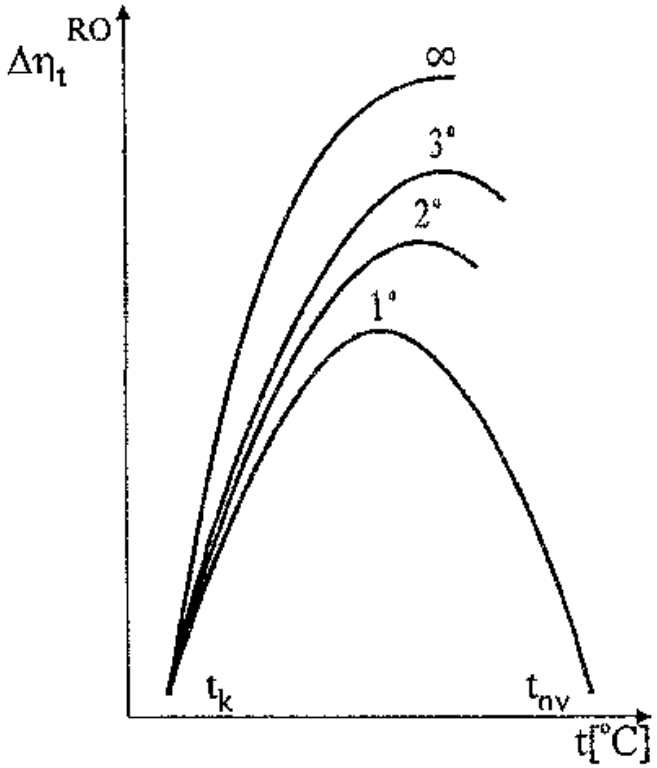
$$t_{ki} = t'_{oi}$$

Calculating the number of necessary regenerative heaters (RH)

- As the number of recuperation heaters increases, the thermal efficiency of the RC cycle increases.

Optimal feed water temperature

- In practice, for economic reasons, a **maximum of 8 recuperation heaters**
- The thermal efficiency gain is highest in a single stage regeneration
- **Multi-stage recuperation** - the highest efficiency if the heating of the feed water in each regenerative heater would the same
- Heating in **1 stage should not be greater than 40 °C**



Number of recuperative heaters (regenerative heaters)

$$t_{nv} = t_k + \frac{z}{z+1} \cdot (t_s - t_k)$$

Saturated steam temperature at pressure p_a

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Calculation of regenerative heating

Calculation of feed water temperature t_{nv}

$$t_{nv} = t_k + \frac{z}{z+1} \cdot (t_s - t_k)$$

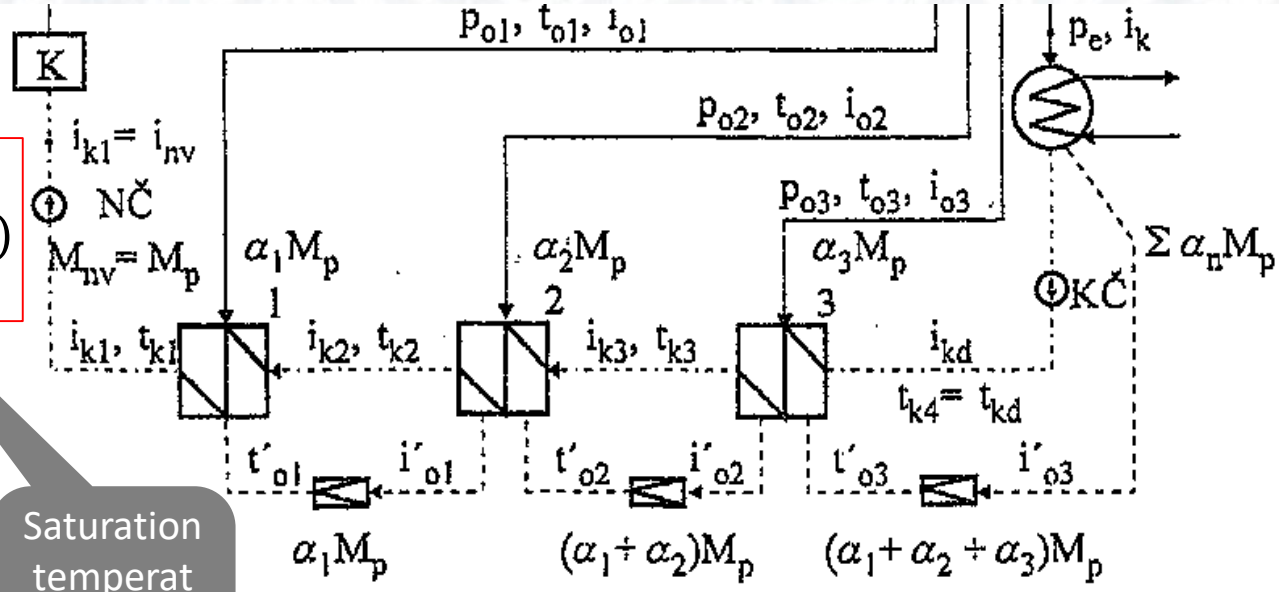
Required heating of feed water

$$\Delta t^{RO} = t_{nv} - t_k$$

Feed water heating in individual recuperation heaters (RH)

$$\Delta t_i^{RO} = \frac{\Delta t^{RO}}{z}$$

Saturation temperature for the pressure at which steam is produced



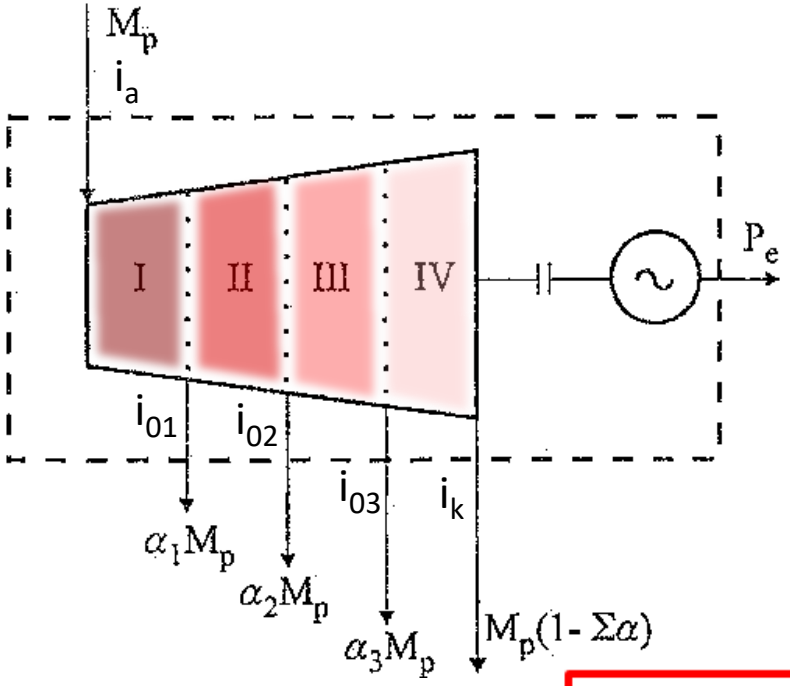
Feed water temperature at the output of individual heaters:

- RH 1 ..... $t_{k1} = \boldsymbol{t_k} + \Delta t_i^{RO}$
- RH 2 ..... $t_{k2} = \boldsymbol{t_{k1}} + \Delta t_i^{RO}$
  -
  -
  -
- RH *i* ..... $t_{ki} = t_{k(i-1)} + \Delta t_i^{RO}$

Balance equations of turbine and generator with unregulated steam extraction

- **Mass flow of steam into the turbine**

= it is determined from the balance equation of the turbine and generator



Thermal gradient

- **Part I** $M_p \cdot i_a = M_p \cdot i_{01} \rightarrow \boldsymbol{M_p \cdot (i_a - i_{01})}$
- **Part II** $\boldsymbol{(M_p - \alpha_1 M_p) \cdot (i_{01} - i_{02})}$
- **Part III** $\boldsymbol{(M_p - \alpha_1 M_p - \alpha_2 M_p) \cdot (i_{02} - i_{03})}$
- **Part IV** $\boldsymbol{(M_p - \alpha_1 M_p - \alpha_2 M_p - \alpha_3 M_p) \cdot (i_{03} - i_k)}$

$$M_p = \frac{3\,600 \cdot P_e}{(i_a - i_k)\eta_g \cdot \eta_m}$$

$$M_p[(i_a - i_{01}) + (1 - \alpha_1) \cdot (i_{01} - i_{02}) + (1 - \alpha_1 - \alpha_2) \cdot (i_{02} - i_{03}) + (1 - \alpha_1 - \alpha_2 - \alpha_3) \cdot (i_{03} - i_k)] = \frac{3\,600 \cdot P_e}{\eta_g \cdot \eta_m}$$

Thermal cycle with unregulated steam extraction

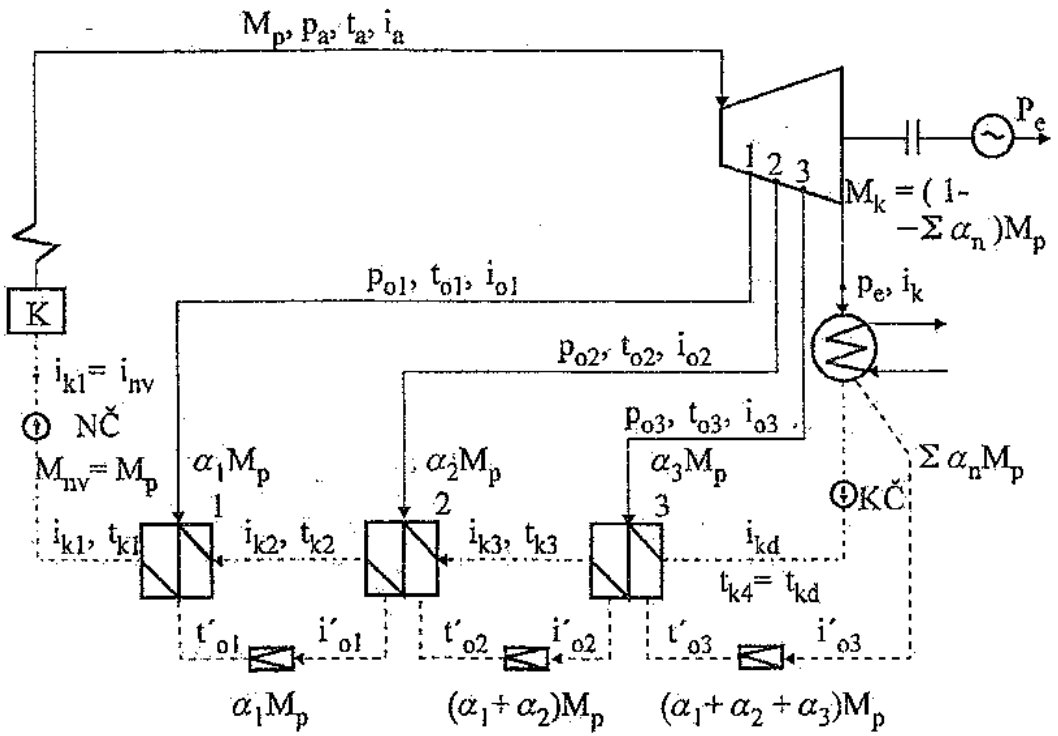
- **Heat consumption for the turbine and generator**

$$Q_T = M_p(i_a - i_{nv})$$

- **Heat consumption for the required electrical power (the whole power plant)**

$$Q_T = Q_p \cdot \eta_k \cdot \eta_p$$

$$\boldsymbol{Q_p} = \frac{Q_T}{\eta_k \cdot \eta_p} = \frac{P_e}{q_p}$$



$$q_p = \frac{Q_p}{P_e} = \frac{3\,600}{\eta_t^{el}}$$

- **Specific heat consumption for the required electrical output power**

- **Amount of the fuel consumed**

$$\boldsymbol{M_{pal}} = \frac{Q_p}{q_{pal}} = \frac{Q_T}{q_{pal} \cdot \eta_k \cdot \eta_p}$$

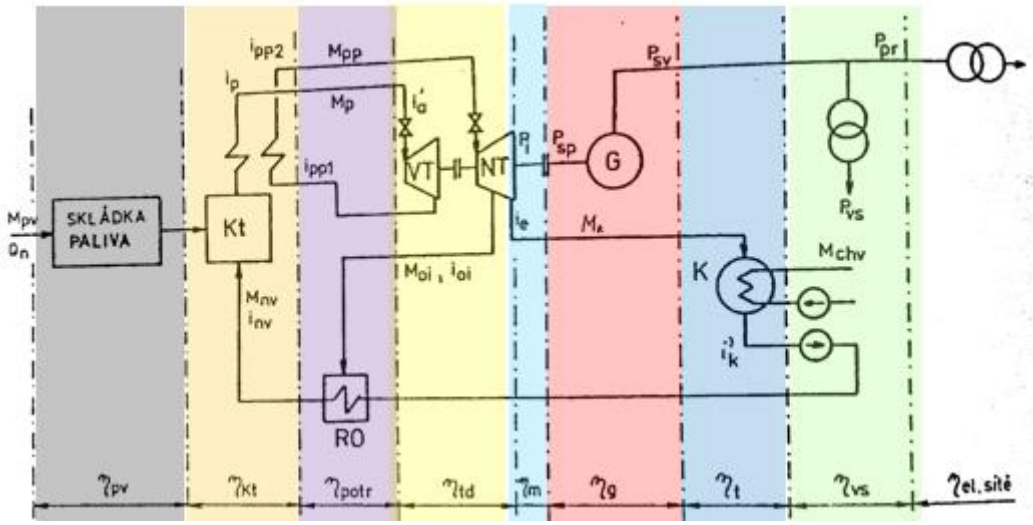
Thermal cycle with unregulated steam extraction

- **Thermal efficiency of the ideal thermal cycle**

$$\eta_{to} = \frac{w_{to}}{q_p} = \frac{M_p(i_a - i_e)}{M_p(i_a - i_{nv})}$$

- **Thermal efficiency of the real thermal cycle**

$$\eta_t = \eta_{to} \cdot \eta_{td} = \frac{w_t}{q_T} = \frac{M_p(i_a - i_k)}{M_p(i_a - i_{nv})}$$



- **Thermal efficiency of the thermal cycle at the generator terminals**

$$\eta'_t = \frac{w_t \cdot \boldsymbol{\eta_m \cdot \eta_g}}{q_T} = \frac{M_p(i_a - i_k) \cdot \boldsymbol{\eta_m \cdot \eta_g}}{M_p(i_a - i_{nv})} = \frac{3600 \cdot P_e}{M_p(i_a - i_{nv})} = \frac{\boldsymbol{3600 \cdot P_e}}{\boldsymbol{Q_T}} = \frac{\boldsymbol{3\,600}}{\boldsymbol{q_T}} = \frac{3600 \cdot P_e}{M_{pal} \cdot q_{pal} \cdot \eta_k \cdot \eta_p}$$

$$\eta'_t = \eta_t \cdot \eta_m \cdot \eta_g$$

- **Thermal efficiency of the power plant at the generator terminals**

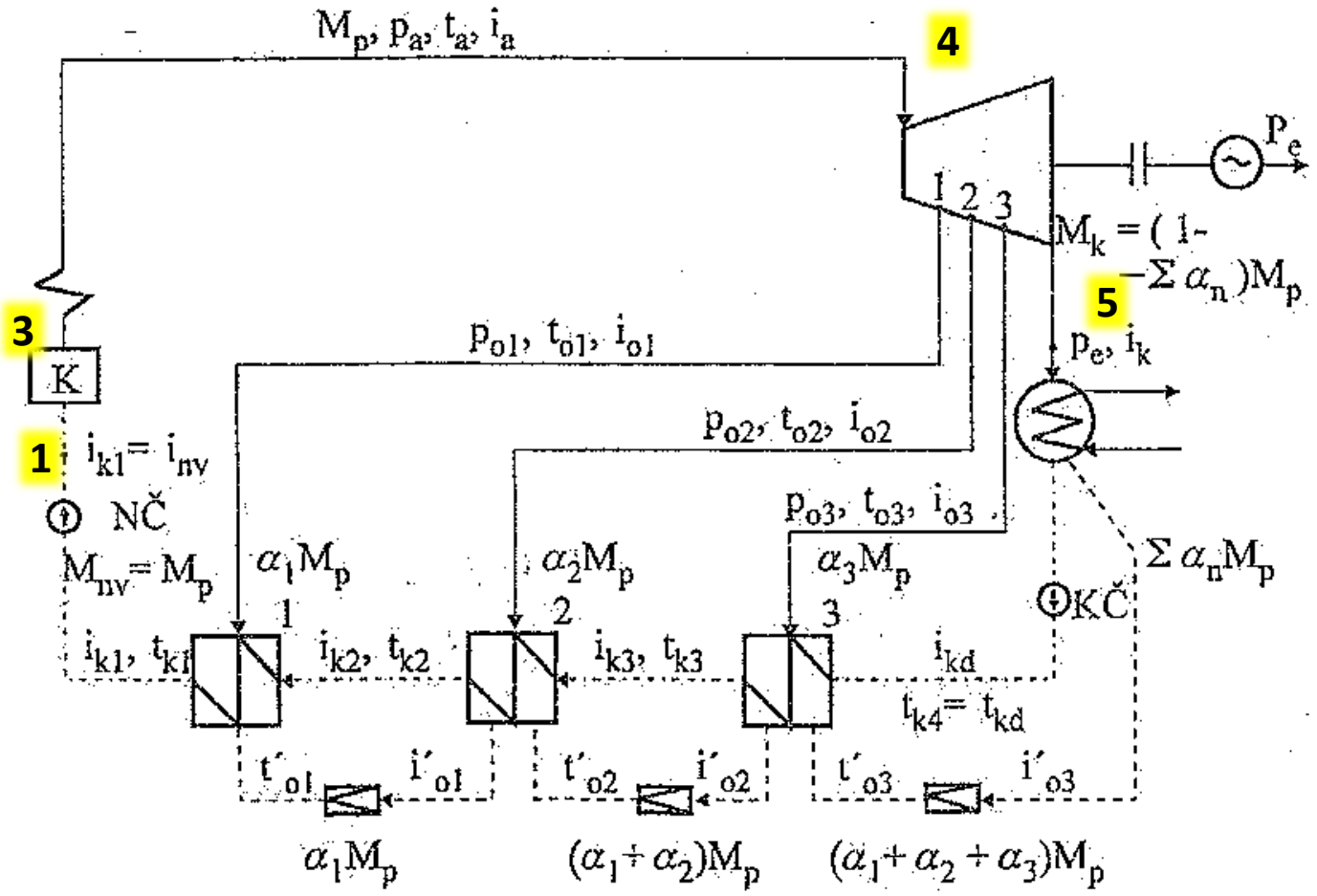
$$\eta_t^{el} = \frac{3600 \cdot P_e}{Q_p} = \frac{3\,600}{q_p} = \frac{\boldsymbol{3600 \cdot P_e}}{\boldsymbol{M_{pal} \cdot q_{pal}}} = \frac{3600 \cdot P_e \cdot \eta_k \cdot \eta_p}{Q_T} = \eta'_t \cdot \eta_k \cdot \eta_p = \eta_{to} \cdot \eta_{td} \cdot \eta_g \cdot \eta_m \cdot \eta_k \cdot \eta_p$$

Example 1

- For a condensing turbine with a generator with an electrical output power P_e = 6 MW , design a thermal scheme with recuperative heating of the feed water and calculate the thermal efficiency and the thermal scheme.The parameters of the block are:
  - p_a = 3.5 MPa, p_e = 5 kPa, q_{pal} = 12 220 kJ.kg^{-1}
  - t_a = 435 °C, t_k = 35 °C.

Efficiency values:

- $\eta_{TZ} = 0.85$
- $\eta_{pot} = 0.99$
- $\eta_{td} = 0.8$
- $\eta_m = 0.96$
- $\eta_g = 0.97$

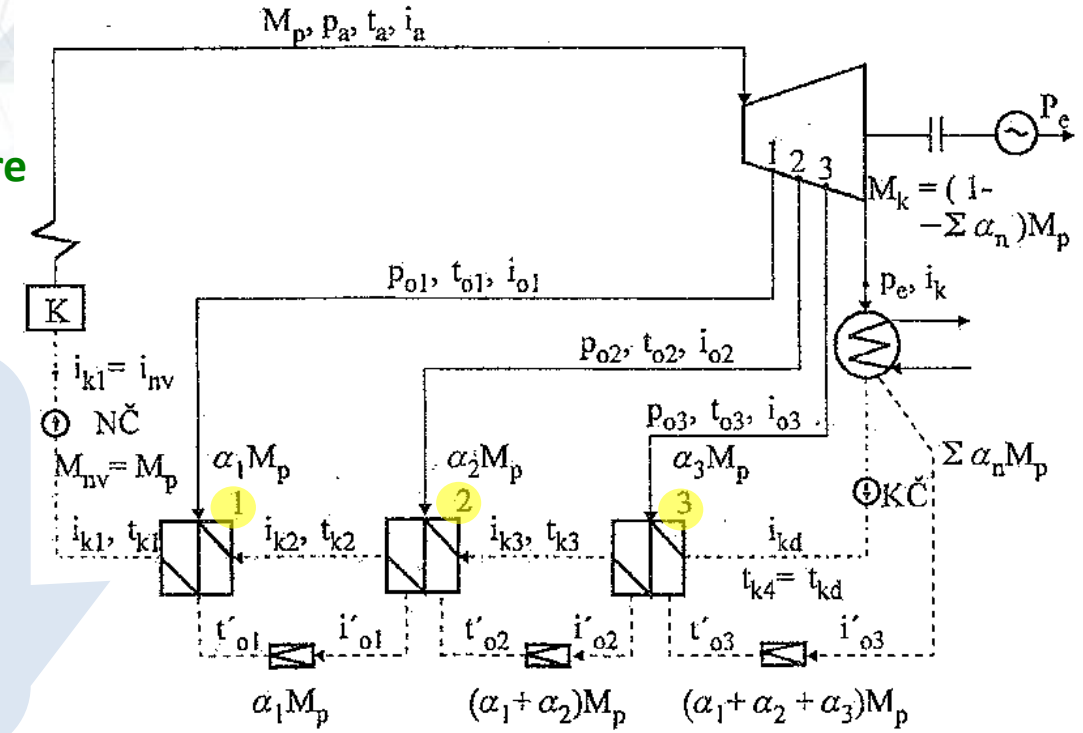


Example 1 - Solution

- **Calculation of feed water temperature**

$$t_{nv} = t_k + \frac{z}{z+1}(t_s - t_k)$$

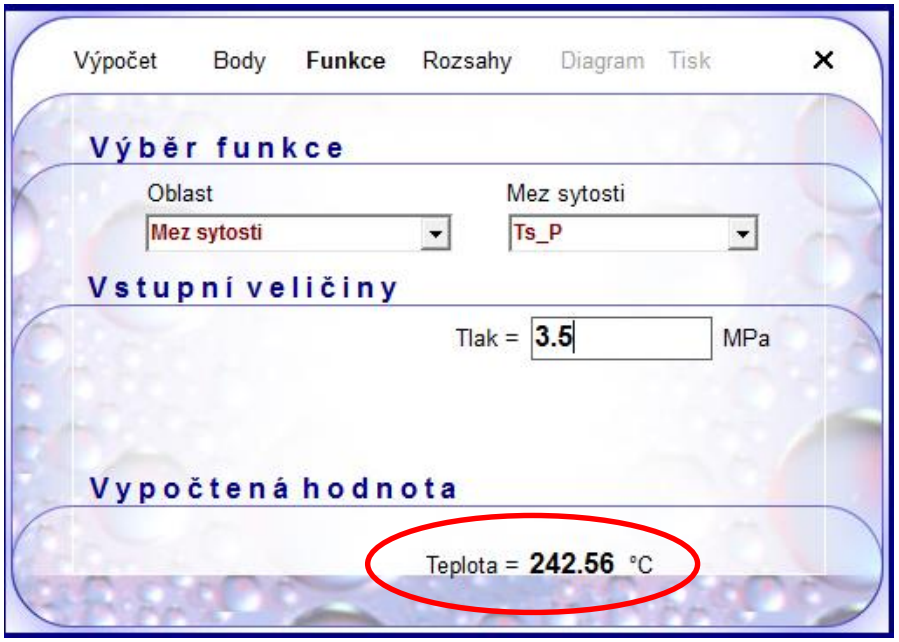
It is clear from the thermal cycle diagram that the 3rd stages recuperative heating of feed water with cascade are selected. → z = 3. To calculate the feed water temperature, we must first determine the saturation temperature.



- **Determination of saturation temperature for pressure p_a**

$$p_a = 3.5\ MPa \Rightarrow t_s = 242.56\ °C$$

$$t_{nv} =$$



Example 1 - Solution

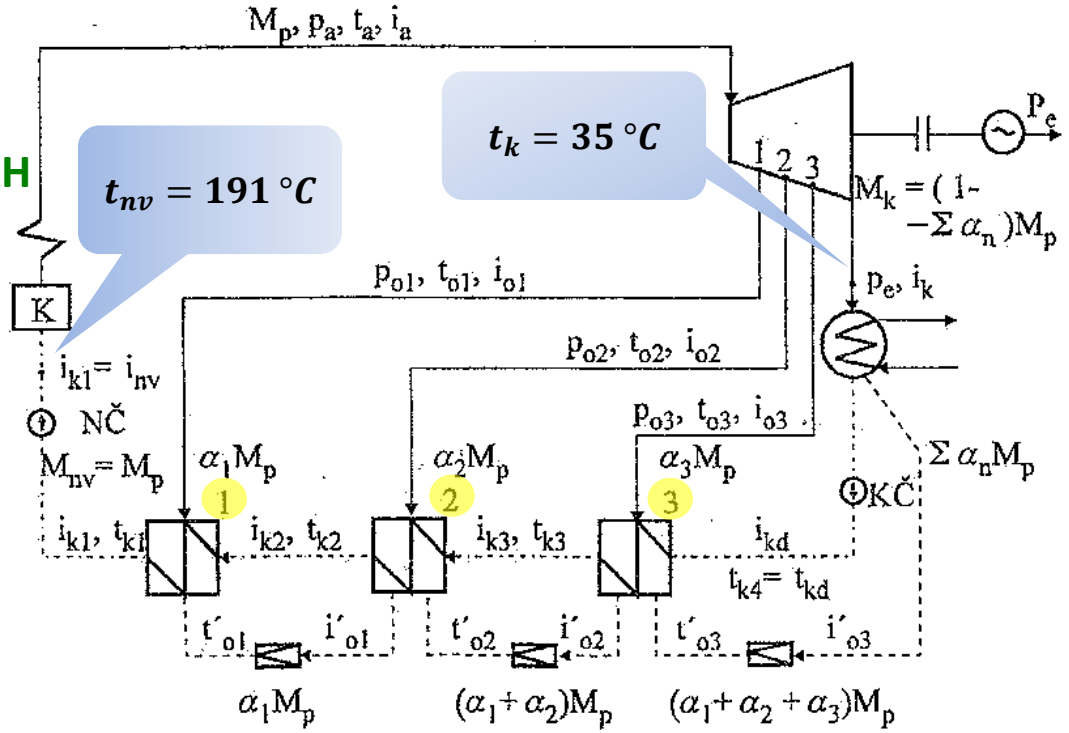
- **Required heating of feed water in all RH**

$$\Delta t^{RO} =$$

$$\Delta \boldsymbol{t^{RO}} =$$

- **Feed water heating in one recuperation heater (RH)**

$$\Delta \boldsymbol{t_i^{RO}} =$$



Proceed from the feedwater pump towards the condenser.

- **Feed water temperature at the output of individual heaters:**
  - RH 1 ..... $\boldsymbol{t_{k1}} =$
  - RH 2 ..... $\boldsymbol{t_{k2}} =$
  - RH 3 ..... $\boldsymbol{t_{k3}} =$

Example 1 - Solution

Given:

p_a = 3.5 MPa
t_a = 435 °C
p_e = 5 kPa
t_k = 35 °C

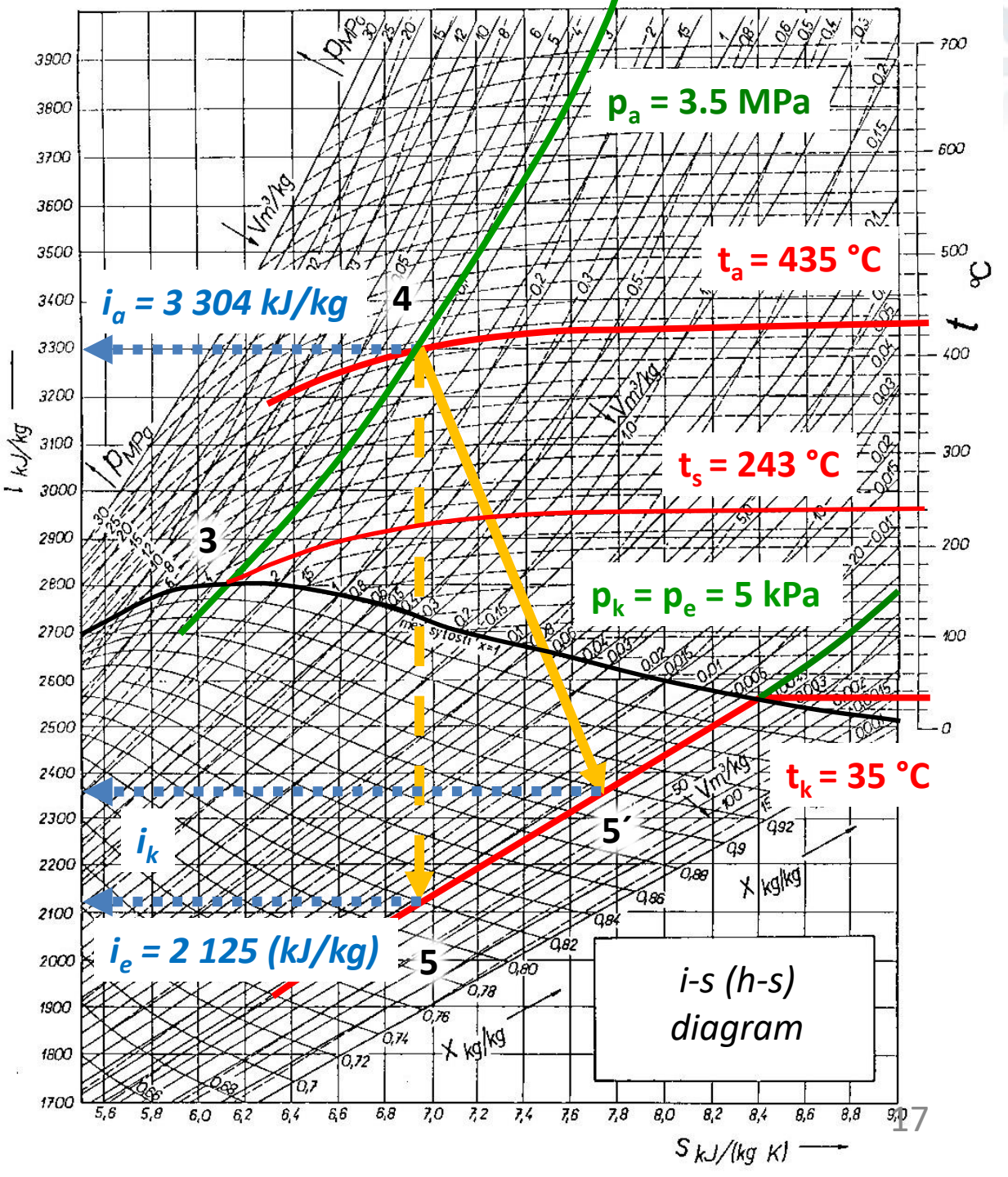
- **Enthalpy i_k**

$$\eta_{td} = \frac{h_s}{h_{ad}} = \frac{i_a - i_k}{i_a - i_e} = 0.80$$

$i_k =$

$i_k =$

$\boldsymbol{i_k} =$



Example 1 - Solution

Given:

$p_a = 3.5\ MPa$

$t_a = 435\ °C$

$p_e = 5\ kPa$

$t_k = 35\ °C$

- **Plotting temperatures $t_{ki} = t'_{oi}$**

$t_{k1} =$

$t_{k2} =$

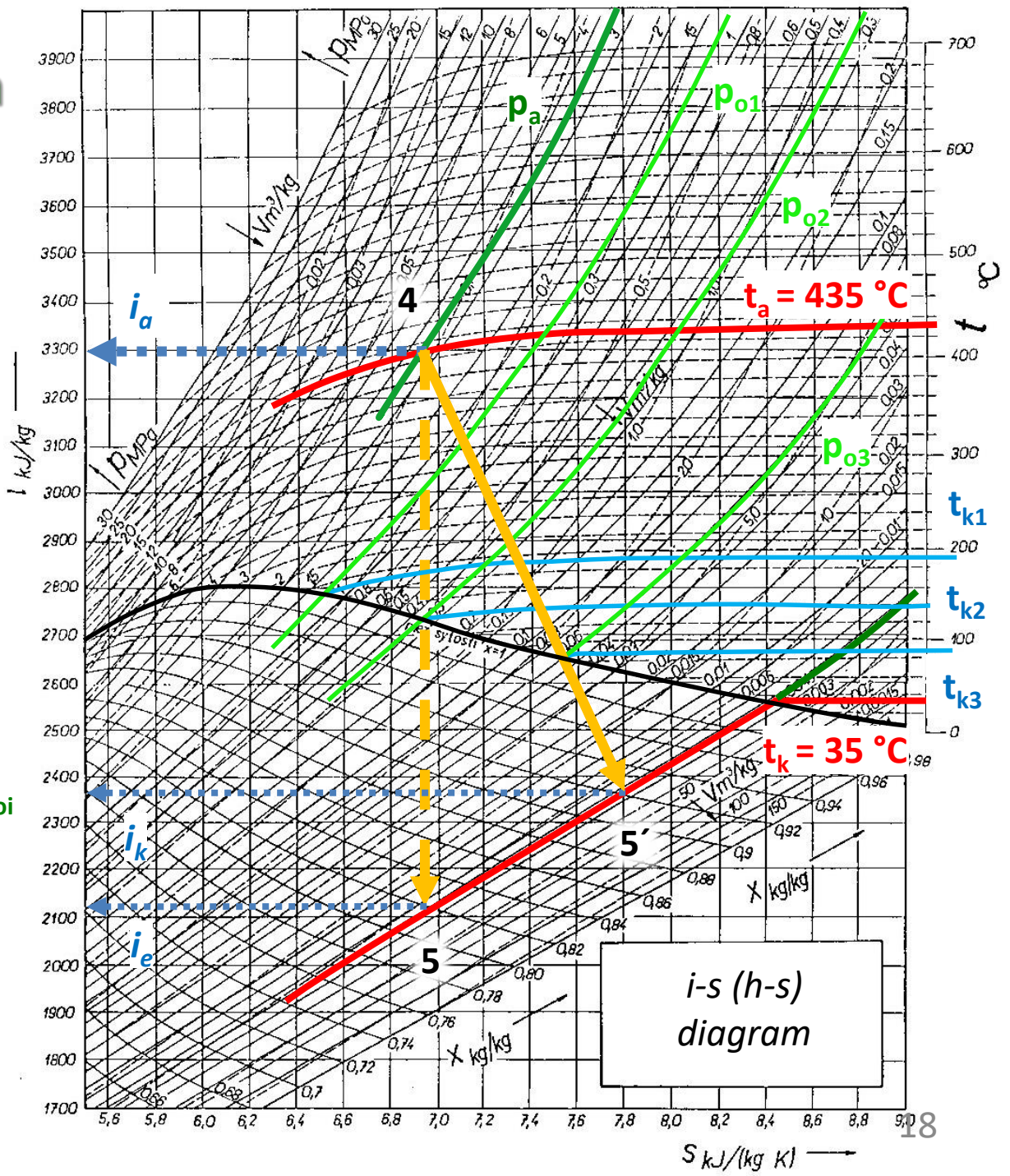
$t_{k3} =$

- **Pressure of extraction steam p_{oi}**

$p_{o1} =$

$p_{o2} =$

$p_{o3} =$

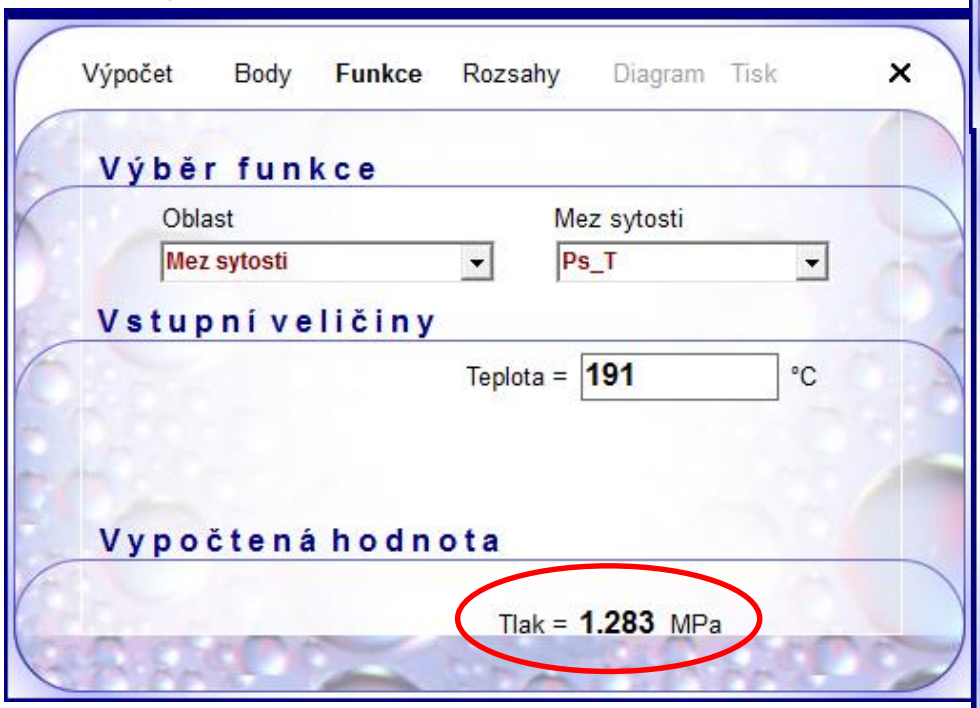


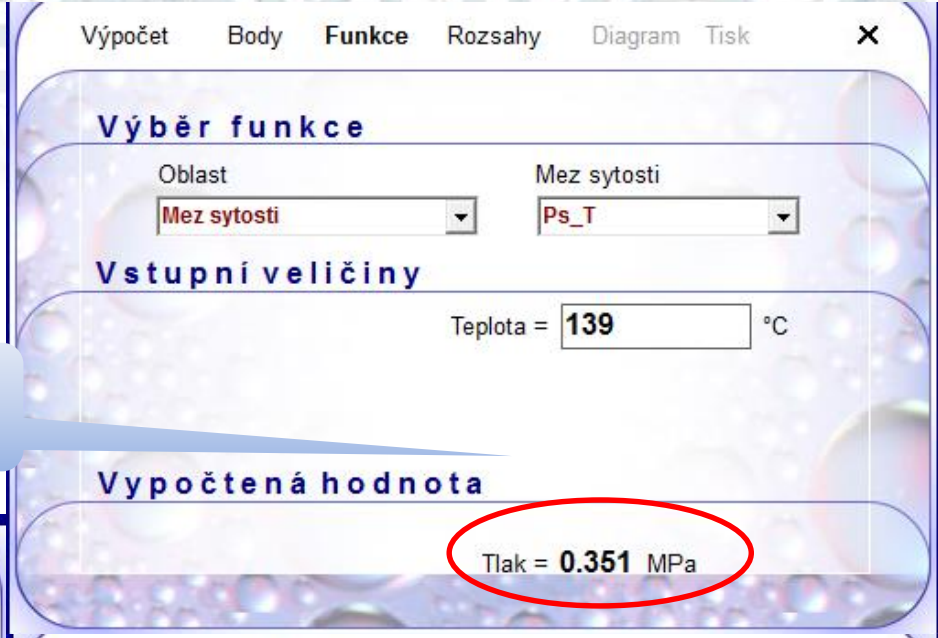
Example 1 - Solution

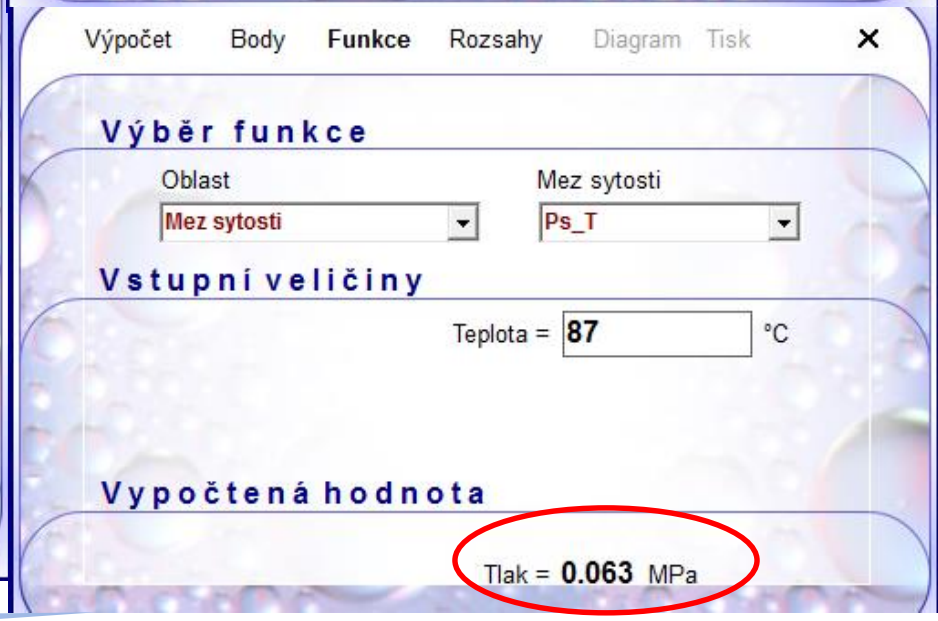
- Determination of the quantities values by means of the software

Pressure of extraction steam p_{o1}

Pressure of extraction steam p_{o2}







Pressure of extraction steam p_{o3}

Example 1 - Solution

Given:

$p_a = 3.5\ MPa$

$t_a = 435\ °C$

$p_e = 5\ kPa$

$t_k = 35\ °C$

- **Temperaure of extraction steam t_{oi}**

$t_{o1} =$

$t_{o2} =$

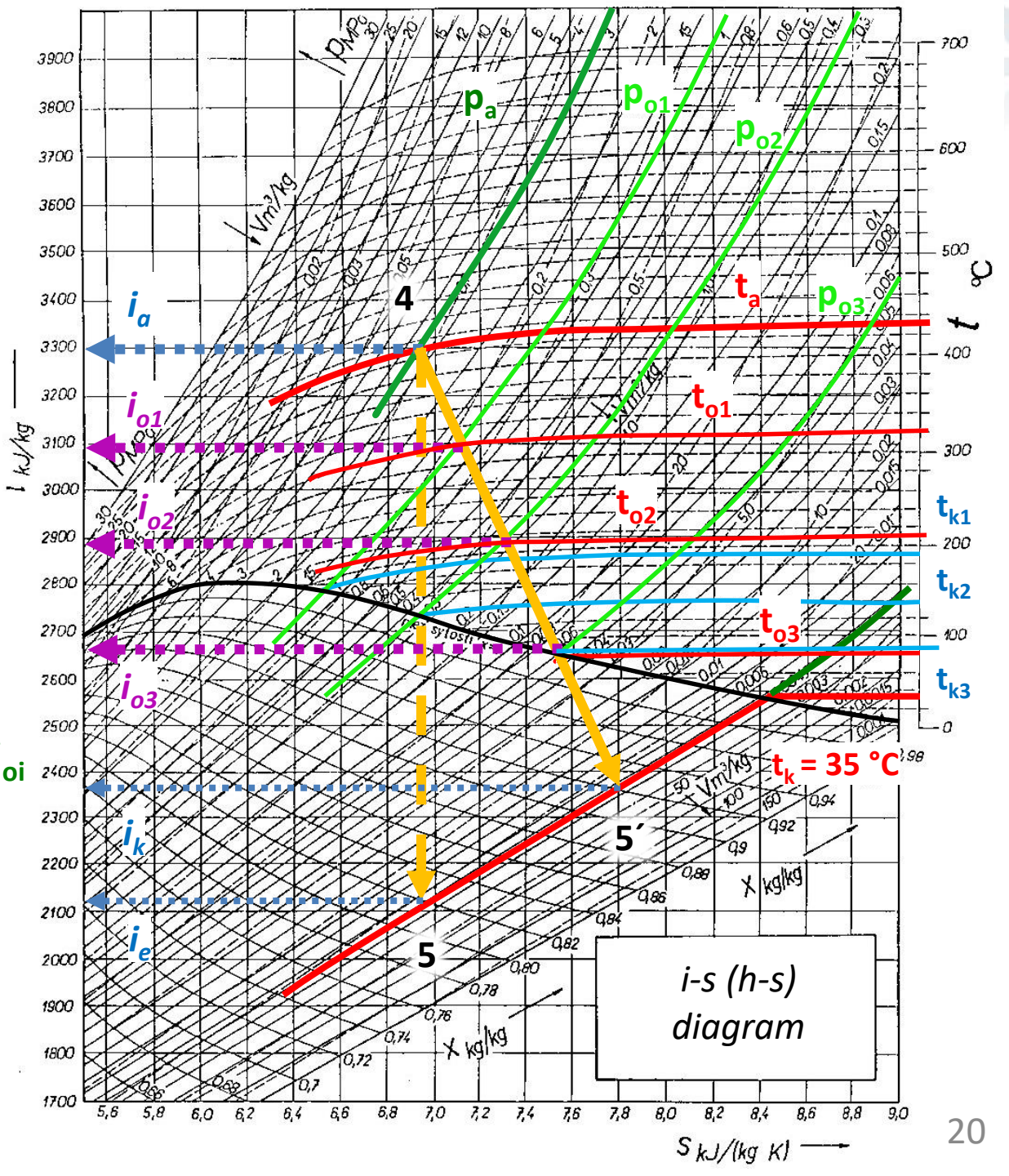
$t_{o3} =$

- **Enthalpy of extraction steam i_{oi}**

$i_{o1} =$

$i_{o2} =$

$i_{o3} =$

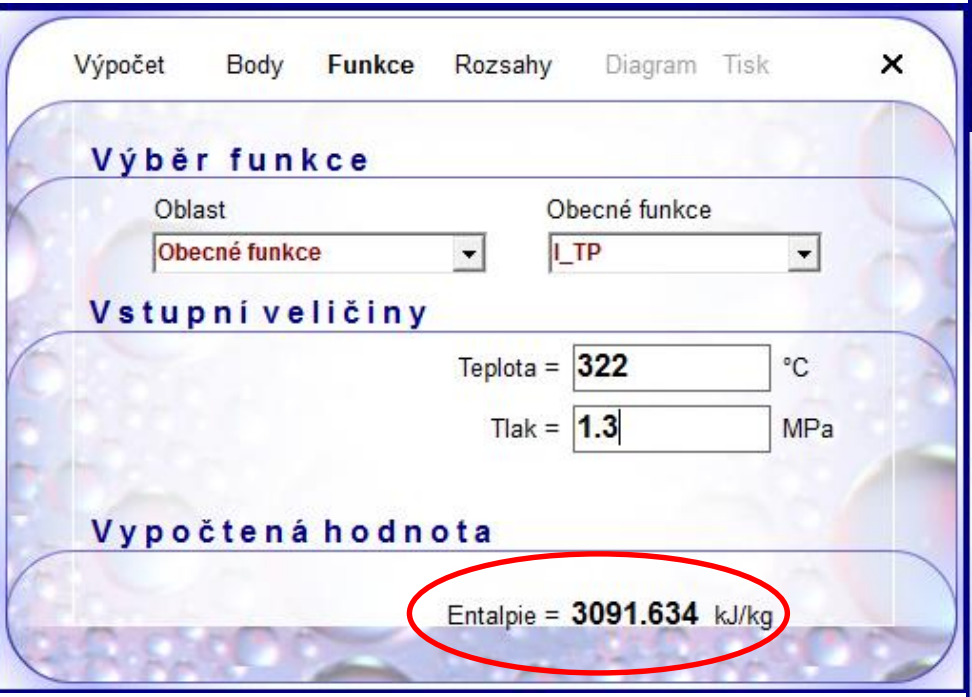


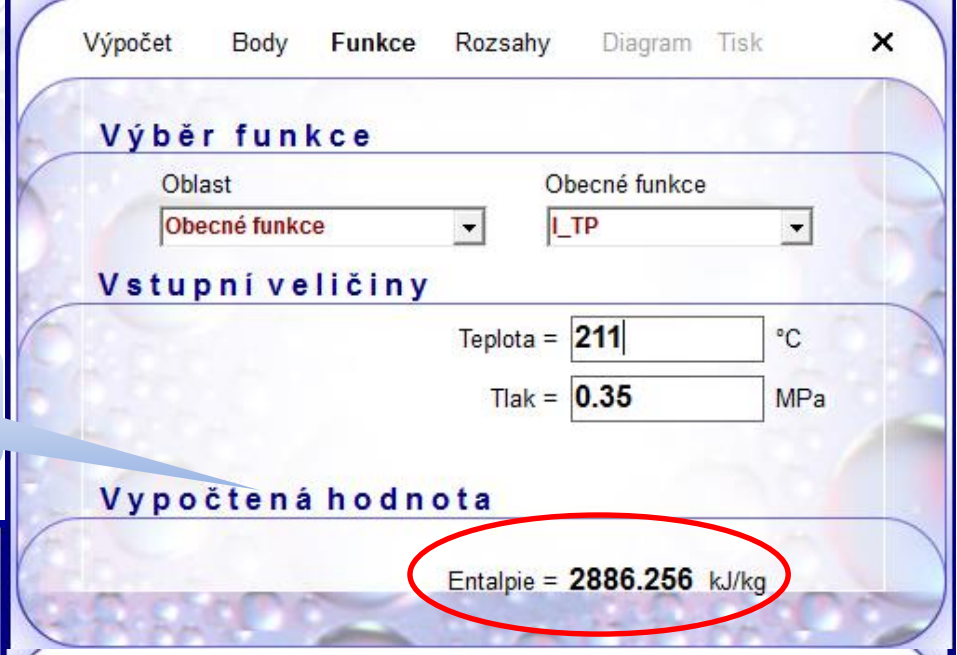
Example 1 - Solution

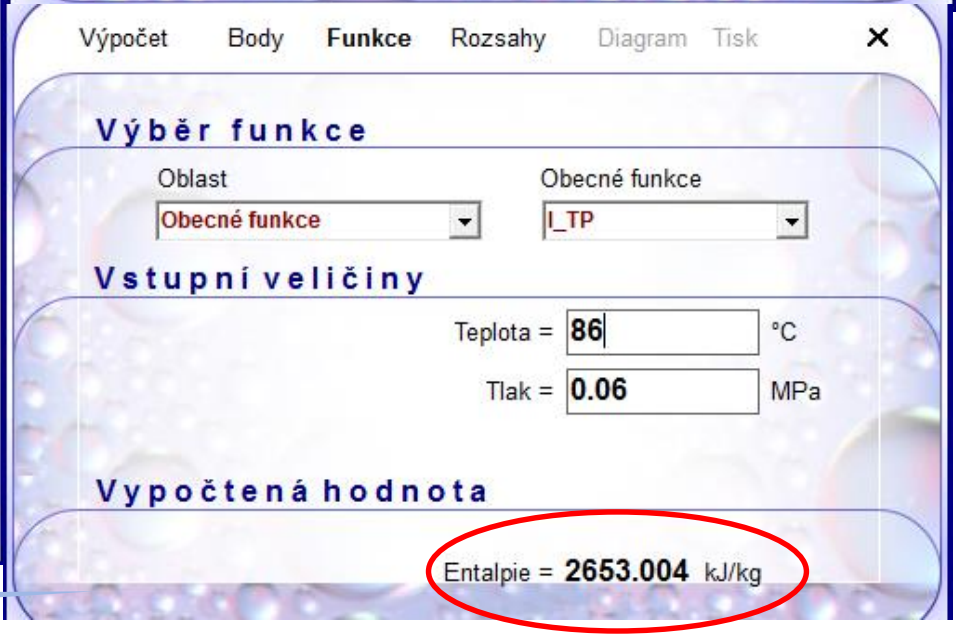
- Determination of the quantities values by means of the software

Enthalpy of extraction steam i_{o1}

Enthalpy of extraction steam i_{o2}







Enthalpy of extraction steam i_{o3}

Example 1 - Solution

- **Condensate enthalpy of extraction (heating) steam i'_{oi}** – by means of t'_{oi} = t_{ki} or p_{oi}

Condensate enthalpy of extraction steam i'_{o1}

Condensate enthalpy of extraction steam i'_{o2}







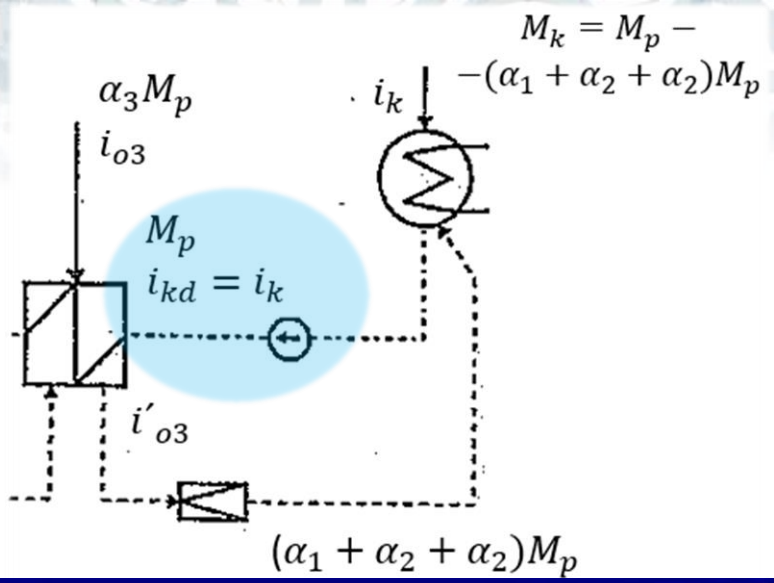
Condensate enthalpy of extraction steam i'_{o3}

Example 1 - Solution

- Condensate enthalpy – by means of t_{kd} = 35 °C

Possibility 1 - calculation

$$i_{kd} =$$

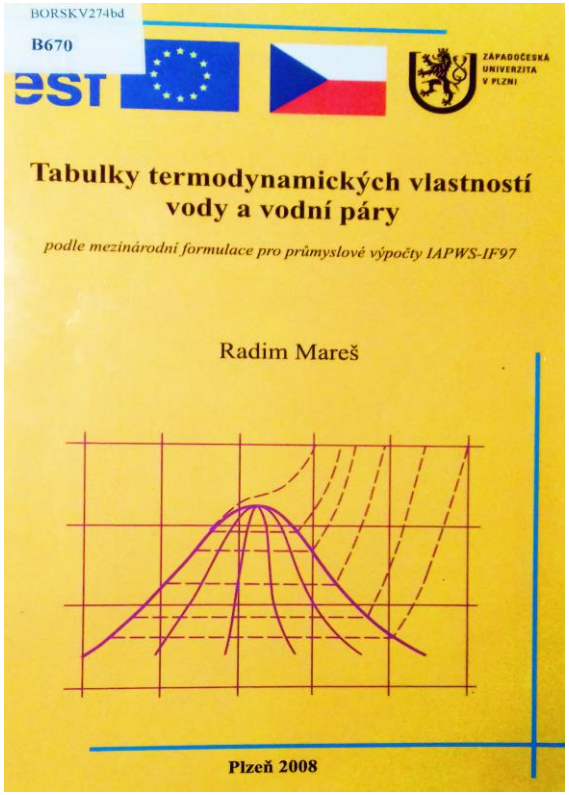


Possibility 2 - from steam tables or software



Example 1 - Solution

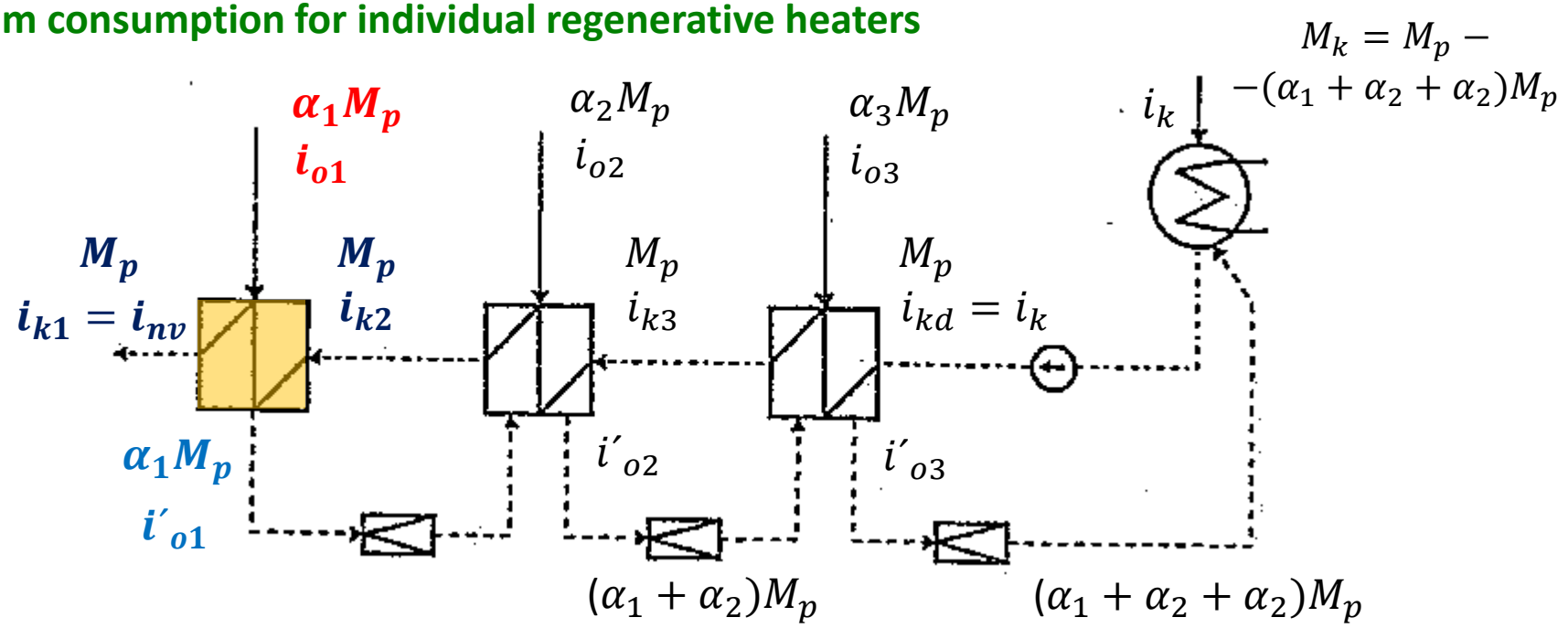
Subtraction of enthalpy and entropy values in the Steam Tables



t °C	p MPa	v' m^3/kg	v" m^3/kg	h' kJ/kg	h" kJ/kg	s' kJ/kgK	s" kJ/kgK
0,00	0,0006112	0,0010002	206,1	-0,04	2500,9	-0.0002	9,1558
0,01	0,0006117	0,0010002	206,0	0,00	2500,9	0.0000	9,1555
1	0,0006571	0,0010001	192,4	4,18	2502,7	0,0153	9,1291
2	0,0007060	0,0010001	179,8	8,39	2504,6	0.0306	9,1027
3	0,0007581	0,0010001	168,0	12,60	2506,4	0.0459	9,0765
4	0,0008135	0,0010001	157,1	16,81	2508,2	0,0611	9.0506
5	0,0008726	0,0010001	147,0	21,02	2510,1	0,0763	9,0249
6	0,0009354	0,0010001	137,6	25,22	2511,9	0,0913	8,9994
7	0,0010021	0,0010001	128,9	29,43	2513,7	0,1064	8,9742
8	0,0010730	0,0010002	120,8	33,63	2515,6	0,1213	8,9492
9	0,0011483	0,0010003	113,3	37,82	2517,4	0,1362	8,9244
⋮							
30	0,004247	0,0010044	32,88	125,75	2555,6	0,4368	8,4521
31	0,004497	0,0010047	31,15	129,93	2557,4	0,4506	8,4317
32	0,004759	0,0010050	29,53	134,11	2559,2	0,4643	8,4115
33	0,005035	0,0010054	28,00	138,29	2561,0	0,4780	8,3914
34	0,005325	0,0010057	26,56	142,47	2562,8	0,4916	8,3715
35	0,005629	0,0010060	25,21	146,64	2564,6	0,5052	8,3518
36	0,005947	0,0010064	23,93	150,82	2566,4	0,5187	8,3323
37	0,006282	0,0010068	22,73	155,00	2568,2	0,5322	8,3129
38	0,006632	0,0010071	21,60	159,18	2570,0	0,5457	8,2936
39	0,007000	0,0010075	20,53	163,36	2571,8	0,5591	8,2746
40	0,007384	0,0010079	19,52	167,54	2573,5	0,5724	8,2557
41	0,007787	0,0010083	18,56	171,72	2575,3	0,5858	8,2369
42	0,008209	0,0010087	17,67	175,90	2577,1	0,5990	8,2183
43	0,008650	0,0010091	16,82	180,08	2578,9	0,6123	8,1999
44	0,009112	0,0010095	16,01	184,26	2580,7	0,6255	8,1816
45	0,009594	0,0010099	15,25	188,44	2582,5	0,6386	8,1634
46	0,010099	0,0010103	14,54	192,62	2584,2	0,6517	8,1454
47	0,010626	0,0010108	13,86	196,80	2586,0	0,6648	8,1276
48	0,011176	0,0010112	13,21	200,98	2587,8	0,6778	8,1099
49	0,011751	0,0010117	12,60	205,16	2589,5	0,6908	8,0923

Example 1 - Solution

- **Steam consumption for individual regenerative heaters**



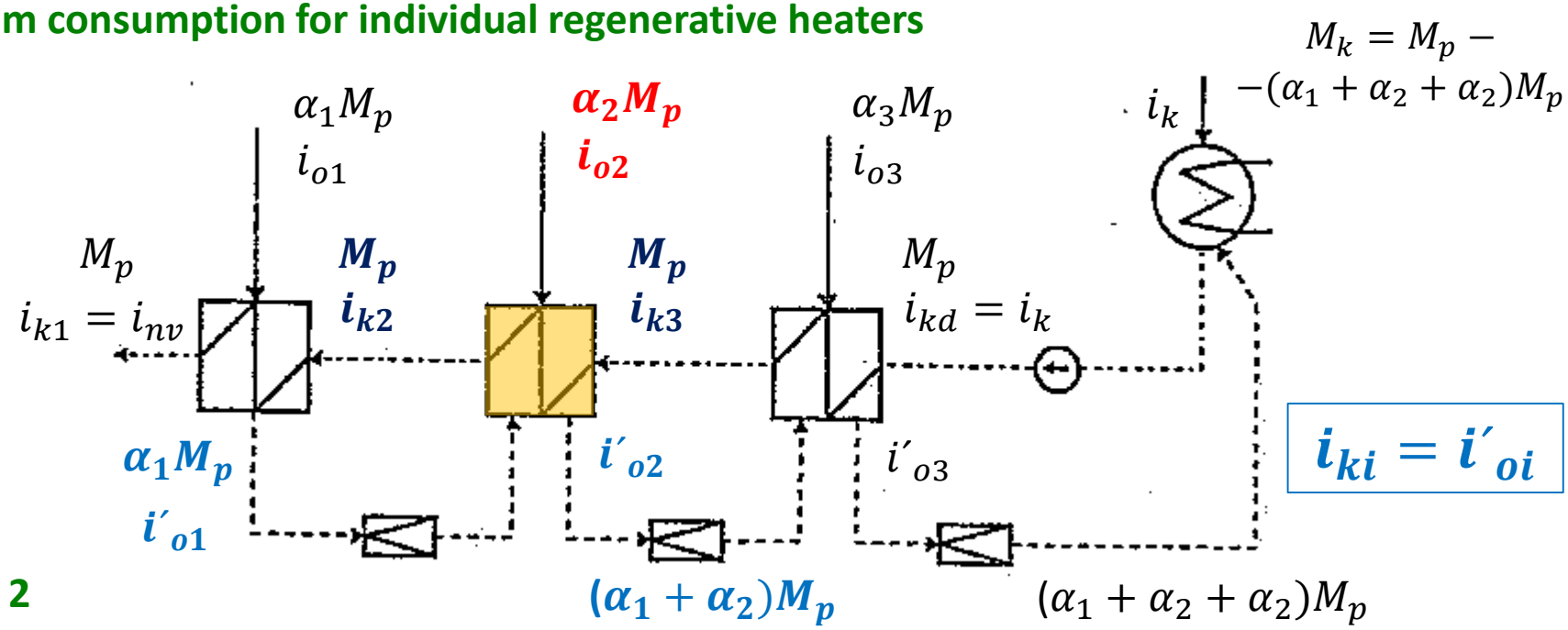
- **RH 1**

$$i_{ki} = i'_{oi}$$

$\alpha_1 =$

Example 1 - Solution

- **Steam consumption for individual regenerative heaters**

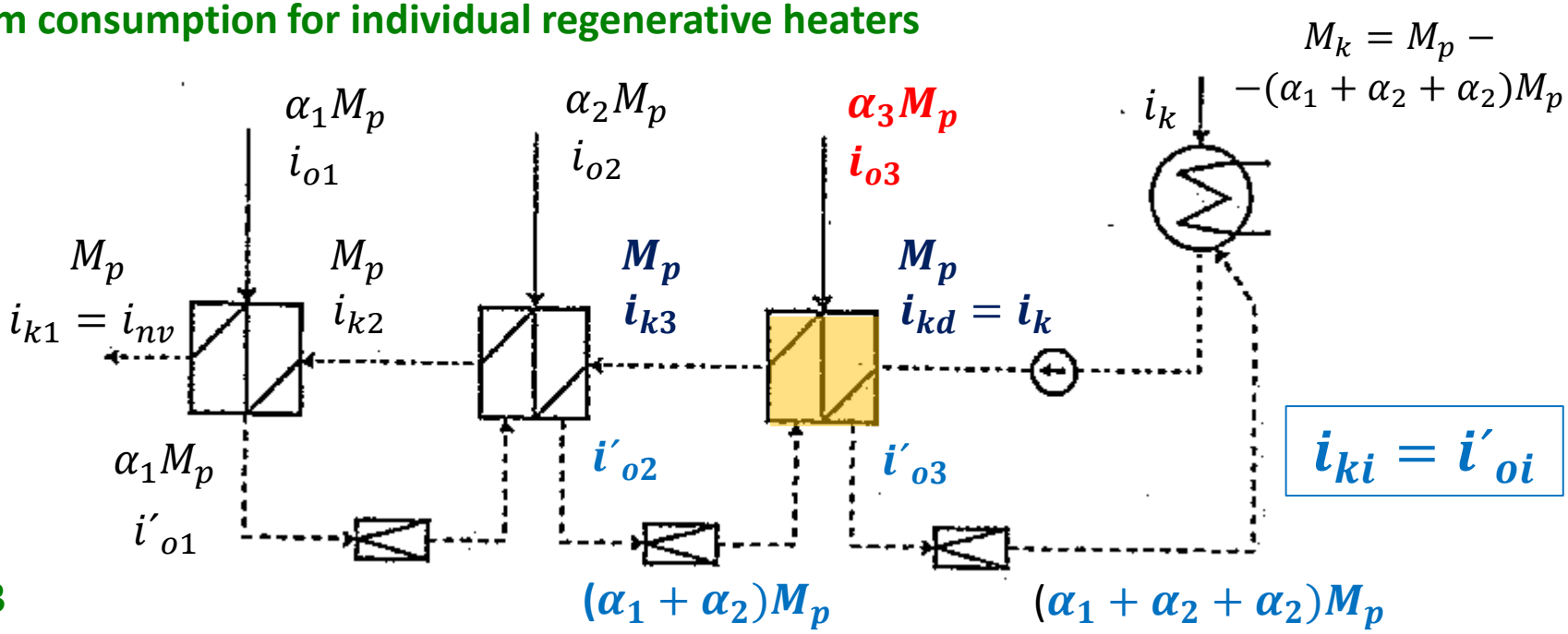


- **RH 2**

$\alpha_2 =$

Example 1 - Solution

- **Steam consumption for individual regenerative heaters**



- **RH 3**

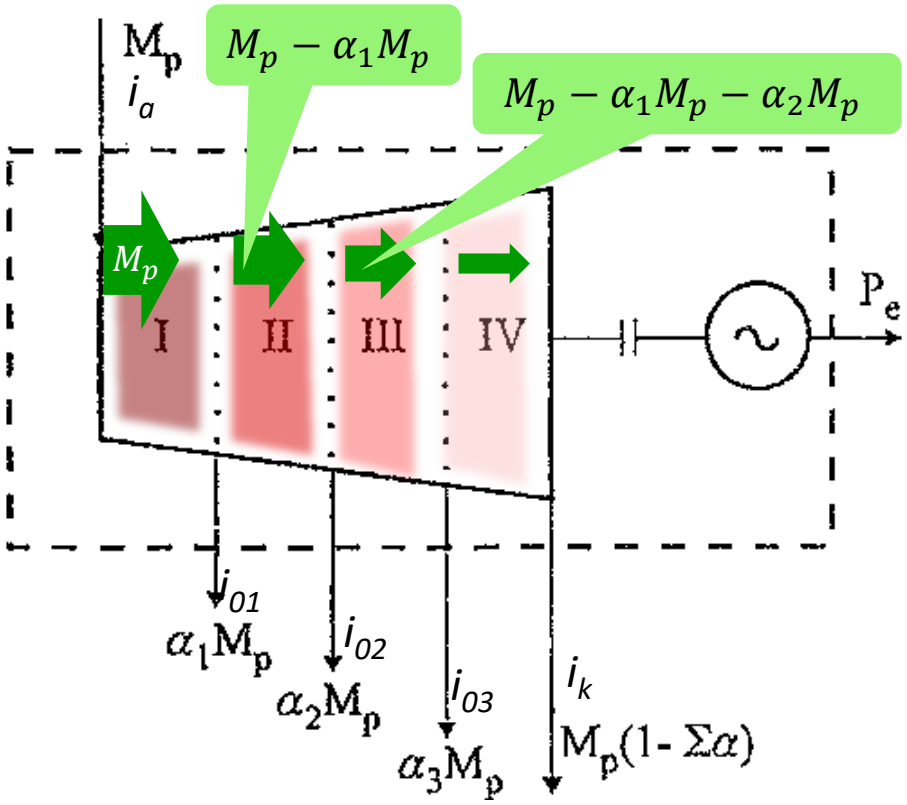
$\alpha_3 =$

$\alpha_3 =$

Example 1 - Solution

- **Mass flow of steam into the turbine**

= it is determined from the balance equation of the turbine and generator



$$INPUT = OUTPUT$$

Thermal gradient

Part I of the turbine

$$M_p \cdot i_a = M_p \cdot i_{01} \rightarrow \boldsymbol{M_p \cdot (i_a - i_{01}) = 0}$$

- **Part II of the turbine**

$$\boldsymbol{(M_p - \alpha_1 M_p) \cdot (i_{01} - i_{02}) = 0}$$

- **Part III of the turbine**

$$\boldsymbol{(M_p - \alpha_1 M_p - \alpha_2 M_p) \cdot (i_{02} - i_{03}) = 0}$$

- **Part IV of the turbine**

$$\boldsymbol{(M_p - \alpha_1 M_p - \alpha_2 M_p - \alpha_3 M_p) \cdot (i_{03} - i_k) = 0}$$

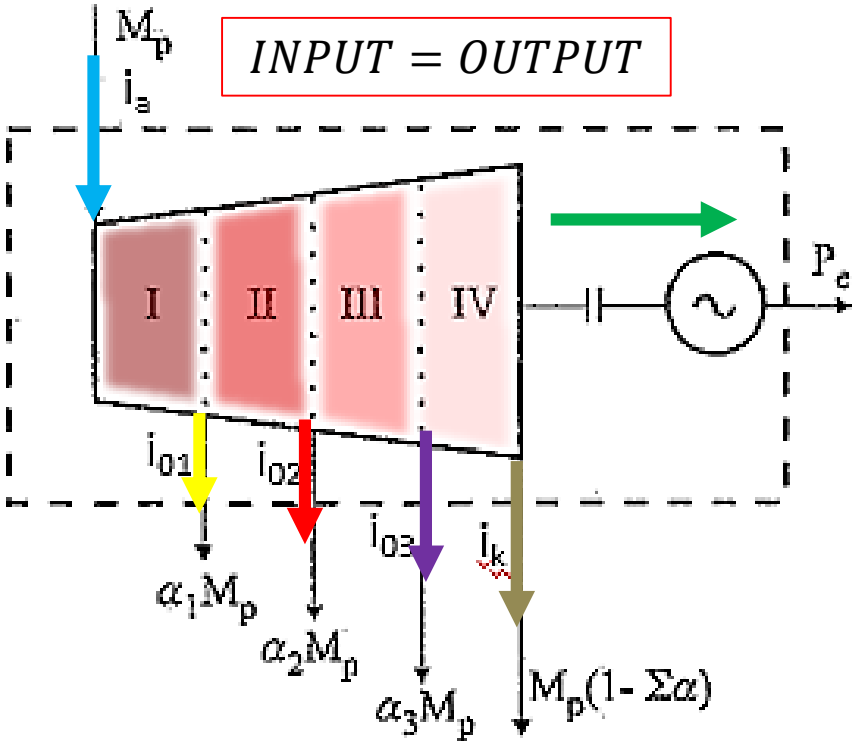
1st variant of calculation = **determination of M_p** from partial balance equations determined for individual parts of the turbine

Example 1 - Solution

- **Mass flow of steam into the turbine**

= it is determined from the balance equation of the turbine and generator

2st variant of calculation = determination of M_p from the balance equation of the whole turbomachinery



$$M_p \cdot i_a =$$

$$= \alpha_1 \cdot M_p \cdot i_{01} + \alpha_2 \cdot M_p \cdot i_{02} + \alpha_3 \cdot M_p \cdot i_{03} + M_p \cdot i_k - M_p \cdot i_k \cdot (\alpha_1 + \alpha_2 + \alpha_3) + \frac{3\,600 \cdot P_e}{\eta_g \cdot \eta_m}$$

$$M_p \cdot i_a - \boldsymbol{\alpha_1} \cdot M_p \cdot i_{01} - \boldsymbol{\alpha_2} \cdot M_p \cdot i_{02} - \boldsymbol{\alpha_3} \cdot M_p \cdot i_{03} - M_p \cdot i_k + M_p \cdot i_k \cdot (\boldsymbol{\alpha_1} + \boldsymbol{\alpha_2} + \boldsymbol{\alpha_3})$$

$$= \frac{3\,600 \cdot P_e}{\eta_g \cdot \eta_m}$$

Example 1 - Solution

2st variant of calculation =
determination of M_p from the balance equation of the whole turbomachinery

- **Mass flow of steam into the turbine**

= it is determined from the balance equation of the turbine and generator

$$M_p \cdot i_a - \alpha_1 \cdot M_p \cdot i_{01} - \alpha_2 \cdot M_p \cdot i_{02} - \alpha_3 \cdot M_p \cdot i_{03} - M_p \cdot i_k + M_p \cdot i_k \cdot (\alpha_1 + \alpha_2 + \alpha_3)$$

$$= \frac{3\,600 \cdot P_e}{\eta_g \cdot \eta_m}$$

$$M_p \cdot i_a - \alpha_1 \cdot M_p \cdot (i_{01} - i_k) - \alpha_2 \cdot M_p \cdot (i_{02} - i_k) - \alpha_3 \cdot M_p \cdot (i_{03} - i_k) - M_p \cdot i_k = \frac{3\,600 \cdot P_e}{\eta_g \cdot \eta_m}$$

$$M_p = \frac{3\,600 \cdot P_e}{\eta_g \cdot \eta_m \cdot [i_a - \alpha_1 \cdot (i_{01} - i_k) - \alpha_2 \cdot (i_{02} - i_k) - \alpha_3 \cdot (i_{03} - i_k) - i_k]}$$

Example 1 - Solution

2st variant of calculation =
determination of M_p from the balance equation of the whole turbomachinery

- **Mass flow of steam into the turbine**

= it is determined from the balance equation of the turbine and generator

$$M_p =$$

Example 1 - Solution

- Heat consumption for the turbine and generator

$$Q_T =$$

- Amount of the fuel consumed

$$M_{pal} =$$

- Thermal efficiency of the thermal cycle at the generator terminals

$$\eta_t^e =$$

- Thermal efficiency of the power plant at the generator terminals

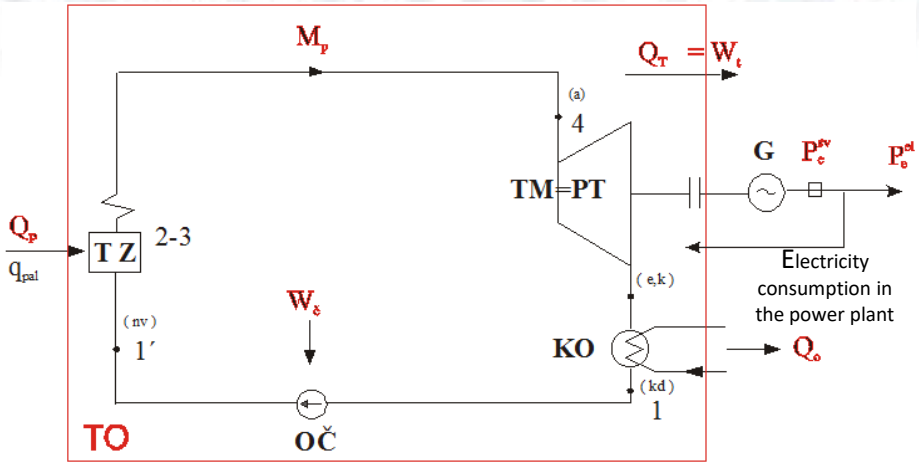
$$\eta_t^{el} =$$

Example 1 – Solution – Comparison of thermal cycle with and without recuperation heaters

$i_a =$

$i_k =$

$i_{nv} =$



- **Mass flow of steam into the turbine**

$\boldsymbol{M_p} =$

- **Heat consumption for the turbine and generator**

$\boldsymbol{Q_T} =$

- **Amount of the fuel consumed**

$\boldsymbol{M_{pal}} =$

Example 1 – Solution – Comparison of thermal cycle with and without recuperation heaters

- Thermal efficiency of the thermal cycle at the generator terminals

$\eta_t^e =$

- Thermal efficiency of the power plant at the generator terminals

$\eta_t^{el} =$

- Comparison of thermal cycle with and without recuperation heaters

	$M_p\ (\frac{kg}{h})$	$Q_T\ (\frac{kJ}{h})$	$M_{pal}\ (\frac{kg}{h})$	η_t^e (%)	η_t^{el} (%)
With					
Without					

Used and recommended literature

- Dvorsky, Emil; Hejtmankova, Pavla; Kocmich, Martin. Elektrarny : zaklady vyroby elektricke energie : priklady. 1. vyd. Pilsen : UWB, 1994. ISBN 80-7082-133-7.
- Dvorsky, Emil; Hejtmankova, Pavla. Elektrarny : zvysovani ucinnosti premen energie v tepelnych elektrarnach : priklady. 1. vyd. Pilsen : UWB, 1999. ISBN 80-7082-523-5.

- …… and other literature listed in the course syllabus on the website Portal of the UWB in Pilsen

Next exercise

Energy calculation of steam generators

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